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Meteorological Research Committee
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655	Checking of Forecasts at 30,000 Feet over Egypt.	C.S. Durst
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M.R.P. 701
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given as

$$E = \frac{bc}{2bc - b - c + 1} \qquad ---(1)$$

This formula indicates that the useful effort depends not only on the forecast accuracy, but also on the frequency of occurrence of the element being forecast. Since one requires $E > b$ to make forecasting worth while, this necessitates $c > \frac{1}{2}$, i.e. the forecast accuracy must exceed 50%.

This argument however assumes that the forecast accuracy is the same for suitable as for unsuitable days, and this is not in general the case. Consider an event about which little is known except that it occurs on 1% of occasions, $b = 0.01$. It might then with some justification be regularly forecast as not occurring, and its forecast accuracy would be zero. On the other hand forecasts of non-occurrence of the event would have an accuracy of 98/99, while the overall forecast accuracy is 98/100. Let then c denote the forecast accuracy of suitable days, and c' that of unsuitable days. The proportions of correct and incorrect forecasts in the two categories may then be set out as in Table I.

Table I. Forecast contingency table

	Forecast	Not forecast	Total
Suitable	bc	b (1-c)	b
Unsuitable	(1-b)c'	(1-b)(1-c')	1-b
Total	bc + (1-b)c'	b(1-c) + (1-b)(1-c')	1

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METEOROLOGICAL RESEARCH COMMITTEE

Note on the usefulness of forecasts.

by
A.F. Crossley (Meteorological Office)

1. The useful effort

The following scheme of assessing the usefulness of a certain type of forecast has been put forward by Mr. Dawes of the Scientific Adviser's Department of the Air Ministry.

The type of forecast concerned is of the "black or white" type and may relate e.g. to the suitability as regards weather of any day for carrying out a specific operation; any one day is therefore forecast as suitable or unsuitable. Dawes defines the forecasting accuracy (c) as the proportion of all occasions which are correctly forecast ($0 \leqslant c \leqslant 1$). The "useful effort" (U.E.) is defined as the proportion of occasions forecast as suitable which in fact turn out to be so. If operations are conducted in accordance with the forecasts, the U.E. represents the proportion of operations which turn out successful as regards weather.

If the proportion of suitable days in a given period is denoted by b, then the proportion bc of suitable days will be correctly forecast and the proportion (1-b)(1-c) of unsuitable days will be incorrectly forecast as suitable. Therefore the proportion of occasions forecast (rightly or wrongly) as suitable will be bc + (1-b)(1-c), and the useful effort is given as

$$E = \frac{bc}{2bc - b - c + 1} \qquad - - - (1)$$

This formula indicates that the useful effort depends not only on the forecast accuracy, but also on the frequency of occurrence of the element being forecast. Since one requires $E > b$ to make forecasting worth while, this necessitates $c > \frac{1}{2}$, i.e. the forecast accuracy must exceed 50%.

This argument however assumes that the forecast accuracy is the same for suitable as for unsuitable days, and this is not in general the case. Consider an event about which little is known except that it occurs on 1% of occasions, b = 0.01. It might then with some justification be regularly forecast as not occurring, and its forecast accuracy would be zero. On the other hand forecasts of non-occurrence of the event would have an accuracy of 98/99, while the overall forecast accuracy is 98/100. Let then c denote the forecast accuracy of suitable days, and c' that of unsuitable days. The proportions of correct and incorrect forecasts in the two categories may then be set out as in Table I.

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Suitable	bc	b (1-c)	b
Unsuitable	(1-b)c'	(1-b)(1-c')	1-b
Total	bc + (1-b)c'	b(1-c) + (1-b)(1-c')	1

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From this, the useful effort (in regard to suitable days) is given by

$$E = \frac{bc}{bc + (1-b)(1-c')} \qquad - - - (2)$$

The condition for worth-while forecasting, $E > b$, then leads to

$$c + c' > 1, \qquad - - - (3)$$

so that the sum of the two forecast accuracies must exceed unity. It is not therefore essential for the forecast accuracy (c) to exceed 50 % in order to make forecasting of some particular event worth while; the accuracy can fall short of 50 %, provided this is counterbalanced by an accuracy above 50 % for forecasts of non-appearance of the event.

Figure 1 illustrates the relationship between useful effort and the forecast accuracies when $b = 0.4$. In this and similar diagrams (not reproduced) for other values of b, the diagonal line $c + c' = 1$ is identical with the isopleth $E = b$. Hence for a useful effort greater than b, the point (c, c') must lie above and to the right of this diagonal. The other diagonal is the line $c = c'$, which is Dawes' over-simplified case. It is seen (e.g.) that to achieve a useful effort of 0.5 it is necessary to have one or other of c, c' in excess of 0.6. However, c may (e.g.) be as low as 0.3 provided c' is at least 0.8. A similar point was made by Dawes in remarking that a useful effort of 0.48 with $b = 0.2$ requires a forecast accuracy of nearly 0.8 (i.e. with $c = c'$).

2. Varying the useful effort

2.1 The useful effort is a function of the frequency (b) of the event forecast and of the accuracies (c,c') with which both the event and its non-occurrence can be forecast. If b can be varied without invalidating the purpose for which the forecast is required, the value of the useful effort will in general be altered. It is a matter for consideration whether the useful effort can be improved. For example, suppose the event for which forecasts are required is that of "cloud base not below 13,000 ft". This is included in "cloud base not below 15,000 ft" and it is conceivable that the useful effort in regard to the latter event is greater than the useful effort in regard to the former. On the other hand, the operations for which the forecasts are required might well be still practicable provided the cloud base is not below 10,000 ft, and again it may be found that the useful effort is in consequence increased. In such cases - where economy of effort is the main consideration, and time of less importance - the limiting situation requires to be selected not only from the point of view of feasibility of a particular operation, but also with regard to the resulting U.E. What at first might appear to be a loss due to restriction or extension of conditions, may be more than compensated by an increase of useful effort.

2.2 Consider again the forecasting of an event of which the frequency distribution consists of a curve with a single hump centred near (but not in general coincident with) the average value, while on either side it tails off towards zero for low and high values. Wind speed (e.g.) has this type of frequency distribution. Let us suppose that forecasts are required of speed greater (or less) than some value V. If V is near the average value, speeds in this neighbourhood will occur with high frequency and it will often be difficult to predict whether the speed will or will not exceed V: on the other hand, occasions of both very strong winds and very light winds will be more readily forecast successfully, since both, being infrequent, depend on exceptional conditions which will be readily recognised in advance. Thus while b decreases, the accuracy of forecasting increases. However, if variations in c' are ignored, the useful effort increases with bc and may be expected to attain its maximum values in the neighbourhood of the two points of inflexion of the frequency curve, i.e. at moderately low and moderately high values of the wind speed.

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2.3 By varying the expression (2) for the useful effort, it may be seen that for small variations in b,c,c' the U.E. is increased provided

$$\frac{\delta c}{c} + \frac{\delta c'}{1-c'} + \frac{\delta b}{b(1-b)} > 0 \qquad \text{- - - (4)}$$

If the forecast accuracies remain unchanged while b is increased, then the useful effort is increased; this increase will be still greater if either or both of c, c' are increased, but some increase in useful effort may still occur even if one or both of c, c' are reduced. On the other hand if b is reduced, there may be increases in c or c' sufficient to ensure an increase of useful effort.

3. Examples

3.1 Some of the above ideas may be illustrated by actual results for certain forecasts. Consider first, a series of trial 4-day forecasts of weather over Great Britain made by the Forecast Research Division at Dunstable over the period February, 1949 to March, 1950. The series included definite forecasts of a change of weather type; such a forecast, to be correct, has to predict a change occurring on the right day of the four. Not more than one change of type occurred, or was forecast, in any of the 4-day periods concerned. The following table shows the results of an analysis of these forecasts, each 4-day forecast period constituting one occasion.

Table II. Forecasts of change of type, Feb., 1949 to March, 1950

No. of cases	Forecast	Not forecast	Total
Change of type	19	17	36
No change	59	13	72
Total	78	30	108

From these figures we have the following -

(i) Frequency of occurrence of "change", $b = \frac{36}{108} = 0.33$*

* This number refers only to changes occurring within the 4-day forecast periods; these did not cover the whole of the period mentioned.

(ii) Accuracy of forecasts of change, $c = \frac{19}{36} = 0.53$

(iii) Accuracy of forecasts of no change, $c = \frac{59}{72} = 0.82$

(iv) Useful effort, or proportion of forecasts of change which are correct $= \frac{19}{19 + 13} = 0.59.$

All these figures indicate that forecasts of a change of type constitute a definite improvement on a wait-and-see policy, according to which a change of type would be expected on average of one occasion in three. At the same time, as might have been expected, there is a much higher forecast accuracy in regard to forecasts of "no change" than there is for forecasts of "change" (82 % against 53 %). Depending on the "operations" concerned, which may of

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course include the use of the forecasts by the public, forecasts of "no change" may be of as much, or more, importance as forecasts of "change". It is equally appropriate therefore to consider the useful effort in regard to forecasts of "no change", which is 59/ (59 + 17) = 0.78. Thus 78 % of the forecasts of "no change" are correct. This compares with the earlier statement that 82 % of the occasions of "no change" are correctly forecast. From the point of view of reliability of the forecasts, the lower figure (the useful effort) is, in this case, the more pertinent.

3.2 Another example is provided by the results of an investigation by Sawyer (1950) into the application of certain criteria for the formation of secondary depressions at points of occlusion. When his results for both warm-occlusion and cold-occlusion secondaries are combined, the following table is obtained -

Table III. Formation of secondary depressions, Oct. 1947 - Sept. 1948.

No. of cases	Criteria correct	Criteria incorrect	Total
Secondary formed	63	14	77
Not formed	137	23	160
Total	200	37	237

Using the same notation as before, we have

$b = 0.32$, $c = 0.82$, $c' = 0.86$.

$E = 0.73$ in regard to secondary forming,

$= 0.91$ in regard to no secondary forming.

These figures show that the criteria have a high degres of validity, especially in regard to non-development of a secondary, in which case 91 % of the expectations are correct. It will be of interest to see if the same degree of accuracy is obtained in actual forecasting.

4. Economic importance

The value of forecasts has been discussed on somewhat similar lines by Bijvoet and Bleeker (1951), but in their case the authors emphasise the economic importance by considering, as an example, the cost of taking precautions against frost damage, and the losses incurred on account of damage if no precautions are taken. Suppose that without preventive measures the average loss on a frost night is L, and that the daily cost of preventive measures is P. Two cases arise according as P is greater or less than bL, in the present notation; i.e. according as the cost of prevention is greater or less than the cost of damage in the absence of preventive measures. If preventive measures are applied only when frost is forecast, it is seen by reference to Table I that the average daily cost is

$$C = \{bc + (1-b)(1-c')\}\ P + b(1-c)\ L$$

Hence if $P > bL$, the average daily saving obtained by use of the forecasts is

$$B_1 = bL - C = bcL - \{bc + (1-b)(1-c')\}P$$

Similarly if $P < bL$, the average daily saving is

$$B_2 = P - C = (b + c' - bc - bc')\ P - b(1-c)\ L$$

From these relationships, Bijvoet and Bleeker give two diagrams showing the relationship between P/L and B_1/L or B_2/L for $b = 1/10$.

5. Conclusion

Such instances as that just discussed, together with the earlier considerations, show that there is little direct relationship between the value and the accuracy of a given type of forecast. The accuracy is a matter depending on the personal ability of the forecaster, the quality of his technique and the meteorological circumstances, while the value of the forecast to the user depends also on the particular circumstances in which it is applied. If the user is unwilling to risk losing an opportunity on account of a bad forecast, then forecasts (for that purpose) are of no use to him, however accurate they may be, short of perfection. Nevertheless it is desirable to have an estimate of the reliability of any given type of forecast, and expression (2) given above for the "useful effort", which is also the chance that a forecast has of being correct, appears to meet this need satisfactorily.

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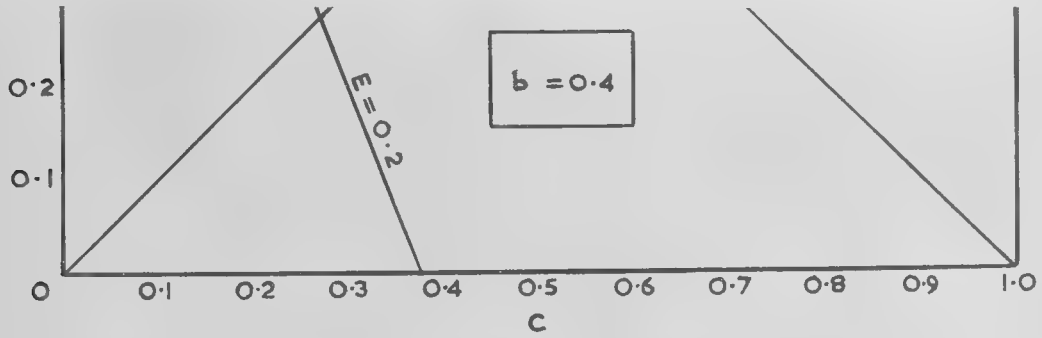

Fig.1. Useful effort(E) in relation to forecast accuracies (C, C') for frequency of occurrence b=0·4.

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An Essay
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C.K.M. Douglas (Meteorological Office)

During the present century there has been more or less continuous progress in synoptic meteorology, due primarily to the steady increase of observational material. Reports from ships in the Atlantic are an obvious necessity for satisfactory forecasting for the British Isles, and these began before the first world war. The first wireless reports from Atlantic liners were in 1909, and by 1914 they were quite numerous, but they ceased in the latter part of the war and were not renewed till 1921. The exploration of the upper air began early in the century, and immediately led to wholly new ideas about the atmosphere, and ever since then there has been a steady growth in our knowledge of the three-dimensional atmospheric processes. Since the sounding balloons were not recovered till some days later they could not be used for forecasting, but the knowledge derived from them affected the interpretation of surface observations and influenced the ideas of the more advanced forecasters from the time of the first world war onwards to an increasing extent. The practical results of increased scientific understanding are difficult to determine. So far they have been rather disappointing, for reasons which I shall attempt to explain later.

The complexity of the atmosphere makes forecasting a very woolly subject, unsuited to description in precise terms. It is impossible to give a detailed history of forecasting techniques, since they are inherently indescribable, and no adequate account of them has ever been published at any time in any country. The present essay only deals with some aspects of the subject, and is mostly based on personal memories. It deals chiefly with the development of the scientific ideas which enter into the interpretation of synoptic charts, but some remarks will be made on their practical applications, limited though these are. No attempt is made to include a bibliography of the subject, which would have to be very long if it were at all comprehensive. A selected bibliography would be rather invidious and would be of little use unless a great deal of time were devoted to it. A few publications are referred to in the narrative but these are either recognised classics or else they are introduced incidentally owing to their relevance to the points discussed.

Forecasting before 1914

Progress throughout the century has been fairly continuous, but the observational advances and the changes in method were specially marked during the two world wars, owing to the demands of the Services. Both the war periods were exceptional and transitional. The three main epochs were before 1914, that between the wars, and the modern period which is only now slowly beginning to take shape. A chronological sequence will be followed, but without any rigid divisions, since a developing idea has to be seen in perspective, and should be assessed in the light of modern knowledge. The main sources of information about the pre-1914 period are Shaw's Forecasting Weather, published in 1911, and the Daily Weather Reports of that time, which contain forecasts for the 24 hours from noon. Shaw's great work, like all others on the subject, deals mainly with synoptic description, though there is a general outline of forecasting procedure. Since the book was intended for the general reader the elements of the subject had to be included, but there is also an account of the physical processes of weather and of the most recent work on trajectories and line-squalls. These portions of the book are of great historical importance in that they link up physical ideas with forecasting problems, but they

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An Essay on the Evolution of Forecasting in the British Isles in the 20th Century

by
C.K.M. Douglas (Meteorological Office)

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Progress throughout the century has been fairly continuous, but the observational advances and the changes in method were specially marked during the two world wars, owing to the demands of the Services. Both the war periods were exceptional and transitional. The three main epochs were before 1914, that between the wars, and the modern period which is only now slowly beginning to take shape. A chronological sequence will be followed, but without any rigid divisions, since a developing idea has to be seen in perspective, and should be assessed in the light of modern knowledge. The main sources of information about the pre-1914 period are Shaw's Forecasting Weather, published in 1911, and the Daily Weather Reports of that time, which contain forecasts for the 24 hours from noon. Shaw's great work, like all others on the subject, deals mainly with synoptic description, though there is a general outline of forecasting procedure. Since the book was intended for the general reader the elements of the subject had to be included, but there is also an account of the physical processes of weather and of the most recent work on trajectories and line-squalls. These portions of the book are of great historical importance in that they link up physical ideas with forecasting problems, but they

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could not have much immediate practical effect. The empirical methods described by Shaw are broadly the same as those in Abercromby's Weather, which is highly praised. There is no doubt that Abercromby did much for synoptic meteorology, and he certainly helped to give it the status of a science, though not of an exact science. Nevertheless the actual practice of forecasting at the beginning of this century was so completely empirical that its character must have been determined by the nature of the available material, and no one man, however able, can have appreciably affected it. Forecasting has always depended on the personal experience and judgement of the forecasters themselves in dealing with each situation as it arises, and this cannot be formulated as invariable rules, though there are certain guiding principles. It is impossible to put into any book just how the difference between good and bad forecasting arises. A good forecaster is one who is more likely to know when not to apply textbook rules, but his decision, often reached after much hesitation and worry, with the low confidence which comes with knowledge when the situation is really difficult, is based on many features on a particular set of charts. No problem is ever repeated in detail. The good forecaster will get better results on the average, but in a particular case he may be unlucky and regret that he did not follow the textbook. No comparison is possible between a forecast problem as seen at the time and when seen after the event.

Forecasting Weather only includes two complete sets of forecasts, one of them on a reproduced Daily Weather Report. Altogether 136 isobaric charts were included in the 1911 edition, many on a small scale. In two cases of unusual rainfall it was mentioned there was a forecast failure. In addition to this a number of charts of unusual interest are reproduced, illustrating gales, snowstorms, and a striking sequence of developments in December 1909. Reference to the Daily Weather Report shows that none of the important developments were in the least foreseen 24 hours previously. Naturally the most striking and important developments are the most difficult to predict, and this is still true today.

Before the 1914 war the routine forecasts issued to the Press were regularly checked, in respect of wind direction, wind force, state of sky, precipitation, temperature and other phenomena, if any. It is stated in Forecasting Weather that the average results were that 56 per cent were wholly successful and another 31 per cent more nearly right than wrong, so that "87 per cent gave a fairly accurate indication of weather for the period to which the forecasts refer". It is stated that in the hands of experienced checkers the system worked mechanically, but that when the checker was inexperienced the figures tended to come out higher. It should be noted that in the absence of "other phenomena" (snow, fog, thunder etc.) the forecast was more right than wrong if wind and temperature were correctly forecast, so that the system of checking was a lenient one. When the 1914 war broke out the increased pressure of work on the small staff of assistants led to the checking being dropped, and it was never renewed.

The visual gale warning system in operation in those days was the same as it is now, apart from the increase in the number of places which fly cones. A "missed" gale was one which started less than two hours after a gale warning message, if any, was dispatched. The following sentence in Forecasting Weather is worth quoting, for the light that it throws on all progress in weather forecasting:-

"It has always been supposed that the facilities for identifying the approach of a depression carrying winds of gale force would be so much facilitated by reports by radio-telegraphy from the Atlantic, that when these were once received, the avoidance of "missed" gales would be an easy matter. This view is doubtless based upon the assumption that gale-carrying depressions pass across from the western region of the Atlantic, the neighbourhood of the transition from warm to cold water, almost with the regularity of an ocean steamer. For that, as for so many other general views of meteorological conditions, some striking instances can be adduced, but it is, unfortunately, not generally true. Since January, 1909, when the transmission of wireless messages from liners began, ... we find that we have still to watch with the same care, and with hardly less anxiety, the course of events off our western shores."

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In 1951 hourly reports are received from our western coasts, and they frequently have to be constantly watched not only in connection with gale warnings, but for all kinds of forecasting.

When the "further outlook" was first introduced it was based on the general weather type prevailing and did not refer to a fixed 24-hour period commencing at the end of the period covered by the main forecast. When there is no marked temperature feature the main point which interests the public is whether the weather is settled, i.e. fair, with no rain, or unsettled, that is, changeable with no prolonged fair weather. Some years after the B.B.C. forecasts had started in 1924, the expression "further outlook unsettled" became a joke, but it had its origin in an attempt to convey to the public some idea of the weather type. This problem has not yet been solved.

The "fine spell" notifications for the hay-making and harvest periods were in operation early in the century. Shaw does not state the degree of success attained, but it is fairly certain that the forecasters waited till an anticyclone was established, so that there should have been a very fair measure of success.

The forecasters of that period attempted less than is done now, particularly as regards timing. If the wind or weather expected within 24 hours differed markedly from that prevailing at the time there was a broad division into "at first" and "later", but more often there was a general 24-hour forecast based on the pressure distribution. On the west coasts, where ample Atlantic information is most necessary for detailed forecasting, the persistence of type leads to a fair measure of success in general forecasting, as there are a good many spells of dry or wet weather. There was a spell of 32 days in the late autumn of 1911 with measurable rain (.01 in. or more) at Valentia, and the forecasts of rainfall were 100 per cent correct, since on every day either "some rain" or "some showers" were forecast. For checking purposes rain and showers could not be distinguished with the information then available; and even now it would sometimes be difficult. In our eastern districts rainfall forecasting is intrinsically more difficult than it is in the west, but if there are no ships' observations there is better material in the east for a wholly synoptic forecast as distinct from a partly climatological one. On the average it takes rain or clearing weather over twelve hours to travel from Valentia to London, and if a short forecast for Valentia can be deduced from the local observations there is material for a successful 24-hour forecast for London if events proceed in a normal way. In the absence of Atlantic information forecasting often resolves itself into a study of the latest observations from our western coasts, especially barometric tendency, wind and cloud. Before 1914 only the general degree of cloudiness was reported, but in some situations, such as a wedge, increasing cloud usually meant an approaching depression, though it could easily be misleading. Lack of observations of cloud forms was a gap in the pre-1914 synoptic information.

Forecasting thunderstorms was one of the most successful branches of pre-1914 forecasting, as it did not often depend on information from ships. At first sight it may seem surprising that much could be done without upper air observations, but there is a fair degree of correlation between upper air and sea level conditions. Thunderstorms are fairly certain if a depression spreads up from France or the Bay of Biscay, and falling tendencies generally give warning of it. Other easily recognisable thundery situations are a trough of low pressure moving east with very warm southerly winds ahead of it, a "col" in warm weather, and the cyclonic showery type of cool weather. In some flat or complex situations there is a difficulty, but there is sometimes a long series of thunderstorm days and then only the first day presents any difficulty. Most of the main thundery outbreaks were forecast.

The four forecasters of that period were men who had started in the Forecast Division as assistants and had a very long experience of charts. For the forecasting of that period experience was more important than science, and its value was shown by the able manner in which the less frequent developments, such as the spreading back of depressions or rain from the Continent,

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were sometimes handled. It is interesting to note that these forecasters had no belief in the value of individual past cases or "analogues" as a guide to forecasting. It was said that as far back as the 1890s this method had been tried and found wanting, and subsequent attempts have confirmed this conclusion. The experience on which the individual forecaster builds up his technique is based on the normal modes of behaviour shown in a large number of somewhat similar situations. The abnormal case is never repeated. In spite of some good individual forecasts, it would be difficult to prove that the pre-1914 forecasts for 24-hour periods were on the whole better than could have been attained by statistical methods, based on persistence with some reversion to the normal. By "persistence" is meant recent trends and not merely a repetition of today's weather. If there is one dry day in the middle of a wet spell, forecasts based on mere repetition would contain two errors, but fully developed statistical methods would in some cases indicate a continued probability of rain, especially in the west with winds from any westerly point, so that there would only be one error. The pre-1914 forecasting organisation should not be judged solely by the forecasts produced but also by what it afterwards developed into, particularly in the 1914-1918 war, and this principle should be kept in mind whenever comparisons are made between synoptic and statistical methods of forecasting.

Weather Forecasts in the first World War

The outbreak of the first world war soon led to a greatly increased demand for meteorological information and forecasts, and the B.E.F. in France and the Navy both developed meteorological services. The effect of temperature and wind on shell trajectories was one of the earlier applications, but there was also an important and increasing demand for forecasts. The close contact between the meteorologists and the users of the forecasts led to the development of what was largely a new type of forecast, namely, the detailed short-range forecast. Previously there had been little demand for these, since it was impossible to disseminate them to a wide public, and for the minority vitally interested in the weather the necessity was not comparable to that of war-time. It is interesting to refer to a passage in Shaw's Forecasting Weather which describes a case of outbreaks of rain in a N.N.E. current extending as far as Ireland when the associated depression was over East France. Shaw quotes an extract from C.J.P. Cave's weather diary which describes the movement of thunderstorms from the southeast, with scud below from the north, and adds "such rain is beyond the forecaster's horizon". It was beyond the horizon of the forecasting then carried out in London, but not beyond the horizon of Mr. Cave and other first-class observers of clouds and weather. There are obvious limitations to single observer forecasting, but unless the sky is overcast with thick low cloud it is often possible to foresee the weather for a few hours ahead, and sometimes for the whole day. Clearly the best results are obtained by combining synoptic methods with local observations, and even with the wealth of information available in modern times the study of the sky is still a most useful aid to short period forecasting for a limited area. This does not mean only one place, since the range of vision of clouds often exceeds 50 miles and may exceed 100 miles, and the chart sometimes shows that local conditions are representative of half England.

The existing Meteorological Office organisation provided the essential basis for all the war-time forecasting. A collective synoptic message was telegraphed to the B.E.F. A new code was introduced which included observations of visibility, cloud types, height of low cloud, and much more detail about present weather. At that time only selected stations, mostly with professional observers, used the new code. After the war it was made general, but observations of clouds have always presented difficulties to part-time observers. There never has been a time when all the cloud observations could be accepted at their face value, but forecasters soon get to know which stations can be relied on. The new information naturally affected forecasting, and physical ideas increasingly entered into it, including what were later called "air mass" ideas. The work of Shaw and Lempfert on trajectories and their emphasis on currents of air paved the way for this. Observations of

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temperature and humidity from kite balloons and later from aircraft, obtained primarily for artillery "meteor" corrections, enabled the treatment of fog and low clouds to be placed on a three-dimensional basis. The "Meteor Flight" at Berck was started early in 1918, and soundings up to 14,000 feet became available for forecasting for the first time. Hertz diagrams were used for forecasting thunderstorms, showers, and cloud structures. There were pilot balloon observations on a large scale and the term "thermal wind" was in routine use. As the war progressed, forecasts became increasingly necessary to air pilots. In its earlier stages flying was only carried out in good weather, but later on weather risks had to be accepted, especially for night bombers and for the Independent Air Force with its flights into Germany, which were long for those days. During the whole war 24-hour forecasts and further outlooks were sometimes of great importance for planning purposes. The 0100 chart started in 1915, and with it the round-the-clock rosters.

During this period the war-time needs of a small neutral country, Norway, largely dependent on agriculture and fishing, led to a development of great importance in the history of forecasting. The first discoveries were based on a close network of observations but later developments did not depend on this. They will be the main subject of discussion in dealing with the inter-war period. The Norwegian work was often referred to as the "Bjerknes theory" but the word theory is unsuitable for anything in synoptic meteorology. It is really a technique based on simplified models of atmospheric structure and movement, and its successes have been based on its empirical and not on its theoretical aspects.

The inter-war period - the Norwegian methods

After the first war the Royal Air Force was cut down to an extent which later events proved to have been unwise, and aviation forecasting only continued on a small scale, its most important application being on the civil air routes from Croydon to Paris and Brussels. Later on there was an expansion both of military and civil aviation. It was only aviation forecasting which could be made detailed. Broadcast forecasts by the B.B.C. started in 1924 but for several years they were only issued at 6 p.m. and 9 p.m., and the most important part of them referred to the following day, so that they could not be made detailed even if there had been room for it, which there was not. A few years later land area forecasts were also issued in the morning and more detail could be given. The Norwegian technique is specially suitable for detailed forecasts for the first twelve hours. When the time interval is extended to 30 hours from the last completely analysed chart, as with most Press and B.B.C. forecasts, the difficulties of timing are great and a high degree of precision is impossible, whatever technique is used. As we shall see later, the Norwegian technique is sometimes applicable to 24-hour developments, but good cases are rare.

It is assumed that the main features of the Norwegian ideas are generally known, and it is only proposed to discuss certain points about which there was some controversy and perhaps some misunderstanding. The widespread adoption of the technique was due to its suitability for surface charts, which were the only ones available, but the underlying ideas were essentially three-dimensional. The emphasis on rainfall and the occlusion process meant that the ascent of air was fundamental, and subsidence and dynamical warming, the importance of which had been previously emphasised by W.H. Dines and others, were soon brought into the picture. The interest of the leaders of the Norwegian school was always centred on the main developments, rather than on the details of aviation forecasting. It was soon realised that the frontal discontinuity is often smoother in the upper air than it is near the ground, but this was not considered important. Bergeron talked in terms of isosteric-isobaric solenoids, and this is the same as a baroclinic zone. Indeed the very terms barotropic and baroclinic were introduced by V. Bjerknes. The methods were well publicised and by some subtle process exaggerated hopes came into being, though these cannot be traced to any definite statement by any of the leading members of that school. As a

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reaction against this there was some criticism of excessive claims and over-simplification. Some over-simplification and unwarranted optimism there certainly was, but without these it is doubtful whether so much valuable research would have been done. Sir George Simpson once remarked that the Norwegians had infused new life into synoptic meteorology. The enthusiasm was primarily directed to an increased understanding of the atmosphere, but there was also an underlying assumption that forecasting would benefit to a substantial degree. This has been partially justified, and more than this cannot be said of anything in the whole history of forecasting.

In the summer of 1920 Prof. V. Bjerknes invited Sir Napier Shaw to visit Bergen with a small delegation of British meteorologists to discuss the new ideas. The three other meteorologists were Dr. Goldie, Dr. Richardson and the present writer, who will all remember the enthusiasm which existed at Bergen. The Norwegians present included V. and J. Bjerknes, Bergeron, Rossby, Hesselberg, Solberg and Calwagen, who was afterwards killed on a meteorological flight. Prof. F.M. Exner of Vienna also attended the latter part of the meeting. The Norwegians were entirely open-minded and willing to recognise that the atmosphere was complex, which was certainly the opinion of their visitors. Dr. Richardson was about to publish his book "Weather Prediction by Numerical Process", and Sir Napier Shaw mentioned that Dr. Richardson and Prof. Bjerknes were the only men engaged on a direct attack on the main atmospheric problem. Dr. Richardson certainly thought the problem highly complex both in its theoretical and practical aspects. I showed slides of cloud photographs and of temperature fluctuations at Berck which had a bearing on fronts and associated clouds, and described both some typical altostratus-nimbostratus cloud structures and also complex multi-layer types, an account of which had been published. Not long afterwards Dr. J. Bjerknes visited Paris to co-ordinate his views with the French ideas on travelling "Systèmes nuageux" and the related isallobaric lows. These included alto-cumulus systems which were attributed to weak fronts, and "ciel de marge" often noted in France at the southern flank of rain areas was explained in the same way in many cases. The reason why the Norwegians adhered to the thick mass of cloud in their main warm front model was that to them the most typical warm front was well within a deepening depression, with rapidly falling pressure and substantial rain ahead of it. The question of the frequency of different structures had to await further observations, and even now the observational material is inadequate for statistical purposes.

During the 1920 meeting Prof. Bjerknes showed a slide of fig. 96 of Shaw's Forecasting Weather (1911 Edition) which foreshadowed the Norwegian model of a depression. Shaw remarked that he had come to regard these features as largely superficial and that he looked on depressions as vortices, mentioning Dines's correlation coefficients in support of this view. Bjerknes replied that in his opinion the vorticity was concentrated along the fronts in the earlier stage of development and afterwards spread over the depression. There is a good deal of truth in this, but the details of the process are still not fully understood. In the light of later developments it may well be that the Norwegians would have done better to have paid less attention to surfaces of discontinuity and more to the generalised problem of divergence and vorticity, but this would only have affected the work of a few mathematicians. It could not have influenced practical developments, since vorticity could not be applied to the main three-dimensional problem till upper air charts were available. Later on J. Bjerknes took a leading part in the development of the newer outlook, and a paper he published in 1932 included the first dynamical relationship between upper divergence and the flow round an upper trough. The precise status of the surface of discontinuity is still in doubt. Some young depressions have such a surface extending to the tropopause, though with complexities, but others have not, and there is no obvious difference of behaviour. This is the main reason for the modern preference for the baroclinic zone.

The Norwegian school attached much importance to the ideas of Margules on the conversion of some small part of the potential and thermal energy of the atmosphere into kinetic energy. Margules worked with highly idealised models

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but he introduced the principle that the vertical circulation with suitable horizontal temperature differences and lapse-rates releases the kinetic energy required for the horizontal motion. Gravitational and thermal energy are both drawn on as they change together, due to the elementary fact that when a column of air is warmed its centre of gravity rises. As Sir Charles Normand has pointed out, he obscured the importance of the latent heat of condensation by only considering cases when a complete spreading of the cold air under the warm could take place even when the rising air is dry. If in such a case one merely substitutes rising saturated air in neutral equilibrium for rising dry air, the condensation merely warms up the top layer without adding to the kinetic energy. In the conditions actually observed in the atmosphere only a partial overturning takes place, and the condensation is vitally important, since without it the amount of vertical circulation would be small. If the rising warm air is convectively unstable kinetic energy is released in it as irregular motion. The general warming of the upper air in the unstable region may give rise to a horizontal temperature gradient and so to a new source of energy, but the scale of this is usually small in our area. The writer attributed importance to vertical instability as a factor in the development of polar air depressions by 1920. The Norwegian school as a whole attributed great importance to the release of latent heat long before Refsdal published his valuable paper on "Der Feuchtlabile Niederschlag" in 1929, which attached far too much importance to instability in relation to the deepening of occluded depressions. There is now plenty of observational material, which shows that post-occlusion deepening is due in the main to horizontal gradients of temperature, and this also holds for many polar air depressions, though instability is a supplementary factor in some cases. The modern view of the role of instability was well summarised by E.J. Sumner in a recent paper (Q.J. 76, 1950, p.384).

The latent heat makes the energy problem difficult. A recent paper by J. Spar (Journal of Meteorology, 7, 1950, p.48) showed that there was no decrease of potential energy over an area covered by a deepening depression and its periphery, and regards this as evidence against Margules, but this is quite normal and since latent heat was ignored this conclusion does not follow. Condensation reduces the rate of cooling of rising air and the rise of temperature shows up in the descending air, as in the föhn process. In the long run this is compensated by outward radiation. In discussing the energy of the upper currents a very large area would have to be considered, but in the lower troposphere there is rapid development of kinetic energy during cyclogenesis, and the only available source is the Margules type of process acting through the pressure gradients. The release of energy implies that the rising air is warmer on the whole than the descending air. This very important point is obscured if the formation of a cold depression is attributed primarily to ascent. The coldest part of the developing depression is the rear portion where the air is usually descending, and the ascending saturated air can rarely become so cold till it has been cooled by radiation.

In 1928 Bergeron published a classical paper on three-dimensional analysis which included a discussion of frontogenesis. The isotherms are drawn together in the first place by deformation of the air mass in suitable isobaric distributions, the "hyperbolic" flow pattern being the ideal one. The frontal zone is a product of the previously existing depressions and anticyclones. A long fairly straight front is usually a trail left by a moving depression, which can be occluded at the centre, and is best defined when there is a warm anticyclone to south or southeast. Convergence is the main agency in sharpening the front. More recently Sutcliffe has suggested that it is only convergence which causes undercutting of the cold air and therefore an approximate surface of discontinuity, at least low down. It is difficult to see how the undercutting could take place without a vertical circulation and ageostrophic motion, of the type which is set up when the horizontal temperature gradient is increasing. Moreover for statical reasons a sloping boundary surface with the cold air below requires cyclonic shear of the geostrophic wind at the front, and this can only be produced by convergence.

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The recent discoveries about the relation of the "jet stream" to the frontal surface support this view. It is not clear why some frontal surfaces extend right up to the stratosphere, but this may require rather special conditions, such as a high wet bulb potential temperature in the warm air and a really low temperature in the upper troposphere in the cold air. Since convergence is fundamental in producing a real front rather than a frontal (or baroclinic) zone, a line of rainfall normally represents a real front when it is more or less parallel to the isotherms in the lower troposphere. For reasons given below, restraint should be exercised regarding the number of fronts put on the large area working charts.

Practical applications of the Norwegian ideas

The now world-wide system of frontal terminology and of marking in fronts in colour on working charts and in black and white on published charts was largely the work of J. Bjerknes, who has probably had more influence on synoptic meteorology than any other individual. Its continued world-wide use is itself a proof of its value, but against the undoubted gain one has to set the disadvantage of a large increase in the subjective element on a synoptic chart. There never was complete objectivity in the drawing of the isobars, but this defect could be minimised by having enough observations. Before the days of fronts the isobars were drawn too smoothly, and afterwards the frontal discontinuities have often been overdone, especially in areas with few observations. Fronts were not published in the Daily Weather Report till 1933, and there was good reason for this caution, since before that time there was no generally high standard of frontal analysis. Some years earlier a number of countries started publishing fronts, and there was vast diversity between them. A great deal of this was due to sheer bad analysis, which indicated a marked degree of backwardness in the standard of synoptic meteorology. J. Bjerknes informed me that some of the published monstrosities which were supposed to be fronts make him quite ashamed, as he felt some responsibility owing to his international propaganda in favour of marking fronts on the charts. In the last twenty years there has been a notable improvement, due to better training and to the great increase of upper air information, but I cannot pretend to approve of all fronts recently published abroad, including some in scientific journals. It has been clear for some time past that even when avoidable errors are eliminated, complete objectivity cannot be obtained in frontal analysis, though the subjective element can be minimised if the number of fronts is kept fairly low. If only the very obvious and pronounced fronts were put on, analysis would lose much of its value. Nowadays the upper air thickness charts show up the general thermal distribution, but it is important to mark in a front where there is an appreciable chance of weather phenomena of frontal type.

Like so much in synoptic meteorology, fronts defy precise definition. Significant fronts vary greatly in structure and in the magnitude and degree of sharpness of the temperature and wind discontinuities. The part of the thermal field that can be called discontinuous is often embedded in a general gradient. Measurement along a fixed length across the front does not solve the problem, since there is no length that would be suitable for more than a limited group of fronts.

For a long time past it has been evident that the three standard types of front did not cover all cases, and in particular that any set of analysed charts contains more occlusions than are formed by the orthodox process. The term "pseudo-occlusion" has often been used to denote fronts with the structure of occlusions but formed without definite pre-existing warm and cold front. When there is convergence and ascent of relatively warm air, the only variation from the normal process is that the warm front, and sometimes also the cold front, were not sufficiently well-marked to go on the chart. Sometimes also a belt of rain develops in unstable air and a weak warm tongue is formed over or just ahead of it. (Old occlusions are sometimes quite close to the axis of a cold tongue even when formed originally by the normal process). Some other fronts are not easily fitted into the standard scheme owing to a reversal

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of temperature difference with height and it would be an advantage to have "unspecified front" recognised internationally, and it is hoped that it soon will be. At the time of writing the old scheme is being used, apart from a slight addition introduced (in this country only) during the war, namely "warm occlusion" and "cold occlusion" when there is a marked difference of air mass temperature on the two sides of the occlusion. This cannot be coded and is therefore rarely used. The sending out of the frontal positions in code to out-stations started before the outbreak of war and was fully developed early in the war. As regards the number of fronts put on the working charts and the D.W.R., the policy from 1933 to the present day has remained unchanged and accords well with the widespread modern view as to the kind of frontal analysis which a Central Forecasting Office should carry out. There are also minor fronts which are shorter in length and duration than the main ones. There may be advantages in putting these on the chart for very short range forecasting, but not on the main charts of a Central Forecasting Office. The published analyses in the D.W.R. are the only six-hourly series in existence. Prof. Bergeron has expressed a preference for these to any other published set for research purposes.

The main practical application of fronts to forecasting is in respect of precipitation for all types of forecast, and also of clouds, which are specially important for aviation forecasting. Their value is greatest when timing can usually be made fairly accurate, especially the first 12 to 18 hours. There are also forecasting problems, especially of clouds, which depend on the study of air masses. The term "air mass" is the necessary complement of "front", and it can be applied to any region of the atmosphere not containing a front or frontal zone. In the Norwegian scheme fronts were primary, and air masses secondary. The Norwegians classified air masses, and the terms they used (some not new) are still useful for descriptive purposes, but in our area "maritime polar air" is too large a class and there is no agreed system of sub-dividing it. Air mass meteorology is the analysis of the physical properties of the air away from the front. The moisture distribution is of primary importance but this is of course greatly influenced by temperature distribution. It is only the elementary part of the subject which is capable of formulation in general terms, and this is set out in introductory text-books. Apart from the names used the subject was never specifically Norwegian and the useful parts of it were developed independently in various countries, including our own. Its scope is very limited. The atmosphere is never really homogeneous, and though away from fronts or developmental areas the vertical motion is small in absolute magnitude, it is often large enough to affect the stability and other properties of the atmosphere in 24 hours. Air mass analysis was directed mainly to estimating the upper air structure from sea level observations, and now that the upper air observations are available, classification has lost its analytical and prognostic value. Measured quantities belong to a more exact and therefore higher scientific category than classified entities. This does not mean that the forecasting of clouds and other air mass characteristics can yet take the form of precise computation; since as with other forecast problems there is vast complexity and only a limited time available. Each locality has its special problems, and there should still be scope for some empirical investigation on a local basis, but this should deal with the observations directly, without troubling much about the labels to be attached to the air masses.

We have already noted the difficulty arising from accurate timing of fronts, more especially in forecasts extending beyond 12 to 18 hours from the time of the chart on which they are based, which includes all forecasts covering the following day. This is the most constantly recurring of all the major forecast problems. It depends mainly on obtaining an accurate forecast of the pressure distribution, which is the key to nearly all 24-hour forecasting, and this in turn depends mainly on the movement and development of pressure systems. There is of course a mutual relationship between the motion of air masses and the development and movements of the systems into which the air motions are organised, but in practice it is the systems which are primary. Any notion to the contrary has never had any foundation either in forecast

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practice or in the ideas which emanated from Norway. In the inter-war period these ideas were of real help in estimating the movement and deepening of depressions in a limited number of cases. An important generalisation was that warm sector depressions move in the direction of the isobars in the warm sector. Another was that when a warm sector depression has started to deepen the process normally continues, often at an increased rate, till after occlusion. A third was that when a depression becomes occluded its motion decreases and it may become stationary, at least for a time. Frontal analysis also showed where a new wave depression was most likely to form, and this was of some use even though one had to wait for a more definite indication, particularly in the barometric tendency, before basing one's whole forecast on it. In the aggregate the value of these principles was appreciable, but the number of good cases was not large. This was partly because most depressions are occluded when they reach the British Isles. At about the time of occlusion depressions tend to change their speed and track, and this is one reason why forecasting for the British Isles is specially difficult. This originates in geography and its influence on the temperature distribution, which leads to many difficulties besides the one mentioned above. When it was impossible to apply Norwegian principles, the forecasting of the pressure distribution depended entirely on the extrapolation of existing trends, modified as necessary on the basis of general experience, and was thus similar in essentials to the pre-1914 practice.

The introduction of the tephigram

Observations from aircraft were very irregular till a Meteor Flight was started in 1925 at Duxford. This was too far east and it was not till 1937 that another flight was started at Aldergrove. Shaw and Fahmy published an important paper on the tephigram in 1925, and it was introduced as soon as possible by Mr. J.S. Dines for the routine plotting of the Duxford ascents. The fact that the pressure lines cut the saturated adiabatics almost at right angles makes this diagram very suitable for showing up small variations of stability, and it was a great improvement on anything previously used in this country. It is widely used in the British Commonwealth but most other countries use a version of the Neuhoff diagram. There is of course no difference in principle, and there is a natural tendency to adhere to the diagram in use unless there is a very good reason to change it. In this country the tephigram has developed into a major forecasting tool, used in various ways. Its original and most important purpose is in connection with vertical stability, especially of saturated air. The liability of the cumulus cloud to sustained vertical growth results from its excess of temperature and (to a much less extent) of humidity over that of its environment, and if the growth extends high enough showers and often thunderstorms develop. The thunderstorms are certain if there is marked instability. The comparison of a mass of air rising adiabatically with an observed environmental curve, assumed undisturbed, has been called the "parcel" method. J. Bjerknes and others have examined the effects of downward motion between the clouds, which must be taken into account in any complete discussion of the subject. No doubt it affects the number of storms which can form in a large area for any given value of the mean vertical motion over the area, which generally differs from zero, though there is no way of measuring it. The majority of thunderstorms form in a region of convergence, sometimes on a narrow belt of convergence, but a limited number form in areas which are neutral as regards cyclonic vorticity or its change with time, and in rare cases there is weak anticyclonic vorticity over the area as a whole, though not close to the developing storm. If there is no general convergence and the lapse-rate is along the saturated adiabatic the thunderstorms will be well scattered but they may still develop. Unless there is clear evidence of subsidence, the saturated adiabatic has to be regarded as the criterion of stability of saturated air. There are many marginal cases with a lapse-rate almost exactly saturated adiabatic, and there are small-scale variations in temperature and more especially in humidity, so that a vertical sounding does not always give a decisive indication for or against thunder or showers. Even in more definite situations the validity as regards time and area is

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often very limited. In the inter-war period the soundings were sometimes of value in forecasting thunderstorms for the same day, especially after the Aldergrove flight started in 1937. Some optimists thought that a good network of upper air observations extending into the Atlantic would lead to reliable thunderstorm forecasting for the following day, but as we shall see later this hope has not been fulfilled. The use of the tephigram has been greatly facilitated by the application of Sir Charles Normand's ideas on the wet bulb potential temperature, which were introduced at the instance of Sir David Brunt within a year or two of the production of the tephigram itself. The humidity has to be averaged over a layer about 100 mb. thick and is taken into account over a greater thickness, though there is no precise rule. Recent American ideas about "entrainment" merely confirm the previous practice of taking humidity up to 700 mb. into account in a vague manner. Two separate processes are involved in "entrainment", as discussed by Byers, one the mixing process and the other the convergence into the lower part of a large mass of cumulus or cumulonimbus, up to a height well above the base.

Other advances in the inter-war period

Apart from the short-term use of the tephigram, the application of upper air information to forecasting in the British Isles in the inter-war period was severely limited by geography. Upper winds up to considerable heights were occasionally available, especially in summer, and these were supplemented by nephoscope observations of medium and high cloud. These sometimes helped in analysis, especially when a strong N.W. cirrus motion at Valentia indicated an approaching warm front. They were also sometimes useful in forecasting the movements of rain and thunderstorms, especially in summer.

Empirical ideas about the effect of upper currents on the movement of pressure systems have a long history. In the early pioneering days it was seen that high clouds were carried ahead of the depression and this was vaguely related to their movement. The application of three-kilometre winds to the movement of small isallobaric systems was included in "Les Systèmes Nuageux" by Schereshewsky and Wehrlé in 1920. Later the Germans followed a similar line but used the term "stratospheric steering". For quite a long time the dominant German academic school was obsessed with the idea that the stratosphere controlled the main atmospheric movements, since the "centres of action", i.e. the warm anticyclones and cold depressions, extend to the stratosphere. In effect this treats the problem as one of statics and assumes that the tail wags the dog, and is itself controlled by some unknown factor such as high-level radiation effects. When the increasing demands of aviation led to a more synoptic outlook in Germany their ideas came nearer to earth, but there was no dynamical explanation of steering till Sutcliffe's 1939 paper. In this country geography made it difficult to apply even empirical upper air steering ideas, but it was possible in a few cases.

In the 1930s Germany developed seven or more meteorological aircraft stations as a training school for their secret Air Force, and they played a pioneer part in developing upper air charts, especially of the 500 mb. absolute and relative topography, which we now call contours and total thicknesses. Since they also received observations to westward they were well placed for the purpose. Their best-known work is that of Scherhag on what is now called difluence and confluence, but what he called divergence and convergence. He gave no dynamical explanation, and his best cases of falling sea-level pressure with difluent areas on the upper air charts were due to the association of both with other factors. Nevertheless he was the first to notice the importance of these features. There has been a tendency to under-emphasise the acceleration along the trajectory in comparison with curvature effects, especially at high levels. Actually it may be equally important, though it works in a more complex way than Scherhag's earlier work appeared to suggest.

The paper by Brunt and Douglas in 1928 brought out the importance of acceleration along the trajectory at low levels (where the isallobaric term generally exceeds the acceleration along the stream-lines) in relation to

/frontal

frontal and non-frontal rainfall, the ageostrophic motion of warm and quasi-stationary fronts, and subsidence. It could not be applied numerically but it is reasonable to hope that a better understanding of the dynamical process associated with rainfall was of some use for empirical forecasting, even if there was no way of assessing what this amounted to, or even of proving its existence. This applies to other modern advances, which in the aggregate have now left 1928 far behind. There has been some progress in practical forecasting, but it is impossible to decide how much of it is due to an increase of information and how much to a better scientific understanding. In the decade before the second world war there was a substantial advance made in understanding the atmosphere, and in particular the papers published in the 1930s by J. Bjerknes, Scherhag, and Sutcliffe had outlined the framework in which the modern advance in three-dimensional synoptic meteorology has taken place.

Weather Forecasting from 1939 to 1946

The second world war, like the first, was an exceptional period, when forecasting was handicapped by lack of information from the Atlantic but nevertheless progressed in other ways. The first war initiated detailed short-period forecasting. In the second war this was greatly developed and expanded, but though very important in itself it was based on existing methods and had little permanent effect on forecasts as distinct from communications. The big advance was the development of the radio-sounding balloons and in particular of radio and radar wind observations, leading to a new high-level aerology with an important direct application to high flying. Though the observational methods were worked out before the war, the scale of the operational development of the observations was greatly speeded up by war-time demands. The first radio-wind at Larkhill, in late October, 1939, was made through frontal cloud in a very strong baroclinic zone, and obviously it opened a new era in synoptic meteorology, though its full implications could only be guessed at and are in fact still doubtful. Previously no wind observation had ever been made inside a cloud. High flying also led to a demand for forecasts of cloud and icing at high levels. The only information available was that brought back by the aircraft themselves, which was linked up with other meteorological features for future forecasting purposes.

A combination of better upper air information and close network of hourly reporting stations with no Atlantic information led to a new situation, and obviously there was a net loss except for very short periods ahead. The upper winds sometimes helped in the analysis, especially a strong N.W. wind ahead of a warm front. There was a certain amount of empirical application of the steering principle and of the movements of rain and thunderstorms at middle heights, especially at 700 mb. Isobaric charts for 3000 and 5000 metres appear in the Upper Air Section of the Daily Weather Report for 1940. The German contour and thickness technique was introduced on 1st January, 1941, at the instance of Mr. E. Gold. The D.W.R. included 500 mb. contours and total (i.e. 1000-500 mb.) thicknesses over a limited area from that day. The 700 mb. charts were drawn but not published. The 250 mb. contours and 500-250 mb. "partial" thicknesses started on 1st April, 1941, having been delayed by lack of data. From 1942 there are charts for all three pressures, with partial thicknesses. (This term was used for the sake of distinction from the "total" thicknesses extending right down to 1000 mb.) A chart of tropopause height also appears in 1942. Though only one set of charts per day was published, the working charts of contours and thicknesses were drawn three times daily, becoming four daily later in the war. Their primary use was for forecasting upper winds. Later in the war there were reconnaissance flights in the Atlantic, and from June, 1944 also ascents from fixed ships of the Royal Navy and the U.S. Navy. This led to cross-Atlantic upper air charts and foreshadowed the modern period, but the fixed ships were withdrawn late in 1945 and were not replaced till late in 1947.

The forecasts for the Royal Air Force were supplied by the out-stations. The official Meteorological Office history contains a record of the immense range of activity of the war-time forecasting organisation at home and abroad.

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The great bulk of it was of detailed short-period type and many forecasters contributed to its success. The Central Forecasting Office supplied general guidance on the analysis and the expected developments, and from February, 1942 this was greatly helped by the issue of forecast or "prebaratic" charts. The term "baratic" was already used for the coded messages giving fronts and pressure distribution on the actual chart. This term and "prebaratic" were introduced by Mr. E. Gold, who initiated these issues to out-stations. The coded message can give a good representation if the coding is well done, i.e. if a good set of points on the isobars and fronts is chosen. When they were first introduced the time interval was 18 hours from the last main chart, or 13½ hours from the time of distribution in coded form. The introduction of the prebaratic as a routine was an important landmark in the history of British forecasting. Previously they had only been drawn experimentally. There was a fairly prolonged experiment about 1925-7, largely confined to isobars near the British Isles, but the results were not good. There is no doubt that good progress was made between then and 1942, particularly with regard to forecasting fronts. From the point of view of the actual forecast, especially of precipitation, the front is the most important thing on the prebaratic, but since its motion depends chiefly on the component of geostrophic wind at right angles to it, a good forecast of the position of a front requires a good forecast of the pressure distribution, especially since the time interval was extended to 24 hours in November, 1946. The prebaratic represents the fullest development of the Norwegian methods of forecasting, but nevertheless the most important part of its construction depends mainly on extrapolation of the development of the pressure field, which has always been the chief basis of forecasting. The extent to which the most recent advances are modifying or are likely to modify this situation will be the main subject of the next section, dealing with the modern period.

Some statistics of the mean errors of frontal positions on 18-hour prebaratics are given in M.O. Professional Notes No. 93. The mean error is about 90 miles, but there are large month-to-month variations, according as the month is a disturbed one or relatively quiet. There was a slight decrease in the mean errors in 1945 followed by a slight increase in 1946, due to the loss of the fixed ships and perhaps partly to the unsettled character of the summer. For 24-hour periods the error is about 120 miles, the increase being proportional to the time interval. It is too early to estimate the trend, as there are casual variations even in a period as long as a year.

An error of 120 miles in distance corresponds to about 4 hours in time. This means that in a forecast for the following day it is difficult to give the public the full benefit of the main advance that has been made, which consists of the substitution of a timed sequence for a more general type of forecast. The existence of an appreciable public demand for really accurate forecasts was shown by the popularity of "Airmet" during its limited period of operation, among people of widely different occupations.

Comparison with position in U.S.A. in 1946

A good summary of the position in the United States at the end of 1946, most of it valid here also, was made by H.G. Houghton in his Presidential Address to the American Meteorological Society in December, 1946 (published in the Bulletin for March, 1948). In the extract quoted below, I should be inclined to modify the first sentence and lay the stress not so much on our limited understanding of the physical processes as on the limited prognostic application even of a fair degree of understanding.

The following is the extract:-

"Our physical understanding of atmospheric processes is so limited that it is of little utility in weather forecasting. Consequently, a forecast is made by the extrapolation of the principal pressure systems in accordance with past trends, as evidenced by previous maps and the pressure tendency field.

/To

To this is added the experience factor which is the fusion of a large number of past situations into a general concept of what constitutes a reasonable pressure configuration under the given conditions. This is largely a subjective process as indicated by the frequent divergences of opinion among equally competent forcasters. Numerous attempts have been made to introduce objective methods based on some theoretical grounds, but it appears that none of these is as effective as the subjective methods practiced by an experienced forecaster. In view of this it is not surprising that many forecasters with little or no formal training are able to compete on equal, or better, terms with the products of our best university training.

"Such comparisons as have been made indicate that there has been no significant increase in the accuracy of short-range forecasts in the past 30 or 40 years. Such comparisons must be made on forecasts of temperature or precipitation since airway and upper-level forecasting are modern developments. However, I believe that it is mere quibbling to argue the point as to whether forecasting accuracy has improved. The number of elements forecast, the frequency of forecasts, and the types of forecasts have all increased greatly, but it is clear that there has been no corresponding increase in accuracy.

"There appears to be no immediate prospect of an objective method of forecasting based entirely on sound physical principles. If we are to attain this goal in a reasonable time, it is essential that the subjective nature of present methods be kept clearly in mind. The vast store of empirical data which we have accumulated is essential for the guidance of the scientific attack but the unaided empirical or synoptic method has approached or reached the point of diminishing returns.

"Since we cannot foresee an early solution to the forecasting problem on a purely physical basis, it is reasonable and proper for us to attempt to improve present methods. It must always be remembered that such efforts are in the nature of palliatives rather than cures."

The reference to the absence of significant increase in accuracy in the last 30 or 40 years presumably refers to the Weather Bureau checks, and the statement tallies with what I have heard verbally from other sources. A system of checking of 24-36 hour forecasts fixed long ago is unlikely to do justice to the actual progress made, which is mainly in providing detailed and fairly accurate forecasts for the first 12 hours. In the British Isles we have benefited from an increase of observations to westward, and it would be hopeless even to attempt to assess how much improvement in general forecasting can be attributed to improved methods and not merely to more information.

The Modern Period

The distinguishing feature of the modern period, which is still in its infancy, is the attempt to apply three-dimensional reasoning, based on dynamical principles, to the general development problem. Its origins extend far back, but till recently the observational material was inadequate for anything but rare and sketchy practical applications, especially in the British Isles, where moderately adequate information has only been available since late in 1947. The most immediate effect of the new information is to improve upper wind forecasts and to extend the range of height and the time interval for which they are reasonably good. It might appear to the man-in-the-street that if good forecasts can be made of winds across the Atlantic it should be possible to tell without fail whether tomorrow will be wet or fine, but whereas the wind forecasts depend on first-order terms in the equations of horizontal motion, the rainfall problem also involves second-order terms. As pointed out by Sutcliffe and Forsdyke in their paper in the Q.J. for April, 1950 the increased information has not been an unmitigated blessing, since "we are rather suddenly provided with more data than we can easily digest, represented on numerous upper air charts and aerological diagrams". The quotation from Houghton in the previous paragraphs is relevant to this, since in much of the U.S. they had upper air information to westward for some years before 1946.

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The atmosphere is in a continuous state of flux and the forecaster has only a limited time at his disposal. If the upper air charts are to be of any real use for 24-30 hour forecasting they must be adequately dealt with in a few hours, since all charts soon become out-of-date. If a forecaster spends too much of his time in examining what he cannot digest he may damage rather than improve his forecast. The upper charts have to be dealt with by a separate roster of forecasters, which makes forecasting more of a group activity and less individual, but someone has to make the final decision so that there is still a subjective element. The amount of subjectivity could only be tested by having forecasters working independently on the same data, but it is generally recognised to be appreciable, though on the average a good deal less than the difference between forecasted and actual events. When the atmosphere behaves in an abnormal way it dupes all forecasters, and though some may be more surprised than others, nobody can really cope with the situation.

The existence of a roster of upper air forecasters does not dispense with the necessity for simplification for the purpose of general 24-hour forecasting The position is analogous to that already mentioned in connection with frontal analysis, with which it is closely related. The complex details, like the very elaborate analyses, have only a very short-term validity. The simplifications advocated by Sutcliffe in 1947 and by Sutcliffe and Forsdyke in 1950 should be judged in the light of practical necessities. For some time past a great deal of research in several countries has been devoted to the main problem of development, of which the most fundamental feature is the vertical motion and the associated field of divergence, which reverses its sign at the level of maximum vertical motion, generally at about 650 or 600 mb. The barometric tendency at sea level represents the integral of the divergence of the whole overlying column of air and has the nature of a small residue, normally the difference between much larger divergences of opposite sign in the upper and lower troposphere, which are themselves second-order effects. Thus the tendency is a third-order product of the air motion, and this makes a direct attack on it almost hopeless. Since the divergence cannot be integrated nearly so well as is done automatically by the tendency field itself, there is no way of adding together the changes in a succession of atmospheric layers, and they are never independent. The key to the whole problem is the vertical motion. In some situations this can be roughly measured by its effect on temperature and humidity, but in the most important cases this is impossible as ascending saturated air is generally in a near-neutral condition. Observations of the rate of rainfall on a sufficient scale might be of use, but over the sea there are observational difficulties. In America a very big effort is being devoted to these problems, and an attempt is being made to use electronic machines in the effort to solve them. This experiment will be watched with interest, but at present the only realistic hope is that the machine may help in forecasting some large-scale developments, leaving the detail to be filled in by other means. If the attempt succeeds it will be a notable advance, more especially for the "further outlook". It is doubtful, however, to what extent the large so-called "waves" (troughs and wedges) at 500 mb., which are the present object of attack, are really independent of ordinary depressions and anticyclones, which can be regarded as atmospheric organisms with a phase of growth and then of slow decay.

On the whole the prospects of computing the future weather, with or without electronic machines, look remote at present. Nevertheless the introduction of quantitative elements, such as have been brought into the study of vorticity and divergence by Sawyer, Sheppard, and others, do represent a real advance, since it leads to a more precise understanding of the atmospheric processes. This must have some effect on forecasting, which is based on what Sutcliffe has called "rational empiricism", that is to say an empiricism based on a partial understanding of the physical and dynamical process underlying the observed developments. All existing techniques are empirical in this sense, and are broadly similar in the temperate zones, though there are differences in detail between various services. The 500 mb. charts are considered the best supplementary ones for general use as the 500 mb. level is representative of the upper troposphere and is normally above the level of maximum vertical

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velocity. The 300 mb. chart is very similar but has steeper general gradients, which makes the detail more difficult to pick up, and it sometimes cuts the stratosphere. Both the absolute contours giving the flow patterns on the isobaric surface, and the thicknesses, which depend on the mean temperature of the column of air, are now in widespread use, and at high levels (500 mb. and above) they have a strong resemblance. Dr. Sutcliffe has stressed the importance of thicknesses on both thermo-dynamical and on dynamical grounds. The fact that the divergence normally reverses sign below the 500 mb. surface means that the divergence of the thermal wind is dominant aloft and it is this, and only this, which is directly related to the sea level pressure tendency, in the sense that withdrawal of air corresponds with falling pressure. Developments of circulation, including not only deepening lows and building highs but much of the movement of pressure systems (that part which is not a pure transport in a general stream) necessarily implies vertical movement. What Sutcliffe called "developmental patterns" have been used at Dunstable since early 1948. (The associated diagrams were not published till Sutcliffe and Forsdyke's 1950 paper in the Q.J.) They are specially relevant to deepening and building, and also to dissipation, though friction also affects this process, which is on the whole less important than development. Models of this type cannot provide solutions applicable to all cases, but are a useful aid to empirical forecasting. I have given an account of their application in a monograph on the construction of prebaratics, and only a very brief summary can be included here. I mentioned that the patterns were "a valuable supplement to the barometric tendency" which represents the highest conceivable praise, since the tendency and the other associated extrapolations of existing trends have been the main basis of forecasting throughout the whole of the half-century under review, and before that. In a forecast of development in the next 6 to 12 hours it is obvious that great weight must be attached to existing trends, so that the problem resolves itself into that of understanding the existing trends and then estimating how much they are likely to be modified during a 24-hour period. Forecasts of changes of thickness are of course made use of as far as possible, but these are largely based on the prebaratic, and the time available for a process of mutual adjustment is small and often nil. Whenever the situation is changing quickly forecasting is a race against the clock. The prognostic value of thermal patterns, and of all empirical applications of upper air charts, is greatest when a small system moves into a different thermal field which will alter its speed and direction and may favour either development or dissipation. Larger systems are much more difficult to deal with but there are cases when even a fairly large system moves into a new field and is bound to be affected by it, though accurate computations are impossible. The reaction of the system of air movement on the thermal field is a serious difficulty. There is a well-known self-generating process of depressions and anticyclones, which create the developmental thermal pattern, and the speed of the system also comes into this problem, and these processes limit predictability.

Among other aspects of modern forecasting, only a few of the most important can be briefly referred to. A good prebaratic chart does not ensure a good forecast though it goes a long way towards it. The forecast of precipitation may be in error even if main features of the prebaratic are right, but in such cases the prebaratic normally contains errors of detail, particularly in the distribution of cyclonic vorticity. This is of course itself largely the result of vertical motion, and until the circle can be broken into there is little chance of a major advance. Convergence and its related vorticity distribution is also very important for the development of thunderstorms and rain of thundery origin, which includes a high proportion of summer rainfall over most of England. Vertical soundings plotted on tephigrams lead to good though not infallible forecasts for a short period ahead, but in a 24-hour period the changes of vertical stability are usually appreciable and are very difficult to predict. Horizontal advection does not provide an adequate basis for a forecast, and J.M. Craddock has recently shown that the advective temperature change is heavily counteracted by other factors and that the resultant change is a residual having a correlation coefficient of 0.6 with the advective component. The relation between

/thunderstorms

thunderstorms and convergence is very obvious, and divergence quickly stabilises almost any air mass, so that for forecasting thunderstorms for the next day a good prebaratic is a most essential tool. Thus forecasting thunder for the next day is still rather similar in principle to what it was early in the century, when fairly good results were obtained, even though a fair measure of understanding has replaced almost complete empiricism.

The location of existing thunderstorms by means of the "sferic" apparatus has its main application to short-period forecasting, but in some situations, particularly when there are outbreaks over the Bay of Biscay or off Portugal, it is valuable for forecasting for the following day. In summer and autumn successive groups of thunderstorms often travel in the currents at medium levels in a baroclinic zone. Each group has a phase of maximum activity of a few hours and is then apt to degenerate into rain without thunder. As with other data used in forecasting, they must be studied in conjunction with other information, and the successive phases must be understood, including such matters as the early morning maximum over sea areas.

Fog forecasting also depends on prebaratics to the extent that except for very short time intervals an accurate forecast of the geostrophic wind is the most difficult part of it. If one has a good wind forecast the problem can be isolated to a greater extent than most of the major forecast problems. The recent memoranda by Mr. Craddock have shown that a good deal of empiricism is still necessary in fog forecasting, and that the physical problem is difficult. If one goes a little higher up and considers low clouds, including stratus, the problem is less isolated and the general vertical motion often comes into it.

Conclusion

The main conclusion from this survey is that the large advance made in observations and communications during the half-century has led to the development of fairly accurate detailed forecasts for the first twelve hours, extending up to high levels, and to some improvement in the standard of 24-30 hours general forecasts. Concurrently there has been a great increase in our understanding of the physical and dynamical atmospheric processes, which has reacted on forecasting and made it more interesting, though there is no means of estimating how much of the forecasting advance can be attributed to it. A good deal of the improvement can be attained by a purely empirical ad hoc application of the new information. Even if the whole of the improvement could be explained in this way, forecasting would be a branch of applied science, since the observations are obtained and transmitted by scientific means. Actually there is no reason to doubt that increased scientific understanding has had some positive result, though it is disappointingly small, especially from the point of view of the scientific forecaster. Any future advance must depend on scientific advance in synoptic meteorology itself, as the possibilities of improvements merely from new data now look small in our area.

Even a limited advance may make a big difference to the real value of the forecasts, as compared with those attainable by statistical methods. The recent great increase in the public interest in the forecasts may be partly due to this, though other factors are helping to produce this result. There are substantial and increasing economic applications over a wide field, which is very satisfactory in itself, though it also throws into greater relief the limitations of present-day forecasting.

We may conclude by a quotation from the last page of Shaw's Forecasting Weather:-

"It may be that in the end the caprices of weather will after all disappoint us, and tomorrow's weather will never be forecasted with sufficient accuracy for all practical purposes. Even in that case the attempt will not have been in vain."

In one sense the doubt has been removed, in that a forecast service is essential in a modern community, But the larger question remains as to whether the forecasts can be made really satisfactory both to the physicists who make them and to the public who use them. After another few decades it should be possible to give a more definite answer to this question.

paper of the Meteorological Research Committee (London).
M.R.P. No..... by Mr.....
A copy is available in the Library of the Meteorological Office."

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AIR MINISTRY

METEOROLOGICAL RESEARCH COMMITTEE

Checking of Forecasts at 30,000 feet over Egypt

by C.S. Durst (Meteorological Office)

OCT 16 1951

1. No. 1 (Middle East) Meteorological Unit has been issuing forecasts of winds at 30,000 for the Delta and Canal Zones as a routine, since March, 1950. They are issued on most days at about 0230 zone time and refer to the period 0400 to 1800 Z. They are based primarily on the upper air charts for 1500 Z. The forecasts were compared at Headquarters M.E.A.F. with actual radar winds determined at Cairo at 1500 Z and with pilot balloon winds observed over the Canal Zone during (a) the morning and (b) the afternoon. The period of comparison was 1st March, 1950 to 28th February, 1951. Unfortunately there were insufficient actual winds in March 1950 to enable more than a few comparisons to be made for that month.

The results were summarised by Mr. J. Harding in the form of a table showing how often there were errors of certain values in each month. These are shown below.

Table I Frequency of various amounts of error in forecast winds over actual winds Cairo, March, 1949 to April, 1950.

	No. of Comparison	No. of occasions of vector error 30-40 kts.	40-50 kts.	50-60 kts.	60-70 kts.	Over 70 kts.
March 1950	No comparison possible					
April "	21	4	1	-	-	-
May "	18	1	4	2	-	-
June 1950	24	2	3	2	-	-
July "	29	2	-	-	-	-
Aug. "	28	1	-	-	-	-
Sept. 1950	28	5	2	-	-	-
Oct. "	25	1	2	-	-	-
Nov. "	26	6	0	0	1	-
Dec. 1950	22	5	2	1	1	1
Jan. 1951	25	3	4	1	0	1
Feb. "	25	3	3	2	0	1

2. The standard vector deviations of wind at 30,000 feet over the Canal Zone are about 42 kts. in winter, 40 kts. in spring, 25 kts. in summer and 32 kts. in autumn. The first three of these figures are based on revised estimates for Geophysical Memoir 85, the revised estimate for autumn had not been made and so the value of 32 kts. was read off the chart in Geophysical Memoir 85.

Interpolating from these we get the following as monthly values of the standard vector deviation.

Table II Standard vector deviation of wind at 30,000 over Canal Zone.

J	F	M	A	M	J	J	A	S	O	N	D
43	43	42	40	37	30	23	22	27	32	37	41

The forecasts were made for a period, the mid time of which was 20 hours after the chart from which the forecast was made but they were compared with winds 24 hours after the chart from which they were made.

/If σ

If σ is the standard vector deviation of the wind and r_t is the correlation between wind at one time and at t hours later then the error which will occur if the regression equation $V_t = \bar{V} - r_t (\bar{V} - V_o)$ is used (where $\bar{V}$ is the normal wind, V_o the wind at time o and V_t the wind at t hours later) is given by

$$\epsilon_t^2 = \sigma^2(1 - r_t^2)$$

Moreover the mean square change in wind in time t is given by $2\sigma^2(1 - r_t)$.

In the present case the forecast is made for a time 20 hours later and the check is made with observations 24 hours later, therefore the errors we should expect would be of magnitude

$$\epsilon'^2 = \sigma^2(1 - r_{20}^2) + 2\sigma^2(1 - r_4)$$

r_{20} may be taken as 0.63 and r_4 as 0.92 so $\epsilon' = 0.87\sigma$
Calculating the frequencies of errors from this expression with the help of Table II we get the following values of the expected errors in comparison with those observed taken from Table I.

Table III Frequency of errors in wind forecasts for the Canal Zone in comparison with those expected from statistical methods.

Error greater than		30 kts.	40 kts.	50 kts.	60 kts.	70 kts.	Number of occasions
D.J.F.	Actual	28	17	8	4	3	72
	Statistical	37	22	11	5	2	
A.M.	Actual	12	7	2	0	0	39
	Statistical	17	10	5	1	0	
J.J.A.	Actual	10	5	2	0	0	81
	Statistical	11	3	1	0	0	
S.O.N.	Actual	17	5	1	1	0	79
	Statistical	21	9	3	1	0	
Total	Actual	67	34	13	5	3	271
	Statistical	86	44	20	7	2	

The improvement of 75% of forecasts with errors less than 30 kts. against the statistical expectation of 68% may not appear to be great, but with the limited information available such an improvement is a real achievement.

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METEOROLOGICAL RESEARCH COMMITTEE

Fluctuations of temperature in clear air as recorded by an ultra-rapid thermometer

by

D. R. Grant (Meteorological Office)

1. Introduction and Summary

A number of flights at heights ranging from 100ft to 30,000ft have been made by the Meteorological Research Flight recording temperature continuously with a thermometer which has a lag of about 4 milliseconds. Although the work is still in its early stages it is thought that some examples of records obtained and a short account of the early work attempted using the instrument would be of interest.

Examples of temperature fluctuations in the friction layer, the troposphere, and the lower stratosphere are shown or described and some examples of fluctuations at or near surfaces of discontinuity (e.g. inversions) are given. Simultaneous records have been obtained of bumpiness on a recording accelerometer and it appears that on every occasion of appreciable temperature fluctuations there is bumpiness, although the amplitude of the bumpiness has a very small correlation with the amplitude of the temperature fluctuations. A frequency analysis of a temperature record has been carried out at the Admiralty Research Laboratory but no marked periodicity was found. Attempts to discover the cause of the fluctuations are described but much further work is necessary before any definite conclusions can be reached.

2. The Instrument

A description of the ultra-rapid thermometer has already been given (Grant 1950) and although a number of minor alterations have been made, the basic circuit remains the same. It is important to remember that the fluctuations may be caused by changes of either air temperature or wind speed. As it would require a 10 knot wind change to give the same deflection as a $\frac{1}{2}^{o}$F temperature change, it is likely that most of the fluctuations, which can have amplitudes of up to 4°F, can be regarded as true air temperature variations.

The instrument has been used in both Halifax and Hastings aircraft and it is proposed to use it in a Mosquito to reach higher altitudes. It is suitable for use in clear air only.

3. Examples of records obtained within the friction layer.

To avoid complications caused by a non uniform land surface, most flights in the friction layer were made over the sea. An example of the type of trace is given in Fig. 1. This was taken at a height of about 150ft on 12th Dec. 1950 about one mile out to sea off Ilfracombe. The wind was north westerly (on shore) 10-15 kt and the air temperature at 150ft was 40°F. As the sea temperature was 47°F conditions were clearly unstable.

It is seen that the fluctuations are of a one sided character showing that there was a general environment of cold air through which patches of

/warm

warm air were rising. The warm air is in places 2°F warmer than its surroundings. The linear dimensions and spacing of the warm patches vary from a few feet to a few hundred feet. The larger warm patches are seen to have small temperature variations within themselves.

On the same flight records were obtained at all heights up to 1500ft and they retained their one-sided character up to 500ft where there was a shallow isothermal layer.

No records have yet been obtained in the friction layer in stable conditions.

4. Examples of records obtained in a "uniform" air mass above the friction layer in the troposphere.

In all air masses and at all heights at present covered by the aircraft of the Meteorological Research Flight (i.e. up to 40,000ft) horizontal temperature variations are known to exist over distances greater than 1,000 yards (Frith 1950). These variations are always present although their amplitude is on occasions less than $\frac{1}{2}$°F. It has been found however that very frequently in a "uniform" air mass above the friction layer there are no fluctuations with an amplitude greater than $^1/_{20}$°F on a scale smaller than 100 yards. It is unfortunately impossible to obtain accurate results over distances between 100 yards and 1000 yards at present as the aircraft speed and height can change appreciably in this distance and corrections for these changes would have to be made before the true amplitude of the temperature changes could be calculated. If recording altimeters and airspeed indicators with lags of less than 1 second were fitted to our aircraft it would be possible to correct for these changes and fill in the gap in the temperature spectrum. There is evidence, however, that, even in this range of 100 yards to 1000 yards, there are quite frequently no temperature fluctuations of measurable amplitude.

In the earliest flights using the ultra-rapid thermometer, the aircraft was flown at a constant height so that the fluctuations would be variations in the horizontal. Quite large fluctuations were occasionally obtained at places where radio-sonde data indicated a steady lapse of temperature, but from later flights obtaining records in a circling climb it was found that only occasionally are there turbulent layers in which the temperature decreases uniformly with height below, throughout, and above the layer. Most of the largest fluctuations are obtained near inversions some of which are too small or too shallow to show up on radio-sonde data. Examples of these are given in section 6. Fig.2 shows an example of fluctuations obtained with no change of lapse rate throughout the turbulent region. It was obtained on 8th June 1950 over Larkhill at a height of 10,000ft. The lapse rate across the turbulent layer which was 300ft thick is 4.8°F/1000ft (allowing for an airspeed variation which was measured at the time). The peak to peak amplitude of the fluctuations is about 0.3°F. This is the most marked case of rapid fluctuations without lapse rate discontinuities we have so far obtained and cases such as this are quite rare.

5. Examples of records obtained in the stratosphere

The ultra-rapid thermometer has been operated in the stratosphere on only two occasions on both of which the tropopause was not well defined and was unusually low. On the first occasion we were at least 3000ft above the tropopause and a level flight showed rapid fluctuations of amplitude up to 1°F. On the second occasion we were in the stratosphere for a very short time but appreciable fluctuations were observed. The equipment is being installed in a Mosquito which has a ceiling of 40,000ft so that records can be obtained in the stratosphere more often.

6. Examples of records obtained at or near surfaces of discontinuity

The places at which rapid temperature fluctuations are most common are at or near surfaces of lapse rate discontinuity, e.g. at inversions or the tropopause. Frontal surfaces have not yet been investigated.

(a) Inversions

(i) Inversions have been examined in a number of ways. When the inversions were obvious from radio-sonde or aircraft ascents, flights have been made ascending or descending through them. The rate of ascent used was usually 1000ft/min. i.e. about 17 feet in one second during which time the aircraft moved through about 300 ft in the horizontal. We know from horizontal flights that changes of the order of 2°F can take place in 300ft and this must be borne in mind when studying the results of these flights. An example of the type of result obtained is given in Fig.3 where the ultra-rapid thermometer trace is shown alongside the temperature profile obtained from the readings of a M.O. flat plate aircraft thermometer. The intensity of bumpiness measured on a recording accelerometer is shown on the left. These results were obtained near Farnborough on 9th June 1950 when the rate of ascent was about 1000ft/min. The actual rate of ascent can be calculated for each interval of height from the 3-second time marks on the film and the heights shown on the graph. The ascent was made in a straight line so that the air at the top of the ascent was not vertically above the air at the start. The first noticeable feature is the small amplitude of the temperature fluctuations in a bumpy layer in which the lapse rate is dry adiabatic. This would be expected if the temperature fluctuations were due to vertical currents, for their amplitude would then depend on $\partial T/\partial z + \Gamma$ (where $\partial T/\partial z$ is the lapse rate and Γ is the dry adiabatic lapse rate) and the depth through which the vertical motion is taking place. Secondly, it is seen from the Ultra Rapid Thermometer record that the increase in temperature with height takes place in a layer of about 60ft. Finally, there are large temperature fluctuations within this layer although the bumpiness decreases suddenly just below the layer. It is probable that the variations within the stable layer are due to convection from a less stable lower layer. This could penetrate into the stable layer but would have its vertical velocity damped out rapidly, resulting in large temperature variations (because of the large value of $\partial T/\partial z + \Gamma$) but relatively small vertical motion, i.e. little bumpiness.

Fig.3 must not be regarded as typical of the conditions near an inversion. In fact records obtained in inversions are never alike and even flights through the same inversion at different points give quite different temperature profiles. The change in temperature is not always confined to a shallow layer or even to a single layer.

(ii) A number of ascents were made circling a radio-sonde balloon so that wind measurements from the balloon could be used to obtain wind shear. It was found that the values of wind shear obtained were not accurate enough over shallow layers but the flights gave useful information about the structure of inversions at all levels. An analysis of the heights at which temperature fluctuations occurred relative to the height of the inversion is given in Table I. This

includes all inversions traversed at heights above 3000ft whose magnitude was greater than $\frac{1}{2}^{o}$F.

TABLE I. FREQUENCY OF OCCURRENCE OF TEMPERATURE FLUCTUATIONS IN THE NEIGHBOURHOOD OF INVERSIONS

	No Flucts	Flucts below only	Flucts below & during inversion layer only	Flucts during inversion layer only	Flucts during & above inversion layer only	Flucts above inversion layer only	Flucts below during & above inversion layer
(i)	When change in temperature takes place within a layer less than about 100ft thick.						
	0	0	0	1	0	0	5
(ii)	When change in temperature takes place within a layer from about 100 to 300ft thick.						
	1	0	2	0	0	0	8
(iii)	When change in temperature takes place within a layer greater than about 300ft thick.						
	11	1	3	2	8	0	10
Totals	12	1	5	3	8	0	23

It is seen that out of a total of 52 cases only 12 had no associated temperature fluctuations and of these 11 had their increase in temperature spread over a deep layer. It is possible that some, if not all, of these 11 cases were not inversions at all but were due to horizontal gradients of temperature. There are 39 cases in which there were temperature fluctuations within the layer in which temperature increases with height. Unfortunately accelerometer records were not always obtained in these flights and when they were, they were confused by the accelerations imposed by the pilot in circling the balloon and accurate timing of the end of the bumpiness was not possible.

(iii) Some flights were made to find the horizontal temperature structure at inversions. On one occasion when flying just above an inversion there was a sudden drop in temperature of 2°F which was followed by rapid temperature fluctuations and bumpiness. This was probably due to the aircraft entering the cold air below the inversion although the change of height of the aircraft was never greater than $\pm$ 20ft. It appears therefore that the height of an inversion may sometimes vary, possibly discontinuously. Attempts to fly for any appreciable time within a layer in which temperature increases with height would probably fail because of this and the fact that the stable layer is often very shallow. This makes investigations of the fluctuations within the layer such as those shown in Fig.3 very difficult.

(b) The tropopause

On the two occasions on which the stratosphere was reached records

were also obtained whilst crossing the tropopause, but as the latter was ill-defined it was impossible to draw any conclusions about the nature of the fluctuations there. It is hoped to obtain better examples when the equipment is installed in the Mosquito.

7. Examples of some other interesting records

As already mentioned a number of flights have been made taking records in the horizontal. The fluctuations obtained in these flights might have been near inversions but as the local lapse rate was not known these records have not been discussed in section (6). There were, however, some remarkable records among them and a few of the most interesting results are given in this section.

Fig. 4 shows an example of a change of temperature of about 0.6°F which takes place within half a second, i.e. in a distance of 50 yards. This record was taken at 18,000ft over Wales on 26th October, 1950. and such discontinuities are quite common.

Fig. 5(a) shows a region of temperature fluctuations which starts very suddenly. The first six seconds demonstrate clearly what was said at the beginning of section (4) that there are areas in which there are no measurable small scale fluctuations. The sudden start of the fluctuations is an unusual feature but has been noticed on one or two other occasions. This record was obtained on 25th April, 1950, over Bradford at 17,000ft.

Fig. 6(a) shows another remarkable record obtained over Bradford at a height of 17,000ft on 25th April, 1950, which was a day of extreme convective instability. The amplitude of the warm "pulse" is about 4°F and its length (just less than 3 seconds) is about 250yds. The cold "pulse" following has an amplitude of $1\frac{1}{2}$° F and length of 450 yds. It is thought that these "pulses" are quite common on days when there is a deep layer of unstable air.

Fig. 7 shows another feature similar to Fig. 6 but there is no warm "pulse" of air corresponding to the cold one shown. The amplitude of the cold "pulse" is 2°F, its length is 1200 yds and the record was obtained at a height of 10,000ft over Wales on 26th October, 1950.

A possible explanation of these cold patches is that they are present just above the tops of cumulus clouds which because of a stable layer have just reached the peak of their development. The air above the cloud top may have been forced up into the stable layer and thus become colder than its environment. On one occasion on 24th April, 1950, a cold patch of $1\frac{1}{2}$°F and length 800 yards was found while flying over the top of a cumulus cloud.

8. Correlation with Accelerometer Records

In a number of the flights using the ultra-rapid thermometer records have been obtained simultaneously on a recording accelerometer. The sensitivity of the accelerometer is such that $\pm$.01g can be measured but its natural frequency is 3 c/s which results in a lowering of sensitivity to the bumps of frequency higher than this. An accelerometer measures the accelerations of the aircraft caused by vertical gusts as well as those imposed on the aircraft by the pilot. The aircraft is comparatively insensitive to horizontal gusts. When the instrument is in use the pilot is asked to fly "hands off" if possible and at all events to use a minimum of control. Because an aircraft's response to a gust depends, among other things, on the gust shape and size and because natural oscillations of the

/aircraft

aircraft are excited by the initial gust, it is not possible to obtain detailed information about vertical air velocities by integrating an accelerometer trace even if the effects of horizontal gusts are negligibly small. It is possible, however, to obtain a measurement of the intensity of the bumpiness.

It has been found that on every occasion when there are appreciable temperature fluctuations there is also bumpiness although the ultra-rapid thermometer can be made very sensitive and may record fluctuations when there are no bumps of measurable amplitude. It is considered that the rapid fluctuations of temperature indicate the presence of turbulence and the ultra-rapid thermometer is, therefore, in certain conditions of environment lapse rate, a very sensitive detector of the presence of turbulence. Neither the intensity of bumpiness nor the amplitude of the temperature fluctuations gives a good indication of the intensity of turbulence as the former probably, as has already been mentioned, depends on the size of the eddies and the gust gradient and the latter on the value of $(\partial T/\partial z + \Gamma)$. Because of this only a small correlation would be expected between the intensity of bumpiness and the amplitude of the temperature fluctuations. The coefficient of correlation found was 0.27 using 50 pairs of observations.

The accelerometer has been useful from the point of view of studying vertical velocities only in cases where the turbulent patch of air has been preceded by a few seconds of non-turbulent air. Even in these cases the only information obtained is whether the initial gust is due to an upward or downward motion of the air.

In Fig. 5(b) is shown the record obtained on the accelerometer at the same time as Fig. 5(a) was obtained on the thermometer. Here it is seen that the initial gust (which is of warmer air) produced a downward acceleration of the aircraft, i.e. the warm air was descending relative to the air surrounding it.

In Fig. 6(b) it is seen that the large "pulse" of warm air shown in Fig. 6(a) must have been ascending as the aircraft was given a large upward acceleration on entering it. There is no marked acceleration in the cold patch. Warm patches of this type may not always be ascending. On the same day two other warm patches of amplitude 3°F, which followed each other in quick succession, appeared to be descending although the accelerometer record was on this occasion confused by an oscillatory motion of the aircraft just before reaching the first of the patches.

9. Frequency analysis

In order to test for periodicity, one of the ultra-rapid thermometer traces was plotted in a form suitable for analysis on an instrument at the Admiralty Research Laboratory (Deacon 1949). The analysis showed no sign of any periodicity. Owing to the amount of work involved in preparing a sample for analysis no further records have been tested for periodicity by this method.

10. Origin of temperature fluctuations

Some work has been carried out in an attempt to find the cause of these fluctuations. The large patches of warm and cold air are almost certainly associated with cloud tops, or large scale convection. Precise observation of clouds and lapse rate in future flights should establish their cause. Some attempts have been made to find the cause of the random rapid fluctuations which we have assumed to be associated with areas of turbulence. Some of the lines on which this problem is being tackled are given in this section.

(a) Decay of large eddies

It is possible that the eddies detected by the ultra-rapid thermometer

and accelerometer are produced by decay of larger eddies. If this is the case and if these larger eddies are being continuously created in a certain area, it would be expected that there would also be small scale eddies in the same area. It has been seen, however, that large scale temperature fluctuations are almost universal whereas there are often areas in which those of a size less than 1000yds. are not present. This may be due to insufficient sensitivity in the ultra-rapid thermometer resulting in the rapid fluctuations associated with the smallest amplitude of large scale fluctuations being of too small amplitude to detect. It would still be expected, however, that in places where large scale fluctuations of large amplitude are present, smaller scale fluctuations with large amplitude would also be present. Some flights were carried out to see if the small scale fluctuations appear at places where there are large scale fluctuations of large amplitude. On the few flights made there was some evidence of such a relationship. It was found, however, that if large amplitude large scale fluctuations are present they extend over a distance much longer than the length of any turbulent area shown by the ultra-rapid thermometer in the same region. If they are the source of the small eddies, it would be necessary to explain why the decay takes place in such a way that the energy spread over a large area in the form of large eddies is dissipated into small eddies at only a few places within the area, and, of course, it would still be necessary to explain the formation of the larger eddies.

(b) Dependence on Richardson's Number

It is possible that the rapid temperature fluctuations occur in areas in which the Richardson Number (Ri) is small. In order to test this a number of ascents were made circling a radio-sonde from which the wind shear was obtained. Temperatures and small scale turbulence were measured from the aircraft by means of the ultra-rapid thermometer. Winds were obtained at half minute (approx. 600ft) intervals and Ri calculated for every 600ft layer. Although it was realised that the values for wind shear and temperature gradient obtained might be smaller than those actually present because of the possibility of the changes taking place over a layer shallower than 600ft, it was thought that a statistical analysis of the results might show a significant correlation between Ri and intensity of turbulence. In order to relate the amplitude of the temperature fluctuations to the intensity of the turbulence, it was assumed that

$$\Delta h = \Delta T/(\partial T/\partial z + \Gamma)$$

Where ΔT = peak to peak temperature fluctuation
Δh = vertical extent of turbulence which must to some extent depend on the turbulent vertical velocities, i.e. on the intensity of the turbulence.
$\partial T/\partial z$ = lapse rate in environment
Γ = dry adiabatic lapse rate.

Using this relation, it would have been expected that there would be a correlation between Ri and Δh i.e. between $Ri(\partial T/\partial z + \Gamma)$ and ΔT The correlation found was very small being only 0.13 using 102 pairs of values.

Another analysis was carried out to find the correlation coefficient between Ri and ΔT but this was also found to be small. This is to be expected as ΔT almost certainly depends on the lapse rate.

Examination of the records, however, showed that there was a well marked tendency for the fluctuations to occur at inversions or discontinuities in the lapse rate. It is known that there is frequently a wind shear across an inversion and it was therefore thought that more detailed measurements of wind shear and lapse rate might give quite different results. An attempt to obtain these more precise measurements has been made. The technique used was to launch a large balloon which ascended at about 500ft/min. and which was followed from the ground by a theodolite. The height of the balloon was measured by flying past it in the aircraft as often as possible (every 80 seconds) and recording the pressure in the aircraft as it passed the balloon. Readings of the theodolite were taken every 10 seconds. In the aircraft 10-second readings of airspeed and height were taken as well as the height and time of passing the balloon and continuous readings were taken on the ultra-rapid thermometer. Turbulence was measured by temperature fluctuations and bumpiness. Having in effect continuous observations of wind, temperature and turbulence it was possible to calculate Ri across any layer in which turbulence occurred.

So far only one flight has been carried out on these lines and the only serious difficulty presented was that of measuring from the records the lapse rate across a turbulent layer. This is because of the presence of horizontal temperature gradients. The turbulent patches of air were in layers with values of Ri of 0.1, 3, 0.9, 0.5, 1, and 3. An interesting feature, however, was a layer in which Ri was probably about 0.7 and certainly not greater than 1 or less than 0.1 (depending on the value taken for the lapse rate) in which there was no measurable turbulence. The wind shear within this layer was 16 mph/1000ft and the nearest turbulent layer was 500-1000ft below. This is only an isolated case but it is interesting to find that such large wind shears can exist without turbulence. It might be possible that the turbulence below originated in the upper layer and detached itself from its source. Many more flights are necessary before any significance can be attached to these results.

(c) Dependence on Lapse Rate

It has already been mentioned that very often turbulence is present near an inversion. In an attempt to find out if the presence of the fluctuations depends on the lapse rate two aircraft were flown one 500 or 1000ft above the other, each taking readings of temperature every 15 seconds, and one of them recording temperature fluctuations and bumpiness. The lapse rate was calculated and comparisons made between lapse rate and turbulent regions. Only one short flight has been carried out and it was not possible to find any significant correlation between lapse rate and turbulence from the results.

(d) Association with horizontal temperature gradient

It might be expected that a horizontal temperature gradient would have associated with it a vertical wind shear which might be capable of producing turbulence. The results of one of the flights carried out under section 10(a) were studied to see if the rapid fluctuations occurred at places where there was a gradient of horizontal temperature but there was no evidence of this. They might, however, be associated with a steady trend in horizontal temperature but no ultra-rapid thermometer observations have yet been made in these conditions.

(e) Association with horizontal wind shear

Owing to the difficulty of measuring horizontal wind shear, its effect on turbulence is difficult to establish. On two occasions, however, when seeding of clouds revealed the presence of a horizontal wind shear no reports of bumpiness were made by the observers.

11. Conclusions

Although the amount of data available is small a number of conclusions can be reached of which there can be little, if any, doubt:-

(a) Horizontal variations of air temperature up to at least 1°F can exist over distances of the order of 100ft at all heights up to at least 30,000ft.

(b) These fluctuations are very common near inversions.

(c) There is always bumpiness in the regions in which appreciable temperature fluctuations are present.

(d) There are occasionally present in the atmosphere at heights up to at least 17,000ft patches of warm and cold air with horizontal dimensions of 200-2000ft and temperature 2-4°F different from the surrounding air.

(e) There are sometimes discontinuities of temperature in the horizontal of up to 2°F, particularly near inversions.

The following conclusions must be regarded as speculative and further flights are necessary to confirm them.

(f) The small scale eddies (assumed to be associated with the small scale temperature fluctuations) may obtain their energy from those of larger scale only if there is some mechanism by which decay to smaller eddies takes place in a few parts of the areas in which the large eddies are present.

(g) The small eddies are not of a periodic nature.

(h) They do not always occur at places of low Richardson Number.

(i) They are of a one-sided character in unstable air near the surface (as in Fig. 1).

(j) Large patches of cold air (as in Fig. 7) occur above the tops of cumulus cloud whose growth has been stopped by a stable layer.

(k) Large patches of warm and cold air (as in Fig. 6) occur on days of extreme instability.

(l) Small scale fluctuations occur in the stratosphere as well as the troposphere.

12. Suggestions for future work

There are two main divisions of the work outlined above (1) investigations of temperature discontinuities (2) investigation of turbulence. In light of past experience it is suggested that the following programme be carried out under these two headings.

(1)(a) More flights should be carried out above cumulus clouds to confirm conclusion (j).

(b) More flights should be made in unstable conditions to confirm conclusion (k).

(c) More flights should be made using two or more aircraft in and near inversions to obtain their detailed structure.

(d) There should be some flights to see if similar discontinuities occur in the stratosphere and to study the strucutre of the tropopause.

(2) A hot wire anemometer is being developed which will be more suitable for measurement of turbulence theory parameters than an ultra-rapid thermometer, but as the latter gives horizontal and vertical variations of temperature as well as some indication of the intensity of turbulence it is a very suitable instrument for use in investigations of the cause of turbulence. Further work on turbulence using this type of thermometer is therefore considered worth while and the following is suggested:-

(a) Further flights circling a balloon on the lines of that described in section 10(b) should be made.

(b) More flights should be made measuring the large scale fluctuations at the same time as the small ones and an attempt made to work out a correlation coefficient between the two.

(c) Further examples of low level fluctuations should be obtained under different conditions.

(d) Examples of temperature fluctuations should be obtained in the stratosphere and at the tropopause.

(e) Flights should be made (using two aircraft to obtain the lapse rate) through cloudless frontal surfaces and in places where there is a large horizontal temperature gradient.

(f) Further flights should be made similar to that described in section 10(c) using two aircraft to correlate turbulence and lapse rate.

In the two-aircraft flights, an ultra-rapid thermometer in each aircraft would be an advantage.

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Frith R. M.R.P. 583 (1951)
Deacon G.E.R. Q.J. R.Met.Soc. V.75 No.325 (July 1949) p.227.

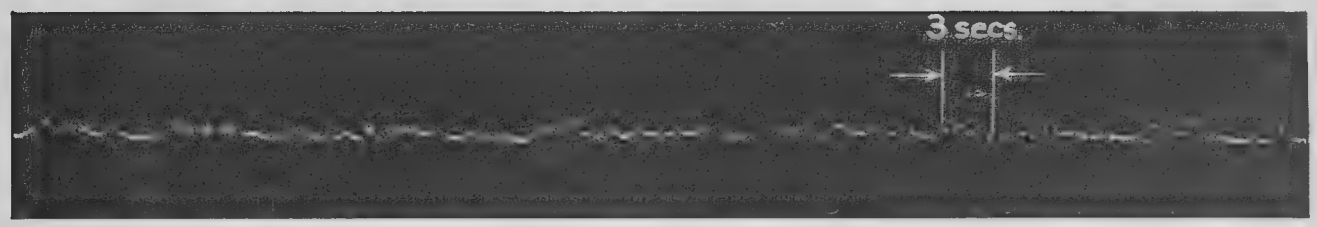


Fig.1. Unstable conditions over the sea at a height of 150 ft in level flight. Sensitivity 1" = 11°F.

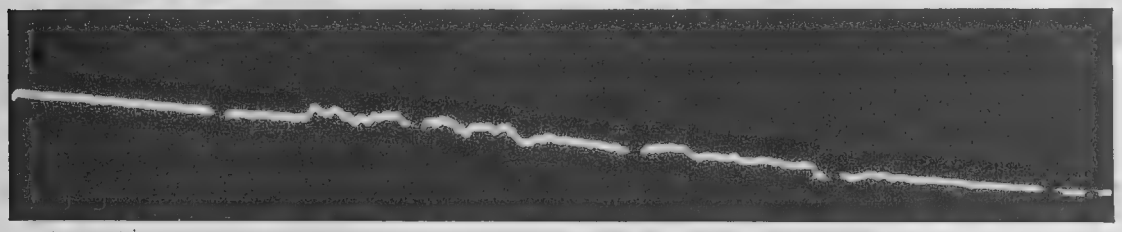


Fig.2. Fluctuations without lapse rate discontinuity at a height of 10,000 ft while climbing at a rate of 20 ft/sec. Sensitivity, 1" = 3.5°F.

For Fig. 3, please see next page.

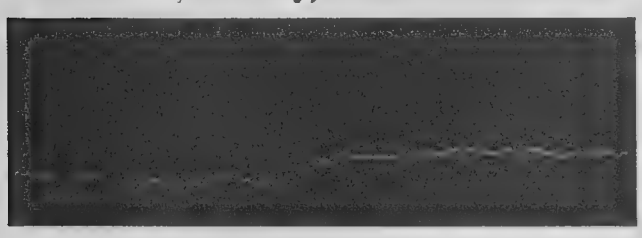


Fig.4. Temperature discontinuity in horizontal at a height of 18,000 ft. Sensitivity 1" = 5°F.

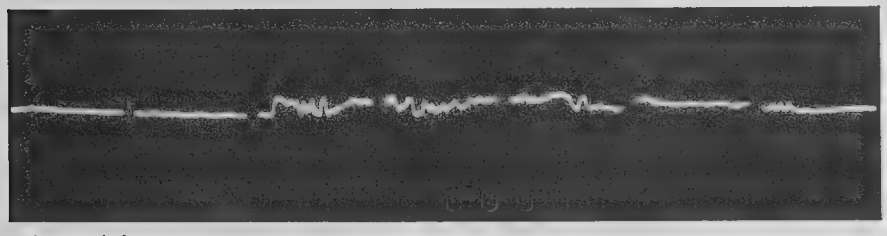


Fig.5(a). Fluctuations in the horizontal starting very suddenly at a height of 17,000 ft. Sensitivity 1" = 9°F.

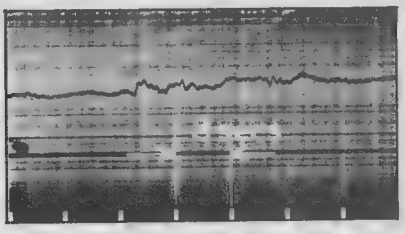


Fig.5(b). Accelerometer record during same period as Fig.5(a).

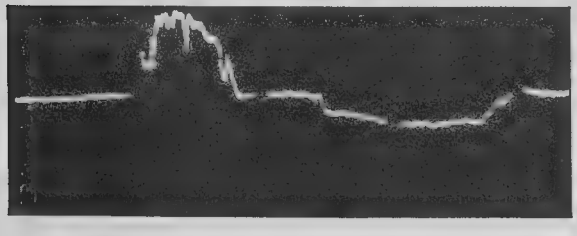
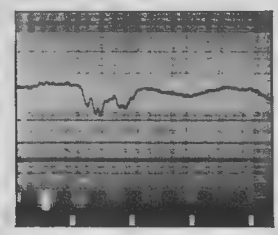


Fig.6(a). Example of large "pulses" of warm and cold air at a height of 17,000 ft. Sensitivity 1" = 9°F.



Accelerometer record during same period as Fig.6(a).

Fig6(b).

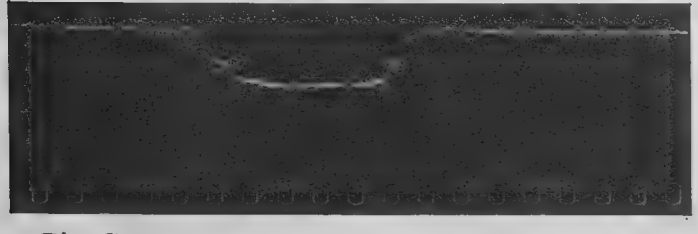


Fig.7. Isolated cold patch of air at 10,000 ft. Sensitivity 1" = 7°F.

Note: In all the above photographs of the ultra-rapid thermometer traces there are gaps at 3 second intervals (approx. 900 ft) as marked on Fig. 1 and temperature increases upwards from the lower edges of the illustrations. In the accelerometer traces, Fig.5(b) and 6(b), an upward acceleration of the aircraft is shown by a deflection of the trace downwards and the 3 second time marks are shown at the lower edges of the illustrations. The sensitivity of the accelerometer is 1" = 0.9g.

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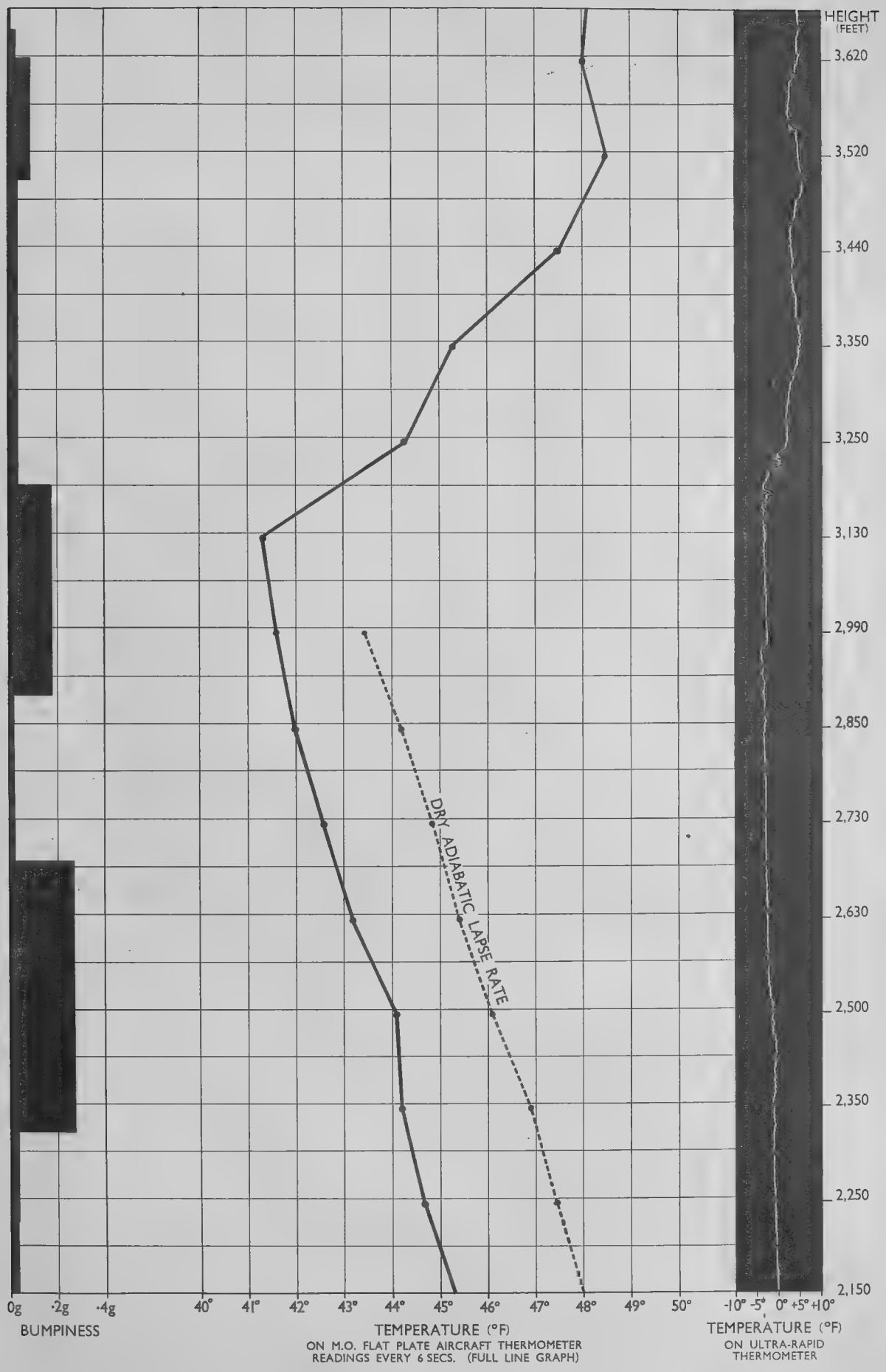

FIG.3. ASCENT THROUGH AN INVERSION USING THE ULTRA-RAPID THERMOMETER

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paper of the Meteorological Research Committee (London),
M.R.P. No..... by Mr.....
A copy is available in the Library of the Meteorological Office."

M.R.P.660
S.C.III/104
6th July, 1951.

AIR MINISTRY

METEOROLOGICAL RESEARCH COMMITTEE

AD-23024

Turbulence at high altitude: a further meteorological analysis

by

J.K. Bannon (Meteorological Office)

Summary

135 turbulent layers observed between 18,000 and 40,000 ft. on flights by the Meteorological Research Flight over a radio-sonde station are analysed using simultaneous temperature and humidity observations and the wind observations obtained from the ground observing station within a few minutes of the aircraft observations. It is found that Richardson number is small (order 1 or 2) when the turbulent layer is also a layer of general shear in the vertical. It is also confirmed that some (about one third) of the turbulent layers occur when there is no general shear of wind in the vertical. Both types of turbulence appear to be associated with isentropic shear significantly above average. A cause of the turbulence occurring in the absence of general shear in the vertical is suggested but without evidence either way. Temperature and humidity distribution with height in the turbulent layers is discussed.

1. Introduction

In a previous paper (1) it has been shown that the occurrence of turbulence in the upper troposphere and lower stratosphere of such a nature as to cause noticeable abnormal movement of an aircraft (bumps) is in many cases associated with small values of the meteorological parameter the Richardson number; the occurrence of such turbulence also seems to be associated with abnormally large shear of wind in the horizontal. Neither of these parameters appears to be uniquely associated with the occurrence of bumps, however, and as far as could be judged such turbulence may occur when the Richardson number is large or when the horizontal shear is small.

In previous meteorological analyses of high altitude turbulence (references 1, 2, 3, 4 and 5) suitable observations of wind and temperature were seldom available at the same time and place as the occurrence of the bumps so that interpolation in space and time was necessary between upper air observations to obtain appropriate values of the meteorological parameters. However, marked changes can occur in the upper air within a short distance and in a short time; turbulence also can vary rapidly in space and time and is often of a patchy nature. Lack of meteorological measurements at the same time and place as the observations of turbulence was, therefore, a handicap in the proper meteorological analysis though it is likely that the general features of the relation between occurrence of turbulence and the physical state of the atmosphere were only slightly blurred by the necessary interpolations.

During 1950 most of the high altitude ascents made by the Meteorological Research Flight, primarily to observe humidity, were made in the vicinity of one or other of the upper air observing stations at Larkhill, Downham Market and at about the same time as observations of upper wind were made at those stations. Qualitative observations of bumps were made on these ascents so that a series of observations of turbulence was obtained for which almost simultaneous observations of wind and temperature were available for analysis.

/This

This paper gives the results of a meteorological analysis of this series of observations of turbulence. It has generally been assumed that the most important factor causing turbulence in the upper troposphere and lower stratosphere, and therefore usually in cloudless air, is wind shear in the vertical; previous work (e.g. references 1, 4, 5) has confirmed that it is usually important. The analysis has been separated into two parts, therefore, according as a general shear of wind in the vertical was present or not. The definition of 'general shear' was necessarily somewhat subjective; observations of wind usually show changes from one height to the next (height intervals are approximately 1,400 ft.) but in the majority of cases it was quite simple to decide whether a change was associated with a significant change of wind with height or whether it was merely a fluctuation about a general mean current. As in previous work it was found that bumpiness did occur in many cases when no vertical shear was present.

The main analysis concerned the association of small Richardson number with turbulent conditions, the turbulence, of course, being defined by its noticeable effect on an aircraft, i.e. it was of a particular linear scale and intensity. The wind shear in the isentropic surfaces, which are usually almost horizontal but sometimes slope appreciably, was also investigated; isentropic shear was preferred to horizontal shear as lateral mixing, if existing, will tend to take place along isentropic surfaces. (See e.g. Haurwitz, (6)).

As observations of frost point temperature were made on these flights it was possible to analyse the distribution of water vapour with height in the turbulent layers to see if there was any indication of abnormal mixing. An attempt was also made to examine the reported temperature lapse rates in the same way.

2. Description of data.

(a) Observations of turbulence.

Qualitative assessments of turbulence were made by the meteorological observer in the Mosquito aircraft which made the high altitude ascents. The ascents usually reached 38,600 ft. (I.C.A.N.) i.e. a pressure of 200 mb. and occasionally reached 40,000 ft. (187 mb.). Bumps were experienced above the 500 mb. level (approximately 18,300 ft.) on 58 flights during the period April to December, 1950 for which simultaneous wind observations were available. In all, 135 different turbulent layers were encountered.

The severity of the bumps was usually estimated. A recording accelerometer was fitted during August and on four occasions after that date, when the observer estimated that the bumpy accelerations were sufficiently large to warrant recording, this accelerometer was used. In the great majority of cases the bumps were noted as only slight or perceptible. From a comparison of the qualitative observations at high altitude and the accelerometer readings it would appear that on these flights the observers classified accelerations greater than 0.3g as 'moderate to heavy'. The maximum acceleration recorded (2nd October, 1950) was between 0.3 and 0.4g at 160 kt. equivalent air speed.

The plan of each flight was as follows; the aircraft first climbed to maximum height over the upper air observing station, usually Larkhill but occasionally Downham Market. Observations of temperature and frost point were then made at 1,000 ft. intervals on the spiral descent over the station. Observations of any other phenomena, including turbulence, were also made at these 1,000 ft. intervals when the aircraft was flying straight and level at 160 to 180 kt. equivalent air speed. The descent from one observational level to the next was made as quickly as possible and probably with the aircraft banking steeply so that it was seldom possible to make any observations at intermediate levels. Thus an observation of turbulence

/at

at one height only indicates that the bumpy layer may have been say 1,900 ft. thick or it may have been very shallow. In this analysis turbulence at one height only was taken to mean turbulence through a layer 1,000 ft. or less in thickness. When bumps were noted at two or more successive observation levels the turbulence was assumed to have been continuous between these levels.

The bumpiness experienced was not always continuous in the horizontal; this is quite a usual feature of high altitude turbulence, especially when slight. In many cases the turbulence was continuous though perhaps varying in intensity and this was usually the case when it occurred through a layer of 2,000 ft. or more.

In this report the word turbulence is usually synonymous with bumps; i.e. the turbulence is of such a scale and intensity that it produces accelerations on an aircraft which are perceptible to the crew.

(b) Meteorological measurements.

Temperature and frost point were measured at 1,000 ft. intervals on the general descent, the aircraft flying level for this purpose for a short time. The temperature observations are estimated to be accurate to within $\pm$ 0.5°F and the frost point temperatures to be accurate to within $\pm$ 3°F at values below - 100°F, the error decreasing to less than $\pm$ 1°F at values above - 40°F. (Reference 7).

The wind measurements were made by the standard method of tracing the ascent of a balloon by radar on the ground. Observations were made every minute, that is every 1,400 ft. approximately; (rates of ascent of these balloons vary from about 1,100 to 1,800 ft./min.). Thus measures of wind were available at height intervals of approximately 1,400 ft. Strictly such measures are measures of the mean wind in the layer between plotting points. These wind measures are subject to instrumental errors which increase with height and which also depend on the state of efficiency of the radar set so that errors are not known precisely. It is estimated (reference 8) that the root mean square vector error (r.m.s.v.e.) in measuring the wind over 1 minute intervals at altitudes from 20,000 to 40,000 ft. is of the order of 1 to 3 kt. for mean winds (from surface to height in question) of 30 kt. and of the order 3 to 5 kt. for mean winds of the order of 60 kt.

The aircraft flights were planned so that the aircraft and the sounding balloon should be in roughly the same position, the former on descent, the latter on ascent, somewhere between the levels of 200 and 300 mb. More than once the sounding balloon was seen from the aircraft at this level but in general the aircraft must have been of the order of 10 miles from the balloon which was travelling with the wind. In general also, though one observation of wind coincided exactly with one level of observation from the aircraft, most of the wind observations differed by up to 20 minutes from the time of the observations of turbulence by the aircraft.

(c) Accuracy of derived meteorological parameters.

The parameters used in this report were the temperature lapse rate, the frost point lapse rate, the shear of wind in the vertical, the Richardson number and the isentropic shear. Temperature lapse rates were usually calculated over height intervals of 1,400 ft. (temperatures interpolated from the aircraft observations at 1,000 ft. intervals). Assuming that the range "within $\pm$ 0.5°F", the accuracy of a temperature observation, includes 90 per cent of all the errors, it is estimated that the calculated lapse rates are subject to errors of root mean square 0.3°F/1,000 ft.

/Similarly

Similarly the r.m.s.e. of frost point lapse rate may be estimated as 0.4°F/1,000 ft. to 1.2°F/1,000 ft. according as frost point is high or low.

The errors in the vertical shear of the wind can be estimated only very approximately. Assuming a r.m.s.v.e. of 3 kt. in a measure of the wind (see above) then the r.m.s.v.e. in shear, measured over a 1,400 ft. layer, is about 3 kt./1,000 ft. Measured over a layer of about 4,000 ft. the r.m.s.v.e. in shear is 1 kt./1,000 ft.

The Richardson number is defined as

$$Ri = \frac{g}{T} \frac{(\partial T/\partial z + \Gamma)}{(\partial V/\partial z)^2}$$

where T = temperature (°F, absolute)

Γ = adiabatic lapse rate of temperature (dry - for cloudless air, wet - adiabatic for cloudy air) (°F/1,000 ft.)

z = height (1,000's of ft.)

V = vector wind velocity (kt.)

g = acceleration of gravity.

An error of 0.3°F/1,000 ft. in $\partial T/\partial z$, providing $-\partial T/\partial z$ is less than 4.3°F/1,000 ft., results in a percentage error in Ri of less than 25. The most serious errors in Ri, however, will arise from errors in the wind shear; when the shear is large Ri will be little affected but when $\partial V/\partial z$ is small the errors in Ri may be large, e.g. if $\partial V/\partial z$ is measured as 1 kt./1,000 ft. over a depth of 1,400 ft. then errors of anything up to 2 kt./1,000 ft. may well be present, i.e. the true value of Ri is likely to be anything between a value one ninth of that calculated and infinity. The technique adopted in this analysis to minimise the effect of errors when $\partial V/\partial z$ was small was to take $\partial V/\partial z$ = 2 kt./1,000 ft. whenever $\partial V/\partial z$, measured over a layer 1,400 ft. thick, was found to be less than 2 kt./1,000 ft. When $\partial V/\partial z$ was measured over approximately 4,000 ft., $\partial V/\partial z$ = 2/3 kt./1,000 ft. was taken as the minimum permissible shear. These precautions had the effect of restricting Ri to less than 25 in the troposphere and to less than 60 in the stratosphere; measured over deeper layers Ri could take larger values. Without such restrictions mean values of Ri for a number of occasions would probably be unduly affected by one or more very large values of Ri which might have arisen from errors in the wind measurement.

It is not possible, of course, to give a general estimate for the accuracy of all determinations of Ri though estimates may be made for any particular occasion. Usually, however, a measure of Ri should not have an error of more than a few units and, in the case of small values of Ri, of order 1, the errors are probably less than 1.

Values of the isentropic shear were estimated by the formula

$$\partial V/\partial n - V/r$$

(see reference 1) where $\partial V/\partial n$ represents the rate of change of wind speed within the isentropic surface and at right angles to the wind and r is the radius of curvature of the stream line, neglecting curvature of the earth and any curvatures due to vertical motion of the air. $\partial V/\partial n$ was estimated from the wind observations made at the various British upper air observing

/stations

stations which are some 150 - 200 miles apart; errors in wind will thus lead to errors of $\partial V/\partial n$ with root mean squares of the order of 0.02 hours^{-1}. The radius of curvature r cannot be assessed with much accuracy and the term V/r is often as important as the term $\partial V/\partial n$. It was sometimes impossible to obtain a value for $\partial V/\partial n$ which could be applied to the district of the flight. Accordingly values of the isentropic shear were analysed only on those occasions when it could be assessed with fair confidence; this unfortunately introduced a subjective element into the analysis and it is possible that the criterion of what is fair confidence, being largely personal, may unwittingly have influenced the results. The errors in the isentropic shear for those cases of fair confidence are estimated to be probably of the order 0.05 hours^{-1}.

3. General shear present.

In this section all cases of turbulence which occurred within a general shearing layer are examined. Those cases where a general vertical shear was present over part only of the turbulent layer have been excluded; they will be mentioned in section 5.

It was quite a simple matter to decide if the turbulent layer was within a general shearing layer; any change of wind with height which was not a fluctuation about the general current was classed as a general shear. To a certain extent this division of cases may have been subjective but the classification appeared so obvious that the writer is confident that another analyst would have reached the same results with perhaps only one or two differences.

Table I sets out the main results, separating troposphere and stratosphere cases. The turbulent layers were divided according as the bumps were observed at one level only (shallow layers, mean depth probably 1,000 ft. or less) or at two or more consecutive levels (deep layers).

The mean of the Richardson numbers (Ri) for the turbulent layers is given in column 4 and also the mean Ri for the layers just below and just above the turbulent layers, in columns 3 and 5. As an indication of the spread of the distributions of Ri the median values of Ri are also given in columns 3, 4 and 5 in brackets. Except in a few cases the Ri were calculated over the smallest possible depth, viz. the height between successive wind observations. (If, however, bumps were observed at the same level as a wind observation then the shear, and hence Ri, had to be calculated over double the usual depth). As outlined in section 2(c), large values of Ri, being liable to large errors, were reduced by assuming a minimum shear of 2 kt./1,000 ft. For shallow layers the Ri calculated strictly applied to a layer of approximately 1,400 ft. which included the level at which the turbulence was noted; the quoted mean value of Ri therefore probably does not apply to the turbulent layers alone but also covers parts of the smooth layers above and below. For the deep layers it was sometimes necessary to interpolate in the wind variation with height to obtain the appropriate Ri; layers of about 1,400 ft. thickness were used and when there were two or more such layers in the turbulent layer the mean of the several values of Ri was taken as applying to the whole bumpy layer. The mean of the Ri for the deep turbulent layers calculated as above was found to be almost exactly the same as the mean of the Ri calculated over the whole of each turbulent layer.

The number of cases in which Ri for the turbulent layer was less than the Ri for the layers just above and below is also given in Table I, column 6. It seems clear from all these figures for Ri that there is a high correlation between turbulence (bumps) and small Richardson number when the turbulence is in a general shearing layer. Exact correspondence between observed turbulence and values of Ri calculated in the manner described would not be expected even if an exact relationship existed; apart from observational errors, turbulence must by the transfer of heat, momentum and water vapour affect the characteristics of the layers concerned so that exact simultaneity of all observations, turbulence

and the meteorological parameters, is required for an exact study. The wind observations in the present investigation though close in space and time were necessarily some miles away and at a time some minutes different from the other observations.

The mean of the isentropic shear observed, regardless of sign is also tabulated for those occasions for which isentropic shear could be estimated with fair accuracy (column 7); the smallest value is given in brackets. For comparison purposes the other statistics (mean Ri etc.) are also given in Table I for this restricted set of occasions. In the stratosphere the isentropic shear was in every case cyclonic; in the troposphere the shear was more frequently cyclonic (ratio 10 : 3) for the deep layers and anticyclonic (ratio 9 : 6) for the shallow layers. The mean horizontal shear regardless of sign over the Midlands is about 0.08 hours^{-1} in the upper troposphere and 0.06 hours^{-1} in the lower stratosphere (reference 1). In most cases the isentropic surfaces are almost horizontal; in the great majority of cases when the isentropic surfaces are not horizontal the isentropic shear is less than the horizontal shear. Thus the isentropic shears associated with these turbulent layers were in the mean significantly greater than normal, especially in the case of the deep layers.

Table I also lists the number of cases in which the frost point distribution with height indicated mixing and interchange of air between different levels in the neighbourhood of the turbulent layers (column 8). In the upper troposphere and lower stratosphere complete mixing in the vertical would result in constant mixing ratio, that is, a lapse rate of frost point of between 0.5 and 1°F/1,000 ft. The frost point distribution was taken to indicate mixing when its lapse rate lay between 2 and -1°F per 1,000 ft. It often happened that in, say, one third of a deep layer the frost point lapse was small but was large over the remainder; in such a case one third was counted as indicating mixing. Thus some of the figures in this column of Table I are not whole numbers. It is seen that the proportion of cases with frost point distribution indicating mixing under the present definition is not large.

For shallow layers the temperature lapse rate, calculated over a layer 2,000 ft. thick, was compared with the lapse rate over a layer 3,000 - 4,000 ft. thick including the turbulent layer. This gave a means of comparing the lapse rate in the turbulent layer with the general lapse rate in the neighbouring layers. In Table I the mean of the differences between the lapse rate of each turbulent layer and the general lapse rate is tabulated in column 9; it is seen that there is no marked tendency for the turbulence to be associated with a steeper lapse rate than normal. Since considerable changes in temperature lapse rate are expected over depths of 4,000 to 6,000 ft. a similar type of analysis was not attempted for the deep layers; instead the mean of the lapse rates in the turbulent layers is given in column 10. It is noteworthy that the mean lapse rate in the moderately turbulent layers of the troposphere is low, only 2.8°F/1,000 ft., yet only one of these layers appeared to be associated with a frontal surface or zone. Exact mean values of the temperature lapse rate in the upper troposphere and lower stratosphere are not available; it is possible that these mean lapse rates vary with the season and with height. Mean temperatures for the standard levels 500, 400 and 300 mb. over a period of 5 years at Larkhill indicate that the mean lapse rate in the layer 18,000 - 24,000 ft. is about 3.8°F/1,000 ft. and just over 4°F/1,000 ft. in the layer 24,000 - 30,000 ft. The mean temperature lapse rates quoted in column 10 of Table I for the troposphere are apparently no greater and probably less than normal lapse rates.

4. General shear absent.

In a number of cases the wind observations showed that there was no general shear in the vertical in the neighbourhood of the observed turbulence; often the wind fluctuated with height so that shears measured over layers of 1,400 ft. thickness were appreciable yet there was little shear over a depth

/of

of 3,000 - 4,000 ft. As with the category 'general shear present' discussed in section 3, the designation 'no general shear' to a particular case was to some extent arbitrary as there was always some shear present, however slight. The Richardson number over a depth of 4,000 ft. or over the depth of the turbulent layer, whichever was the greater, was tabulated for each occasion; the least value was 7.4, arising with a shear of 1.7 kt./1,000 ft., but usually the values were much larger, and this is an indication of the stringency of the criterion 'no shear'.

Table II gives mean values of Ri associated with these turbulent layers and other details as in Table I; a description of the various statistics is given in section 3. The mean of the Ri calculated over a depth of about 4,000 ft. or over the depth of the turbulent layer, whichever was the greater, is also given in column 6. Cases where a large shear was observed just below or just above the turbulent layer were excluded in this analysis.

Although Ri measured over the smallest depth possible (about 1,400 ft.) was often small in the neighbourhood of the observed turbulence, though not usually at the exact level of the turbulence, the mean values of Ri over 4,000 ft. or more show what was also immediately obvious from the wind observations that these small values arise from the wind fluctuations with height and not from any general shear.

The Richardson number in the turbulent layer was less than that above or below this level on only two occasions out of 25 (column 7); it was greater than above or below, i.e. a maximum, on 19 occasions.

The mean isentropic shears (regardless of sign) given in column 8 were again appreciably greater than normal, as in the cases of general shear. In the troposphere, for shallow layers, the shear was anticyclonic in one of the six cases; for deep layers shear in three of the five layers was anticyclonic. The only stratospheric isentropic shear measurable was anticyclonic.

On the average the frost point distribution in the vertical shows very little mixing effect in these cases of turbulence, even less than was shown in the cases occurring in a general shearing layer (column 9).

The temperature lapse rates in the turbulent layers also give little indication, in the mean, of modification by turbulence (columns 10 and 11).

5. Other cases.

Section 3 dealt with 73 turbulent layers (one counted twice in Table I) which occurred in a general shearing layer and section 4 discussed 25 turbulent layers which were not in a general vertical shear. Of the remaining 37 bumpy layers observed three were at the tropopause and it was thus impossible to estimate representative values of Ri; in another five cases the turbulence was reported at a discontinuity in temperature lapse rate and estimation of Ri was therefore doubtful.

In eight of the cases not analysed in sections 3 or 4 part of the turbulent layer was in a general shear but not all and in seven cases, though the turbulent layer was not, apparently, in a general shear, a general shear was observed just above or just below the bumpy levels. The remaining turbulent layers were not analysed because of lack of data.

6. Discussion

(a) General shear

It has been established (section 3) that in many cases turbulent layers occur in a layer of general wind change with height and that the associated Richardson number is small in these cases and, moreover, considerably smaller than at levels above and below the turbulent layer.

/Since

Since turbulence must tend to alter the properties of the layer it is likely that sometimes the difference in space and time between the two observations, turbulence and wind, may have caused a significant change in the shear so that the shear measured may not have been quite appropriate to the turbulence observations; e.g. a spate of eddies might have temporarily eliminated much of the momentum differences in the bottom half of a general shearing layer so that a particular wind observation might show the shear as extending over only half the depth of the general shearing layer. Also the path of the balloon by which the winds are deduced is influenced by any turbulence present; if the turbulence includes eddies of dimensions of the order of 1,400 ft, and we may assume that this is possible in the atmosphere, then the shears calculated over depths of 1,400 ft. from the track of the balloon will not always be representative of the mean shear. Thus many of the cases not analysed (section 5) may well have been associated with a general wind shear with height for in 15 cases shear, as shown by the independent wind observations, was present either in part of the turbulent layer or just above or below it.

It seems a reasonable deduction, therefore, that in many cases, probably the majority, the eddies which cause bumps to an aircraft at high altitude are formed in a general shearing layer in a manner resembling the way eddies are formed in the shearing layer near the earth's surface in the absence of convection.

Calder (9) has elaborated Richardson's (10) original equation for the rate of increase of turbulent energy in a stratified atmosphere with a general wind shear with height so that it becomes, (Calder's equation 41)

Rate of change of turbulent energy per unit volume

$$= \bar{\rho}\left[K_m \left(\frac{\partial \bar{u}}{\partial z}\right)^2 - \frac{gK_H}{\bar{T}}\left(\frac{\partial \bar{T}}{\partial z} + \Gamma\right)\right]$$

- (rate of dissipation of energy by molecular forces)

- (terms covering the convective change of energy associated with the mean motion, the diffusion of energy associated with the eddy motion and the rate of working of the fluctuating gradients of static pressure on the eddying motion)

(1)

where $\bar{\rho}$ = mean density at a fixed point. (All means are with regard to time).

$\bar{u}$ = mean horizontal velocity. (Mean horizontal flow in one direction)

$\bar{T}$ = mean temperature

K_m = coefficient of vertical diffusion for momentum

K_H = coefficient of vertical diffusion for heat.

Calder says that the second and third terms on the right hand side of equation (1) are important near the ground. With present knowledge it is impossible to assess their significance in the upper air away from boundary surfaces. However, it seems very unlikely that mean shears in the upper air approach anything like the large values found in the layers near the ground with which Calder was mainly concerned and it is very likely, therefore, that though these terms may not always be negligible they are not usually the main terms in the equation and the most important term is the first, which may be written, assuming $K_m = K_H$,

$$\bar{\rho} K_m \left(\frac{\partial \bar{u}}{\partial z}\right)^2 (1 - Ri)$$

/If

If the large scale forces, i.e. developments arising from depressions and anticyclones etc., can maintain a general shear in the face of turbulence so that

$$1 - Ri \geq \delta \quad \text{where} \quad \delta < 1,$$

then from equation (1), on the above theory and assumptions, the turbulence should continue.

This rough argument[7] is borne out by the figures in Table I; the mean of Ri in the turbulent layers is of the order 1, in spite of errors of observation some of which are biassed towards increasing the value of Ri (Random errors in shear will, when the shear is larger than its random error, as is usual when Ri is small, give a distribution of Ri with a mean larger than the Ri corresponding to the true shear. And again since turbulence in the upper air is usually absent we may argue that if turbulence and conditions associated with it are present in a certain intensity at one place and time then on the average turbulence and associated conditions are less likely to be so accentuated some distance away, e.g. where a wind observation is being made.)

(b) No general shear.

It is difficult to find a ready physical explanation of the turbulence which occurs when there is no general change of wind with height, (section 4). This type of turbulence would seem to occur less frequently than that associated with a general shear (in a rough ratio 1 to 3) and in the present data the intensity of the bumps was always slight. An interesting point is that such turbulence sometimes occurs in broad wind maxima.

The evidence is consistent with the hypothesis that this type of turbulence is associated with larger eddies which often are of dimensions in the vertical which show up in the wind observations at 1,400 ft. height intervals as fluctuations about a general mean current. The interactions of such larger eddies may give rise to large shears, locally, and hence to eddies of dimensions small enough (of order 200 ft. across) to give bumps and this local breakdown would, presumably, be governed by the type of law in equation (1). Such turbulence if occurring in this fashion would probably be 'patchy'; the observations of turbulence were not sufficiently detailed, however, (the aircraft remained only a short time at each level of observation) to be able to decide whether this was a significant feature of this type of turbulence.

The association of turbulence and abnormal isentropic shear leads to the speculation that large isentropic shear causes eddies rotating mainly about axes normal to the isentropic sheets and of dimensions perhaps several miles across. Durst (12) has shown that eddies of such dimensions exist. These eddies might be comparatively shallow, of order 500 to 2,000 ft. thick, and might be the larger eddies postulated above which cause the smaller bumpy eddies on their margins.

There is the possibility that large anticyclonic isentropic shear may lead to dynamic instability (see e.g. Sawyer, (13)) which might appear as turbulence of the bumpy scale. Many of the cases analysed in section 4, however, were associated with cyclonic shear.

It is possible and indeed probable that in many cases of bumpiness in a general shearing layer this other unknown cause is also operative.

/(c)

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[7] It is not certain that $K_m = K_H$ e.g. Priestley and Swinbank, (11), maintain that K_H in the sense used here may take negative values on occasions. It is also assumed here that the motion is uniform across the stream.

(c) Transfer of heat and water vapour.

In the cases of turbulence analysed in sections 3 and 4 there is little evidence of the turbulence causing much change in the temperature or humidity (frost point) distribution. This may be an indication that such turbulence is not a powerful factor in the transfer of heat and water vapour but this is by no means a necessary deduction. The bumps are an indication of turbulence in a certain size range, say 50 to 500 ft. across, but an observer in an aircraft cannot with present instruments observe turbulence outside this size range. If, for example, bumpiness in a certain layer X is the only turbulence occurring then we would expect the temperature and humidity characteristics to be modified quite rapidly assuming that turbulence noticeable as bumps must have vertical turbulent velocities (see reference 1). If, however, there is other turbulence present, above and below layer X, which is outside the bumpiness size range, then the temperature and humidity structure need not necessarily be changed. For example there may be a steady heat flux down through layer X and surrounding layers leaving the temperature lapse rate unaltered.

(d) Stratopshere turbulence.

From Table I it is seen that for deep layers the associated mean Richardson numbers are very similar in both troposphere and stratosphere. For shallow layers the mean Ri is appreciably smaller in the troposphere than in the stratosphere. However the general values of Ri are larger in the stratosphere than in the troposphere (see, e.g., columns 3 and 5 of Table I) and, as pointed out in section 3, the Ri in this analysis associated with each shallow layer was computed over a layer deeper than the turbulent layer. It is probably safe to say, therefore, that in the case of general shear there is little difference in the general values of Richardson number associated with the turbulence in troposphere and stratosphere and from this it is a reasonable deduction that the mechanism causing the turbulence is similar in the two regions.

When the turbulence did not occur in a general shearing layer most of the stratospheric cases had an interesting similarity; of the six stratosphere incidents analysed in Table II, five were in or on the edge of an upper low or upper trough with a depressed tropopause surface and four of the cases were within 200 miles of an apparent discontinuity in the tropopause surface. A discontinuous tropopause surface is usual in association with an upper low or trough; see, e.g. Sawyer, (14). The remaining case of the five occurred in an upper low which had a low tropopause surface over a large area and the tropopause discontinuity on its edge, if it existed, was some hundreds of miles away. There were four other occurrences of bumps in the stratosphere through layers of 2,000 ft. or more in which there was no general vertical shear but which were not analysed in Table II as shearing layers occurred just above or just below them; three of these cases were also in upper lows or troughs with low tropopause surfaces.

It may be argued that such a similarity is to be expected since the observations were limited to a height of 40,000 ft. and the greatest time spent in the stratosphere by the observing aircraft would have been when the tropopause was abnormally low which is almost always in the situation of upper lows or troughs. However, of the 17 stratosphere cases analysed in Table I only eight were in or near upper troughs or lows so that it is seen that the tendency for the 'no general shear' cases of turbulence in the stratosphere to be associated with upper lows or troughs is probably significant.

Unfortunately it is in cases of large curvature of the streamlines, as is found in lows and troughs, that isentropic shear is often impossible to estimate; the one case in Table II in which the isentropic shear was assessable was not in the vicinity of an upper low or trough. In regions of discontinuous or steeply sloping tropopause surface there are often

/large

large horizontal temperature gradients resulting from the close juxtaposition of warm stratospheric air above the low tropopause and cold tropospheric air under the neighbouring high tropopause. These large temperature gradients are often associated with large isentropic shear of wind, as, e.g. on the cold side of a typical jet stream. It is possible, therefore, that the isentropic shears were high (and cyclonic) in these cases of turbulence occurring in the stratosphere in or near upper lows or troughs.

7. Conclusions.

This analysis with more detailed observations and data than have been used previously confirms that turbulence (bumps) often occurs in layers of general shear with height and also indicates that in such conditions the Richardson number is of the order 1 or 2 (sections 3, 6). The isentropic shear is also usually significantly greater than normal in these cases.

It is also confirmed that turbulence (bumps) occurs when no general shear in the vertical is present; the way in which this turbulence is generated is still unknown though a possible mechanism is proposed. Again the isentropic shear is greater than normal when such turbulence occurs. It appears that this type of turbulence occurs in the lower stratosphere in the region of upper lows or troughs with low tropopause surfaces and near discontinuities in the tropopause surface. Such regions are often close to a jet stream (cold side) or a wind configuration in the upper air which is similar to that in a jet stream though the wind velocities may not be large. It is suggested, therefore, that upper lows and troughs and the cold side of jet streams are suitable regions for further investigations of the unknown cause of turbulence which occurs without associated general shear in the vertical. It may be relevant that for flights by Comet aircraft the only occasion of severe turbulence (qualitative assessment) which was not associated with a jet stream was on the edge of an upper low, near the tropopause which was probably discontinuous at no great distance from the incident and, as far as could be judged, in winds which were below average strength and which had no great shear in the vertical. (The incident occurred about 16 G.M.T., 23rd October, 1950 over Northolt at about 35,000 ft.).

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Table I

Turbulence in a general shearing layer: mean Richardson numbers (Ri), isentropic shears and other data.

	No. of layers	Mean depth of layers (1,000's ft.)	Mean Ri Below turbulent layer	Mean Ri At turbulent layer	Mean Ri Above turbulent layer	No. of cases with Ri a minimum in turbulent layer	Mean isentropic shear ($hours^{-1}$)	No. of cases with F.P. L.R. indicating mixing	Mean L.R. minus general L.R. ($^{o}F/1,000$ ft.)	Mean L.R. ($^{o}F/1,000$ ft.)
	1	2	3	4	5	6	7	8	9	10
Shallow Layers										
Troposphere										
All cases	30	1	5.5 (3.3)	2.7 (1.8)	9.4 (6.4)	20	-	11	0.1	-
Cases with complete observations.	15	1	3.5 (2.9)	2.2 (1.8)	11.0 (7.3)	10	0.14 (0.0)	2	0.1	-
Moderate intensity	1	1	2.6	0.5	1.3	1	-	0	-0.6	-
Stratosphere										
All cases	6	1	9.5 (3.2)	5.7 (4.3)	15.7 (5.7)	1	-	1	0.0	-
Cases with complete observations.	4	1	12.5 (3.9)	6.6 (4.3)	21.5 (18.3)	1	0.15 (0.06)	1	-0.4	-
Moderate intensity	1	1	5.8	4.8	4.1	0	0.13	0	0	-

/Deep Layers

Table I (Continued)

	1	2	3	4	5	6	7	8	9	10
Deep Layers										
Troposphere										
All cases	27	3.1	7.0 (3.2)	2.1 (1.4)	14.2 (2.8)	20	-	9	-	3.8
Cases with complete observations	14	3.3	4.5 (2.3)	1.6 (0.5)	8.3 (2.3)	11	0.28 (0.05)	7.3	-	3.7
Moderate intensity	8	3.3	10.9 (4.1)	1.5 (1.1)	15.1 (11.0)	7	0.29 (6 cases) (0.16)	3.5	-	2.8
Stratosphere										
All cases	11	2.5	10.4 (5.5)	2.3 (1.8)	16.8 (6.4)	8	-	3	-	-0.6
Cases with complete observations	7	2.4	11.7 (7.5)	1.8 (1.0)	12.3 (5.8)	7	0.24 (0.14)	3	-	-0.3
Moderate intensity	1	3.0	2.2	0.6	2.5	1	0.45	1		-1.5

Notes: (i) The figures in brackets in columns 3, 4 and 5 are the median values of the observed distributions of Ri.

(ii) The figures in brackets in column 7 are the minimum values of isentropic shear observed in the various turbulent layers.

(iii) In one case a turbulent layer extended across the tropopause; it was counted as two layers, in this analysis, one in the troposphere - moderate intensity - and one in the stratosphere - slight intensity.

(iv) F.P. = Last point. L.R. = Lapse rate of temperature.

Table II

Turbulence without general vertical shear: mean Richardson numbers (Ri), isentropic shears and other data.

	No. of layers	Mean depth of layers (1,000's ft.)	Mean Ri Below turbulent layer	Mean Ri At turbulent layer	Mean Ri Above turbulent layer	Mean Ri over the greater of 4,000 ft. or turbulent layer	No. of cases with Ri a minimum in turbulent layer	Mean isentropic shear (hours^{-1})	No. of cases with F.P. L.R. indicating mixing	Mean L.R. minus general L.R. (^{o}F/1,000 ft.)	Mean L.R. (^{o}F/1,000 ft.)
	1	2	3	4	5	6	7	8	9	10	11
Shallow Layers											
Troposphere											
All cases	13	1	3.3	9.5	6.4	53.1	1	-	3	0.1	
Cases with complete observations	6	1	2.9	7.5	3.8	53.8	0	0.17 (0.05)	0	-0.1	
Statosphere											
All cases	2	1	19.8	43.2	10.3	15.7	0	0.20 (1 case)	1	-0.7	
Deep Layers											
Troposphere											
All cases	6	2.5	7.1	8.5	5.2	63.7	1	-	1	-	3.4
Cases with complete observations	5	2.4	3.7	5.0	4.4	59.6	1	0.19 (0.10)	1	-	3.5
Stratosphere											
All cases	4	2.0	36.7	21.1	14.5	80.5	0	-	3	-	0

Notes: (i) Turbulence was slight in all cases.

(ii) The figures in brackets in column 8 are the minimum values of isentropic shear observed in the various turbulent layers.

(iii) F.P. = Frost point L.R. = Temperature lapse rate.

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S.C.I./55
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AIR MINISTRY

METEOROLOGICAL RESEARCH COMMITTEE

A Method of Detecting Thin Ice Films on Metal Surfaces

by D.D. Clark (Meteorological Office)

Introduction

The effect of a metal reflecting surface on polarised light has been known since the early 19th Century but it is only in the last 10-12 years that the use of photo-electric methods have made it possible to use these principles in the study of the optical properties of metal surfaces.

The following method is based on recent work, along these lines, by Professor A.M. Taylor and his associates at University College, Southampton, on the measurement of thin films on metal surfaces.

Theory.

When a plane-polarised beam of light is reflected at a metal surface then the two components, one parallel to, and the other at right angles to, the plane of incidence, experience a change in phase by reflection, but the change is less for the component polarised at right angles to, than for the component polarised in, the plane of incidence. Therefore, in the reflected beam, the components in and perpendicular to the plane of incidence will differ in phase and the reflected beam will be elliptically polarised. The difference in phase increases with increasing angle of incidence. The intensities of the two components are also changed after reflection.

If a thin transparent film, such as ice, is added to the metallic surface, the phase difference is increased or decreased accordingly as the refractive index of the metal is greater or less than that of the ice.

Using a chromium plated surface (chromium has a much higher index of refraction than ice) and an angle of incidence of 45° the intensities of the two reflected components are almost the same and the phase difference between them is 171.94°. The addition of a film of ice 10 Å thick to the chromium surface increases the phase difference to 173.04°, but does not affect the intensities.

In the laboratory, phase differences of 20' can be measured. It should be possible therefore with quite simple apparatus to detect an ice film of a few Å thickness.

Since only detection and not measurement is required, white light, provided it has a constant colour temperature, can be used in preference to mono-chromatic light to give increased sensitivity by increasing the light level.

The colour effect of the ice film should be negligible compared to the change in phase difference.

Suggested Method.

The optical arrangement is shown in figure I. A beam of light from a source of white light of constant colour temperature is collimated by lens system L. After plane polarisation at an azimuth of 45° by passage through a polaroid screen, the light falls on the plane polished surface of the chromium plated metal disc M, whose temperature can be controlled and measured. After reflection, the elliptically polarised light passes

through the long focus lens L_2, and falls on the half-silvered glass plate S. The transmitted and reflected portions of the beam pass, respectively, through polaroid screens p_1 and p_2, whose polarising axes are set at 45° to the plane of incidence and at 90° to each other, and finally converge on the photocells C_1 and C_2.

With this arrangement the two photocells pick out the components of the reflected beam corresponding with the two axes of the ellipse of polarisation. The output of the two photocells therefore will change as the phase difference between the components of the reflected beam, normal to, and parallel to, the plane of incidence, increases when an ice film forms. If the outputs of the two photocells are balanced for the condition of a clean mirror surface, the resultant out-of-balance produced by the presence of an ice film can be used to operate an indicator.

By this means the formation of ice on a cooled metal surface could be detected at an early stage and, what is important, the methods should be thoroughly effective in the detection of thin layers of glassy ice at temperatures below - 80°C.

The principles and method outlined above would appear to offer a sound basis on which to construct a frost point hygrometer for low humidities.

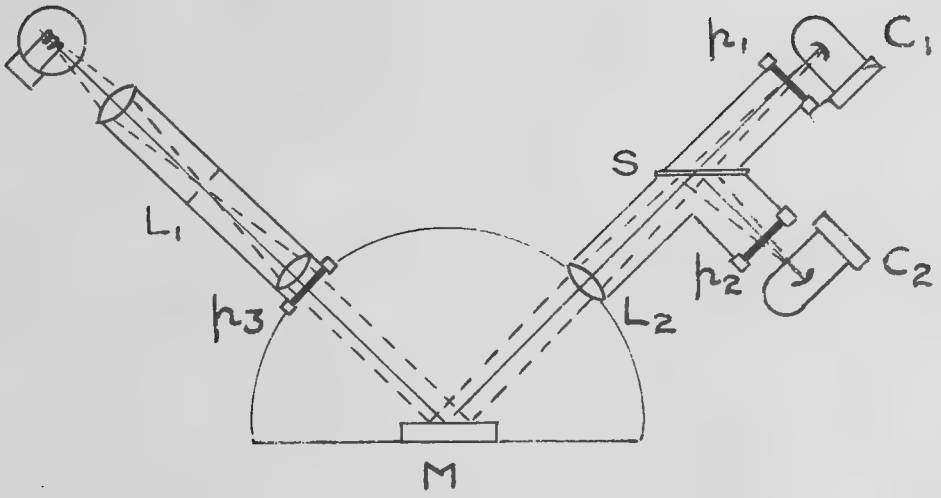

Fig. 1

(9)	24 hour forecast error	20	27	33	23
(10)	Error due to erroneous forecasting of contours ((9) less (8))	17	23	26	17
(11)	24 hour wind variation	25	35	48	27
(12)	Standard vector deviation from seasonal mean wind	28	36	50	40

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METEOROLOGICAL RESEARCH COMMITTEE

Practical Value of the Contour Chart as a Method of Representing Upper Winds

Summary of Results

by R. Murray (Meteorological Office)

1. Introductory remarks.

The established method of forecasting upper winds is by reading off geostrophic values from forecast contour charts, extrapolating and interpolating as necessary. The method introduces many sources of error, and an attempt has been made to estimate the overall standard error and to analyse this according to the different contributory factors. The work involved has been fairly heavy; a full account running to some 30 pages has been prepared and is available in typescript. However, it is thought that this brief summary may be a better basis for preliminary discussion.

The main results are presented as root mean square vector errors and variations (knots) in Table I, based on a trial period. The several rows of entries are commented on briefly below. Roughly, the relative importance of the entries is proportional to their squares.

2. Discussion of main results

Table I. Magnitude of Errors in 24 hr. forecasts of wind from forecast charts analysed according to the contribution of various factors and compared with the variability of the wind at selected levels. (All figures are root mean square vector errors or departures in knots).

	700mb	500mb	300mb	200mb
(1) Errors due to contour heights as derived from observations	3	4	10	14
(2) Errors due to estimation of geostrophic wind	5	7	8	6
(3) Errors due to personal element in chart construction	4	5	6	4
(4) Errors due to measurements of wind	2	3	4	4
(5) Errors due to small scale fluctuations	4	4	4	4
(6) Overall inherent technique errors ((1) to (5) combined)	8	11	15	16
(7) Geostrophic departures	6	9	13	8
(8) Apparent departure from geostrophic value on working charts	10	14	20	18
(9) 24 hour forecast error	20	27	33	25
(10) Error due to erroneous forecasting of contours ((9) less (8))	17	23	26	17
(11) 24 hour wind variation	25	35	48	27
(12) Standard vector deviation from seasonal mean wind	28	36	50	40

Row (1). Errors due to errors in observations contour heights.

It is not easy to estimate how much the errors of upper air temperature observations contribute in fact to the errors of contour gradients on constructed charts. Where observations are numerous the analyst can minimise the errors by smoothing. Where observations are widely scattered the entire field must be interpolated and a small error in observation is relatively unimportant. However, an estimate of the contribution can be made by considering conditions over the British Isles, where the network is sufficiently dense to define the field fairly definitely but not so dense as to permit of much smoothing. Then the errors of gradient are broadly those given by following the erroneous observations faithfully. The estimates given are based on Sheppard's (1945) estimates of standard errors of contour heights from British radio-sondes. The error is almost negligible at lower levels, but at 200 mb. it is the largest contribution amongst the various factors given in rows (1) to (5). (It may be mentioned that from other work it appears that at 100 mb. the computed contour heights are of practically no value in defining the contour gradient, except on a very broad scale.)

Row (2). Errors due to estimation of geostrophic wind.

The measurement of geostrophic wind at a definite point from a contour chart is not an objective process. The contour height isopleths are often far apart, they are curved and non-parallel, etc., and in practice the forecaster allows for the variations as seems best to him. The figures given are based on a test whereby two experienced analysts took the same charts and made their estimates independently. It is difficult to see how these errors can be decreased to any considerable extent.

Row (3). Personal element in chart construction.

Two analysts using the same basic observations will undoubtedly construct somewhat different charts. The estimated "personal errors" given are the result of a comparison of a set of upper air charts drawn at C.F.O. with a similar set constructed independently at London Airport. All that need be said is that the two sets of charts are in good agreement with each other; and that the "personal errors" appear to be acceptably small.

Row (4). Wind errors.

Bannon (1948) and Harrison (1949) discussed the errors of wind observations made by radar equipment at British land stations and Ocean Weather Ships respectively. The estimates given in row (4) are based on their papers, taking account of the number of observations in each category in the sample used in this investigation. In point of fact, the wind errors of British land stations are slightly less than, whilst the wind errors of Ships are perhaps 2 - 3 times greater than the figures given. The overall wind errors given in row (4) are clearly negligibly small.

Row (5). Small scale fluctuations.

Turbulent wind fluctuations, which cannot be determined from the contour pattern, occur in the free atmosphere. The very small fluctuations tend to be smoothed out in the process of obtaining the "observed wind", which is in fact a mean wind over a layer of the atmosphere. From a consideration of observational data contained in papers by Durst (1948), Durst and Gilbert (1950) and Loeser (1941) an estimate has been made of the order of magnitude of the wind fluctuations which are too large to be smoothed out in obtaining "observed winds" yet too small to be represented in the contour pattern. The figures given are manifestly quite small and, in any case, nothing can possibly be done to make allowance for them in synoptic work.

/Row (6)

Row (6). Overall errors inherent in technique.

Estimates of wind from contour gradient, quite apart from the inadequacy of the geostrophic approximation, must contain errors due to the combined effect of the factors (1) to (5). On the assumption that errors (1) to (5) are independent, the figures in row (6) are obtained. These overall errors are inherent in the technique, analysis and observations. It cannot be said that the figures given are negligibly small, particularly at 300 and 200 mb. Some reduction in the overall errors at high levels would result from an improvement in the accuracy of the basic temperature observations (see row (1)); but otherwise it is difficult to see how these basic errors can be decreased.

Row (7). Geostrophic departures.

The figures for geostrophic departure given in the table are obtained by subtraction (root of difference of squares) of the figures in rows (6) and (8) on the assumption that the apparent departures are a combination of real departures with errors due to technique. Investigations by Neiburger et al (1948), Bannon (1949), Godson (1950), Hartman (1950) and Durst and Gilbert (1950) show that geostrophic departures are in general of the order of 10 knots - broadly in agreement with these estimates. Several of the writers quoted above also examined the value of the "gradient wind" approximation and came to the conclusion that it is scarcely significantly better than the simple geostrophic approximation in practice. Much earlier Sutcliffe (1938) demonstrated theoretically that such a result might be expected, since the clyclostrophic component is only one of several factors associated with a lack of balance between wind and pressure gradient. However, there is no method available for direct measurement of ageostrophic winds on contour charts, although perhaps on some occasions a dynamical appreciation may allow a qualitative estimate to be made. It will be noted that the geostrophic departures given in row (7) are rather less than the overall errors of technique in row (6).

Row (8). Apparent departure from geostrophic value on working charts.

A set of working charts drawn at C.F.O. were copied, omitting all basic observations. The figures given result from a comparison of observed winds with measured geostrophic winds on these replica working charts. The apparent geostrophic departures indicate the disagreement between wind and contour gradient due to the limitations both of the geostrophic approximation and of the charting technique. Charts forecast exactly, that is identical with subsequently drawn "actual" charts, are likely to contain errors in terms of wind of the order of magnitude indicated by the apparent geostrophic departures.

Row (9). 24 hour forecast error.

The figures arise from a comparison of observed winds with forecast (geostrophic) winds obtained from forecast charts. Remembering that the squares of r.m.s. values are to be compared in considering the relative importance of individual factors, it is clear that the apparent 24 hour forecast errors at all levels are considerably larger than any of the figures given in rows (1) to (8). It will be noted, furthermore, that the largest forecast errors occur at 300 mb., i.e. in the upper troposphere.

Row (10). Errors due to erroneous forecasting of contours.

These figures indicate more accurately than row (9) the magnitudes of the errors arising purely from erroneous forecasting of the contour patterns. They are the differences between the values given in rows (9) and (8). It is patent that these figures at all levels up to 300 mb. are greater than the geostrophic departures or the individual errors inherent

in the technique and analysis; but at 200 mb. the forecasting error is of the same order of magnitude as the apparent geostrophic departure. Thus there is considerable scope for improvement of the forecasts by improving the prontours (forecast contour charts), and accepting the real or apparent geostrophic departures.

Row (11). 24 hour wind variation.

This row indicates the sort of errors that would occur from the use of observed winds as 24 hour forecast winds. Obviously the forecasting errors in row (10) at all levels are markedly less than the 24 hour wind variations. Except at 200 mb., the total 24 hour forecast errors in row (9) are also considerably smaller than the figures of row (11). It appears that the 24 hour forecast error at 200 mb. is only slightly smaller than the 24 hour wind variation mainly because the apparent geostrophic departure is large; and the apparent geostrophic departure assumes a large value which, it is inferred, arises mainly from the basic contour height errors.

Row (12). Standard vector deviations from seasonal mean wind.

These figures are based on Geophysical Memoir No. 85 by Brooks and others (1950); they give an idea of the errors likely to occur if the seasonal mean wind is used as a forecast wind. Obviously forecast errors in rows (9) and (10) are substantially less than the standard vector deviations.

3. Miscellaneous points.

(1) Route forecasts.

It is well known that the errors associated with forecasting mean winds over routes are less than those applicable to "point" forecasts. For a 1200 mile long route at latitude 55°N the r.m.s. vector errors in 24 hour forecasts of mean wind are about 13 kt. at 700 mb. and 17 kt. at 200 mb. These particular errors may be compared with row (9) of Table I. It is estimated that they could be reduced by about 5 kt. if the forecast errors associated with the prontours could be practically eliminated; then further reduction would occur only by dealing with the geostrophic departures and inherent technique and analysis errors.

(2) Errors due to transmitting charts to outstations.

When contour and prontour charts are re-drawn at outstations from the directive messages transmitted from C.F.O. additional wind errors are introduced. A test, made by comparing original charts with those prepared at an outstation, showed that the errors were very considerable, being of about the same magnitude as the apparent geostrophic departures; they are considerably less but still important in connection with mean winds over routes or areas. A photographic method of transmission of the C.F.O. contour and prontour charts would eliminate these errors completely.

(3) Comparison with forecasts by statistical regressions.

The wind errors of contour and prontour charts may be discussed briefly in relationship to some statistical results, obtained by Durst (1950), connecting the variation of wind with time and distance. The regression equation giving the variation of wind with time may be used as a method of forecasting wind at a point, but the standard errors of such forecasts appear to be a little greater than the forecast errors associated with the use of prontours, at least for periods between 6 and 24 hours. Contour and prontour charts are useful for interpolating or forecasting winds at points where no recent observation has been made. Moreover, the use of statistical regression equations connecting the variation of wind with distance and with time as a means of interpolating or forecasting wind at such points produces errors that

are greater than those of the contour or prontour charts in most cases. Only for very short period forecasts (less than about 6 hours) of wind not farther away than about 2 - 300 miles from a wind observing station can the prontour technique be regarded as unjustified. It must also be remembered that a prontour chart enables winds over a wide area to be easily computed, so that it has considerable practical convenience.

4. Conclusions.

The main conclusions are:-

(a) A considerable contribution to the errors of forecasting by the contour technique is made by errors inherent in the technique which cannot, by any obvious means, be avoided.

(b) Errors due to the wind observations are negligibly small.

(c) Errors due to the temperature observations become significant at 300 mb. and serious at 200 mb.

(d) Errors due to geostrophic departures account for only a small fraction (about 10%) of the total root mean square error of forecasts.

(e) Errors due to "forecasting" are definitely the greatest, accounting for some 70% of the total error, except at 200 mb. where errors of observation and technique become equally large.

(f) The errors are generally much less than the 24 hour variation except at 200 mb. where the difference is small.

(g) Errors arising from communicating prontours to outstations are very considerable.

M.O. 21/9/M.O.15

To all recipients of M.R.P. 663

Addendum

With reference to Section 2, Row (11), please add footnote to page 4 as follows:-

"There are considerable differences in the wind variability in the same month from year to year; in particular the 24 hour wind variation at 200 mb given in Table I is some 10 kt less than the normal variability for the Season. This does not affect the general results".

A. C. Best

Secretary,
Meteorological Research Committee

M.O.15

22nd January, 1952

are greater than those of the contour or prontour charts in most cases. Only for very short period forecasts (less than about 6 hours) of wind not farther away than about 2 - 300 miles from a wind observing station can the prontour technique be regarded as unjustified. It must also be remembered that a prontour chart enables winds over a wide area to be easily computed, so that it has considerable practical convenience.

4. Conclusions.

The main conclusions are:-

(a) A considerable contribution to the errors of forecasting by the contour technique is made by errors inherent in the technique which cannot, by any obvious means, be avoided.

(b) Errors due to the wind observations are negligibly small.

(c) Errors due to the temperature observations become significant at 300 mb. and serious at 200 mb.

(d) Errors due to geostrophic departures account for only a small fraction (about 10%) of the total root mean square error of forecasts.

(e) Errors due to "forecasting" are definitely the greatest, accounting for some 70% of the total error, except at 200 mb. where errors of observation and technique become equally large.

(f) The errors are generally much less than the 24 hour variation except at 200 mb. where the difference is small.

(g) Errors arising from communicating prontours to outstations are very considerable.

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The distribution of humidity at sea

by C.S. Durst and A.H. Gordon (Meteorological Office)

In the study of radio ducts over the ocean there is need for much more information in regard to the distribution of water vapour at different heights over the open sea. One of the great difficulties in getting such information in the past has been the uncertainty of the accuracy of observations of temperature and particularly of wet bulb temperature when made in a fixed or portable screen on ships.

The Ocean Weather Ships should give opportunity to get far better observations of temperature than have been made before because in them it is possible to devote sufficient time to enable an observation to be made with an aspirated thermometer with all precautions taken as to the cleanliness of the cotton on the wet bulb.

As a preliminary step to see what type of information can be obtained from the data at present received from the O.W.S.S. two calculations have been made and are described below. It must be emphasised that they were made solely to see whether more elaborate investigations were worth while and to act as a basis for obtaining advice how those more elaborate investigations should be made.

(1) The first of these produced Tables I and II. These tables were compiled by simply extracting the observations of wind direction and force, sea temperature and wet and dry bulb temperatures from the records made at station Jig during the periods October 8th to 28th, November 24th to 30th and December 6th to 17th, 1947. (O.W.S.S. Weather Recorder and Weather Observer). The saturation vapour pressure at sea temperature was then taken from the tables and the observed vapour pressure computed from the wet and dry bulb. The difference between the two gives a measure of the probable lapse in the lowest 28 - 30 feet (provided that the assumption that the air is saturated at the sea surface itself is justified).

The differences were then summarised according to wind direction and force.

The results are crude and unchecked and only broad conclusions can be based on them. The data used were those made in the early days of weather ships and more modern observations are probably more accurate.

It is to be noted that the sea temperature readings are only made to the nearest ^{o}F.

The tables show that at station Jig (Latitude 53°50'N, Longitude 18°40'W) the decrease in vapour pressure with height in the lowest 28 to 30 feet is most marked with winds from North, Northeast and East and least with winds from South and Southwest. Since Northeasterly winds are likely to be associated with cold dry air masses and Southwesterly with warm humid air masses these effects are not unexpected.

Table I also shows a tendency for the very light winds (forces 0 and 1) to have a comparatively smaller lapse rate than forces 2 and 3. There seems to be a fairly regular decrease in the lapse rate as the wind increases above force 2.

It may be suspected that with very light winds the air has moved slowly over the sea and hence even with Northeast and East winds the moisture has had time to diffuse up to 30 feet; with winds of a few knots the air at 30 feet still keeps some of the characteristics of another locality. Above force 3 turbulence increases and stirs the moisture upwards.

/Such

Such conclusions must await a reasoned discussion after more data have been analysed to be sure that the factual information is representative.

Table I. Saturation vapour pressure at temperature of the sea surface minus observed vapour pressure (mb.)

October 2 - 28
November 24 - 30 } 1947 O.W.S. Jig 53°50'N, 18°40'W.
December 6 - 17

Direction	Beaufort Force 1	2	3	4	5	6	7	8	All Forces	Calm
N (N'W - NNE)		5.6	5.3	4.9		5.7			5.3	2.8
No. of obs.		2	4	3		4			13	2
NE (NE'N - ENE)	3.5	6.3	5.2	5.6	6.0				5.4	
	2	1	4	6	3				16	
E (E'N - ESE)	4.3	6.9	5.1	5.6	3.2	2.5			4.6	
	2	1	6	6	2	4			21	
SE (S'E - SSE)	1.3	2.9	2.2	2.1	1.6	2.2			1.9	
	1	4	9	14	32	7			67	
S (S'E - SSW)	1.6	-	0.9	0.8	0.4	1.3	-0.6	0.1	0.7	
	1		9	28	13	8	7	3	69	
SW (SW'S - WSW)	1.9	4.4	1.9	0.7	1.0	0.8	2.8	-1.2	1.3	
	3	1	5	10	22	3	3	1	48	
W (W'S - WNW)	2.0	1.5	3.7	3.0	3.1	3.7	6.1	1.8	3.2	
	2	3	8	18	11	11	1	1	55	
NW (NW'W- NNW)	1.1	3.1	2.9	4.3	-	4.7			3.7	
	1	2	7	7		2			19	
All directions	2.4	3.6	3.1	2.5	1.7	2.9	0.9	0.2		
	12	14	52	92	83	39	11	5		

Table II. Frequencies of vapour pressure differences with various wind directions

	more than -1.9	-1.0 to -1.9	-0 to -0.9	0.0 to 0.9	1.0 to 1.9	2.0 to 2.9	3.0 to 3.9	4.0 to 4.9	5.0 to 5.9	6.0 to 6.9	7.0 to 7.9 mb	Total
N							3	2	5	1	2	13
NE					1	-	1	-	8	6		16
E					4	2	2	3	2	5	3	21
SE			1	8	34	16	1	6	1			67
S		3	13	33	12	4	2	1	1			69
SW	2*	3	6	12	9	5	8	2	1			48
W				4	15	8	8	13	5	2		55
NW			1		2	1	6	5	4			19
Calm				1				1				2

* -4.2 and -2.3

(Note - These tables were prepared largely on the initiative of Miss E.E. Austin).

2. One of the main criticisms of these data was felt to be the lack of knowledge of how the vapour pressure varied with height between the surface and 30 feet. Efforts are being made this summer to get regular profiles at the O.W.S.S. from sea surface to mast head using a ventilated psychrometer, but in the meantime the humidity at bridge height was examined in relation to the wind direction and speed. The method pursued was as follows:- The Hollerith Cards of all observations from O.W.S.S. were used, both those made from station Item and station Jig and all months were combined. These observations were sorted for direction and force of wind and recombined into 8 groups of directions and 8 forces, 65 groupings in all; each of the 65 groups were then sorted for temperature and wet bulb temperature and tabulated. These tabulations were then in a form from which frequencies could be obtained of dew point, vapour pressure or relative humidity. Had we selected either of the first two we should not have had any information except for the height at which the observations were made. In the case of relative humidity however we knew that the air in contact with the sea surface would in all probability be saturated, hence frequency curves of relative humidity were made out for each of the 65 groups and plotted. From the resulting curves the modes, the 10 percentile and the 80 percentile were deduced and have been plotted against wind force and direction in Fig. 1 on which have been drawn isopleths of each 5% of relative humidity from 60% to 95%.

3. The features which appear on these diagrams are:-

(a) High relative humidity with Southeasterly and Southerly winds and low relative humidity with Northwesterly and Northerly winds.

(b) An increase in relative humidity with increase in wind speed after a minimum at about force 2. The latter effect is thrown into evidence by Fig. 2 in which the values for winds between E'S and S.W. are grouped together and also between W'N and N.E. This confirms the figures shown in Table I in which the greatest difference between the vapour pressure at 30 feet and at sea surface is when the wind is force 2.

(c) The constancy of the value of relative humidity at higher wind speeds than force 4.

4. The humidity over a point at sea for from shore depends on a number of factors:-

(a) Vertical movement. Descending air is usually dry air and probably the greatest contrast between the vapour pressure of the air at 30 feet and that at sea surface occurs when air has descended in an anticyclone (with incidentally light winds).

(b) Horizontal advection of moister or drier air. Throughout the year near the points Item and Jig the gradients of both the sea and air temperatures are from N.W to S.E. with higher temperatures to S.E. Absolute humidity increases in the same direction so that the tendency is for Northwesterly winds to bring drier air, Southeasterly to bring moister air. Moreover other things being equal the stronger the wind the more marked will be the advection. However at the same time the advection will bring air of a different temperature.

/(c)

(c) Evaporation from the sea surface into the air in contact with it and subsequent mixing upwards by turbulent diffusion. Conversely there may be condensation from the air onto the sea surface in cases where mild humid air is flowing over cold sea. It may be noted that the magnitude of the turbulent diffusion is probably related to the wind speed.

(d) Evaporation from spray flung off the sea surface by breaking waves. Since waves begin to break at about force 5 this effect will be greater in strong winds than light winds.

5. These factors are interacting and appear to be too complex for their effects to be easily distinguished in such diagrams and tables as are shown above, nevertheless if the problem of evaporation over the ocean and its allied problem of the distribution of humidity with height is to be solved, data such as those we give above must be required.

Fig.1. Relationship of Relative Humidity to Wind force for various directions.

Modal frequency

Wind Direction (points) \ Wind Force	0	1	2	3	4	5	6	7	≥8
01-04	82	85	76	84	81	80	82	85	90
05-08	82	84	85	83	86	84	85	84	91
09-12	82	80	84	86	86	91	91	89	92
13-16	82	86	86	86	88	92	92	92	92
17-20	82	81	81	86	87	86	90	90	87
21-24	82	81	81	81	82	84	83	82	83
25-28	82	79	80	80	81	79	78	80	80
29-32	82	83	77	79	79	80	77	77	84
01-04	82	85	76	84	81	80	82	85	90

HIGH

LOW

10 percentile

01-04	66	60	61	68	63	62	64	69	73
05-08	66	64	67	68	67	65	67	65	69
09-12	66	66	63	71	68	75	73	69	75
13-16	66	67	63	67	74	79	78	78	78
17-20	66	58	60	69	70	70	72	72	70
21-24	66	63	63	64	68	68	68	65	67
25-28	66	60	59	63	67	64	63	65	63
29-32	66	68	57	60	67	65	63	59	67
01-04	66	60	61	68	63	62	64	69	73

HIGH

LOW

LOW

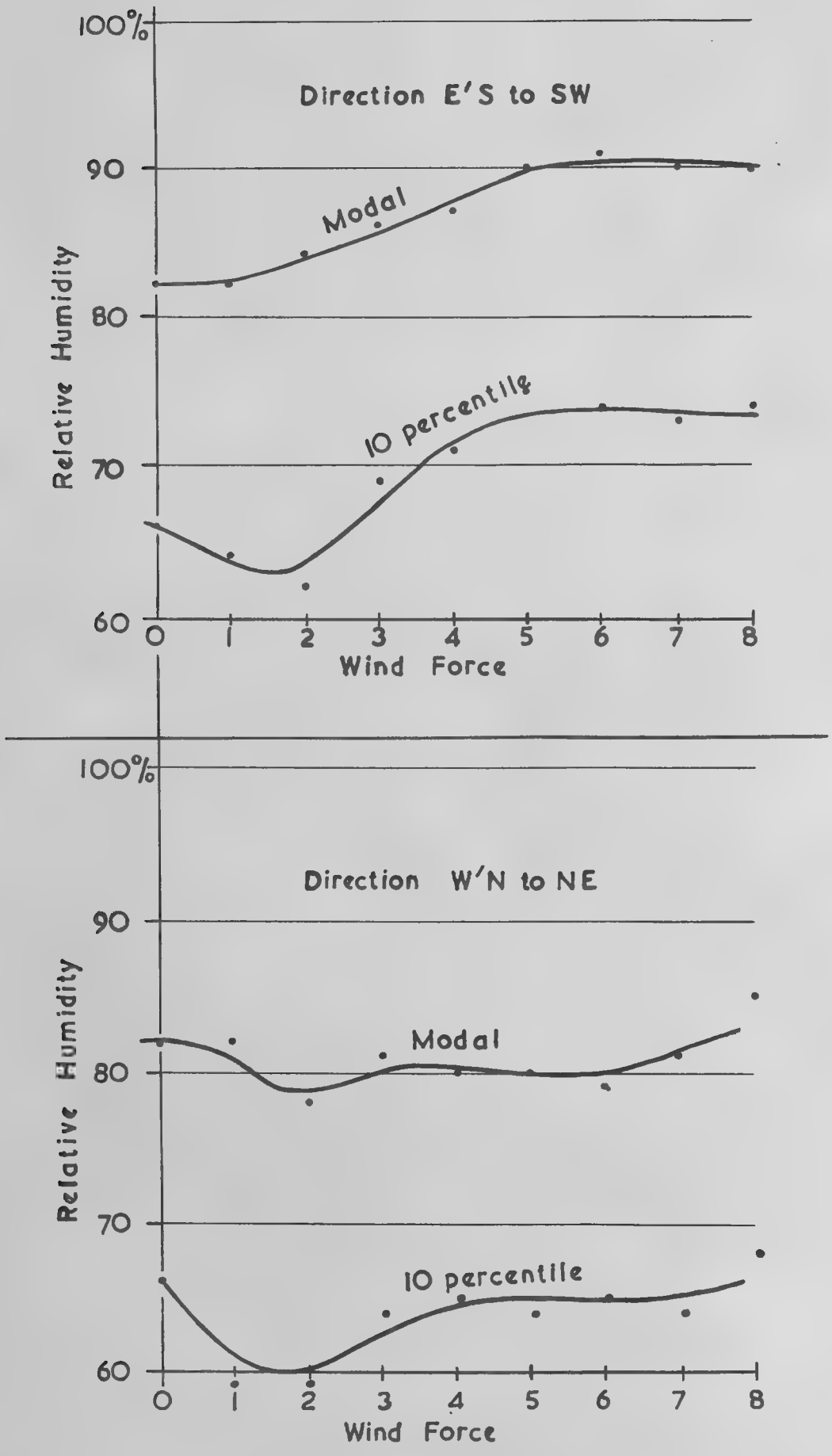
80 percentile

01-04	93	92	86	93	90	91	92	91	95
05-08	93	92	93	92	93	93	93	93	94
09-12	93	92	93	93	93	94	95	95	96
13-16	93	93	94	93	94	95	95	96	96
17-20	93	94	93	94	94	94	94	94	94
21-24	93	92	93	92	93	93	92	92	92
25-28	93	89	92	91	89	87	91	91	91
29-32	93	95	88	91	90	90	88	88	92
01-04	93	92	86	93	90	91	92	91	95

HIGH

LOW

Fig. 2. Variation of Relative Humidity with Wind Speeds.



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Notes on the wind field of middle latitudes

by R. Murray and D.H. Johnson (Meteorological Office)

SUMMARY

On the basis of 60 cross-sections, two for each day of September, 1950, it is concluded that

(a) the westerlies always contain one or more wind maxima in the upper troposphere with structure broadly similar to that described by other writers for jet streams but with much variety of intensities and gradients.

(b) the mean horizontal wind shear on the warm side is about two-thirds of that on the cold side where it averages 20 kt/100 mi and may exceed 100kt/100 mi.

(c) the geostrophic equation gives a broadly correct representation of the true winds.

(d) routine 300 mb. charts as prepared at present tend to displace the maximum and to introduce spurious double maxima and other irregularities. The axes of wind maxima might profitably be marked on the 300 mb. charts as an aid to continuity in analysis.

(e) there is a broad connexion between fronts and jet streams but the relationship is often less simple than that suggested in published work. Divided jets, distinct double jets and other complexities arise.

Daily charts are given showing the jet axes and intensities at 300 mb. superimposed on the surface analysis.

Some points requiring further study are mentioned.

1. Introduction.

The notes presented here are mainly the outcome of an examination of 60 cross-section charts depicting the wind and temperature fields through the extratropical westerlies during September, 1950.

The positions of the sections are shown in Fig.1. One section extends from near Angmagssalik across the British Isles to near Malta (for brevity, called section AM); the other is from near Lagens across southern England to southeast of Stockholm (called section LS). The upper air stations used in the construction of the sections are also shown in Fig.1. Some of these observations are appreciably off the line of the section. Consideration of the synoptic situation usually enabled a decision to be made as to the representativeness or otherwise of these 'off-the-line' observations. Thus it was necessary on some occasions to neglect and on other occasions to modify the observation reported (this applied to the temperature field). The wind field, represented by the components of the wind perpendicular to the line of the section (positive for flow from the Atlantic through the sections to Europe - negative for the reverse direction), was generally based on wind observations, but occasional measurements of geostrophic wind were made, particularly in the regions Angmagssalik to the Hebrides and Lagens to Cornwall, when actual wind reports were either missing or non-representative.

The sections were drawn daily on the basis of the 1400 G.M.T. upper air observations.

Since the publication in 1947 of the paper "On the general circulation of the atmosphere in middle latitudes" by the Staff Members, University of Chicago (ref. 3), a considerable number of cross-sections have been constructed through pre-selected jet streams. However, the fixed sections AM and LS were chosen in order to study the general wind and temperature fields, although with emphasis on the jet stream phenomenon. Our work with the September, 1950 fixed sections has suggested several aspects of the jet stream requiring detailed and special investigation. In the main, however, the purpose of this paper is to draw attention to some general features which seem of particular interest to forecasters.

2. Broad features.

Figs. 2 and 3 are examples of wind fields which most commonly occurred. One third to a half of the section is dominated by a strong current having a maximum of velocity normal to the section of 80 kt. or more. The rest of the field is weak and featureless by comparison, although it may contain local maxima of the order of 20-40 kt. at any level of the troposphere or, infrequently, in the stratosphere. On all but five of the sections there could be found currents with maximum (normal) speed greater than 60 kt; on these five sections there were local maxima of 45/60 kt. and the complementary sections for the same day showed maxima of 75 kt. at least. Thus the series of cross-sections clearly demonstrated the existence, every day of the month, of wind maxima of the jet stream type in the upper troposphere.

In general terms, we may say that the sections show that every current has a maximum, located somewhere in the central part and near or just under the tropopause. More than one maximum sometimes occur in the upper troposphere; this seems usually to be associated with the splitting of a main current into two subsidiary branches, in a region of confluence or difluence. Distinct maxima often occur in the two branches separated by distances of 500 miles or more but may be continuous with more closely spaced maxima within the core of the main stream.

The number and location of wind maxima above 500 mb. along each section during the month are displayed graphically in Figs. 4 and 5. These values represent the wind speed maxima and not merely the maxima of the normal components. It will be noted that the wind maxima tend to be located at a somewhat higher level in lower than higher latitudes (see Fig. 4); that, in the neighbourhood of the British Isles at least, the wind maxima are most commonly between 300 and 200 mb. with rather few maxima outside these pressure levels and that wind maxima with an easterly component are uncommon. These results are not unexpected.

Further information concerning the frequency of occurrence of wind maxima within different pressure intervals is presented in Figs. 6 and 7 - these figures also contain the mean value of the wind maxima in the pressure interval. In round figures, 70% and 40% of the wind maxima were greater than 80 kt. and 100 kt. respectively.

3. Typical jet stream structure.

The jet stream associated with a frontal zone has been described by Palmén (ref. 2), by Durst and Davis (ref. 1) and by others. Our cross-sections contain jet streams with similar structure (e.g. Fig. 2), although with appreciable difference in detail between individual wind maxima.

The high wind velocity is evidently produced in the main by the integration throughout the tropopause of the thermal wind effect. In most cases the subtraction of the surface velocity field would have little

effect on the general form and magnitude of the isokinetics of the jet stream. Now in the lower troposphere strong horizontal thermal gradients are usually contained within narrow sloping frontal zones and are found in conjunction with a vertical wind shear. However, the thermal wind field associated with the simple model of a sloping boundary layer is not sufficient to produce large wind maxima of the type observed. In point of fact the jet streams are located within zones of the upper troposphere in which the isotherms slope relatively steeply and with marked horizontal thermal gradients extending throughout a deep layer in the vertical. It seems that although two air masses are normally separated in the lower troposphere by a frontal zone (which may be very ill-defined at times), the thermal contrast aloft is usually spread over a much wider vertical band. When there is a well defined sloping frontal layer near the surface it is often difficult to detect above 500 mb. and most commonly merges into the wider vertical column aloft which shows up as a broad inflexion in the isotherms (i.e. strong baroclinity). The tropopause is instrumental in introducing reversed thermal gradients, thus causing a decrease in wind speed above the jet stream. The sharp diminution in wind speed towards the colder (tropospheric) side at the level of the wind maximum is generally closely associated with a very steep tropopause (or possibly with a discontinuity in the tropopause); both factors, almost of necessity, seem to occur together.

4. Wind shears associated with jet streams.

(a) Horizontal wind shears.

The kind of shear that occurs may be illustrated from profiles of the wind speed across the jet streams, based on the September sections.

Fig.8 contains three mean wind speed profiles, constructed from 'cold' jets (i.e. jets which tend to move laterally towards the warmer tropospheric air), from 'warm' jets (opposite of 'cold' type), and from jets of both types. Not all the jets that showed up on the 60 fixed sections were utilised in the construction of Fig.8, but only those well substantiated by actual wind reports. In addition only jets were considered when the jet deviated by less than 45 degrees (in most cases it was less than 25 degrees) from the line normal to the section: this restriction ensured that the major characteristics of the jet stream were given by those of the jet component perpendicular to the section. Altogether 23 separate wind profiles were constructed, although only 21 cases were employed in computing Fig.8, as 2 profiles did not extend sufficiently far on each side of the wind maximum.

From Fig. 8 we see that the three profiles are broadly similar. In each profile the horizontal wind shear is greater on the cold side than on the warm side of the jet axis. The intensity of the mean cold jet is some 20 kt. greater than that of the mean warm jet, yet the horizontal shears are about the same within the main body of the current. On the cold side the wind decreases by about 18 kt. in the first 100 miles, and equals about half the maximum speed some 200 miles from the jet axis, whilst the wind diminution is about 14 kt. in the first 100 miles on the warm side, with the wind falling off to half the maximum value about 320 miles from the jet axis. On the average the wind shear on the warm side is roughly 2/3 of the shear on the cold side.

Fig. 8 depicts average conditions. There is considerable variation in the shape of the wind maximum and the value of the wind shears from case to case. Two interesting cases are presented as profiles (i) (section LS, 16th September, 1950) and (ii) (section LS, 22nd September, 1950) in Fig.9. Profile (i) has a sharp

maximum with a large wind shear, of the order 60/70 kt. in 100 miles, over at least 200 miles. This is an example of two contrary jets quite near each other; the wind shear on the cold side centred about the zero speed averaged 100 kt. over 100 miles at the 300 mb. level. A different type is shown as profile (ii); here the wind maximum is quite flat and the wind shears small in comparison with those of profile (i).

It has already been mentioned that the wind profiles are based on wind components normal to the sections. It we had varied the position of the sections in order to have them more nearly perpendicular to the individual jets slightly greater maxima and shears would be recorded. Our estimate is that the maxima given in Fig.8 should be increased by 10 kt. or so, with the mean shears perhaps 1/10th greater than given.

(b) Vertical wind shears.

The vertical wind shears are rather variable from jet to jet. The greatest shears normally occur in the troposphere below and on the cold side of the jet. If the frontal zone is marked, as in Fig. 2, there is usually an associated concentration of shear with values of the order 10/15 kt. per 1000 ft. Sometimes, however, the thermal contrast is spread over a wide zone and then the shear may be rather uniform in the troposphere, perhaps not exceeding 5 kt. per 1000 ft. anywhere. On the warm side of the jet the shears tend to be rather uniform throughout the troposphere, being often only a few kt./1000 ft. The vertical shears above the jet in the stratosphere vary considerably from case to case. As has been suggested by Durst and Davis (ref. 1) the total change of wind is often spread more evenly over a deep stratospheric layer (of the order 12000 ft.) than in the troposphere. But this is not always the case. Quite often a very strong horizontal thermal gradient opposed to that in the troposphere sets in immediately above the tropopause, associated with vertical shears of about 10/15 kt./1000 ft. as indicated in Fig.2. On occasion the shear just above the tropopause may even be two or three times greater than the shear anywhere in the troposphere (e.g. case of jet near Aldergrove on 24th September, 1950).

5. Jet streams in relation to 300 mb. charts.

Experience with the cross-sections shows that the general structure of the wind is given by the geostrophic equation. The wind maximum at 300 mb. is normally located just under the axis of the jet stream while the wind profile at 300 mb. is broadly similar to that at the level of jet maximum. If, therefore, the 300 mb. contour charts are correctly drawn they should give a good indication of the location and intensity of the jet stream.

In routine work however the 300 mb. contour charts must be constructed rapidly, there is rarely time to make a detailed cross-sectional study and the contours may not give a very close representation of the true structure.

This point was examined by comparing the wind profile at 300 mb. (i) from cross-sections - the "cross-section wind", and (ii) from routine 300 mb. contour charts - the "300 mb. contour wind".

On each of the occasions of jet stream given by the cross-sections used in section 4 (see Fig.8) the "300 mb. contour" wind component normal to the appropriate section was measured at suitable points (by measuring the mean geostrophic wind over 400 ft. contour intervals which overlapped by 200 ft.). Individual and mean "300 mb. contour" wind profiles, as well as individual and mean "cross-section" wind profiles at 300 mb., were constructed.

The mean profiles of "cross-section" and "300 mb. contour" wind for the cold jets, the warm jets and both types together are shown in Figs. 10, 11 and 12 respectively. It will be noted that in spite of the small number of cases the mean "cross-section" and "300 mb. contour" wind profiles are remarkably similar, indicating that, in the mean, the supposition that the wind maximum obtained by geostrophic measurement of 300 mb. contours should be a good representation of the position and intensity of the jet stream axis is justified. However, examination of the individual profiles gives a less satisfactory picture.

If the "300 mb. contour wind" maximum is taken as the actual wind maximum, then the mean distance error is about 75 miles and the mean error in the speed of the jet is 17 kt. (nearly all the "300 mb. contour" jets being stronger). These mean errors were obtained by considering only the most pronounced "300 mb. contour" wind maximum when, as often happens, more than one maximum is shown in the "300 mb. contour" wind profile. Smoothing the "300 mb. contour" wind profiles in order to eliminate apparently random fluctuations of wind speed reduced the mean distance error to 67 miles and scarcely affected the mean speed error. Two cases may be quoted by way of illustration: Fig. 13 shows a case of fairly good agreement and Fig. 14 a case of poor agreement.

It has been mentioned that a common "300 mb. contour" wind profile (based on working charts) exhibits two maxima as in Fig. 15. More often than not these maxima merely arise as an accident in the drawing of the chart. Smoothing the profile will generally produce something more comparable with reality, although on a few occasions two wind maxima do in fact exist, so that the smoothing process may seriously increase the discrepancy between the "300 mb. contour" and "cross-section" wind profiles.

It is believed that the main reasons for the differences between the "300 mb. contour" and "cross-section" wind maxima are:-

(i) contour height errors.

(ii) unintentional introduction of unreal gradients.

(iii) the geostrophic approximation.

We consider that (i) contributes the main error. An examination of some 300 mb. charts has clearly shown that unreal gradients are often introduced by too slavish acceptance of height observations as correct (marked fluctuation in contour heights over the period 6 hours before until 6 hours after the time of the chart often indicates that the contour heights are in error). The errors introduced by (ii) are less important, but could be reduced if sufficient time is spent on the preparation of the charts - hasty work is probably the main cause of such errors. There is no reason to attempt to improve on the geostrophic approximation so long as errors of the type (i) and (ii) remain.

The conclusion reached is that the 300 mb. contours as drawn under working conditions give a rough indication of the position and intensity of the jet stream axis. Experience with upper air charts and cross-sections suggests, however, that the additional aids arising from continuity in the charts and maximum utilisation of observed winds enable reasonably accurate positioning of the jet stream axis to be made. It seems desirable that estimated positions of the axis of the jet stream should be marked on 300 mb. charts in much the same way as fronts are marked on surface charts. This simple procedure materially helps in maintaining continuity from chart to chart and prevents the introduction of unreal gradient.

6. Association between jets and surface fronts.

When warm and cold air masses are brought closely together there are created strong horizontal temperature gradients with which are normally

associated fronts and jet streams. Generally speaking, a well marked (in a thermal sense) surface front is sufficient but not necessary evidence for the existence of a jet stream aloft. Quite stromg wind maxima may exist in association with weak surface fronts. This arises by virtue of the fact that strong currents aloft can be produced by broad thermal contrasts over a sufficiently deep layer, whereas the thermal contrast is normally concentrated in a rather narrow band at a well-marked front. The position of the jet stream axis in relation to frontal surfaces is generally in the uppermost part of the warm troposphere approximately above the intersection of the front with the 500 mb. surface, as has been pointed out by the Staff Members etc. (ref. 3) and by other writers. This empirical rule is a fairly good approximation to reality in cases of strong fronts and jets. The rule implies that the jet stream axis should often be 2-400 miles behind a surface cold front and 4-800 miles in advance of a surface warm front. In many cases much greater complexity exists in the relationship between surface fronts and jet streams as can readily be observed from Figs. 16 to 45, which give the estimated positions of the jet stream axes and surface fronts daily at about 1500 G.M.T. in September, 1950, in the neighbourhood of the British Isles. The positions of the jet streams were obtained from consideration of the series of cross-sections and the 300 mb. charts (available at 6-hourly intervals). Afterwards the surface fronts and 1000 mb. contours (copied from the working charts prepared at the Central Forecasting Office) and the jet streams were put on the same charts. It should be mentioned that the lines representing the axes of jet streams have generally been terminated where the wind maxima become ill-defined, although in some cases west of the British Isles no jet stream is indicated simply because of uncertainty in locating it in that region. An indication of the variation of wind speed from one jet to another and at different positions on the same jet is also given in Figs. 16 to 45 (the estimates of speed referring to the 300 mb. level).

Many examples (e.g. Figs. 28 and 41) show jet streams in advance of warm fronts and in the rear of cold fronts in approximately the position that one might expect. In most cases the jets are parallel to the surface fronts only over a limited distance; generally the jets tend to approach that part of the front which is near to the cyclonic centre and to recede from the other end of the front, as in Fig.28.

Small running waves on a slow moving cold front affect the associated jet hardly at all. If the wave develops, the jet becomes distorted in sympathy with the increasing amplitude of the wave-like surface disturbances. As the occluding process proceeds the jet tends to weaken in the neighbourhood of the surface centre, perhaps lying almost perpendicular to the surface occlusion, as in Fig. 42. On other occasions the original jet separates out into two distinct jets - a warm and a cold jet - as in Fig. 39. Warm and cold occlusions, having the character of warm and cold fronts, may sometimes be associated with warm and cold jets respectively.

These remarks are in the nature of broad generalisations. On occasion there appears to be little or no association between surface fronts and jets, as in Fig.18. This state of affairs is likely to arise when the surface fronts are thermally weak, or when a new strong jet is overtaking an older, degenerating frontal system, as in Fig.18. The remnants of degenerating jets may also complicate the picture. These complexities are inevitable in an atmosphere essentially dynamic rather than static in its behaviour. However, over a short period, say of the order of 12 hours, there is considerable conservatism in the relationship of the jet with its front; and this fact can be of some use in forecasting. For instance, different warm jets may be differently aligned in relation to warm fronts, but a particular relationship, once established, is likely to persist, at least, for a short period.

7. Some points requiring investigation.

As has been stated in section 1, this paper purports only to describe and discuss certain broad features of the wind field in the westerlies that

appear to be of significance in synoptic practice. However, there are a number of questions (not all of which were suggested by the work of the September cross-sections), which deserve brief mention here, although firm answers cannot be given with any degree of confidence at this stage. Further investigations, entailing the construction of many cross-sections at frequent intervals in different positions, are called for.

(a) Movement and distortion of jets.

A common tenet is that a jet propagates itself along its length with only slight lateral motion. It is clear that this is only a tendency at the best. Intense jet streams quite often behave in this manner, but there are great deviations in particular cases, with some jets, especially smaller ones, often moving quite rapidly normal to themselves. Long and nearly straight jet streams have a certain persistency, but ultimately they become very distorted with associated great changes in the synoptic type, which are fundamentally important in the forecasting problem.

(b) Double jets and splitting jets.

Double jets and splitting jets are indubitably real phenomena, but there are many details about them that are not clear. In particular, the type of wind profile near the region of 'difluence' in the flow is uncertain. In the case of jets which have branched off from a parent jet the relative intensity (and change in relative intensity) of the two branches (usually one with cyclonic and the other with anticyclonic curvature) may be important in synoptic development.

(c) Effect of subsidence.

Markedly different rates of subsidence in neighbouring air columns may set up a horizontal temperature gradient which may increase or diminish the pre-existing temperature gradient and hence modify the jet stream structure. It is not clear just how important is the effect. An example of the part played by subsidence is the tendency for some cold jets to lag behind cold fronts, possibly due to pronounced subsidence in the troposphere just in the rear of the surface front.

(d) Tropopause structure.

The structure of the tropopause near jet streams is complex. There appear to be three possibilities at least:- (1) the tropopause is discontinuous, or (2) it is continuous but very steep or (3) it is 'folded'. There is evidence - albeit inconclusive evidence - that all these states can exist, and it may be that they fit into the life-cycle of a jet stream in a manner requiring elucidation.

References

(1) Durst, C.S. and Davis, N.E. 1949 J. Inst.Nav., Vol.2, No.3, pp. 210 - 218.

(2) Palmen, E. 1948 J. of Met., Vol. 5, No.1, pp. 20 - 27.

(3) Staff Members., Dept. of Met., University of Chicago. 1947 Bull. Amer. Met. Soc., Vol. 28, pp. 255 - 280.

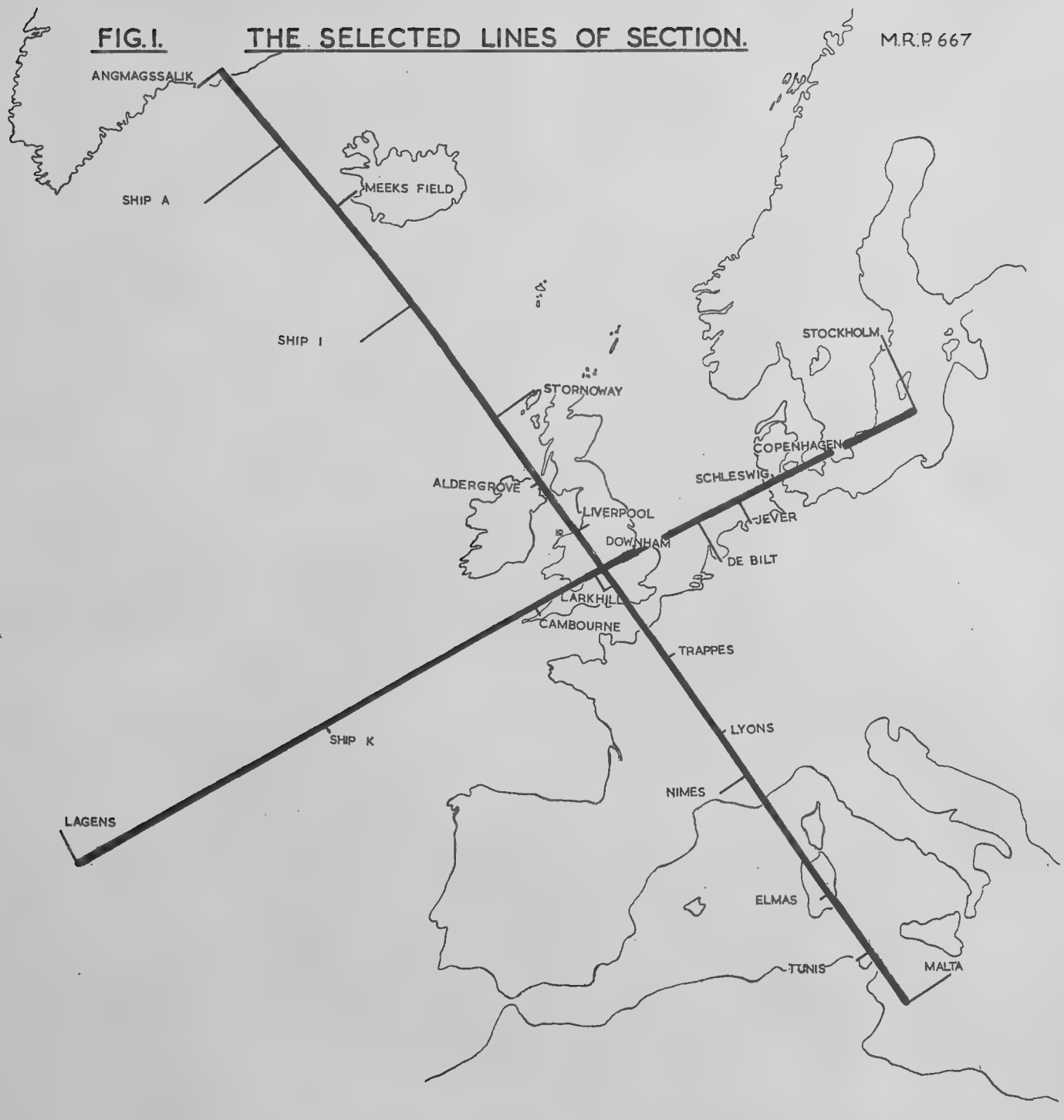


FIG.I. THE SELECTED LINES OF SECTION.
M.R.P. 667
ANGMAGSSALIK
SHIP A
MEEKS FIELD
SHIP I
STORNOWAY
STOCKHOLM
COPENHAGEN
SCHLESWIG
ALDERGROVE
LIVERPOOL
JEVER
DOWNHAM
DE BILT
LARKHILL
CAMBOURNE
TRAPPES
LYONS
SHIP K
NIMES
LAGENS
ELMAS
TUNIS
MALTA

FIG. 2 CROSS-SECTION FROM AZORES – BALTIC 1400Z. 4th SEPTEMBER 1950. M.R.P. 667

FIG.3

CROSS-SECTION FROM GREENLAND – MALTA 1400Z. 30th SEPTEMBER 1950.

M.R.P. 667

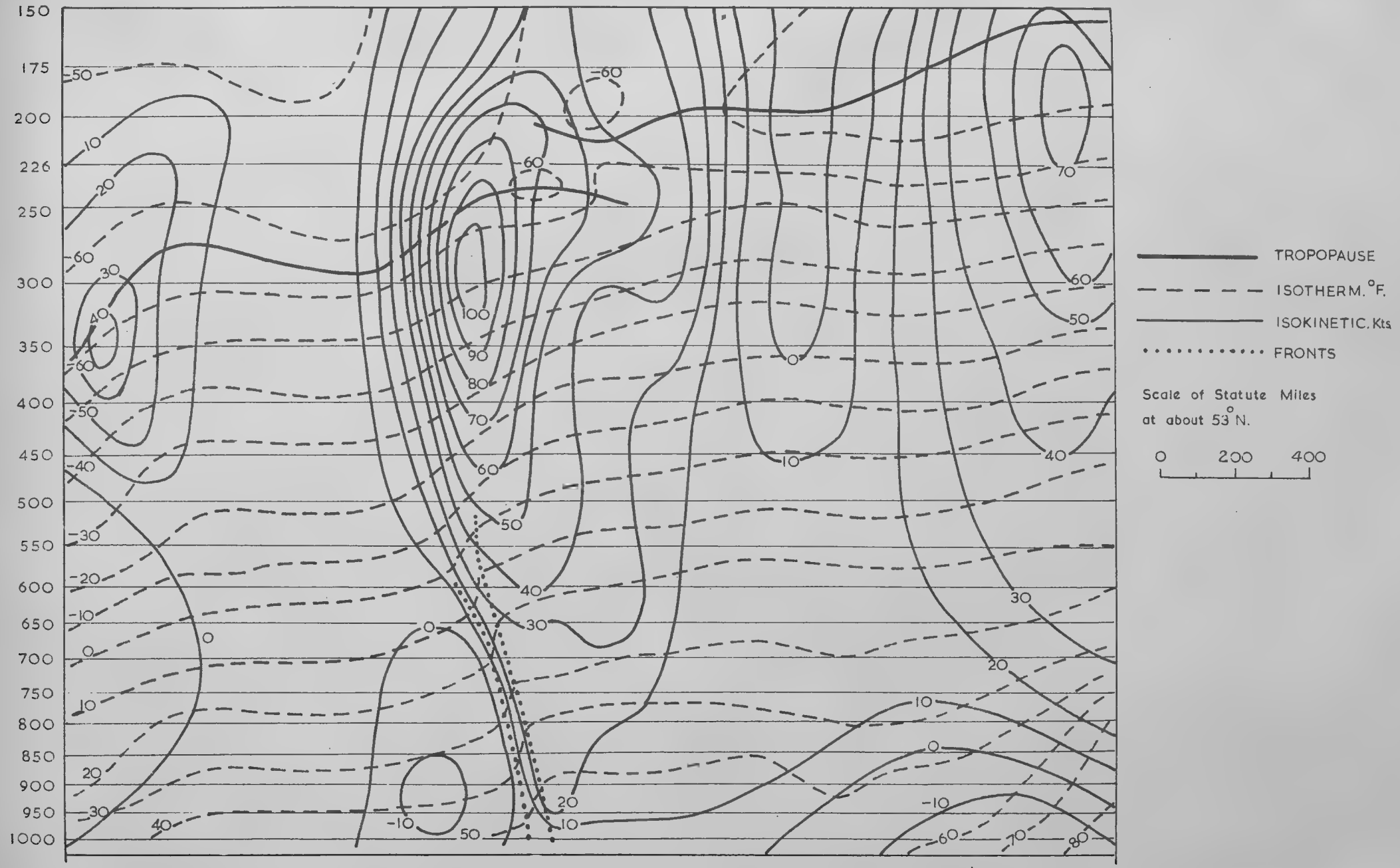


FIG. 4.

MRP 667

POSITIONS AND MAGNITUDES (AT 1400 hrs.) OF CENTRES OF MAXIMUM VELOCITY IN AIRSTREAMS CROSSING THE SECTION ANGMAGSSALIK-MALTA IN SEPTEMBER 1950

THE FIGURES GIVE THE VELOCITY AT THE CENTRES OF THE MAXIMA IN TENS OF KNOTS.

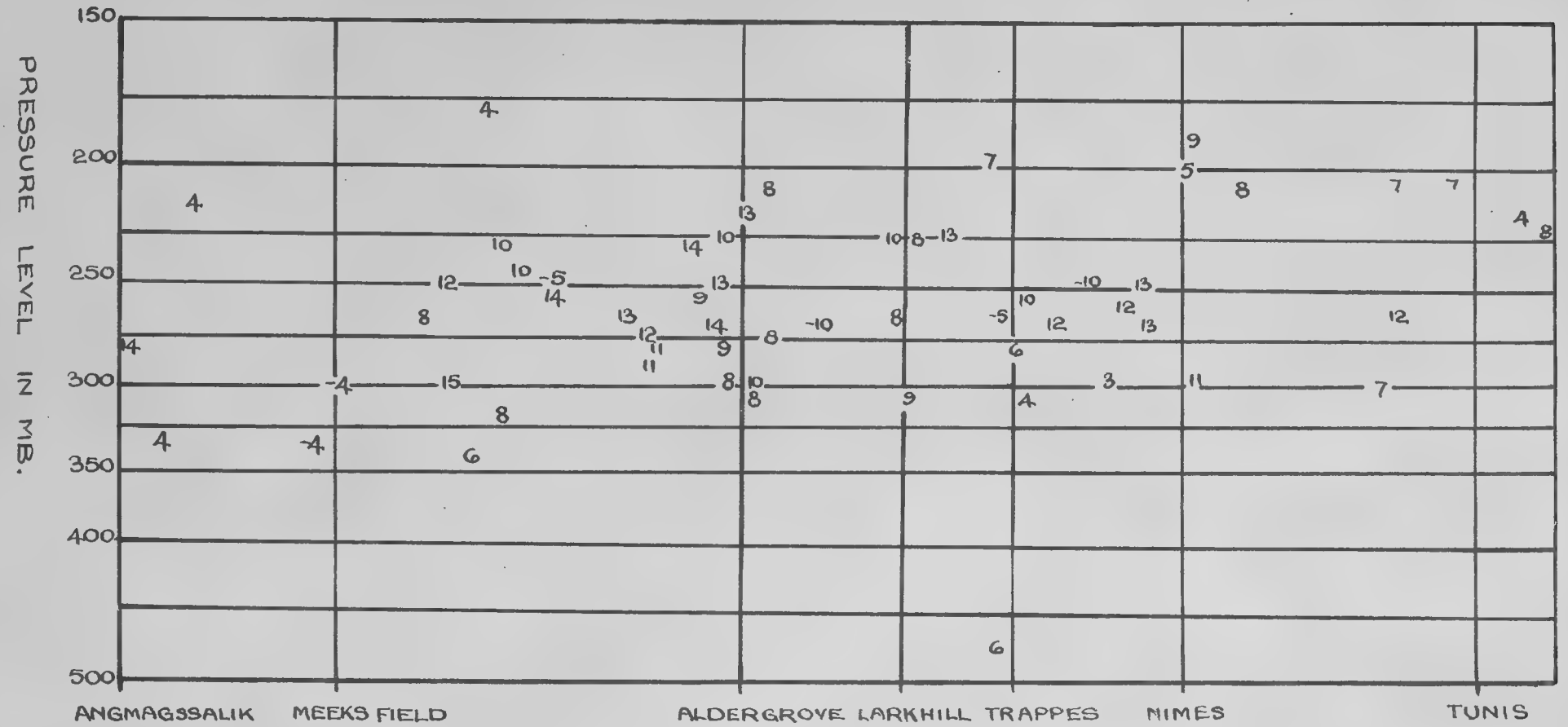
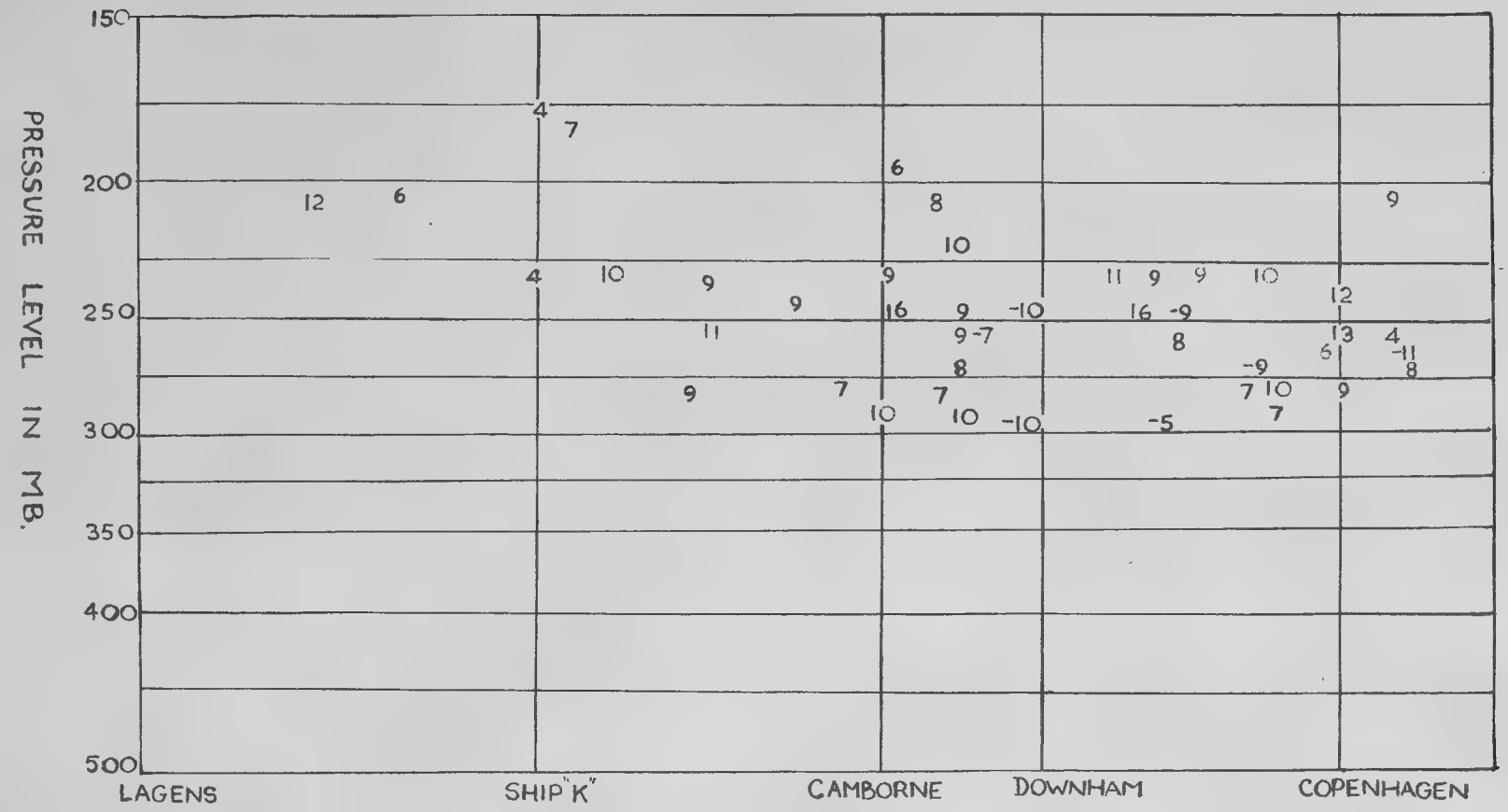


FIG.5. M.R.P. 667

POSITIONS AND MAGNITUDES (AT 1400hrs) OF CENTRES OF MAXIMUM VELOCITY IN AIRSTREAMS CROSSING THE SECTION LAGENS COPENHAGEN IN SEPTEMBER 1950.

THE FIGURES GIVE THE VELOCITY AT THE CENTRES OF THE MAXIMA IN TENS OF KTS.



FREQUENCY DIAGRAMS OF THE WIND MAXIMA FOR SEPTEMBER 1950.

FIG.7.

SECTION LAGENS-STOCKHOLM.

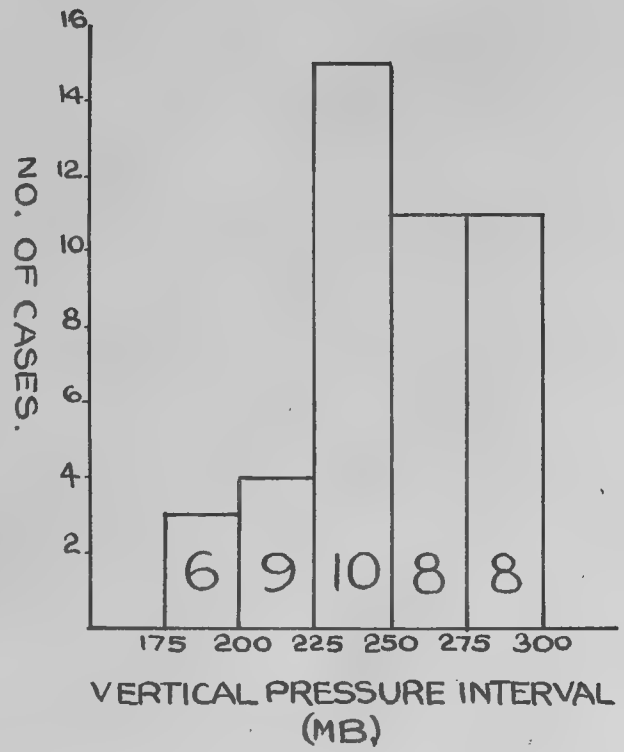
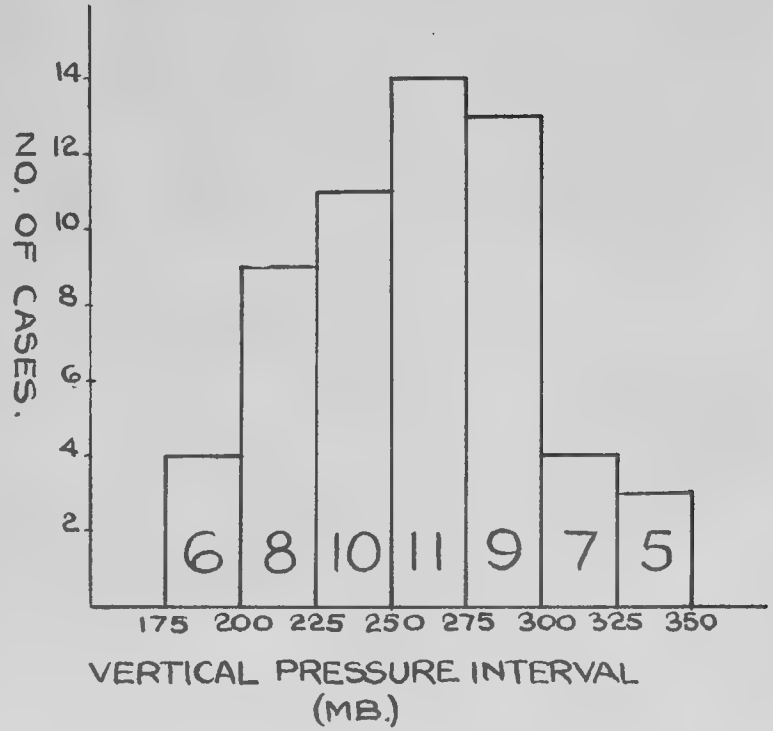


FIG.6.

SECTION ANGMAGSSALIK - MALTA.

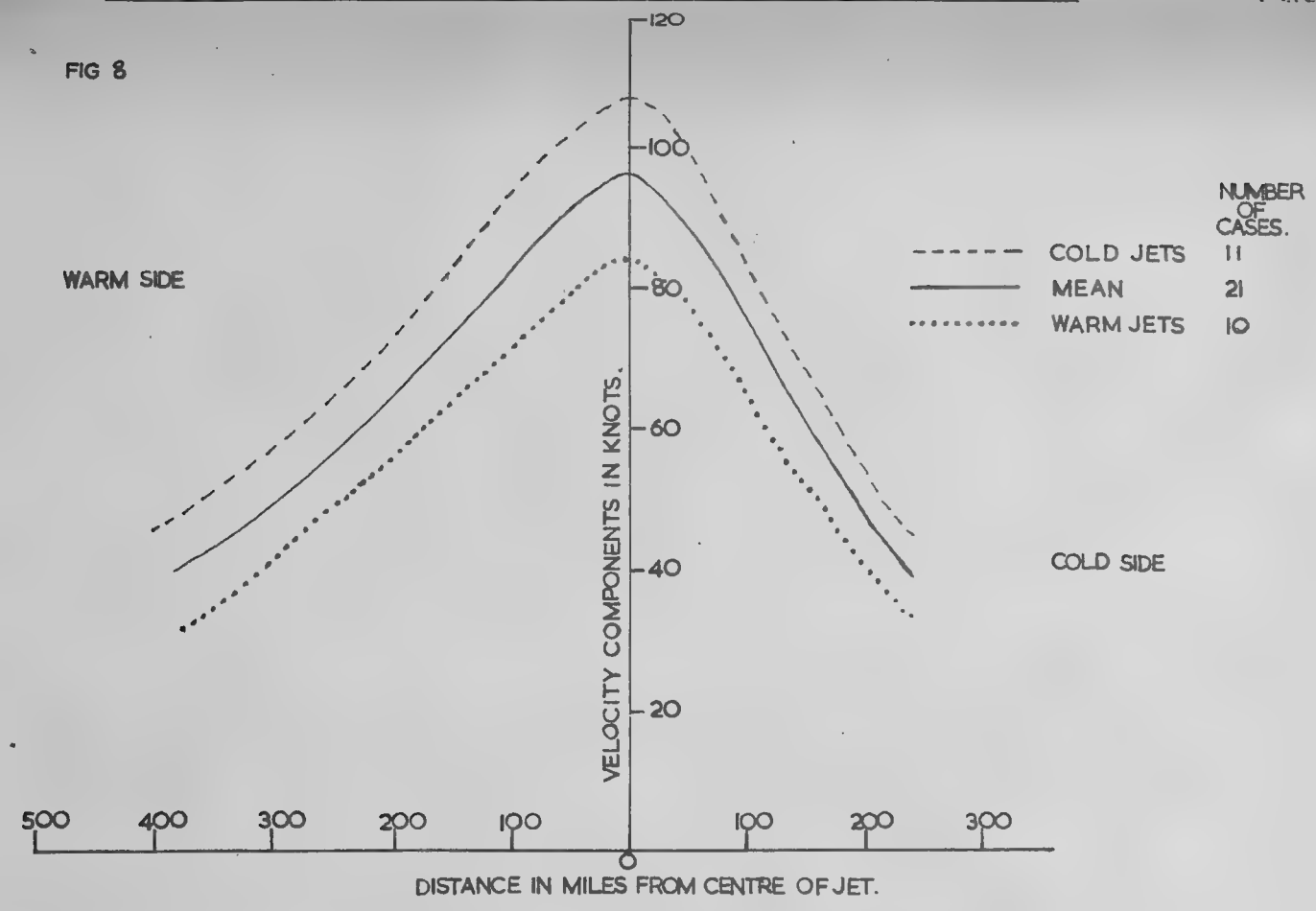


N.B. FIGURES INSIDE THE HISTOGRAM GIVE THE MEAN VALUE OF THE WIND MAXIMA (TENS OF KNOTS) OCCURRING IN THE GIVEN PRESSURE INTERVAL.

MEAN VELOCITY PROFILES AT THE LEVELS OF THE CENTRES OF JET-STREAMS.

M.R.P. 667

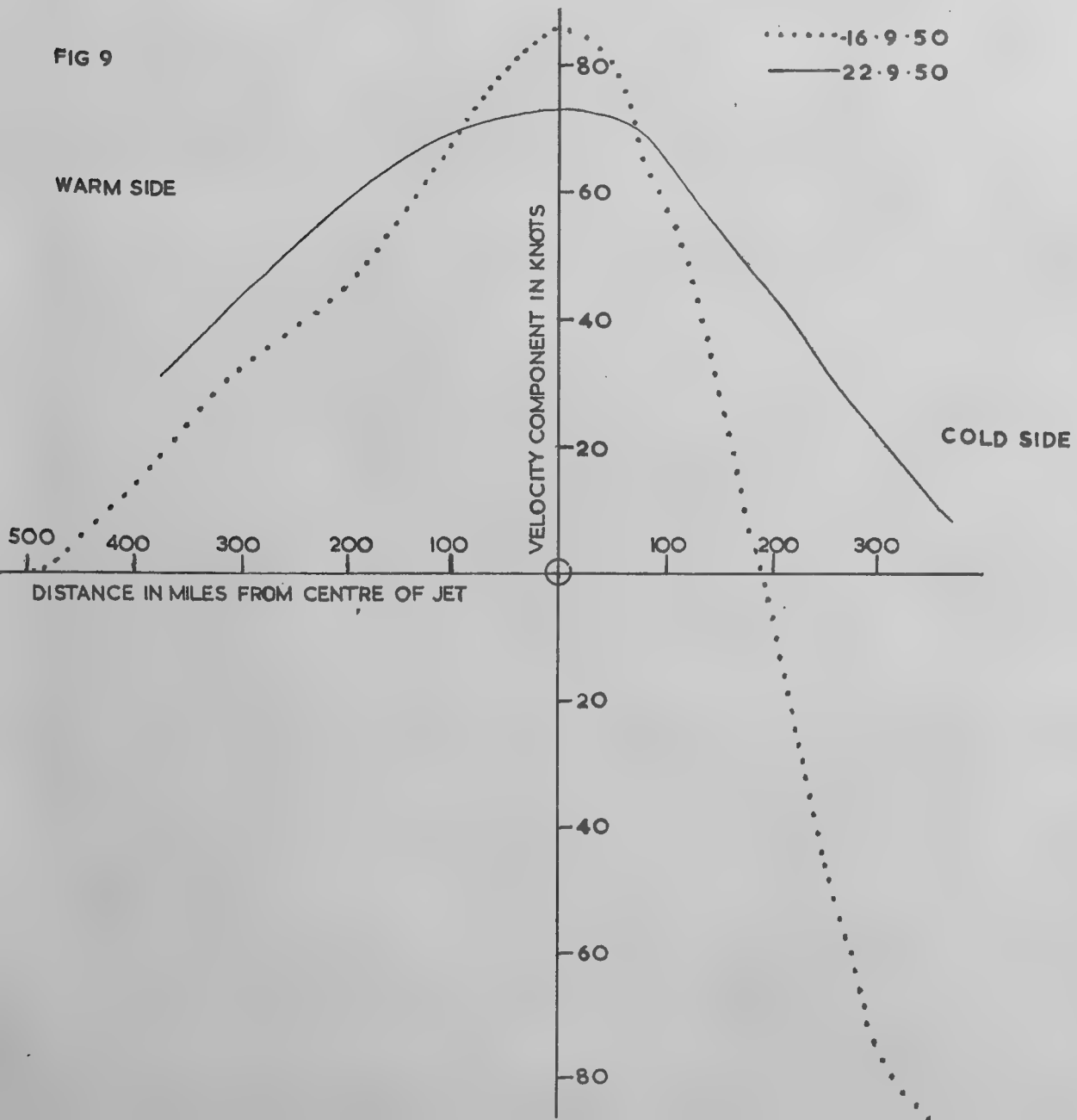
FIG 8



VELOCITY PROFILES AT THE LEVELS OF THE CENTRES OF INDIVIDUAL JET-STREAMS

MRP 667

FIG 9



MEAN 300mb WIND PROFILES FOR SELECTED WIND MAXIMA OF SEPT. 1950. FIGS. 10–12. M.R.P. 667

FIG. 10.- COLD TYPE.

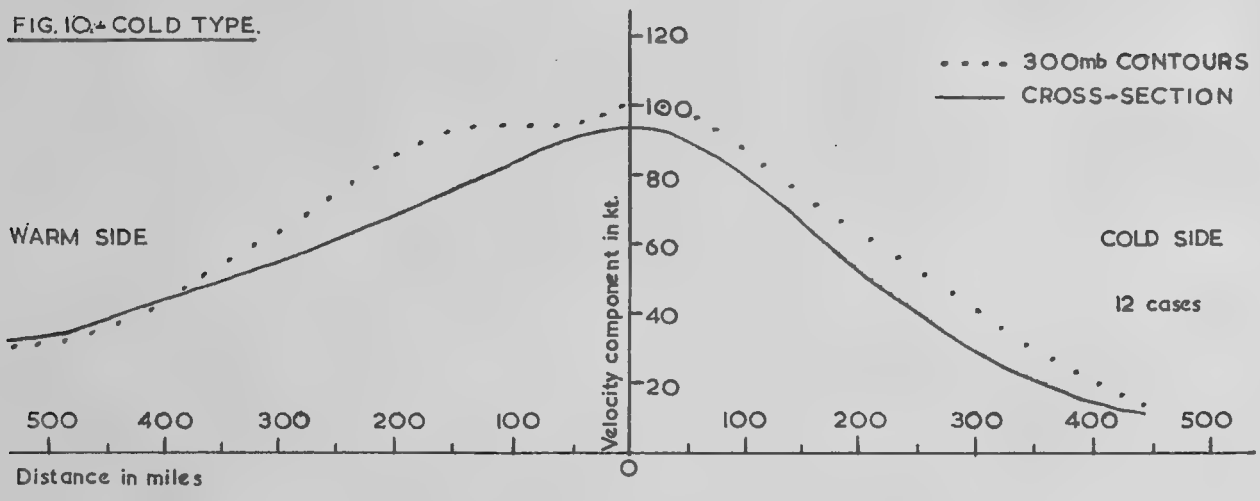


FIG. 11- WARM TYPE.

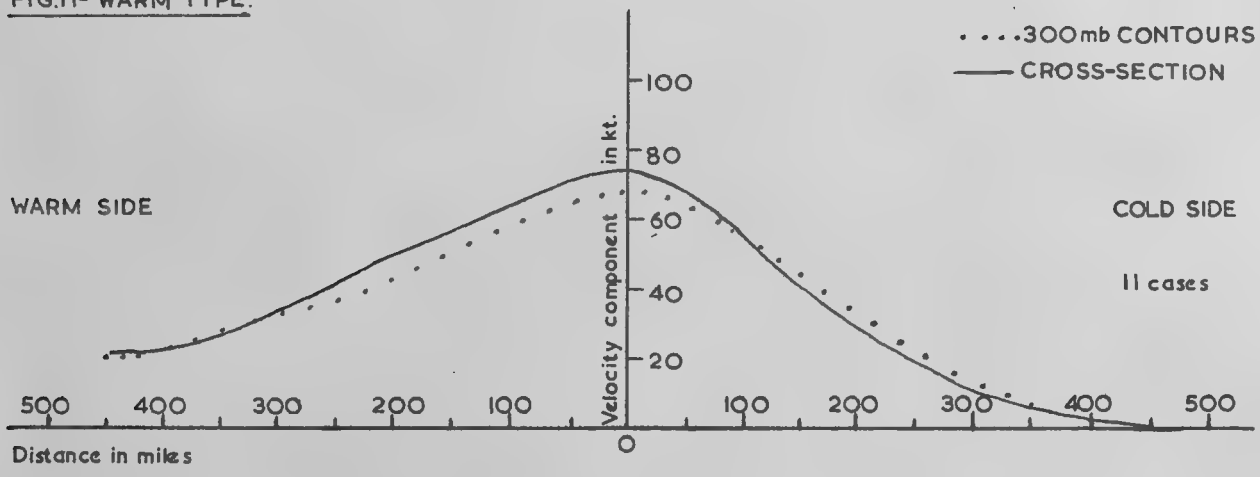


FIG 12- OVERALL MEAN

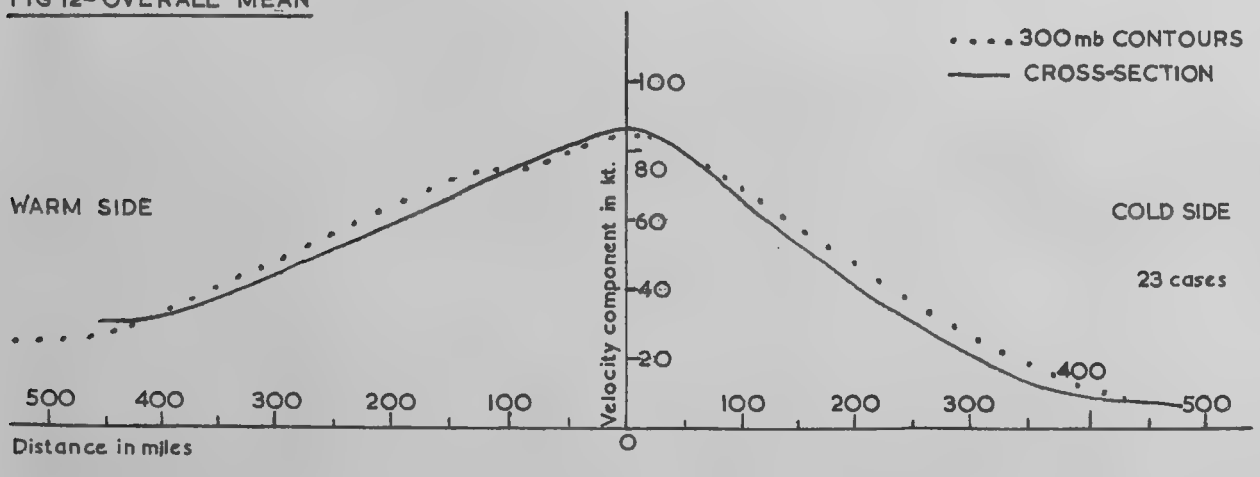


FIG. 13

23 SEPT. 1950 - WARM TYPE

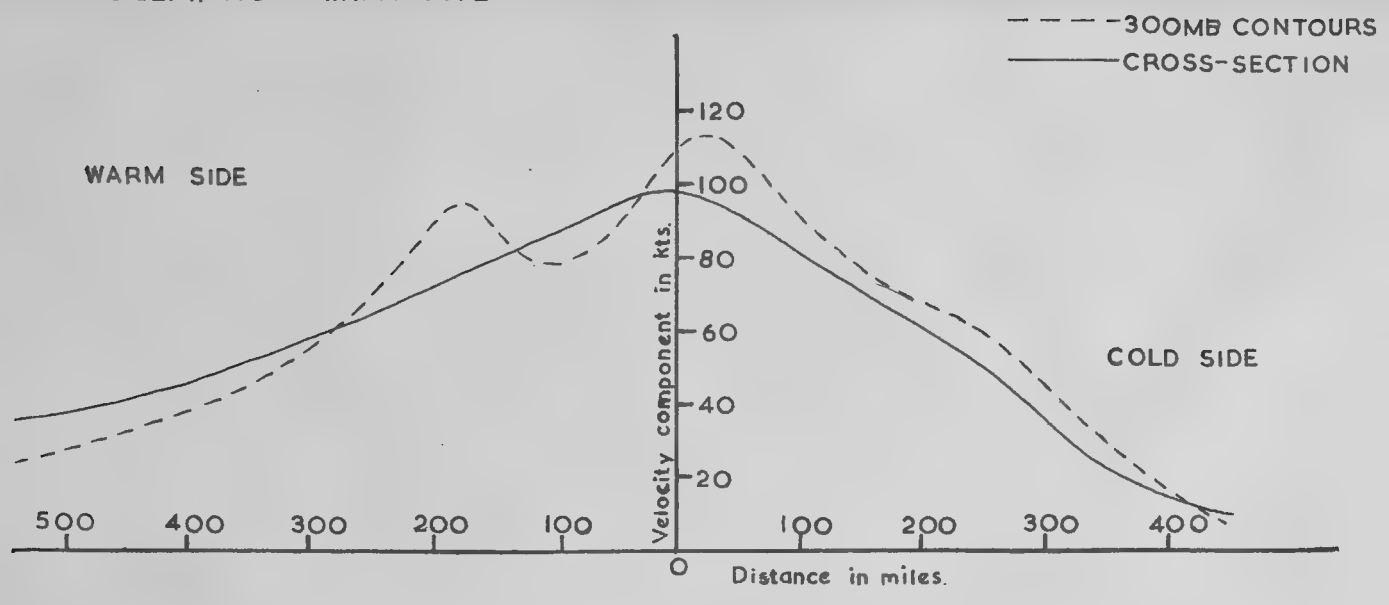


FIG. 14

28 SEPT 1950 - COLD TYPE

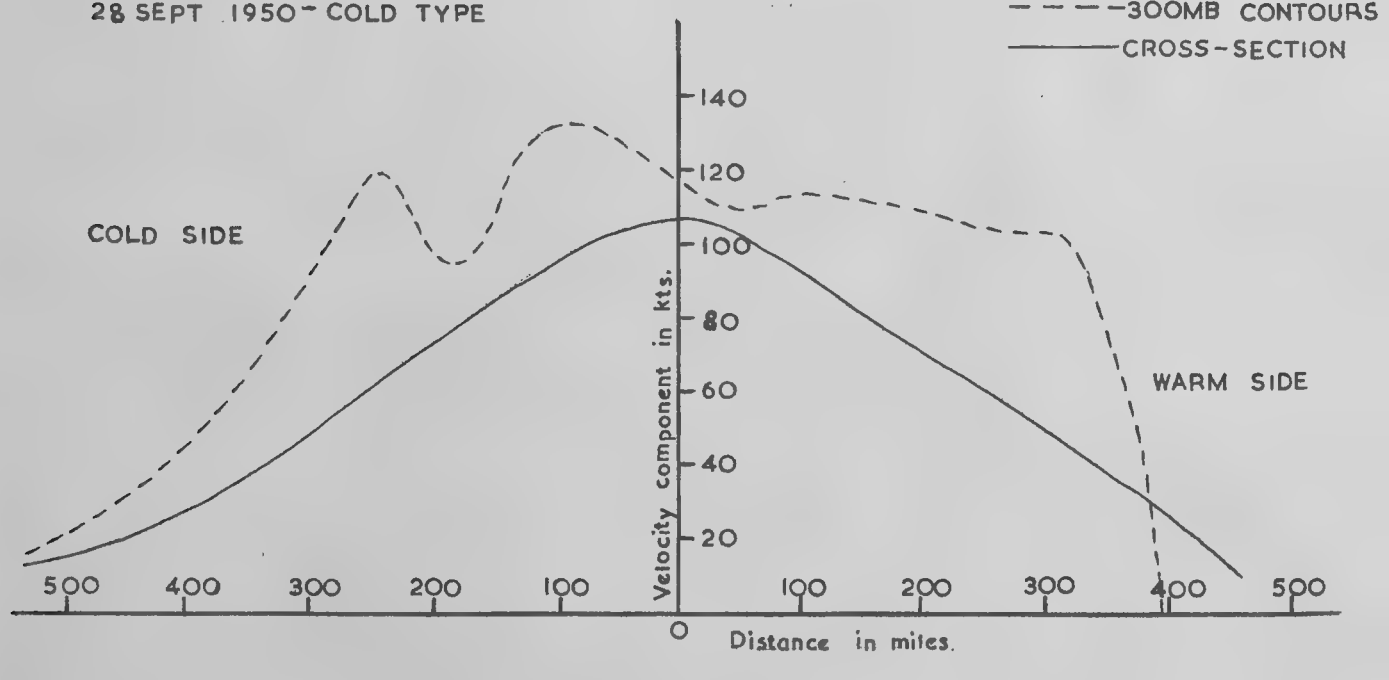
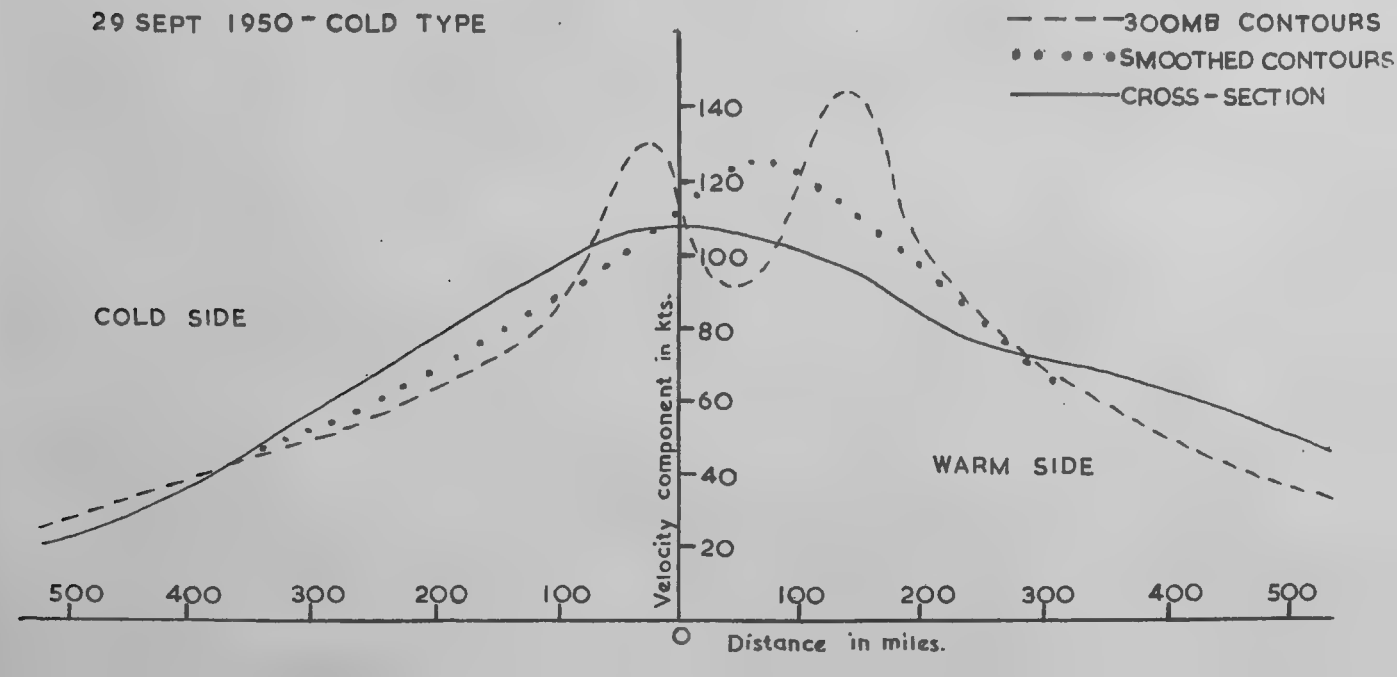
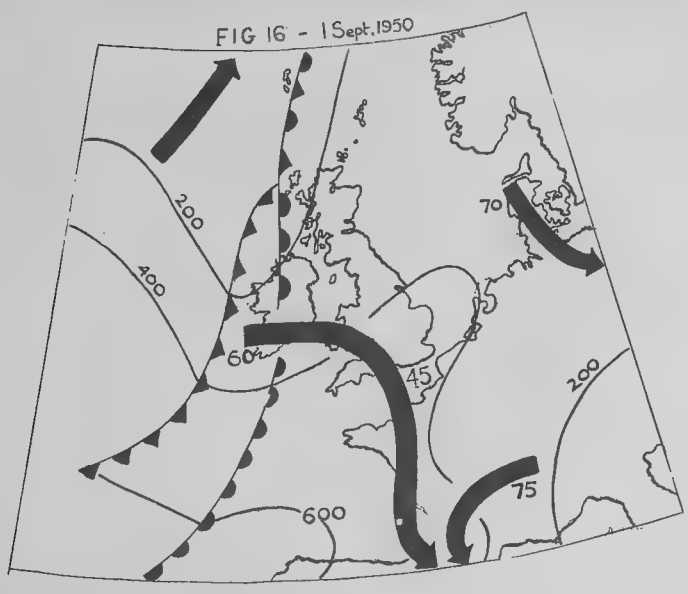


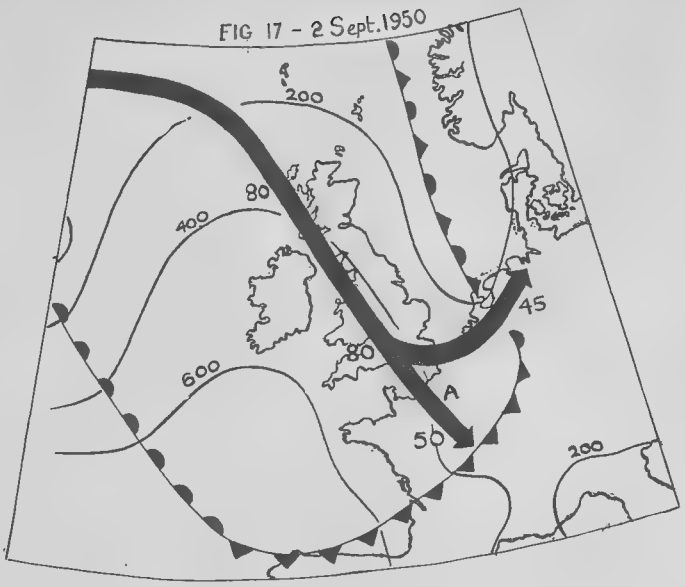
FIG. 15

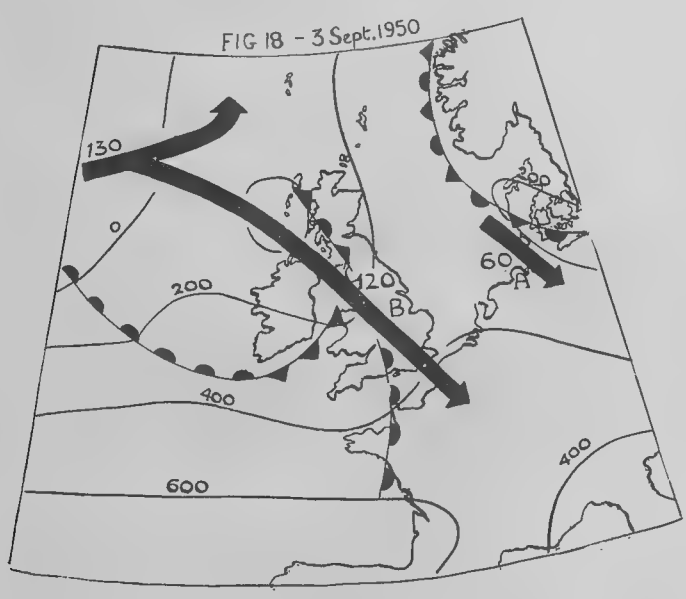
29 SEPT 1950 - COLD TYPE

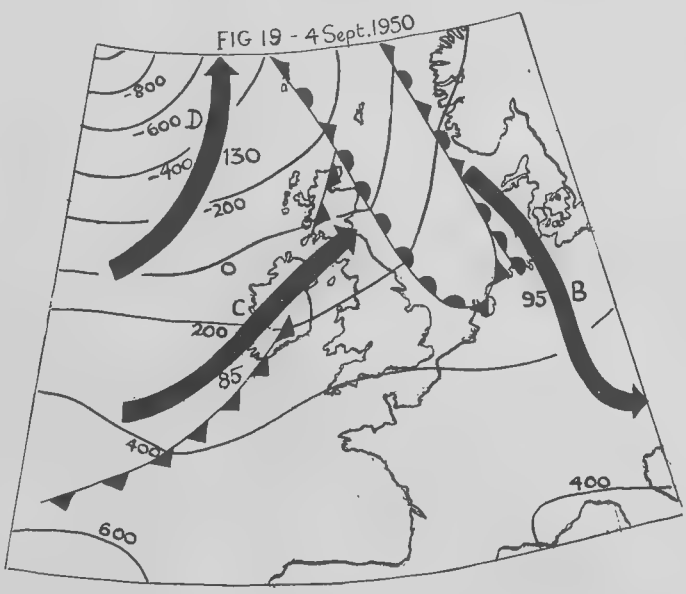


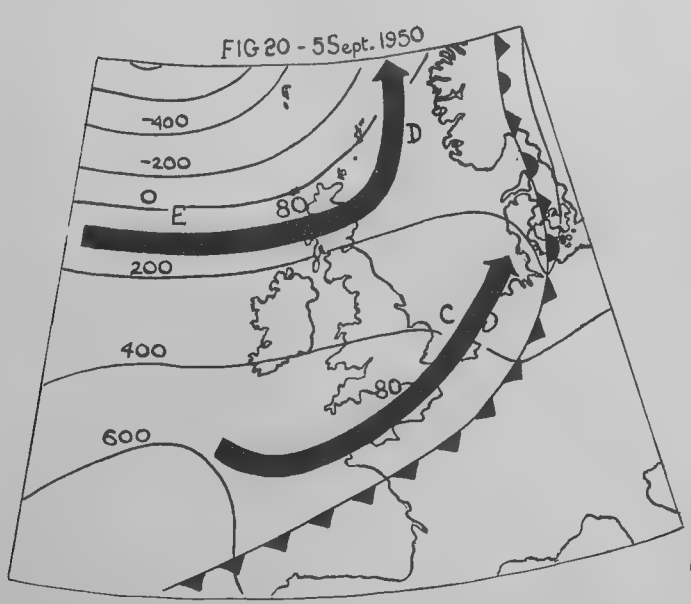
Association between Jets and Surface Fronts. Figs. 16 - 21.

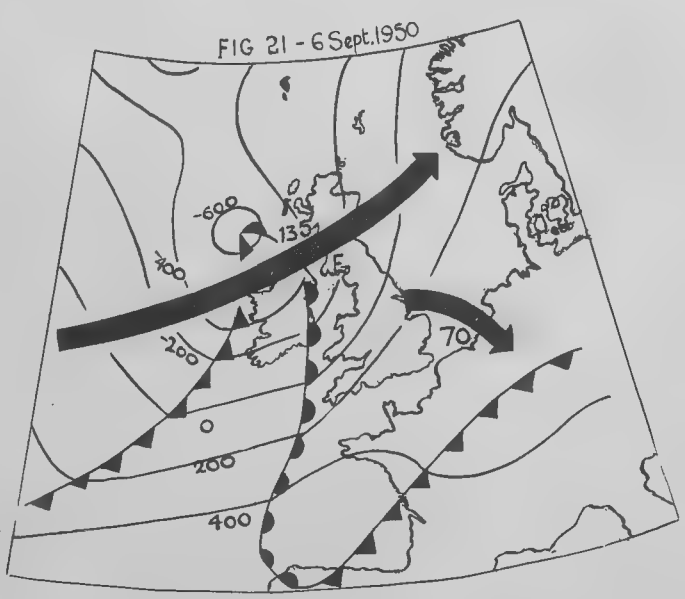












N.B. Fronts indicated in conventional manner.

Jet as ➡ with 300mb. speed in kt. at selected points.

1000mb. contours (ft.) as ——

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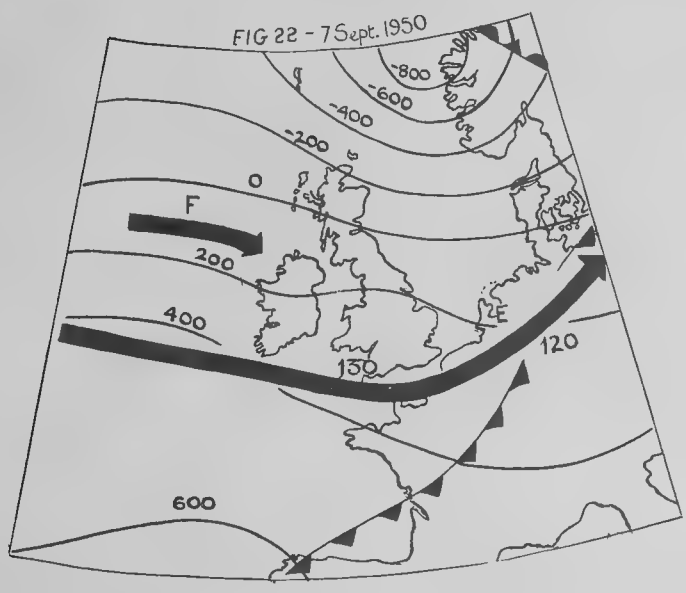
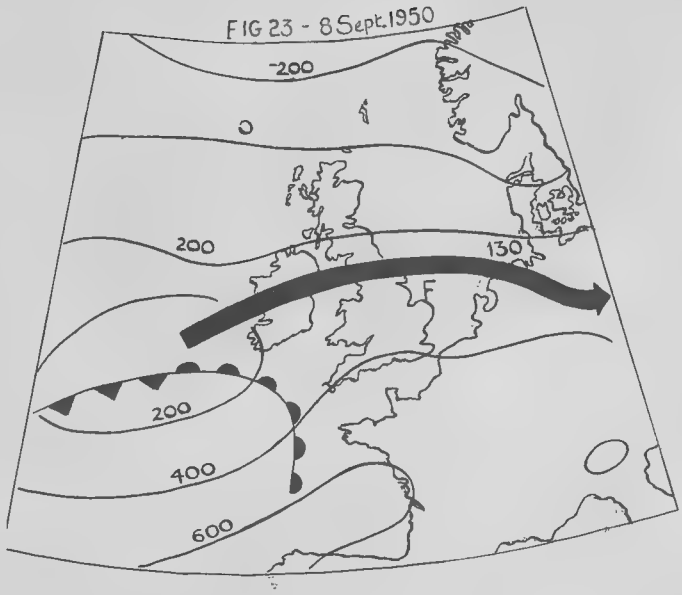
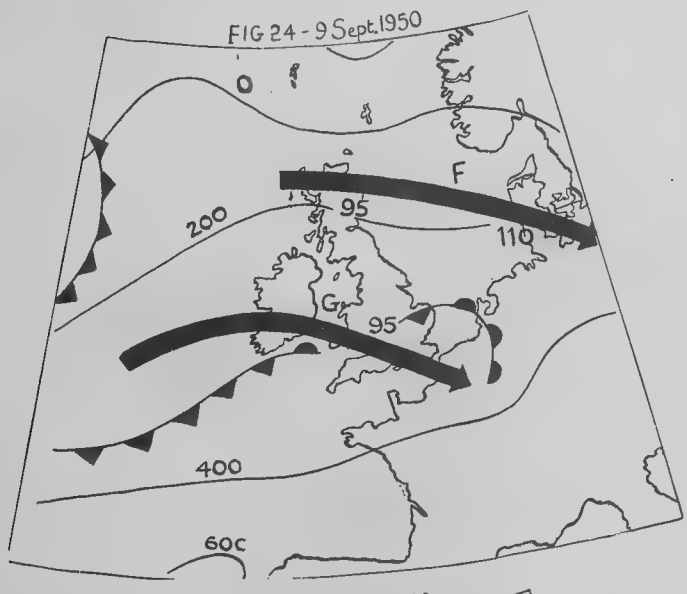
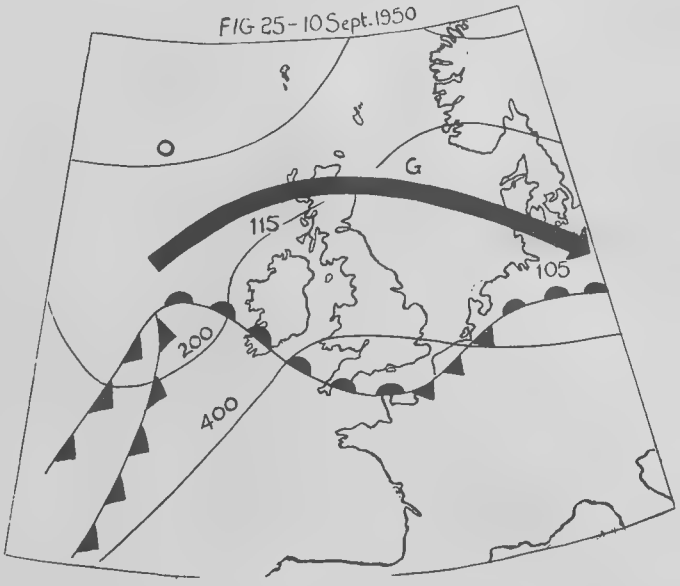
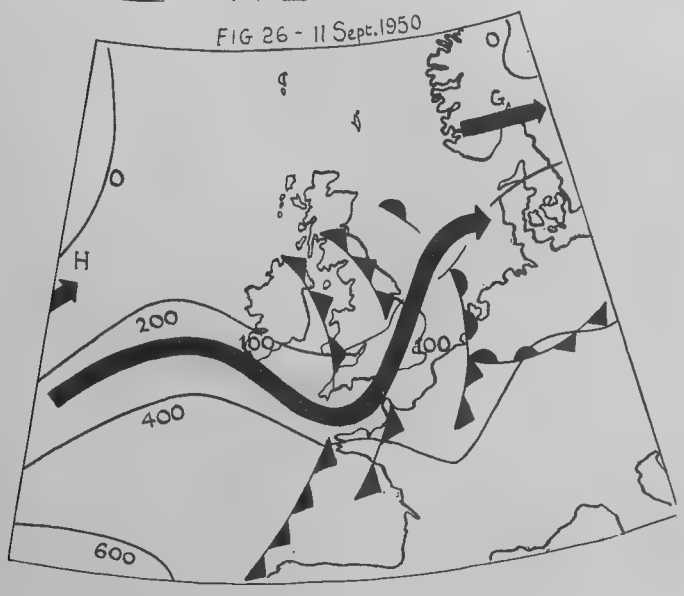
Association between Jets and Surface Fronts. Figs 22-27.

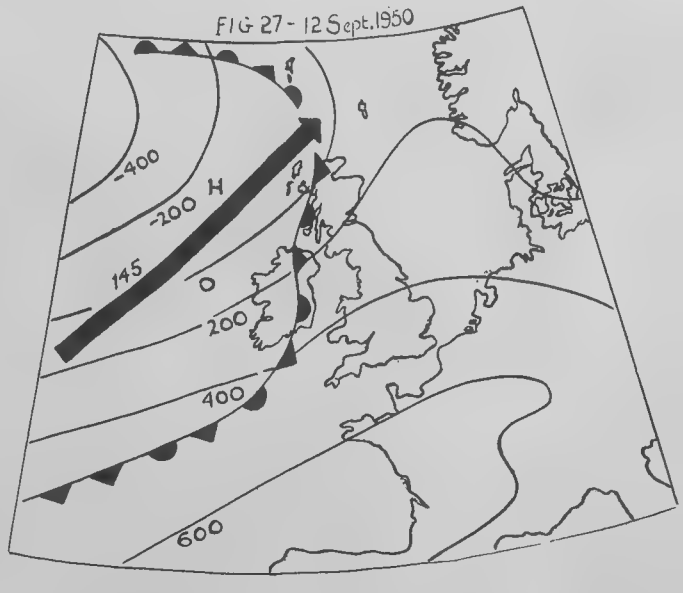












NB. Fronts indicated in conventional manner.

Jet as ➡ with 300 mb. speed in kt at selected points.

1000 mb contours (ft.) as ——.

MRP 667

Association between Jets and Surface Fronts. Figs. 28 - 33.

FIG 28 - 13 Sept. 1950

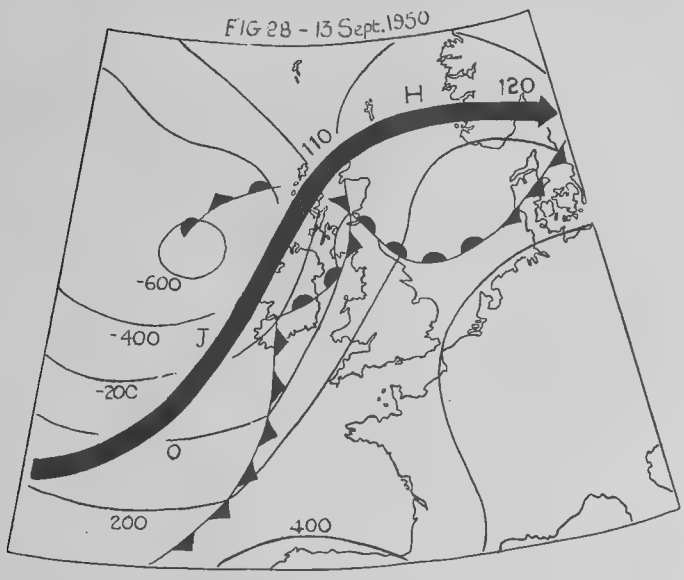


FIG 29 - 14 Sept. 1950

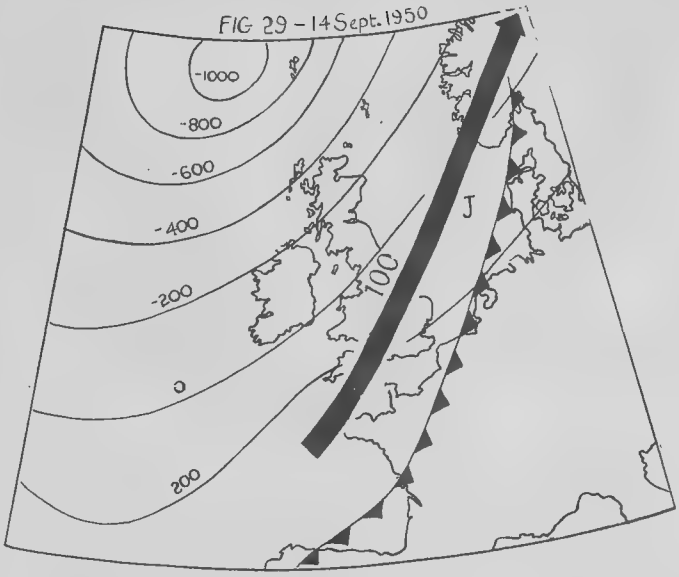


FIG 30 - 15 Sept. 1950

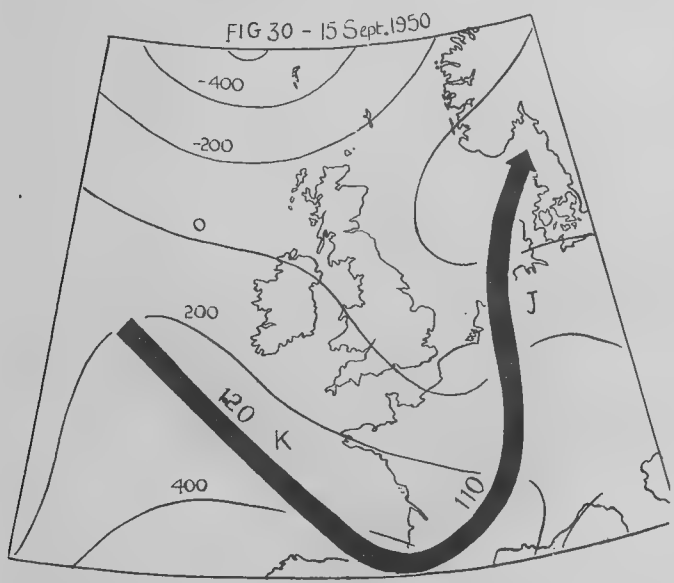


FIG 31 - 16 Sept. 1950

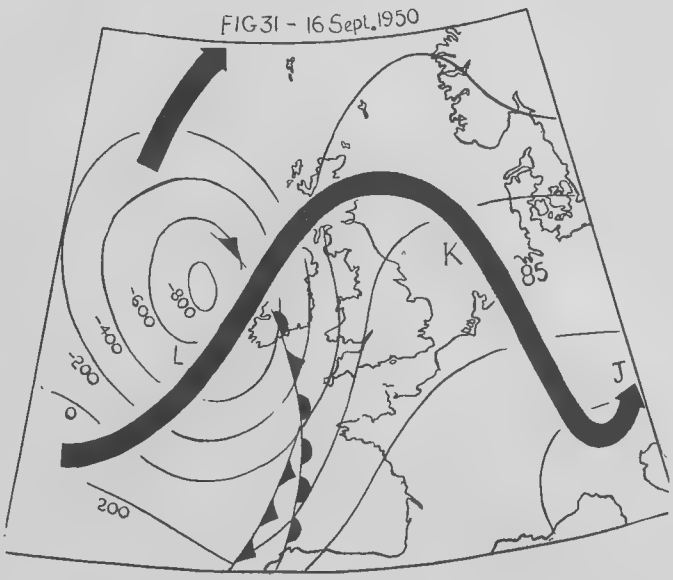


FIG 32 - 17 Sept. 1950

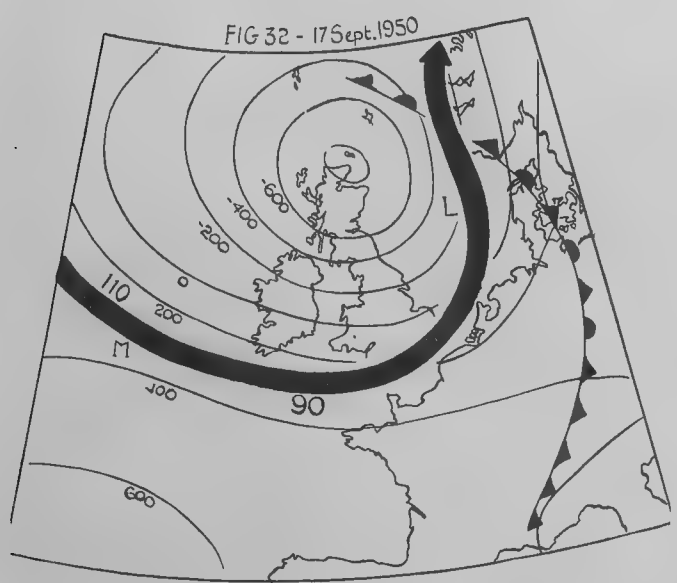
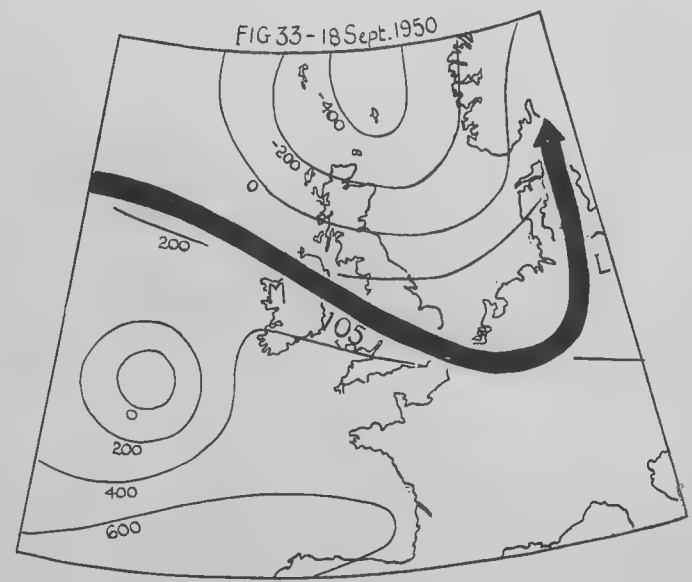


FIG 33 - 18 Sept. 1950



NB. Fronts indicated in conventional manner.

Jet as ➡ with 300 mb speed in kt at selected points.

1000 mb contours (ft.) as ——.

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Association between Jets and Surface Fronts. Fig 34-39.

FIG 34 - 19 Sept. 1950

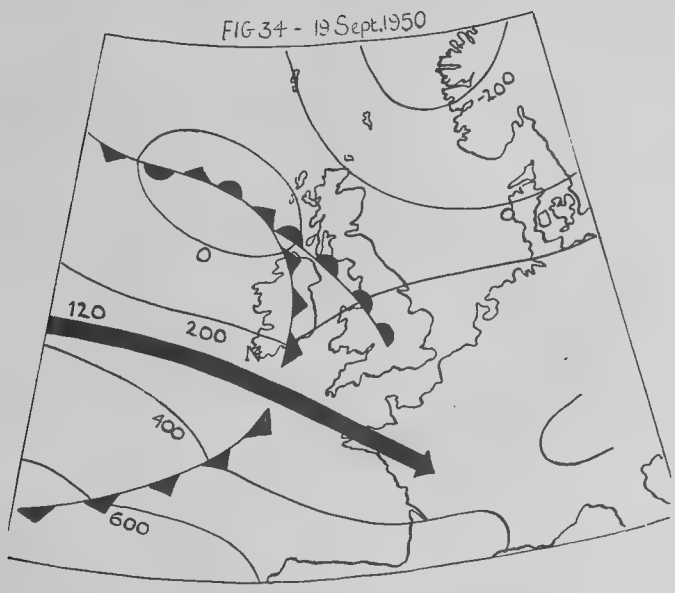


FIG 35 - 20 Sept. 1950

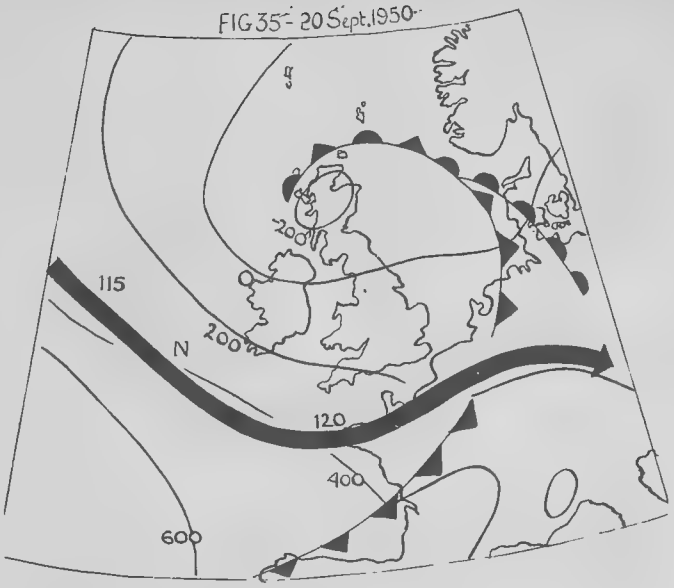


FIG 36 - 21 Sept. 1950

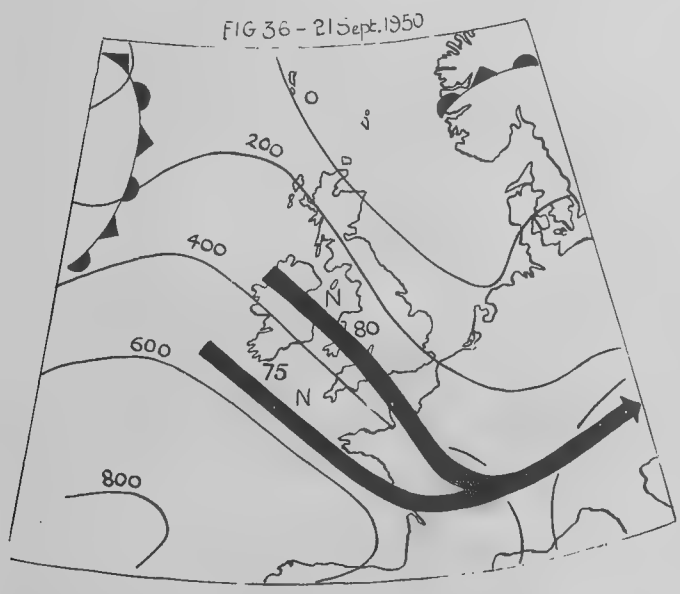


FIG 37 - 22 Sept. 1950

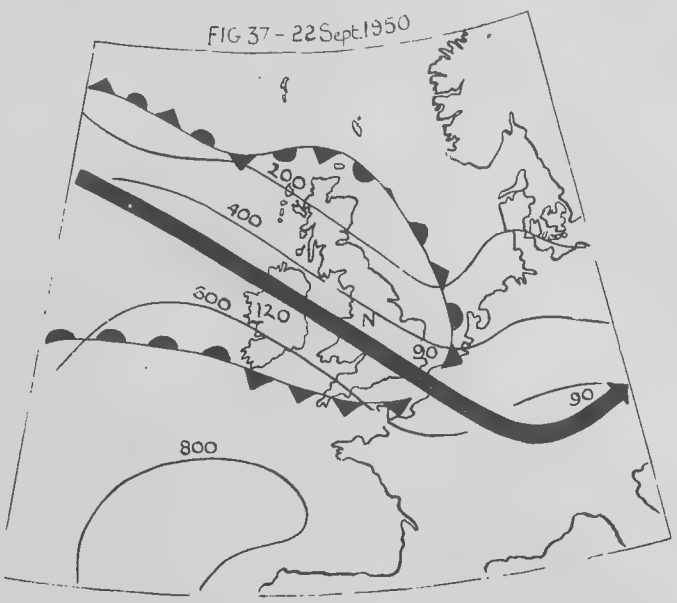


FIG 38 - 23 Sept. 1950

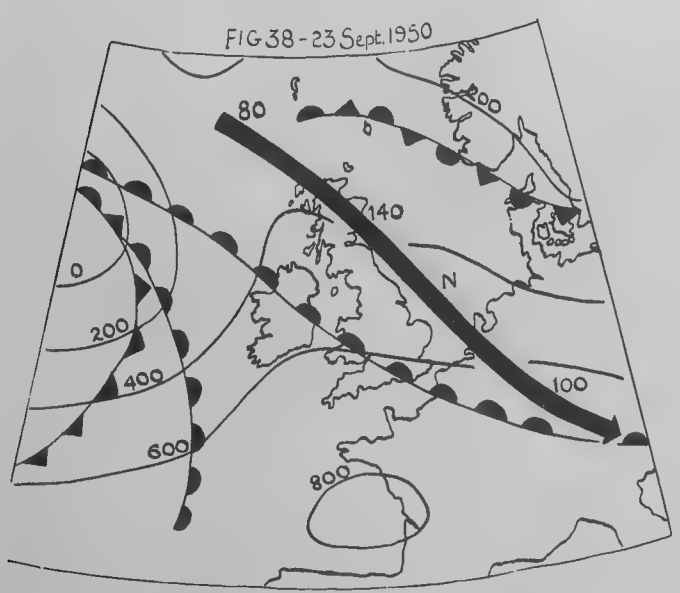
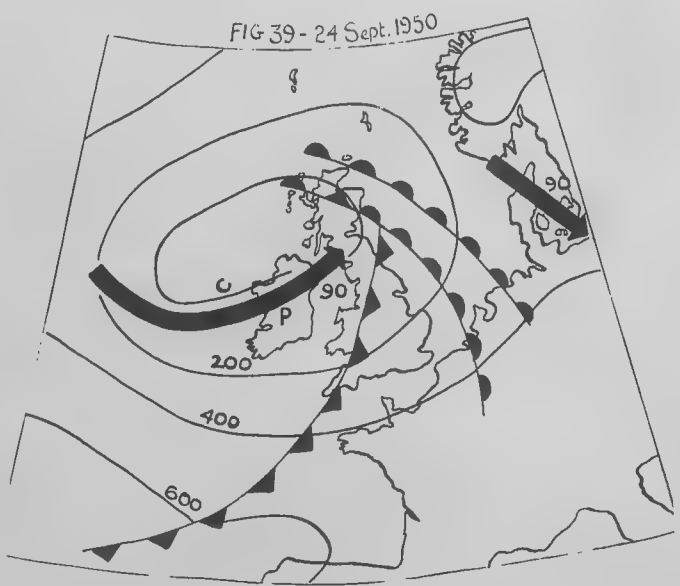


FIG 39 - 24 Sept. 1950



NB. Fronts indicated in conventional manner.

Jet as ➡ with 300 mb. speed in kt at selected points.

1000 mb. contours (ft.) as —·

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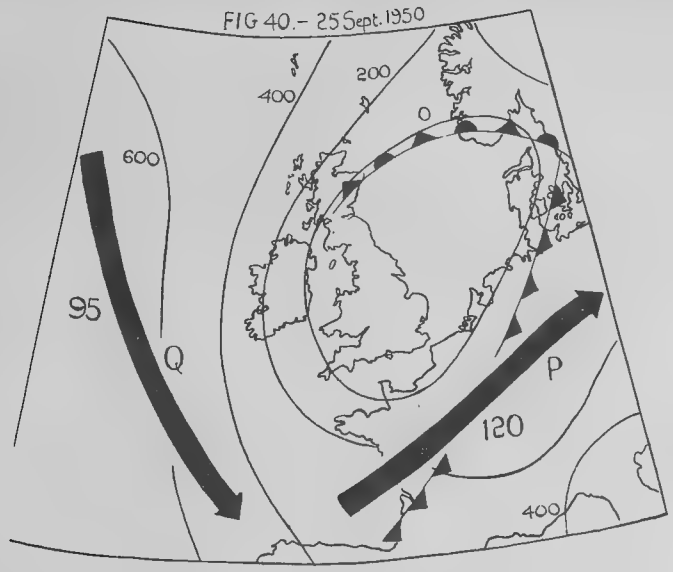
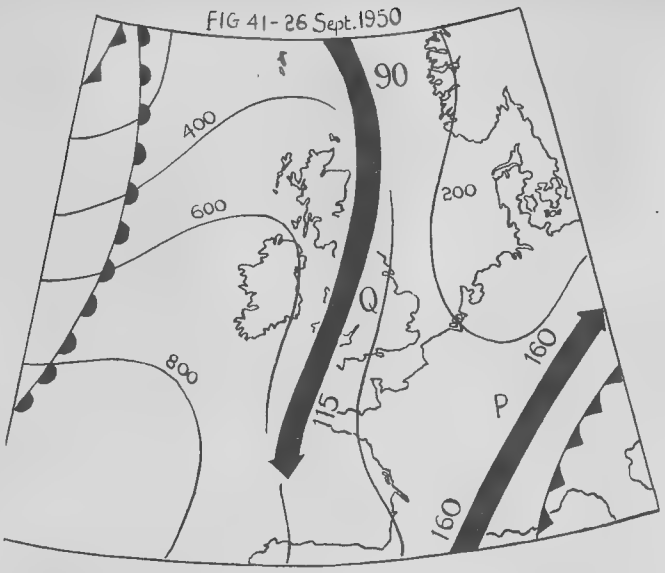
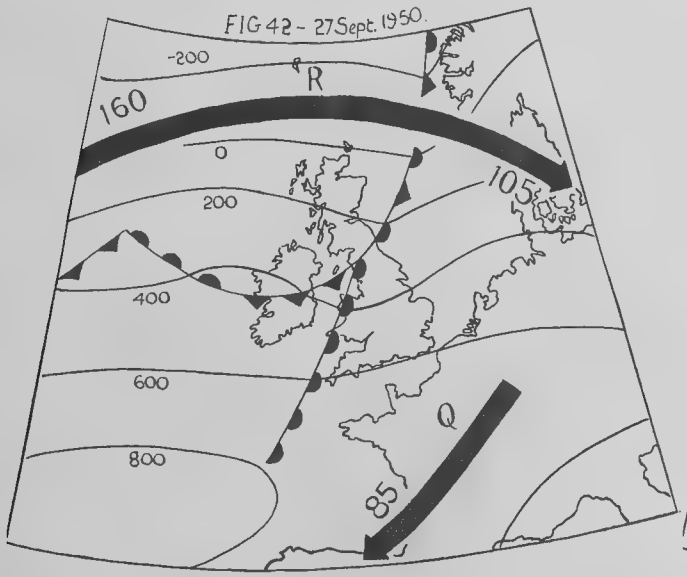
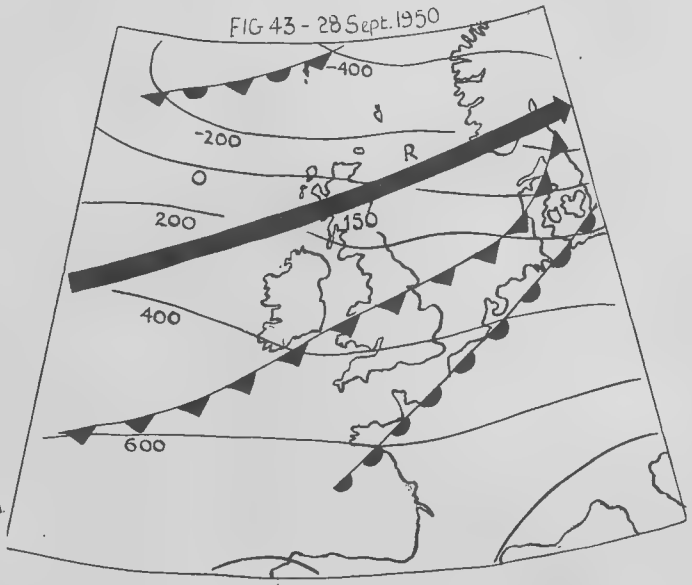
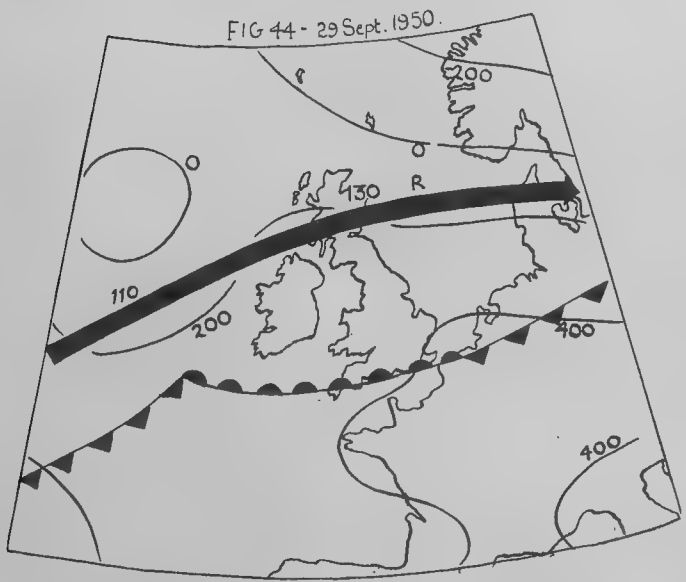
Association between Jets and Surface Fronts. Figs. 40-45

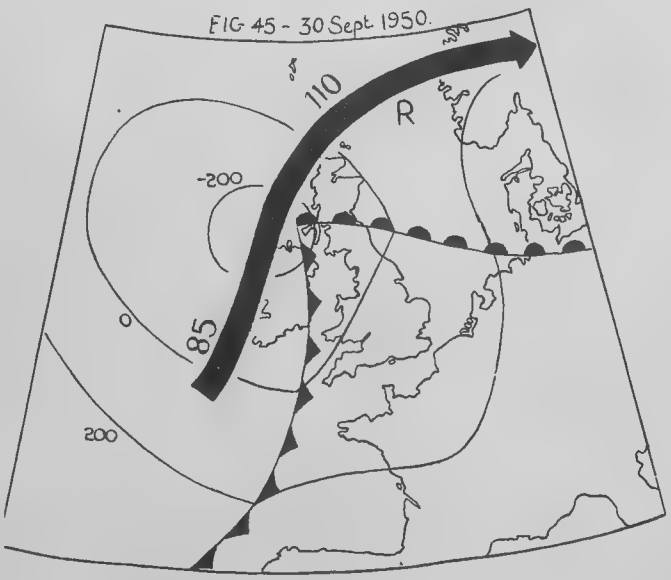












NB. Fronts indicated in conventional manner.

Jet as ➡ with 300mb speed in kt. at selected points.

1000 mb contours (ft.) as ——.

MRP 667

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M.R.P. 668
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15th August, 1951.

AIR MINISTRY

METEOROLOGICAL RESEARCH COMMITTEE

Statistics of Upper Air Temperature

by C.S. Durst (Meteorological Office)

1. Introduction.

Since information regarding the variations of temperature are required by aircraft operators, particularly for the use of jet propelled aircraft, some statistics have been compiled of the variation of temperature with time place and height. They are given in terms of the standard deviations and correlation coefficients but it must always be remembered that the distribution of temperature in the upper atmosphere is not normal. This is particularly so in the region where the tropopause is most frequently encountered. To emphasise this two diagrams are reproduced from M.R.P.511, Fig.1 showing the frequency distribution of temperature at Larkhill and Fig.2 the variation with height of the standard deviation at that place. It is clear that the increase in the standard deviation at 200 mb. (Fig.2) is due in no small measure to the two-humped type of frequency shown in Fig.1. The consequential effects of this abnormality will be apparent in what follows.

2. Variation of Temperature with Time.

In Table I are set out the standard deviations of temperature (from monthly means) and the correlation coefficients with respect to time of the observations made at Larkhill in 1945.

The correlation coefficients for the year as a whole are plotted in Fig.3.

The most notable feature is the smallness of the correlation coefficients at 300 mb. - where the standard deviation of temperature is least - due no doubt to the effect of the passage of the tropopause through the pressure surface of 300 mb.

It has been pointed out to me by A.F. Crossley that if account is taken of the observational error of the temperatures it tends to raise the values of the correlation coefficients, particularly where the standard deviations of the observations are small. The true correlation coefficient is

$$R'(1 + \bar{\epsilon}^2/\sigma^2)$$

where R' is the uncorrected coefficient $\bar{\epsilon}^2$ the mean square error of the observations and σ the standard deviation. Sheppard (M.O. SDTM.75) gives as probable errors of radio-sonde temperatures $\pm$ 0.76°F. at 750 mb, $\pm$ 1.16°F. at 500 mb. and $\pm$ 1.60°F. at 300 mb. which correspond to standard errors of 1.1°F, 1.7°F. and 2.4°F. respectively. If we introduce a correction on this account to the annual values of R_6 given in Table I we get 0.75 at 300 mb., 0.89 at 500 mb. and 0.88 at 700 mb. So that the allowance for observational errors does not make a very great difference.

/Table I

Table I.

Relation between temperature (°F) at Larkhill at a given hour and the temperature 6, 12, 18 and 24 hours later (period 1945).

Height (mb.)	feet	Standard Deviation °F	°C	6 hours R_6	12 hours R_{12}	18 hours R_{18}	24 hours R_{24}
Spring							
200	39000	10.5	5.8	.83	.82	.73	.65
300	30000	4.3	2.4	.63	.50	.42	.42
500	18000	7.9	4.4	.91	.84	.63	.57
700	10000	7.9	4.4	.89	.83	.79	.71
Summer							
200	39000	9.5	5.3	.90	.67	.57	.45
300	30000	5.3	2.9	.67	.54	.32	.31
500	18000	6.0	3.3	.79	.61	.43	.27
700	10000	5.8	3.2	.80	.70	.58	.50
Autumn							
200	39000	8.4	4.7	.83	.72	.62	.56
300	30000	5.8	3.2	.60	.47	.37	.32
500	18000	6.0	3.3	.78	.63	.51	.43
700	10000	5.6	3.1	.88	.78	.70	.61
Winter							
200	39000	10.6	5.9	.78	.63	.49	.43
300	30000	6.0	3.3	.53	.42	.31	.25
500	18000	8.6	4.8	.89	.73	.58	.46
700	10000	7.9	4.4	.89	.76	.63	.52
Year							
200	39000	9.8	5.4	.83	.71	.61	.52
300	30000	5.3	2.9	.62	.49	.36	.32
500	18000	7.3	4.1	.85	.72	.60	.47
700	10000	6.9	3.8	.86	.78	.69	.60

In Tables II and III there are given similar data for Habbaniya, Bahrein and Nairobi. Features which seem to be noteworthy are that at both Bahrein and Nairobi there is a decided decrease in the correlation coefficients in summer (and probably autumn). The correlation at Nairobi even in winter is considerably less than at Bahrein. These features are no doubt associated with the situation at Bahrein in relation to the belt of strong westerly winds which are so marked in winter and the comparatively light and fitful winds which prevail over Nairobi. When (and where) the light winds prevail the temperature changes due to advection are small and uncertain but when in winter the strong westerlies are established a change of wind direction (which may persist for 24 hours or more) brings to the station a marked change of temperature.

Table II

Relation between temperature (°F) at Habbaniya at a given hour and the temperature 12 and 24 hours later.

Height (mb)	feet	Standard Deviation °F	No. of Obs.	12 hours R_{12}	No. of Obs.	24 hours R_{24}
		Jan., Feb. 1948, 1949		Dec. 1948.		
200	39000	7.5	33x	0.32	131	0.34
300	30000	6.0	33x	0.69	139	0.50
500	18000	6.1	34x	0.32	151	0.44
700	10000	6.7	34x	0.75	145	0.53

x Feb. 1948 only.

/March,

Table II (Cont'd)

Height (mb.)	feet	Standard Deviation °F	No. of Obs.	12 hours R_{12}	No. of Obs.	24 hours R_{24}
	March, April, May 1948, 1949.					
200	39000	8.6	211	0.69	211	0.53
300	30000	5.8	212	0.47	215	0.40
500	18000	6.6	217	0.66	219	0.55
700	10000	6.5	218	0.84	178	0.76
	June, July, Aug. 1948.					
200	39000	6.1	58	0.64	71	0.42
300	30000	7.4	66	0.80	84	0.57
500	18000	5.4	69	0.71	90	0.38
700	10000	3.5	69	0.67	89	0.43
	Sept., Oct., Nov. 1948					
200	39000	5.9	45	0.53	83	0.57
300	30000	5.6	51	0.52	84	0.73
500	18000	5.1	59	0.78	90	0.70
700	10000	4.6	59	0.89	88	0.62
	Total					
200	39000	7.7	347	0.65	499	0.48
300	30000	6.0	362	0.60	522	0.52
500	18000	5.9	379	0.63	550	0.52
700	10000	5.7	380	0.80	500	0.61

Table III

Relation between temperature (°F) at Bahrein and Nairobi at a given hour and the temperature 24 hours later.

Height (mb.)	feet	Bahrein Standard Deviation Jan. Feb. 1949.	No. of Obs.	24 hours R_{24}	Nairobi Standard Deviation Dec. 1948, Jan. Feb. 1949	No. of Obs.	24 hours R_{24}
200	39000	5.7 3.2	53	0.50	5.2	50	0.32
300	30000	7.8 4.3	53	0.48	4.1	50	0.54
500	18000	5.5 3.1	57	0.59	2.5	57	0.21
		Mar. Apr. May 1949.			Mar. Apr. May 1949.		
200	39000	6.9 3.8	79	0.65	4.2	34	0.39
300	30000	6.4 3.6	89	0.60	3.3	36	0.46
500	18000	5.3 2.9	89	0.71	2.8	41	0.07
		June, July, Aug. 1949.			June, July, Aug. 1949.		
200	39000	3.9 2.2	76	0.23	4.5	43	-0.01
300	30000	5.4 3.0	76	0.40	3.0	50	-0.07
500	18000	4.9 2.7	89	0.55	2.6	56	-0.15

Table III (Cont'd)

Height (mb.)	feet	Bahrein Standard Deviation	No. of Obs.	24 hours R_{24}	Nairobi Standard Deviation	No. of Obs.	24 hours R_{24}
		Sept. 1949.			Nov. 1948, Sept, 1949.		
200	39000	4.0	27	0.02	4.9	26	0.14
300	30000	3.6	27	0.32	3.8	26	0.17
500	18000	5.4	29	0.35	2.1	30	0.01
		Total			Total		
200	39000	5.8	235	0.55	4.8	153	0.21
300	30000	6.7	245	0.55	3.6	162	0.34
500	18000	4.4	264	0.62	2.8	184	0.12

3. Variation of Temperature with Distance

In Table IV there are given the correlation coefficients of the simultaneous temperatures at Larkhill and Downham Market at 700, 500, 300 and 200 for the year 1945 and at 100 mb. for the year 1949.

Table IV Standard Deviations (S.D.) and Correlation Coefficients (R) of Temperatures at Larkhill and Downham Market (130 miles) at various heights (1945 and 1949) (based on four observations a day for 1945 and 1500 GMT for 1949)

Height mb.	feet	Winter S.D.	R.	Spring S.D.	R.	Summer S.D.	R.	Autumn S.D.	R.	Year S.D.	R.
100	53000	(61) 6.5	.82	(67) 6.9	.85	(66) 5.8	.78	(67) 6.9	.79	6.5	.80
200	39000	10.8	.80	10.7	.86	9.7	.83	8.5	.82	9.9	.83
300	30000	5.9	.60	4.5	.59	5.5	.66	5.7	.69	5.4	.63
500	18000	9.2	.92	8.2	.93	6.4	.87	6.2	.84	8.2	.90
700	10000	8.1	.88	8.0	.83	5.9	.74	5.9	.87	7.0	.84

(The figures in brackets show the number of observations at 100 mb.)

In Table V are given the correlation coefficients of simultaneous temperatures at 200 and 300 mb. between Hanover, Downham and Aldergrove in winter and summer (Dec. 1949 to Aug. 1950.)

Table V Standard Deviations (S.D.) and Correlation Coefficients (R) of Temperatures at Hanover, Downham Market and Aldergrove at 200 and 300 mb.

Height mb.	No. of Obs.	Standard Deviation Hanover	Downham	Aldergrove	Correlation coefficients H - D (375 miles)	D - A (320 miles)	H - A (675 miles)
Winter							
200	299	11.2	11.6	12.8	.57	.62	.19
300	336	5.7	5.3	6.6	.46	.51	.34
Summer							
200	336	9.6	10.4	10.7	.46	.68	.21
300	348	5.4	6.0	6.2	.31	.57	.08

/In

In Table VI there is given a collection of correlation coefficients between temperatures at various heights and at various distances.

Table VI Standard Deviations (S.D.) and Correlation Coefficients (R) of Temperature between different places at various heights.

Period	Height	Places	Distance (miles)	Number of Obs.	SD M	SD WF	R
June - Aug. 1949	200 mb.	Malta-Wheelus Field	220	23	3.6°F	3.5°F	0.44
Sep. - Nov. 1949	200 mb.	" " "	"	96	3.7	3.8	0.26
Dec.49 - Feb.1950	200 mb.	" " "	"	87	4.6	4.3	0.60
June 49 - Feb. 50	200 mb.	" " "	"		4.0	3.9	0.46
June - Aug.1949	300 mb.	" " "	"	37	2.5	3.3	0.62
Sep. - Nov.1949	300 mb.	" " "	"	115	2.6	3.2	0.36
Dec.49 - Feb.1950	300 mb.	" " "	"	112	2.8	3.2	0.49
June 49 - Feb.50	300 mb.	" " "	"		2.6	3.2	0.50
June - Aug.1949	500 mb.	" " "	"	67	2.2	2.3	0.37
Sep. - Nov.1949	500 mb.	" " "	"	121	2.3	3.3	0.34
Dec.49 - Feb.1950	500 mb.	" " "	"	129	2.8	3.9	0.59
June 49 - Feb.50	500 mb.	" " "	"		2.5	3.2	0.46
					SA	B	
Nov.48 - Feb.49	200 mb.	S.Antonio - Brownsville	230	77	2.4	1.9	0.48
Mar. - May 1949	200 mb.	" "	"	62	3.7	3.8	0.39
June - Sept. 49	200 mb.	" "	"	69	1.5	1.7	0.13
Nov. 48 - Sept.49	200 mb.	" "	"	208	2.7	2.7	0.37
					M	T	
Nov.48 - Feb. 49	200 mb.	Miami - Tampa	185	71	1.9	2.5	0.44
Mar.- May, 1949	200 mb.	" "	"	67	3.3	2.7	0.46
June - Sep. 1949	200 mb.	" "	"	80	2.1	1.4	0.19
Nov. 48 - Sep. 49	200 mb.	" "	"	218	2.5	2.3	0.40
					H	B	
Dec.Jan.Feb. 1949	100 mb.	Habbaniya - Bahrein	720	45	4.9	5.4	+ 0.27
Mar.Apl.May 1949	100 mb.	" "	"	60	5.0	4.9	+ 0.30
June, July, Aug. 49	100 mb.	" "	"	47	5.4	3.4	+ 0.17
Sep.Oct.Nov. 1949	100 mb.	" "	"	44	4.7	5.7	+ 0.27
Jan. to Dec. 1949	100 mb.	" "	"	196	5.0	4.9	+ 0.25
Dec.Jan.Feb. 1949	200 mb.	" "	"	79	7.9	5.4	+ 0.35
Mar.Apr.May 1949	200 mb.	" "	"	82	8.4	6.5	+ 0.50
June,July,Aug.1949	200 mb.	" "	"	70	7.1	6.4	0.03
Sep.Oct.Nov.1949	200 mb.	" "	"	82	4.1	4.2	0.11
Jan. to Dec.1949	200 mb.	" "	"	313	7.1	5.7	0.28
Dec.Jan.Feb.1949	300 mb.	" "	"	83	5.6	6.3	0.41
Mar.Apr.May 1949	300 mb.	" "	"	91	6.7	6.3	0.40
June,July,Aug.49	300 mb.	" "	"	76	3.8	4.4	0.15
Sep.Oct.Nov. 1949	300 mb.	" "	"	86	4.7	4.6	0.33
Jan. to Dec. 1949	300 mb.	" "	"	336	5.3	5.5	0.35
					H	N	
Dec.Jan.Feb. 1949	200 mb.	Habbaniya - Nicosia	620	78	7.7	9.7	+ 0.18
Mar.Apr.May 1949	200 mb.	" "	"	82	8.4	9.0	+ 0.51
June,July,Aug.49	200 mb.	" "	"	66	4.0	4.0	+ 0.17
Sep.Oct.Nov. 1949	200 mb.	" "	"	87	4.1	4.4	+ 0.27
Jan. to Dec. 1949	200 mb.	" "	"		6.4	7.2	+ 0.33

These correlation coefficients are plotted on Fig.4 and 5 for 200 mb. and 300 mb. respectively.

/The

The outstanding features of these tables are

(a) over the British Isles and Western Europe the correlation coefficients at 300 mb. are decidedly less than at either 500 mb. or 200 mb.; on the other hand the standard deviation is also least at 300 mb.; for the year as a whole the root mean square deviations from linear relationships between Larkhill and Downham are 5.4°F at 200 mb., 4.2°F at 300 mb., 3.6°F at 500 mb. and 3.8°F at 700 mb.

(b) At 200 mb. the correlation coefficient appears to fall off regularly with distance over N.W. Europe at a rate very similar to that found for wind (vide M.R.P.556). Over the Southern U.S.A. and Mediterranean, however, the decrease is much more rapid and is even more rapid than for wind. The correlation coefficient between Habbaniya and Bahrein is unexpectedly higher.

(c) At 300 mb. the decrease over the N.W. Europe is more rapid than at 200 mb. and is about as rapid as over the Mediterranean. Again the Habbaniya - Bahrein combination is anomalously high.

The impression is left that these correlation coefficients are controlled primarily by the normal position of the tropopause.

The high correlation coefficients between Habbaniya and Bahrein and between Nicosia and Habbaniya are remarkable and need further investigation.

An investigation was made of the variation of temperatures with distance by using the maps of thickness of the layer between 225 and 96 mb. given in the German D.W.Rs. for 1943. The 9 positions at which the topography was estimated were Latitude 54°N, Longitudes 0°, 5°, 10°, 15°, 20°, 25°, 30°, 35° and 40°E and measurements were made once a day for the six months January, February, June, July, August and December. Further estimates were also made for the positions 60°N 10°E for the same three months for comparison with the position 54°N 10°E.

The result of this investigation can be summarised as follows. (Table VII).

Table VII Mean thickness (in dynamic metres) of layer 225 mb to 96 mb.(T.) and Standard Deviation (S.D.)

Position	A		B		C		D		E		F		G		H		J		X	
Latitude	54°		54°		54°		54°		54°		54°		54°		54°		54°		60°	
Longitude	0°		5°E		10°E		15°E		20°E		25°E		30°E		35°E		40°E		10°E	
	T	SD	T	SD	T	SD	T	SD	T	SD	T	SD	T	SD	T	SD	T	SD	T	SD
December 1943	5218	115	5222	110	5229	108	5236	112	5240	109	5246	102	5251	91	5261	84	5261	90	5221	96
January 1943	5290	79	5280	85	5273	96	5259	102	5248	106	5241	106	5237	101	5237	90	5230	82	5236	99
February 1943	5231	145	5240	146	5241	144	5245	131	5252	123	5252	115	5258	117	5257	121	5267	86	5208	131
Winter 1943	5246	119	5247	118	5247	118	5247	115	5247	113	5246	108	5249	104	5252	99	5253	88	5222	107
June 1943	5442	77	5455	76	5467	76	5469	76	5462	85	5464	-	5470	78	5469	-	5472	65	5502	81
July 1943	5411	75	5422	85	5447	78	5477	80	5492	69	5506	-	5519	57	5523	-	5511	59	5503	96
August 1943	5454	75	5447	84	5437	92	5429	92	5426	99	5427	-	5435	79	5439	-	5442	50	5522	86
Summer 1943	5436	80	5441	83	5450	83	5458	86	5460	89	5466	-	5475	80	5477	-	5475	64	5509	88

/A thickness

A thickness of 5,500 dynamic metres corresponds to a temperature of -45°C. and a variation in thickness of 24.4 dynamic metres corresponds to a variation of 1°C., so that the standard deviations derived from thickness are about 4 or 5°C (about 8°F) in winter and 3 or 4°C (about 6°F) in summer. This compares with a mean standard deviation for the layer 100 to 200 mb. of about 10°F. at Larkhill in winter and 7½°F in summer, so that the effect of using thickness as a substitute for temperature is to give a reduction in standard deviation of about 20 per cent.

Since the information at position A was not as accurate as at others further east, correlation coefficients were calculated between position B and C, B and D, B and E also A and G and C and X with the results set out in Table VIII.

Table VIII. Correlation coefficients between partial thickness at various distances at the level 225 - 96 mb. 1943.

Positions	B C	B D	B E	A G	C X	Number
Distance (miles)	200	400	600	1200	415	of Obs.-A
Winter	+ 0.94	+ 0.80	+ 0.55	+ 0.14	+ 0.83	90
Summer	+ 0.85	+ 0.53	+ 0.24	- 0.17	+ 0.64	92
Mean	+ 0.91	+ 0.70	+ 0.44	+ 0.03	+ 0.75	182

There seems to be a decrease in the correlation coefficient in summer as compared with winter.

These coefficients are illustrated in Fig.6.

4. Variation of Temperature with Height.

In Tables IX and X there are set out the standard deviations and correlation coefficients of the temperature-height relationship at Larkhill.

Table IX. Seasonal correlation coefficients between temperatures (°F) at various heights up to 40,000 feet, Larkhill, 1945.

Pressure Height	No. of Obs.	Standard Deviation 700 mb.	500 mb.	300 mb.	200 mb.	Correlation Coefficients 700-500	700-300	700-200	500-300	500-200	300-200
Dec. Jan. Feb.	360	7.9	8.8	6.0	10.6	0.877	0.470	-0.681	0.576	-0.655	-0.298
Mar. Apr. May	368	7.9	7.9	4.3	10.5	0.902	0.432	-0.722	0.499	-0.717	-0.435
June July Aug.	368	5.8	6.0	5.3	9.5	0.753	0.521	-0.520	0.659	-0.666	-0.418
Sept. Oct. Nov.	364	5.7	6.0	5.8	8.5	0.792	0.506	-0.583	0.583	-0.577	-0.334
Jan. to Dec.	1460	6.9	7.3	5.4	9.8	0.850	0.469	-0.643	0.566	-0.670	-0.362

Table X. Seasonal values of standard deviations and correlation coefficients between Temperatures (°F) at various heights up to 53,000 feet, Larkhill, 1949.

Pressure Height	No. of Obs.	Standard Deviation 500mb.	400 mb.	300mb.	250mb.	200mb.	170mb.	150mb.	100mb.
Dec. Jan. Feb.	360	7.8	7.7	5.9	6.9	11.2	11.4	9.3	7.7
Mar. Apr. May	368				6.6	10.2	10.2	7.6	
June July Aug.	368	6.5	6.8	6.2	5.0	8.4	9.1	8.6	6.8
Sept. Oct. Nov.	364				8.6	9.0	9.2	8.7	

Pressure Heights	Correlation Coefficients 500) 400)	500) 300)	500) 200)	500) 170)	500) 150)	500) 100)	400) 200)	400) 150)	300) 100)	250) 200)	200) 170)	200) 150)	200) 100)	170) 150)
Dec. Jan. Feb.	+0.900	+0.500	-0.525	-0.556	-0.578	-0.543	-0.482	-0.557	-0.187	+0.631	+0.836	+0.783	+0.565	+0.917
Mar. Apr. May	-	-	-	-	-	-	-	-	-	+0.633	+0.901	0.817	-	+0.878
June July Aug.	+0.932	+0.730	-0.582	-0.655	-0.695	-0.585	-0.591	-0.690	-0.421	+0.319	+0.792	0.718	+0.581	+0.931
Sept. Oct. Nov.	-	-	-	-	-	-	-	-	-	+0.454	+0.852	0.758	-	+0.937
Mean	+0.916	+0.615	-0.553	-0.605	-0.637	-0.564	-0.537	-0.623	-0.304	+0.520	+0.842	+0.764	+0.573	+0.913

The values in Tables IX and X follow broadly the same trends and are illustrated in Fig.7. This brings out how the correlation between temperatures at a fixed difference in height is least at about 35,000 feet, i.e. where the tropopause is most commonly situated. High positive correlation coefficients occur where the atmosphere is definitely in the troposphere or in the stratosphere. On the other hand there is a region of comparatively high negative correlation: the maximum value appears to be in the correlation of temperatures at 500 mb. with those at 150 mb.

In Tables XI and XII are given corresponding correlation coefficients for Habbaniya and Nairobi.

Table XI. Seasonal values of standard deviations and correlation coefficients between temperatures (°F) at various heights over Habbaniya, 1948 and 1949.

Pressure Height	Year	No. of Obs.	Standard Deviations 700mb.	500mb.	300mb.	200mb.	150mb.	100mb.
Dec. Jan. Feb.	1948	82	6.2	6.1	6.3	7.4		
	1949	59	6.1	5.7	4.8	7.9	5.6	5.2
Mar. Apr. May	1949	71	6.9	7.2	9.1	9.0	6.4	6.6
June July Aug.	1948	80	3.8	5.3	6.8	6.1		
	1949	67	3.5	4.9	3.7	4.0	4.2	5.0
Sept. Oct. Nov.	1949	74	3.2	3.1	4.7	4.2	4.4	5.3

Table XI (Cont'd)

Pressure Heights	Year	Correlation Coefficients														
		700) 500)	700) 300)	700) 200)	500) 300)	500) 200)	300) 200)	700) 150)	700) 100)	500) 150)	500) 100)	300) 150)	300) 100)	(200 (150	200) 100)	150) 100)
Dec. Jan. Feb.	1948	+0.85	+0.28	-0.40	+0.38	-0.39	+0.01									
	1949	+0.69	-0.18	-0.51	-	-	-	-0.45	-0.05	-0.31	-0.20	-0.11	-0.29	+0.65	+0.06	+0.44
Mar. Apr. May	1949	+0.85	+0.56	-0.34	-	-	-	-0.50	-0.55	-0.47	-0.57	-0.21	-0.45	+0.57	+0.09	+0.51
June July Aug.	1948	+0.43	+0.28	+0.29	+0.50	+0.37	+0.81									
	1949	+0.09	+0.27	+0.06	-	-	-	+0.19	+0.17	-0.03	-0.21	+0.29	+0.11	+0.69	+0.44	+0.47
Sept. Oct. Nov.	1949	+0.38	+0.45	+0.20	-	-	-	-0.13	-0.11	-0.13	-0.01	-0.16	-0.20	+0.28	+0.10	+0.52

Table XII. Seasonal correlation coefficients between temperatures (°F) at various heights, Nairobi, 1949.

Pressure Height	No. of Obs.	Standard Deviation 700 mb.	500 mb.	300 mb.	200 mb.	Correlation Coefficients 700-500	700-300	700-200	500-300	500-200	300-200
Winter	68	1.9	2.4	5.0	5.0	+0.00	-0.01	-0.13	+0.35	+0.43	+0.71
Summer	64	2.9	2.6	3.0	4.5	-0.11	-0.07	+0.01	+0.34	+0.40	+0.74

The values for Habbaniya are illustrated for summer and winter in Figs. 8 and 9. Clearly the winter regime is much akin to that over temperate latitudes whereas the summer regime is quite different. It is notable from the comparison between 1948 and 1949 that there are considerable differences in the correlation coefficients in the two years and that there are also big differences in the standard deviations at 500mb., 300mb. and 200mb. in the two years. The standard deviations in June, July and August in the two years are shown in Table XIII.

Table XIII. Standard Deviation of Temperature at 1400 (G.M.T.) at 300mb. at Habbaniya.

	Year	June	July	August
200 mb.	1948	7.9°F(26)	5.5°F(26)	4.3°F(28)
	1949	3.9°F(23)	4.1°F(25)	4.1°F(19)
300mb.	1948	9.4°F(26)	4.8°F(26)	5.1°F(28)
	1949	4.4°F(23)	3.4°F(25)	3.0°F(19)
500mb.	1948	5.4°F(26)	4.9°F(26)	5.4°F(28)
	1949	4.1°F(23)	5.5°F(25)	5.2°F(19)

The big standard deviation in June, 1948 was due to a large extent to a very cold spell in the first 7 days of the month. It is notable also that in May, 1949 there was a standard deviation of 9°F due also to very cold air at the beginning of the month and a sharp increase in temperature later. It has been noted by Goldie that at Habbaniya in the early part of the summer there is a very marked change in the height of the tropopause which suddenly rises and does not oscillate back to its former level. No doubt these big temperature variations are due to the same phenomenon.

If Nairobi is typical of the tropics, clearly the correlation with height is much less noticeable than in temperate latitudes at any rate below 20,000 or 30,000 feet. The synoptic significance of this is that air masses in the tropics are not nearly so well defined by temperature as in temperate latitudes. It should however be noted that the standard deviations of temperature in the low levels are much smaller in the tropics than in temperate latitudes. The

effect of this would be to increase the influence of casual errors on the correlation coefficients with the result of reducing them to small values. Indeed if the same value of errors in radio sonde observations are applicable in the tropics as are quoted by Sheppard the true standard deviations at Nairobi are reduced to between 2 and 3°F. Actually with stronger radiation it is probable that the random errors are greater in the tropics than over the British Isles and the true standard deviations may be even less.

5. Correlation of Temperature and Wind

Since the relationship of temperature to wind might have a considerable effect on the statistics of operation of aircraft it was necessary to obtain some representative correlation coefficients. This has been done and the results are presented in the following tables.

Table XIV. Correlation of Temperature with Wind at Larkhill, Dec. 1948 - Aug. 1949.

		Winter					Summer				
	Height	Standard Deviation			Correlation Coefficients		Standard Deviation			Correlation Coefficients	
mb.	feet	Temp. °F	N.Comp. kt.	E.Comp. kt.	R_N	R_E	Temp. °F	N.Comp. kt.	E.Comp. kt.	R_N	R_E
200		9.8	30	29	-0.32	-0.23	8.1	27	25	-0.27	-0.14
300		5.0	42	33	+0.01	+0.01	6.1	25	29	-0.10	+0.13
500		6.5	30	27	-0.12	-0.02	6.9	19	17	+0.16	+0.06
700		3.9	12	12	-0.18	-0.06	5.9	13	13	+0.05	+0.10

All the coefficients are small but the winter correlation coefficients with the northerly component are certainly significant and show that a northerly wind brings colder air even at 200 mb. The table brings out that at that level in summer as well as winter the north easterly winds are the cold winds but at 500 and 700 mb. north and east winds are cold in winter but warm in summer. However the most notable feature is the smallness of the coefficients, so small indeed that the relationship at 500 and 700 mb. is scarcely worth recording.

Correlation between temperature and wind had also been made for 300 mb. at Habbaniya and 200mb. at Nairobi. The results are set out in Tables XV and XVI.

Table XV. Correlation of Temperature with Wind at Habbaniya at 300 mb. (1948)

Season	Number of Obs.	Standard Deviation Temp. °F	N. Comp. kt.	E.Comp. kt.	Correlation Coefficient R_N	R_E
Winter Dec. Jan. Feb.		6.2	23	30	-0.06	-0.51
Spring Mar. Apr. May		5.2	21	21	-0.11	-0.21
Summer June July Aug.		6.9	11	15	+0.12	-0.12
Autumn Sept. Oct. Nov.		6.1	16	21	-0.26	-0.09

/Table XVI.

Table XVI. Correlation of Temperature with Wind at Nairobi at 200 mb. (January to September, 1949)

Season	Number of Obs.	Standard Deviation Temp. °F	N. Comp. kt.	E.Comp. kt.	Correlation Coefficient R_N	R_E
Jan. Feb.	42	5.5	14	14	0.35	-0.06
Mar. Apr. May	53	4.2	10	18	0.09	-0.17
June July Aug. Sept.	84	4.5	10	14	-0.05	-0.09

Again the correlation coefficients are very small. The only ones which are possibly important are the cold easterlies at 300mb. over Iraq in winter and the warm northerlies over East Africa at 200 mb. in the early months of the year.

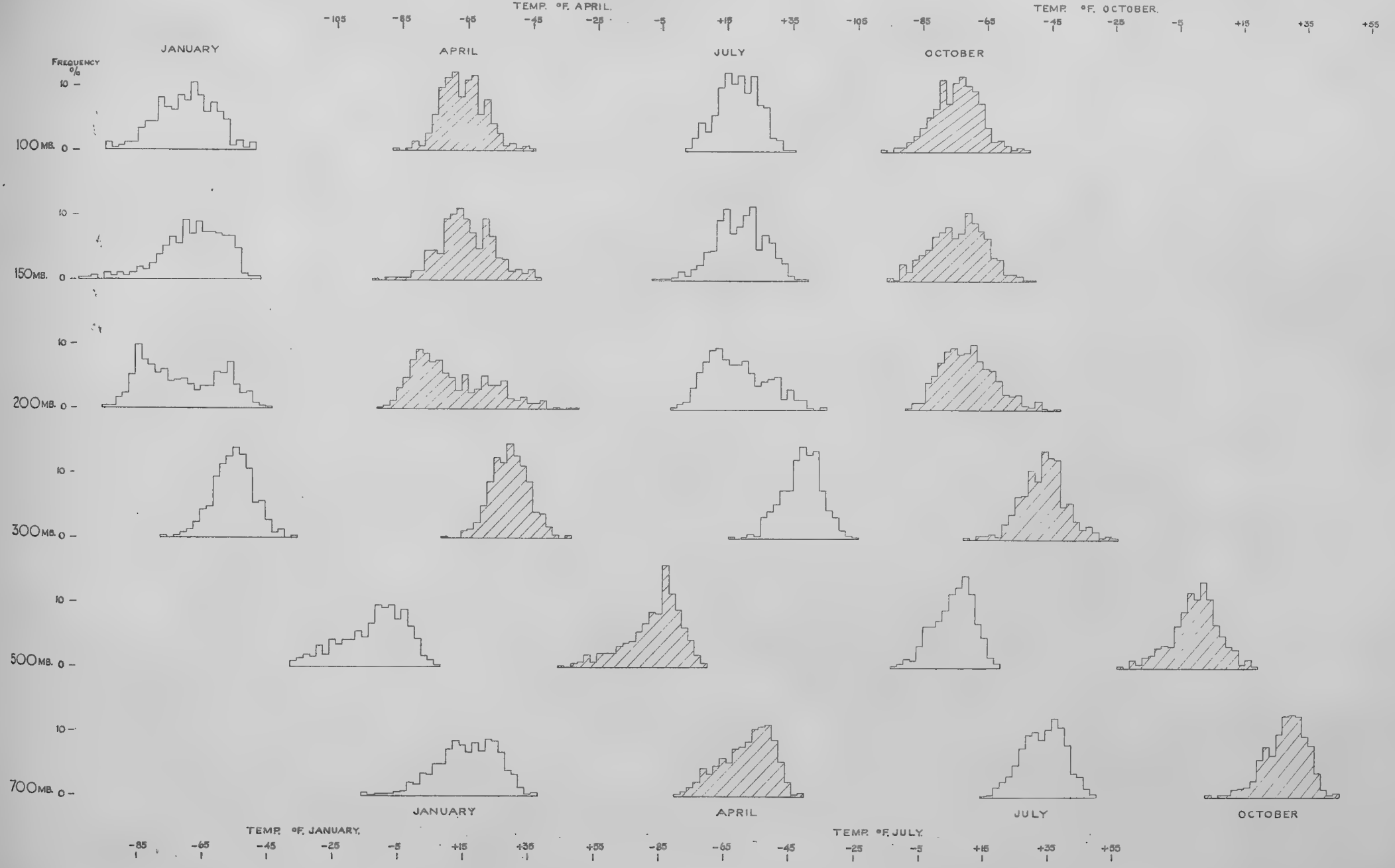
The really important point about these figures is that none is sufficiently great to necessitate inclusion in the consideration of Equivalent Head Winds compensated for temperature.

FIGURE 1.

FREQUENCIES OF 2-DEGREE RANGES OF TEMPERATURE °F. AT LARKHILL

MRP. 668.

PERIOD: JULY 1945 – JULY 1949
ALL HOURS OF OBSERVATION.



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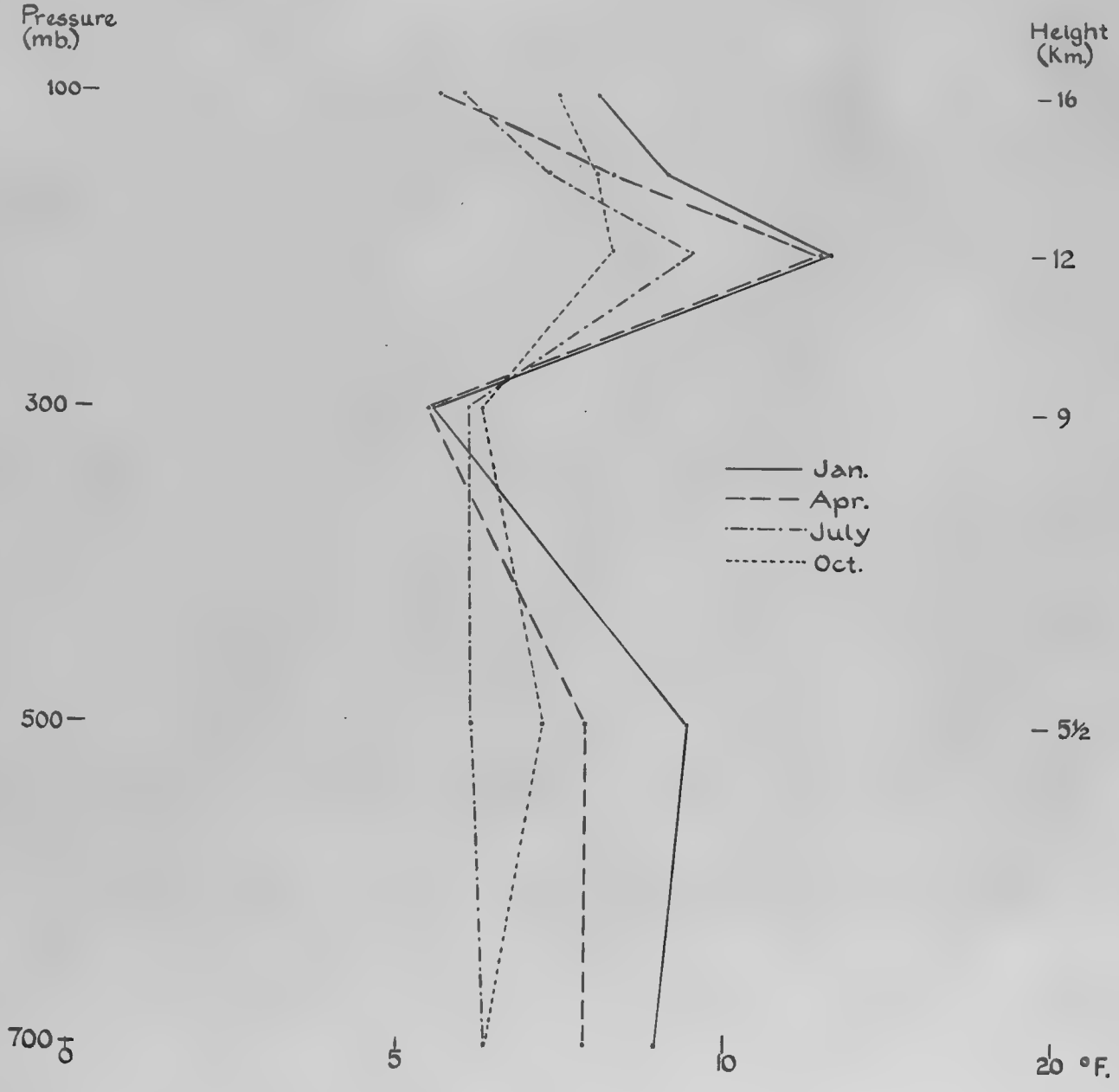


FIGURE 2.

Standard deviation of Upper air temperature at 0000 or 0300 hrs G.M.T. 1946–49.

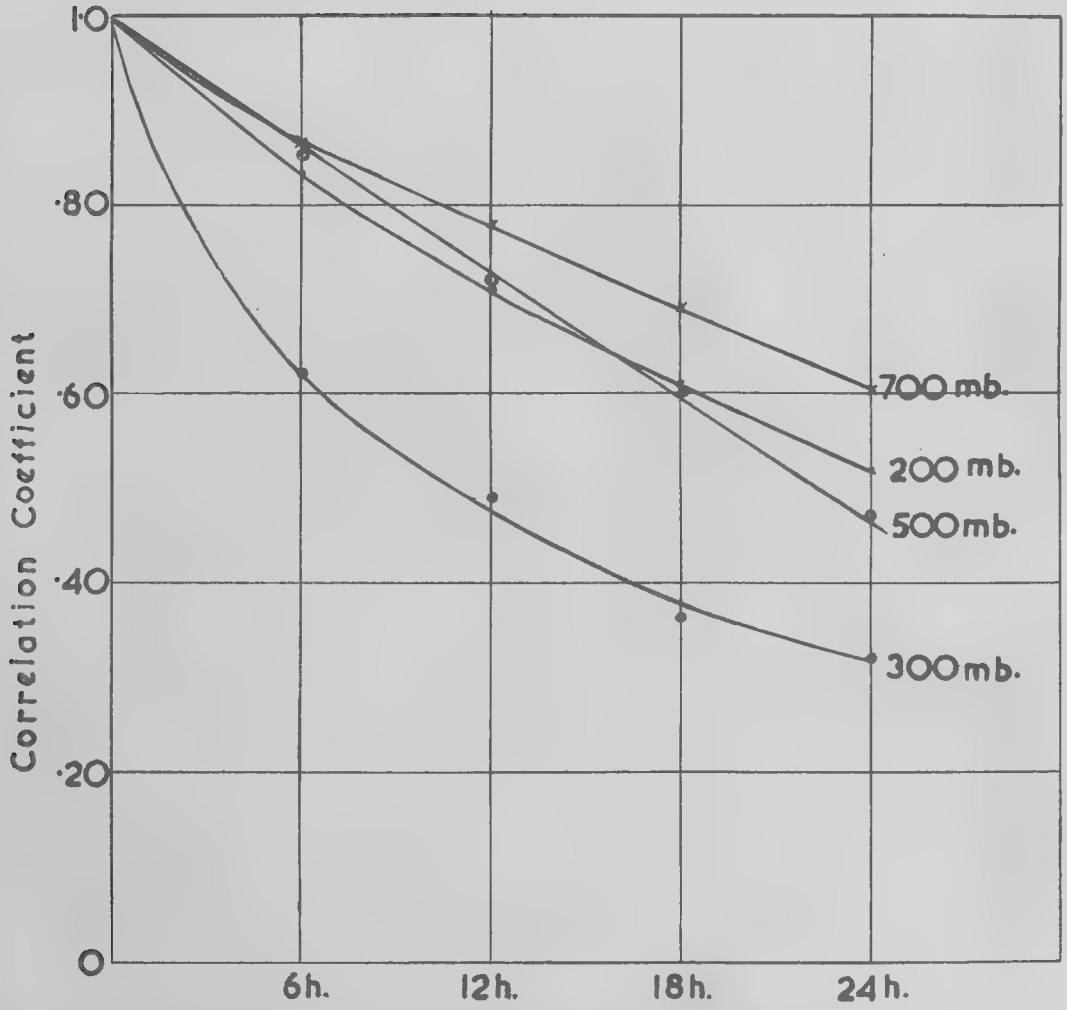


Figure 3. Variation of Correlation of Temperature with Time.
LARKHILL 1945.

Figure 4. Variation of Correlation of Temperature with Distance.

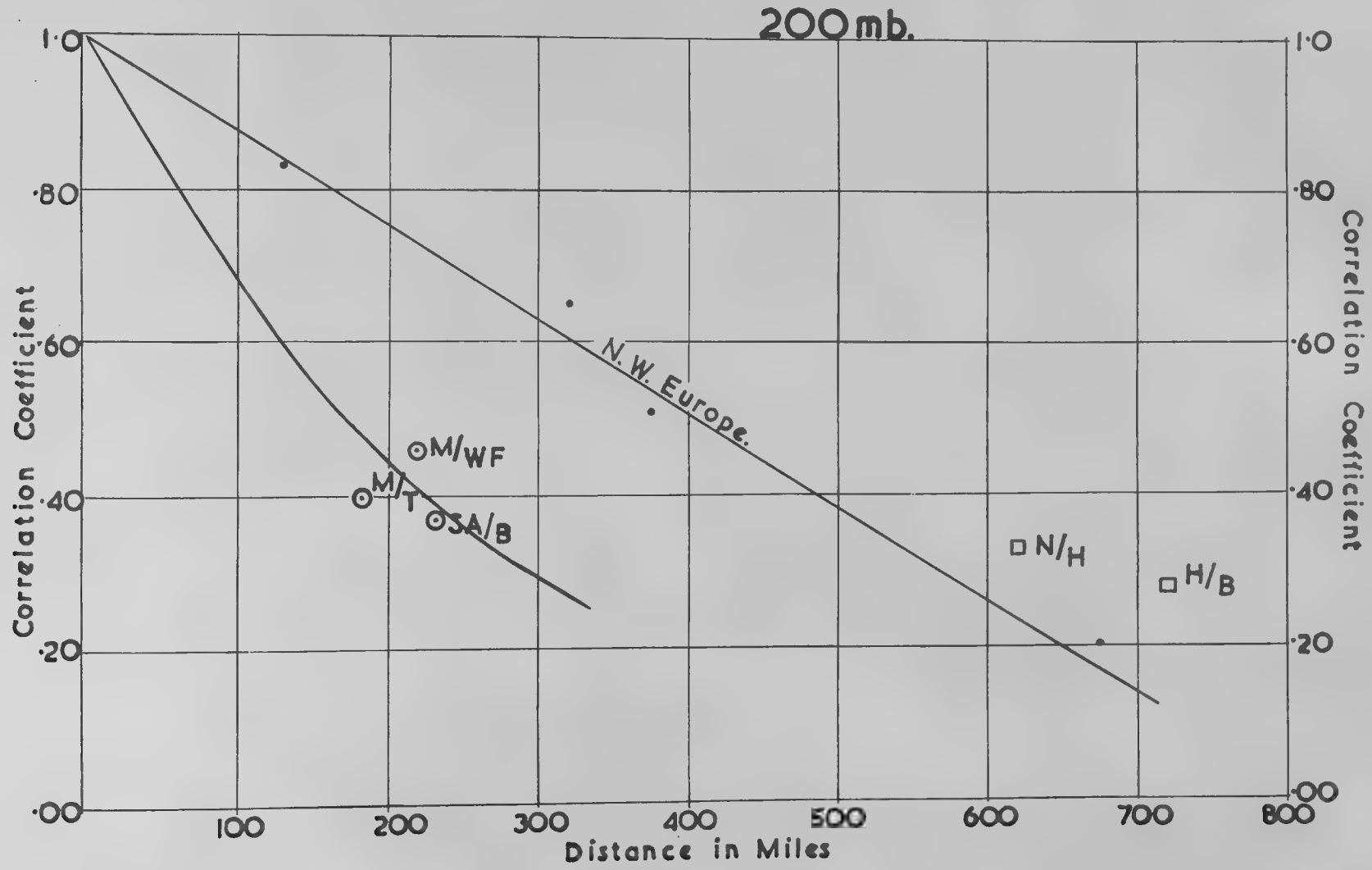


Figure 5. Variation of Correlation of Temperature with Distance.

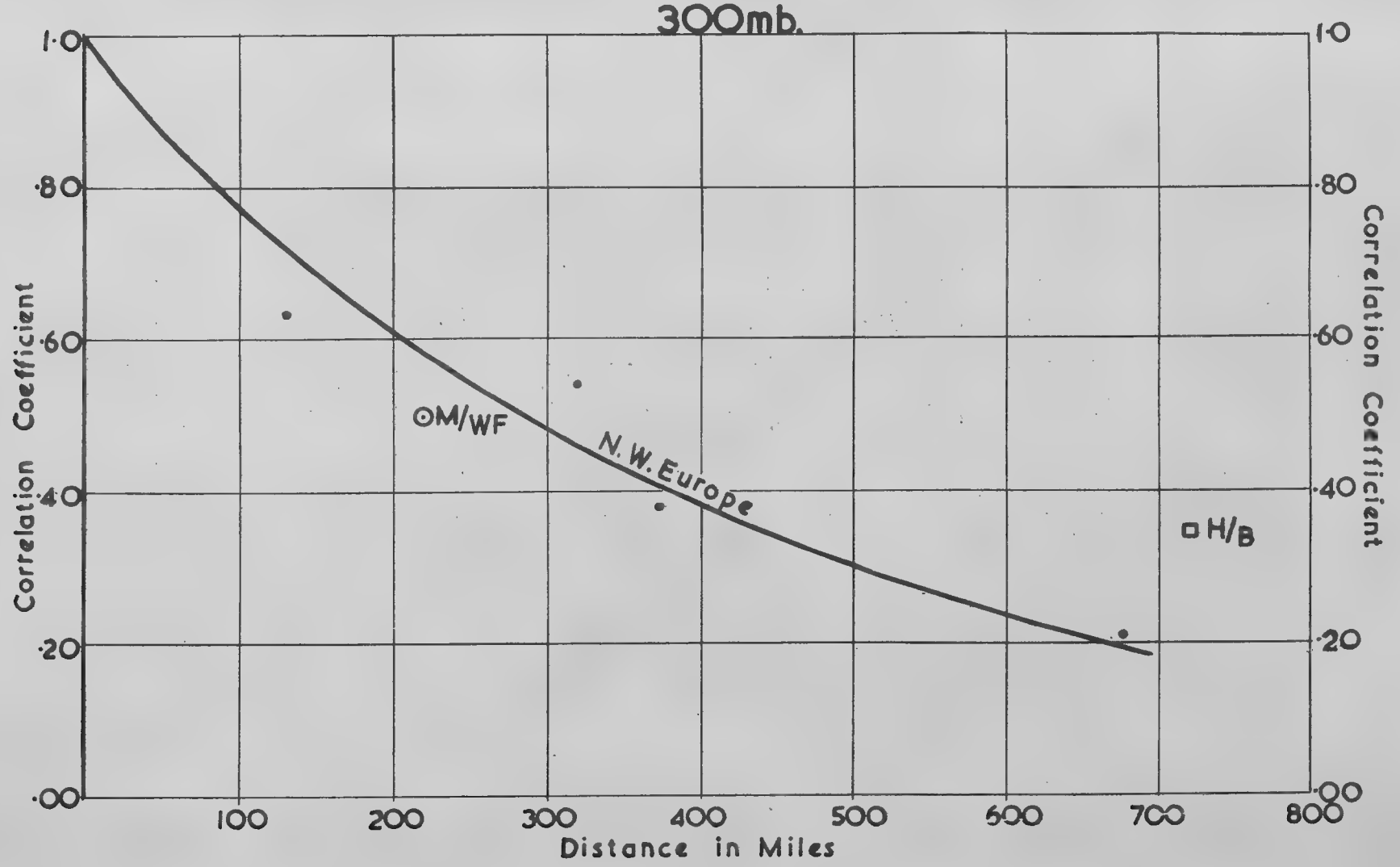
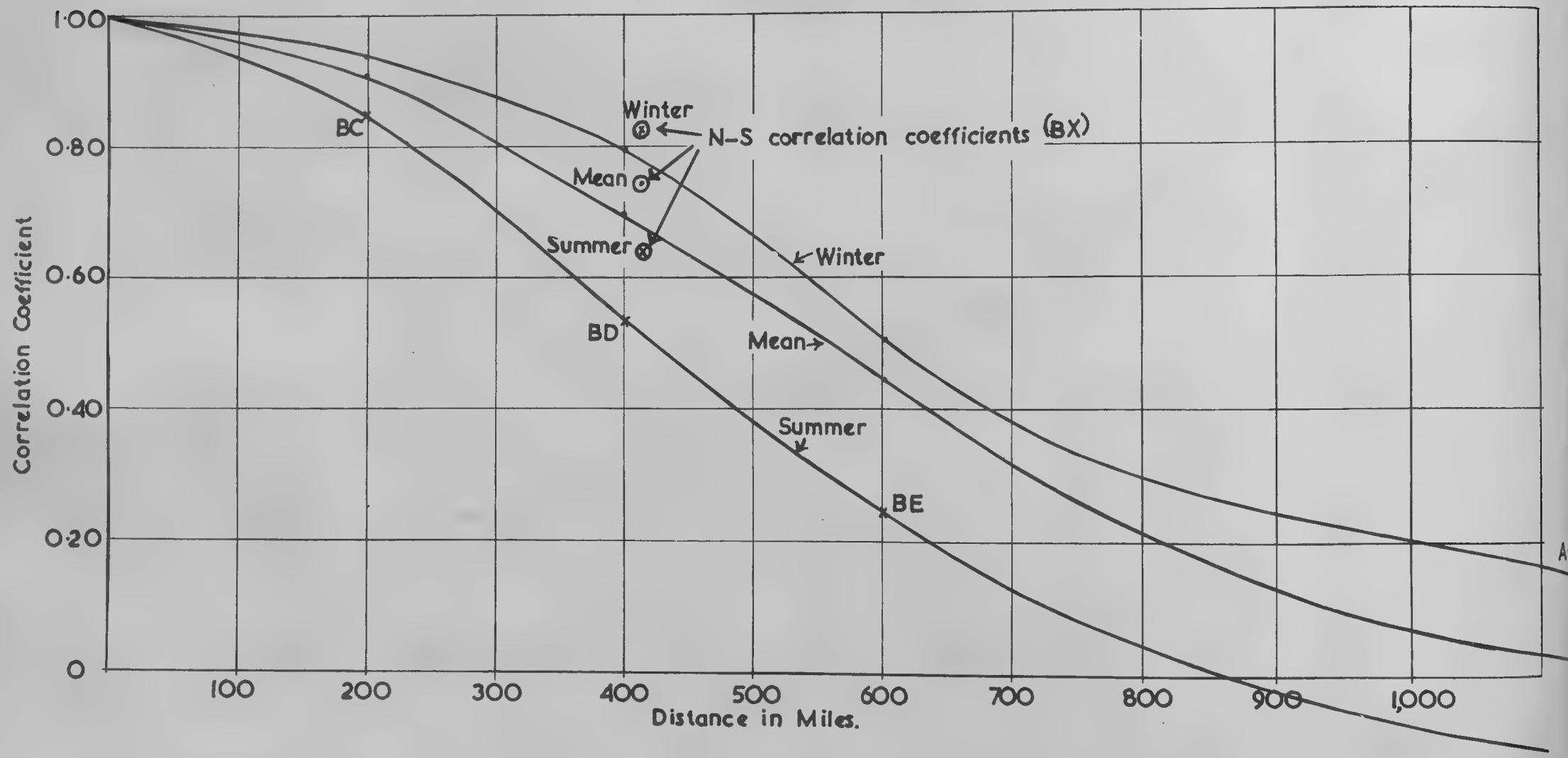


Figure 6. Variation of Correlation with Distance over NW.Europe from thickness 225–96 mb.



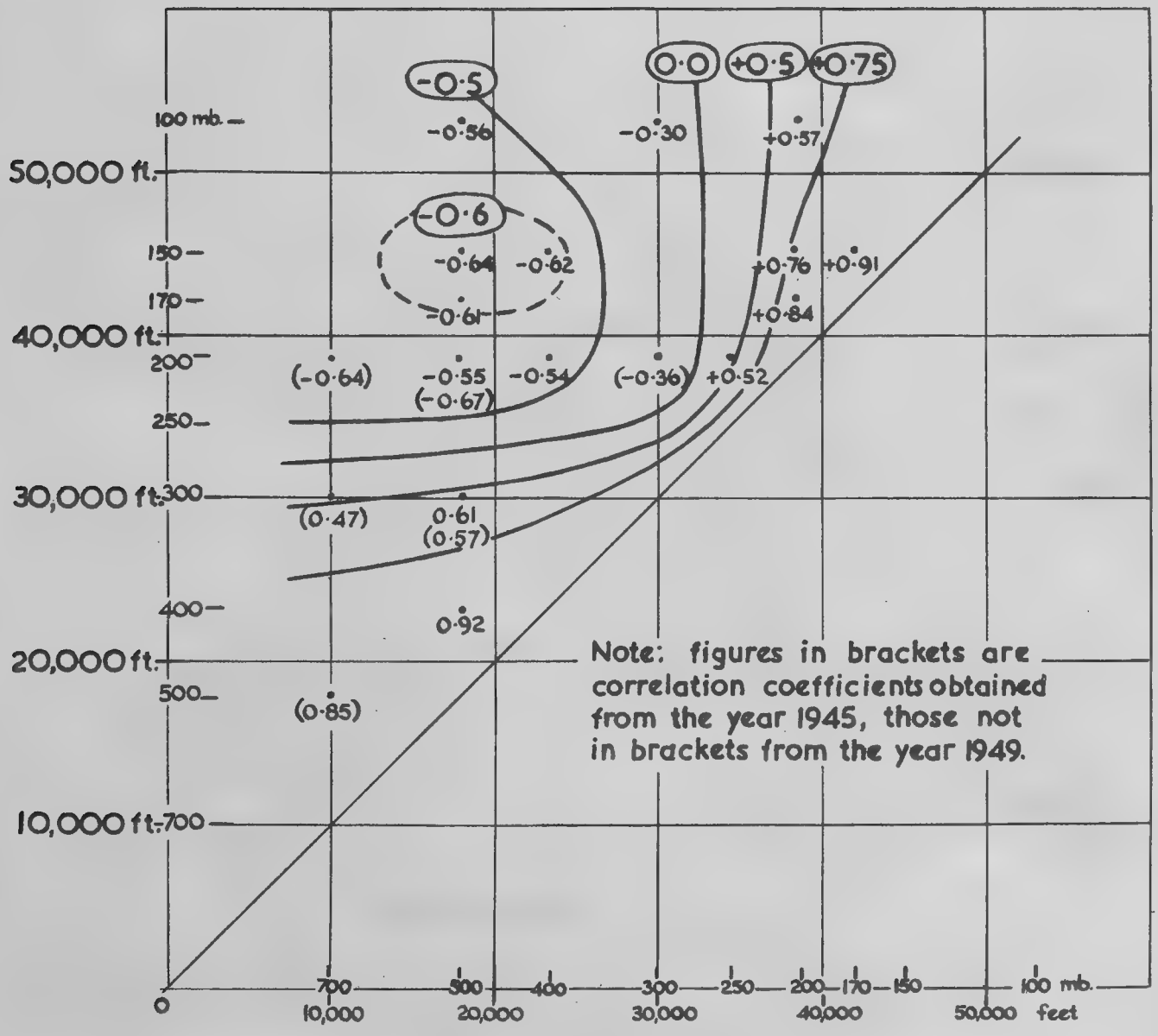


Figure 7. Correlation of Temperature with Height

LARKHILL

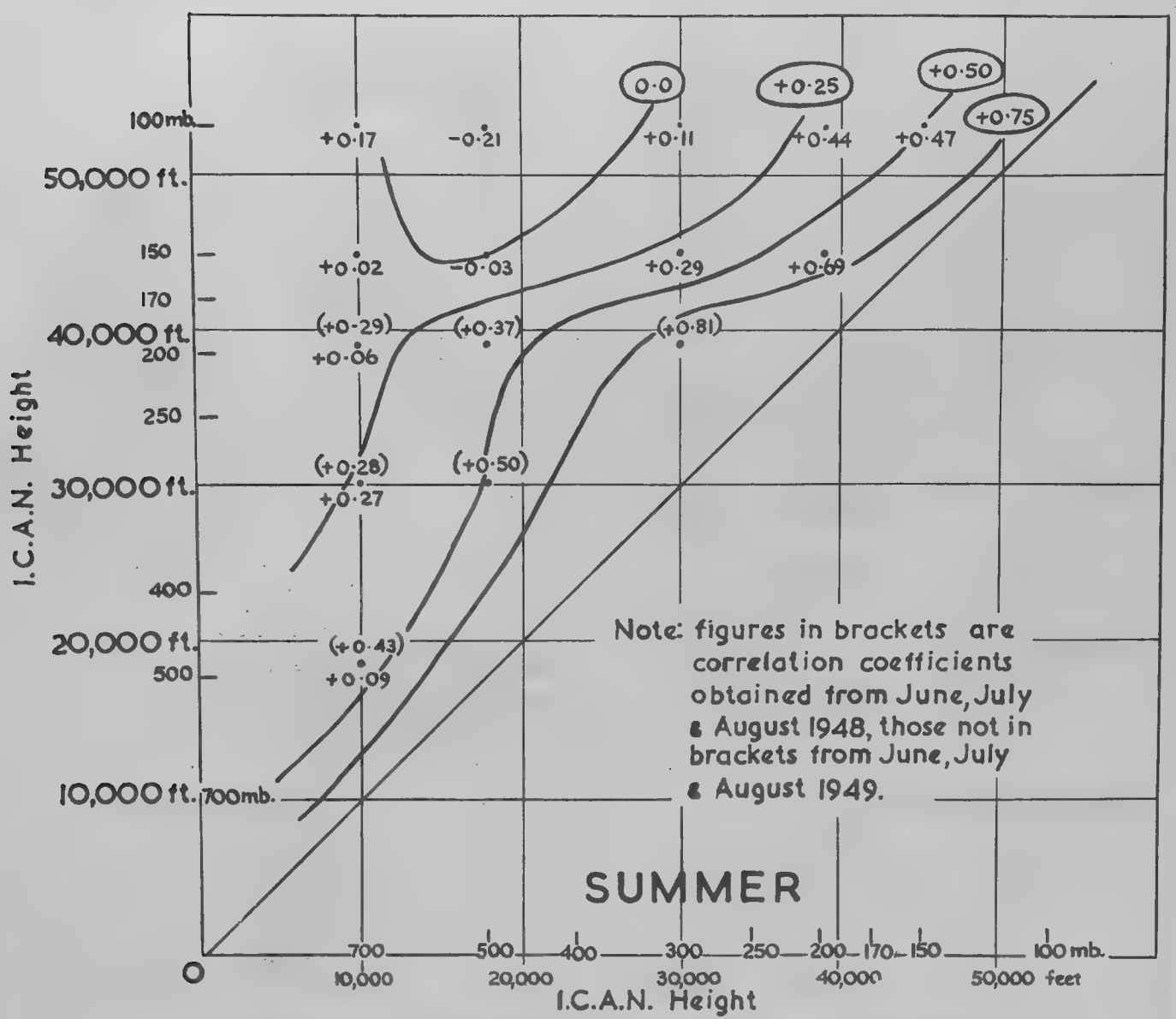


Figure 8. Correlation of Temperature with Height.

HABBANIYA

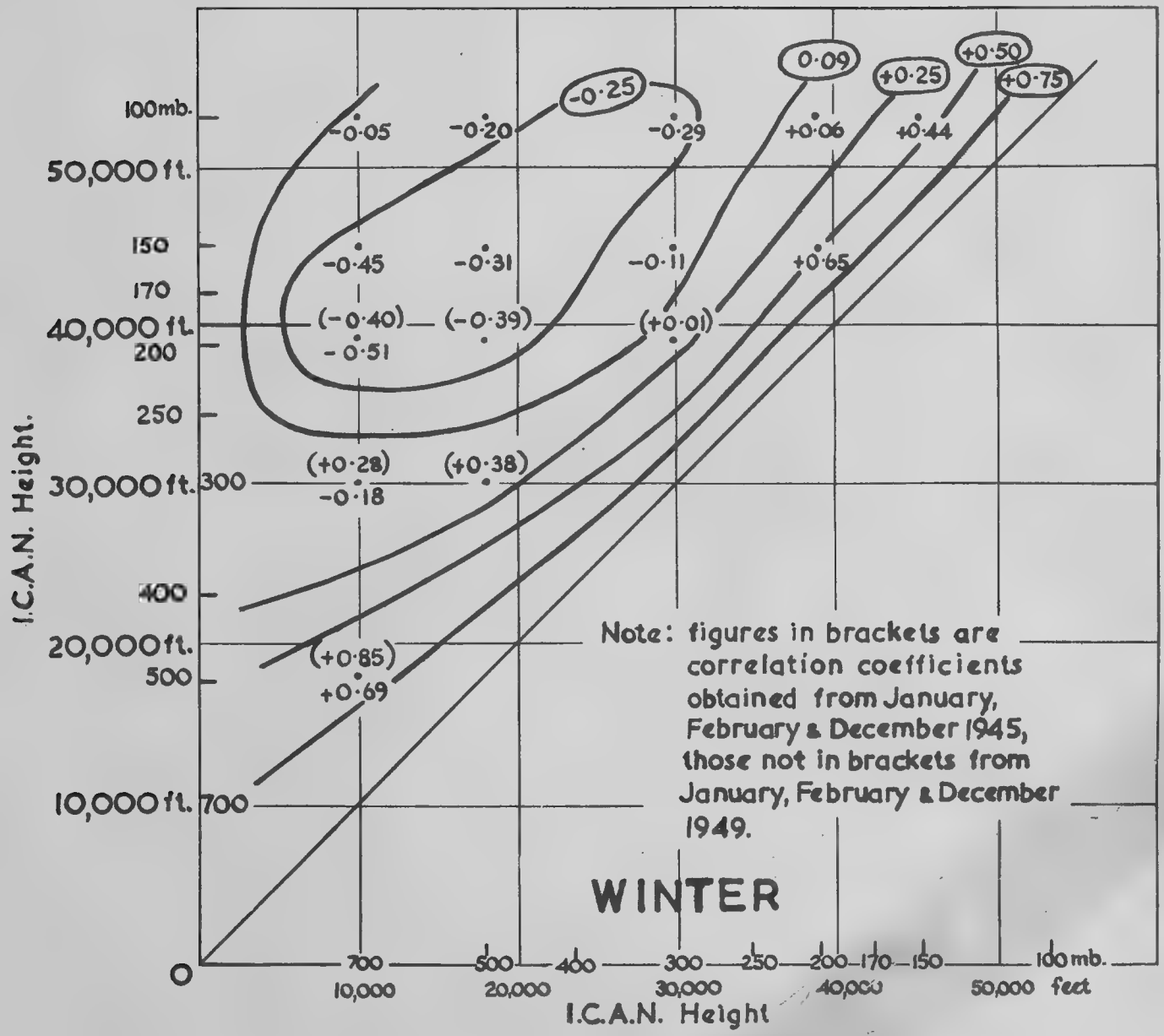


Figure 9. Correlation of Temperature with Height.
HABBANIYA

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M.R.P. 669
S.C.III/107
3rd September, 1951

AIR MINISTRY

METEOROLOGICAL RESEARCH COMMITTEE

Weather systems associated with some occasions of severe turbulence at high altitude

by
J.K. Bannon (Meteorological Office)

Summary

92 cases of severe turbulence reported at altitudes above 20,000 ft. over Great Britain were analysed with respect to broad features of the upper air pattern. 67 per cent of the occasions were associated with a jet stream, less than 7 per cent were unclassifiable and the remaining cases were associated with upper lows, upper troughs, strong winds or discontinuous tropopause surfaces. The cases occurring near jet streams were found to be grouped in particular zones relative to the jet stream axis.

Introduction

In previous papers (1, 2, 3 and 4) attempts were made to analyse occasions of turbulence experienced by aircraft at high altitude and mainly in clear air. Various meteorological parameters were investigated and the most closely related to the turbulence were found to be the Richardson number and the horizontal or isentropic shear of the wind. It became clear in these investigations that for a proper analysis it was necessary to have meteorological observations at the same time and place as the gustiness observations; interpolation in time and space of wind and temperature observations in the upper air could not give the necessary detail. From reference (2) and also from Hislop's (5) description of the results of British European Airways' investigation it appears that high altitude turbulence can occur in almost any type of weather situation.

The severity of high altitude turbulence is described as slight or moderate in the majority of cases but occasionally it is described as severe, violent, heavy or even frightening. Hislop (5) has estimated that the severe turbulence met by a Comet aircraft at 35,000 ft. on 14th November, 1949, gave equivalent gust velocities of about 35 ft./sec. which is of the same order of magnitude as the gusts sometimes encountered in cumulonimbus clouds over England, though not as great as the most severe gust (equivalent gust velocity 54 ft./sec.) yet measured in such conditions. (Jones (6)).

This note describes a brief analysis of all cases of high altitude turbulence classed as severe which have been reported over the British Isles at heights above 20,000 ft. Slight or moderate turbulence is not of great practical importance in the operation of aircraft and it therefore seemed best to restrict the investigation to the well marked severe cases, especially as any relationship between turbulence and meteorological features would be expected to be strongest in these cases. Because of the uncertainties introduced by interpolation of wind and temperature values, mentioned above, the analysis is merely an attempt to associate the severe turbulence with broad features of the general weather picture, lows and troughs in the upper air, jet streams and discontinuities in the tropopause surface. As was expected, jet streams are the weather features most frequently associated with severe turbulence and the analysis in this note is mainly concerned with the position of the turbulence relative to the jet stream axis.

2. Weather situations associated with severe turbulence

Instrumental observations of turbulence were made during the period

July, 1946 to November, 1948 between the levels 20,000 and 40,000 ft. by means of accelerometers fitted on Spitfire aircraft of the Royal Aircraft Establishment. (Port, (7)). British European Airways also made flights for the observation of high altitude turbulence using accelerometers (5). Accelerometers were also used on a few high altitude flights by the Meteorological Research Flight since the latter part of 1950. For the purposes of this analysis the turbulence was classified as severe when the measured acceleration imparted to the aircraft by the gust was greater than or equal to 0.4 g, (see reference (1)); there were six such cases in the R.A.E. series, one in the B.E.A. series which was over Britain and at a height above 20,000 ft., (other cases not over the British Isles were not analysed), and one from the M.R.F. flights.

Many other qualitative observations of high altitude turbulence were made by aircraft of the Royal Air Force, the Meteorological Research Flight and the De Havilland Aircraft Company≠ over or near Britain since 1948. 84 of these observations at heights above 20,000 ft. specified the turbulence as severe, heavy or violent. Thus, in all, 92 cases of what may be broadly classed as severe turbulence were available for analysis.

Unfortunately the qualitative assessments of turbulence are probably not comparable in all cases; there is the risk that a pilot may describe turbulence as severe at, say, 30,000 ft. because he considers it severe compared with the usual level of turbulence at that level rather than severe on an absolute scale independent of height.

Table I gives the numbers of cases occurring in various weather situations. The majority were associated with jet streams. These together with those classed as "probably near jet stream" constitute 67 per cent of the total. The six unclassified cases occurred in comparatively featureless upper air situations.

TABLE I

Weather situations associated with occasions of severe turbulence above 20,000 ft.

Weather situation	No. of cases
Near jet stream	56
Probably near jet stream (i.e. jet stream almost certainly present but details of windspeed, breadth etc. not available)	6
Upper trough	8
Upper low	2
Discontinuity in tropopause surface at same levels as turbulence and within 100 mi. of it.	2
Ditto and near upper trough	4
Ditto and near upper low	2
Strong upper winds but not a well defined jet stream	6
Unclassified	6
Total	92

≠ Readings from an accelerometer mounted in the cockpit of Comet aircraft were noted on several occasions; these readings however are not considered reliable as indications of the accelerations experienced at the centre of gravity of the aircraft. See Hislop (5).

3. Jet streams

The 56 cases of severe turbulence occurring above 20,000 ft. and in the vicinity of jet streams have been analysed with regard to their position relative to the jet stream axis. The method adopted was that used previously (reference (8)). Figure 1 shows a schematic vertical cross section at right angles to the general flow of a composite jet stream, obtained by taking a mean of three typical jet stream cross sections. The horizontal unit of measurement is the distance from the centre of the jet stream, on the low pressure side, over which the wind velocity at the same level as the jet axis falls to half its maximum value; the vertical unit is pressure relative to the pressure at the jet axis. As a guide to general characteristics the diagram shows isopleths of wind speed normal to the section and expressed as a percentage of the maximum, and also the mean position of the tropopause surfaces, all referring to the composite jet. (This same composite jet stream was discussed in reference (8)). On this diagram is plotted the position, relative to its jet axis, of each of the 56 turbulent zones associated with jet streams.

The definition of a jet stream is necessarily somewhat arbitrary. The criteria used in this analysis are best illustrated by the mean values and extremes of various jet stream parameters given in Table II. Estimates of the maximum wind speed and the position of the jet axis were made by a scrutiny of the wind observations from the British upper air observing stations and from those observations from continental stations which are plotted in the Daily Aerological Record. Since observing stations are of the order of 200 miles apart and observations are made at six-hourly intervals, there is necessarily some doubt as to the exact position of a jet stream axis at a particular time and its exact height (pressure) and the wind speed there, though errors in these have been reduced as far as possible by considerations of continuity. It is thought, however, that the accuracy is sufficiently good to ensure that the general zones of occurrence of turbulence shown in Figure 1 are reasonably reliable; the number of observations should be sufficiently large to counteract the effect of casual errors.

TABLE II

Mean values and extremes of various parameters of 56 jet streams in which severe turbulence occurred

	Maximum speed (kt.)	Pressure at axis (mb.)	Horizontal scale (n.mi.)
Mean	115	293	141
Extremes	80 (minimum)	-	54 (min.) 240 (max.)≠

The horizontal scale is the distance from the axis at the same level on the low pressure side at which the wind speed is half the value at the axis.

≠ The maximum scale value occurred with a jet speed of 90 kt.

The most striking result of Figure 1 is that severe turbulence was reported only on three occasions in the quadrant of the diagram below and on the high pressure side of the axis (i.e. lower left of the diagram). The greatest numbers of observations were just to the right of the jet axis in the diagram and nearly all observations to the left of the axis were above the level of the axis, six being in the stratosphere. The distribution of observations shown in Figure 1 is consistent with the scheme that most cases of severe

/turbulence

turbulence near a jet stream are associated with:-

(i) the frontal surface which is almost invariably associated with a jet stream. (Note the large number of cases just to the right of and below the axis and possibly the three cases to left of and below the axis in the diagram. For typical positions of the frontal surface relative to the axis see references (2) and (3)).

(ii) the zone near which the tropopause is usually discontinuous. (Note the large number of cases just above and to right of the jet axis in the figure).

(iii) the zone above the jet axis on the anticyclonic side (i.e. on the left of the diagram) associated with the rapid decrease of wind speed with height.

Regarding (iii) above, it is a fact of observation that the height of the level of maximum wind usually increases with distance from the jet axis on its anticyclonic side; correspondingly the zone of rapid decrease of wind with height is usually higher the further from the jet axis on this side. There is a tendency for the observations of turbulence shown in Figure 1 to slope upwards in a corresponding manner.

Zones (i) and (iii) above are usually associated with large shear of wind with height but zone (ii) is not always in a region of wind change with height. Well to the low pressure side and above the jet axis there is usually little change of wind with height though from Figure 1 it is seen that severe turbulence sometimes occurs there; in this region, however, there is considerable shear in the horizontal. It is impossible to argue from the general to the particular case in this connection, of course, as individual jet streams do vary significantly from each other. All that can be said is that most of the observations of severe turbulence near a jet stream were probably associated with large wind shear in the vertical but that it seems likely that a few cases were not associated with large vertical shear but with large horizontal shear. This would be consistent with the general results in reference (4) where it was found that turbulence, not necessarily near a jet stream and usually of slight intensity, was associated with general shear of wind in the vertical in three cases out of four and that both the cases of vertical shear and those of no general vertical shear were associated with large isentropic or horizontal shears.

Ten cases of severe turbulence occurred near the entrance and eight near the exit of jet streams. Four of the eight cases near a jet exit were on the anticyclonic side of and above the jet axis whereas none of the 10 cases near a jet stream entrance was in this zone. However, as the numbers of cases involved were so small, this apparent difference between entrance and exit of a jet stream may not be significant. No other peculiarities or variations in the occurrence of turbulence with respect to position along the jet stream were noted.

4. Discontinuity in tropopause surface

It is noted from Table I that eight cases of severe turbulence were associated with probable discontinuities in the tropopause surface at the same levels and within 100 miles; six of these cases were in the stratosphere and another was probably at or near the discontinuity. The decision as to whether the tropopause surface was discontinuous or not was made after scrutiny of available temperature soundings and using continuity considerations of both time and space. This analysis was necessarily somewhat subjective and there is the possibility that another analyst would have reached different conclusions in some cases.

/Many

Many of the other cases of severe turbulence plotted in Figure 1 were near and at the same level as the discontinuous tropopause surface which is a feature of most well-defined jet streams.

In reference (4) it was pointed out that the neighbourhood of a discontinuity in tropopause surface may often be a region of large wind shear in the vertical or large isentropic shear because of the proximity of cold tropospheric and comparatively warm stratospheric air at the same level on either side of the discontinuity of tropopause, respectively. Presumably the six cases of severe turbulence discussed above were associated with one or other of these large shears; in reference (4) it is suggested that isentropic shear might be the more likely but there is no evidence either way in these cases.

5. Conclusions

This brief analysis of occasions of severe turbulence occurring over or near Britain at heights above 20,000 ft. has shown that the major number occur in association with well-defined jet streams and that most of the other cases were associated with upper lows, upper troughs, discontinuous tropopause surfaces or in strong winds. Less than seven per cent of cases appeared to have no obvious association with any configuration of the upper air pattern. The cases associated with jet streams appeared to be grouped in particular zones relative to the jet axis, the majority (75 per cent) occurring on the low pressure side of the stream.

Turbulence in the upper troposphere and lower stratosphere, like many turbulence phenomena, appears to be somewhat haphazard in its occurrence. Forecasting such turbulence must therefore remain largely a statement of risks. The few results presented here may, it is hoped, be of use in helping forecasters to be more precise as to those regions in which the risk of severe turbulence is greatest.

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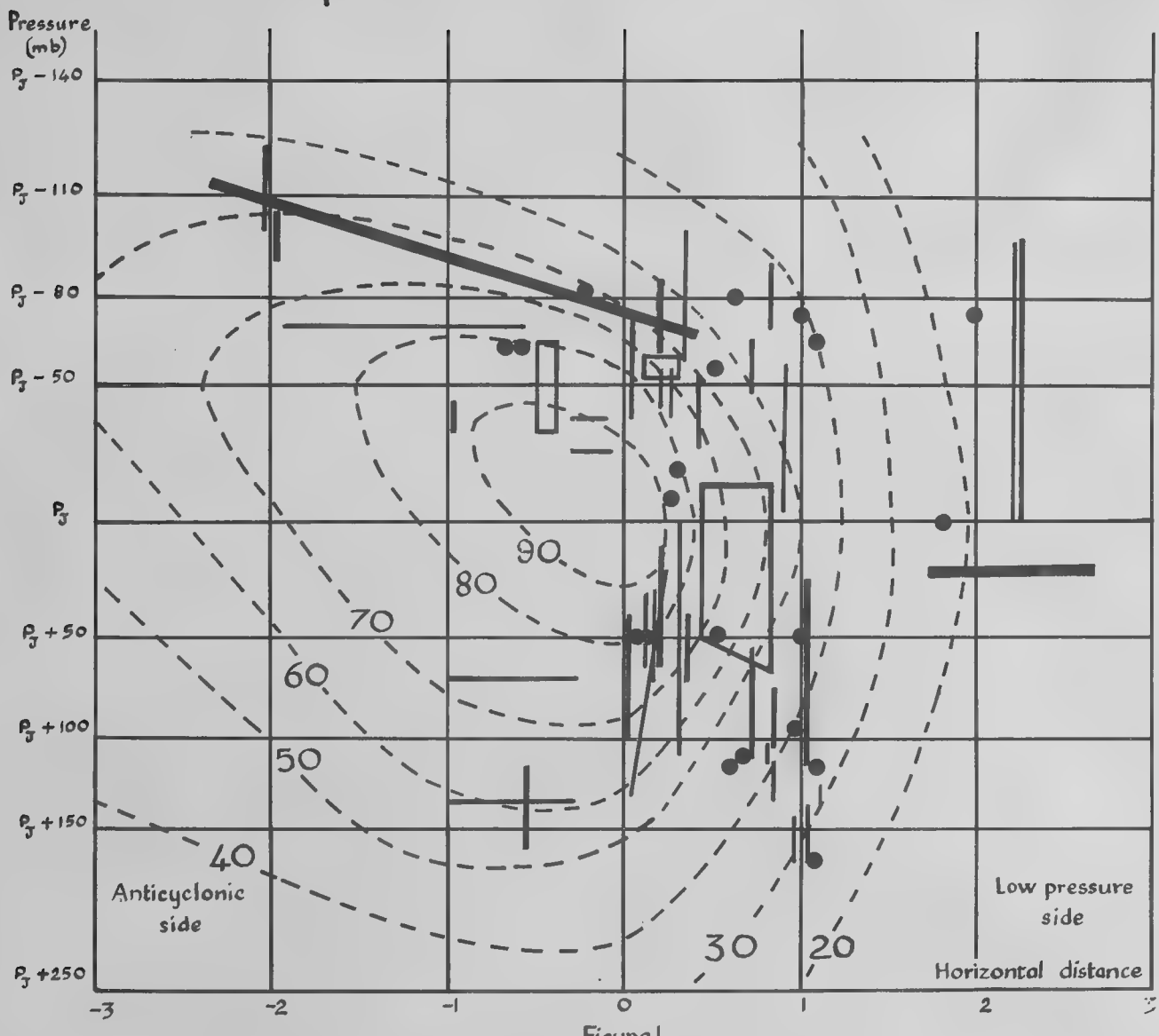

Figure 1.

Positions of occurrences of severe turbulence relative to the jet stream axis.

- - - - - Isopleths of wind velocity normal to section (percentage of maximum); mean of three typical jet streams.

Mean tropopause surface (three typical jet streams).

Severe turbulence observations.

P_J Pressure at axis of jet stream.

For details of horizontal and pressure scales see Table II.

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JUN 16 1952

M.R.P. 670
S.C.II/93

10th September, 1951.

A-11-15/015

AIR MINISTRY

METEOROLOGICAL RESEARCH COMMITTEE

A Report on Charney's two-dimensional method for computing the instantaneous height tendency

by
F.H. Bushby (Meteorological Office)

1. Introduction.

In a recent paper, Charney (1) proposed a numerical method for computing the rate of change of the 500 mb. contour height from the existing configuration of the 500 mb. contours. The method was based on the assumption that the large scale atmospheric motions can be treated as those of a barotropic fluid moving on a rotating globe. If it were possible to derive the height tendency $\partial z/\partial t$ from the existing field of the 500 mb. height z, it would be of fundamental importance as opening the way to numerical integration of the governing equation with respect to time and providing a basis for forecasting the 500 mb. chart. In his paper Charney (1) presented one example which suggested that close agreement between the observed and calculated values of the 500 mb. height tendency was possible, and in view of the fundamental significance of the work it was decided to perform similar calculations in the Forecast Research Division at Dunstable in order to form an opinion of the validity of the method. Calculations of the height tendency have been made on two synoptic charts and compared with the observed values and a study has also been made of some of the separate factors entering into the calculations.

On completion of this work we received a copy of a paper by Charney (2) which showed that similar but much more elaborate work had been carried out at Princetown. The conclusions reached by Charney were very similar to those obtained as a result of the less ambitious experiment conducted at Dunstable. The relevant sections of Charney's papers are discussed in this report, together with a brief summary of the work done in the Forecast Research Division. One of the two synoptic situations used at Dunstable for testing Charney's formula is reproduced in this report to illustrate the points discussed.

2. The Method.

Charney assumes that the horizontal winds ($\underline{V}$) in the large scale atmospheric systems vary according to the law

$$\underline{V}\,(s_1, s_2, p) = A(p)\,\underline{V}\,(s_1, s_2, p_o) \qquad \ldots\ldots\ldots\ldots (1)$$

where s_1 and s_2 are orthogonal curvilinear distance co-ordinates on the sphere, p is the vertical pressure co-ordinate and p_o the mean surface value of p. This implies that the wind direction is the same at all levels.

Charney (1) shows that the vorticity equation may be integrated through a vertical column and then gives the approximate form

$$\frac{\partial \overline{\zeta}}{\partial t} = -\overline{\underline{V}} \cdot \nabla (K\overline{\zeta} + f) \qquad \ldots\ldots\ldots\ldots\ldots (2)$$

where $\overline{\zeta}$ is the mean relative vertical vorticity component through the column, f is the Coriolis parameter and $K = \overline{A^2}/(\overline{A})^2$, the bar representing the vertical mean with respect to pressure.

In the original paper, Charney took K = 1 (computations show K $\doteq$ 1·25), but Charney subsequently shows (2) that by defining $\overline{A}^* = \overline{A^2}/\overline{A}$ a similar equation applies viz:-

$$\frac{\partial \zeta^*}{\partial t} = - \underline{V}^* . \nabla (\zeta^* + f) \qquad \ldots\ldots\ldots (3)$$

where $\underline{V}^*$ and ζ^* are respectively the wind velocity and relative vorticity at the level p^* defined by $A(p^*) = A^*$.

Equation (3) is true at all levels in a barotropic atmosphere, and p^* is said to be the "equivalent barotropic level". p^* also represents the level of non-divergence. p^* is found to be about 500 mbs., approximately 100 mbs. higher than $\overline{p}$. As 500 mb. height observations are made on a routine basis over a large area, the 500 mb. level is a convenient one to take as an approximation to the "equivalent barotropic level". Thus the method consists of treating the motion at 500 mbs. as though the vertical component of the absolute vorticity is conserved following the motion of the fluid.

Substituting the geostrophic approximation into equation (3), one obtains

$$\frac{\partial}{\partial t} \Delta_s z = J_s(\eta , z) \qquad \ldots\ldots\ldots\ldots (4)$$

where η is the absolute vorticity

$$\eta = \zeta + f \qquad \ldots\ldots\ldots\ldots (5)$$

where Δ_s is the surface spherical Laplacian operator and J_s is the Jacobian operator with respect to s_1 and s_2 .

By conformally representing the spherical earth by a stereographic projection from the south pole onto a tangent plane at the north pole, equations (4) and (5) transform into

$$\nabla^2 \frac{\partial z}{\partial t} = J (\eta , z) \qquad \ldots\ldots\ldots\ldots (6)$$

$$\eta = \frac{gm^2}{f} \nabla^2 z + f \qquad \ldots\ldots\ldots\ldots (7)$$

where ∇^2 is the plane Laplacian operator and J is the plane Jacobian operator referred to rectangular coordinates on the plane tangential to the earth at the north pole, and m is the magnification factor $\sec^2\{(\pi - 2\theta)/4\}$ where θ is the geographical latitude.

Given sufficient boundary conditions, equations (6) and (7) can be solved numerically for $\partial z/\partial t$ and η at a finite number of points. It is most convenient to use a rectangular grid. The boundary conditions employed must be sufficient to define the behaviour of z along the boundary. Charney (2) uses a heuristic argument to show that it is probably correct to assert that where fluid is entering the region enclosed by the boundary both z and the relative vorticity must be prescribed, but where fluid is leaving the region it is sufficient to prescribe z. Since these quantities are known only initially, their values on the boundary have to be fixed in some arbitrary manner, but as large scale influences travel with a velocity of the same order of magnitude as that of the wind the effect of these influences is provided for by integrating over a slightly larger area than the forecast area.

Charney (2) has used the Eniac, a high speed computing machine, to obtain numerical solutions to equations (6) and (7). The solution for z may be found iteratively by solving for $\partial z/\partial t$ and extrapolating the motion forward in time. Charney devotes section V of reference (2) to the computational stability of the finite difference equations. For hand computation, equation (6) can be solved for $\partial z/\partial t$ by the relaxation technique (3). This method was used in the Forecast Research Division at Dunstable, but the time integration was not attempted, because of lack of facilities and the poor agreement obtained between the computed 500 mb. tendency and the actual 500 mb. tendency meaned over a 24 hour period centred at the hour used for the computations.

3. The Forecast Research Division Experiment.

The two particular occasions which were investigated were 1500 G.M.T. April 22nd, 1950, and 1500 G.M.T. June 12th, 1950. The grid length chosen was approximately equal to 500 miles in middle latitudes. The experiment consisted of two parts; the differential form of equation (6) was examined, and the equation was integrated to solve for $\partial z/\partial t$. The results obtained in each case were fairly similar, and the method will be illustrated by charts referring to the situation on June 12th. Figure 1 shows the 500 mb. contour surface at 1500 G.M.T. on June 12th, 1950. The absolute vorticity (Figure 2) was computed from this map by using the finite difference approximation to equation (7). $J(\eta, z)$ can then be evaluated, and by multiplying $J(\eta, z)$ by the factor m^2g/f, we obtain the absolute vorticity advection (Figure 3).

The advection of the Coriolis parameter, $J(f, z)\ m^2f/g$, was also computed (Figure 4) to test whether it comprised an appreciable part of the absolute vorticity advection. An examination of Figures 3 and 4 shows that the advection of the Coriolis term is of the same order of magnitude as the absolute vorticity advection, although it is only in regions of strong meridional flow that the advection of the Coriolis parameter is appreciable. This result is only true when we are dealing with large scale atmospheric systems. In this problem the gradients of contour heights and vorticities have been averaged over a distance of the order of 1,000 miles. Similar results were obtained in the April case.

The local rate of change of vorticity (Figure 5) was computed by measuring the change of vorticity over a 24 hour period. By comparing Figures 3 and 5 one can test the validity of equation (6). There is a fair agreement between the regions of maxima and minima, although the magnitudes do not always agree. Certain errors of detail become important when the equation is integrated. Over North Baffin Land there was a bigger increase of cyclonic vorticity than could be accounted for by straightforward advection, similarly to the N.W. of Hudson Bay, whilst advection would have produced a bigger increase of cyclonic vorticity to the S.W. of the Hudson Bay than actually occurred. There were also large discrepancies over the Caspian Sea area, although these may have been due to faulty analysis as a result of lack of reliable observations. Over a limited area it was possible to obtain a value for $\partial\zeta/\partial t$ meaned over 12 hours, but this did not show appreciably better agreement with the absolute vorticity advection than the average over the 24 hour period.

The computed 500 mb. tendency (Figure 6) was evaluated by integrating equation (6) by the relaxation technique, and the results compared with the actual 500 mb. tendency (Figure 7) averaged over a 24 hour period. The boundary condition used was $\partial z/\partial t = 0$, this being a reasonable assumption as the boundary was entirely in the sub-tropics.

Correlation coefficients were calculated between the computed and actual tendencies at intervals of 10° of longitude along latitudes 45° and 60° North. Regression coefficients were calculated for the best fitting straight line giving the actual change (Y feet per hour) in terms of the computed change (X feet per hour) in the form

$$Y = aX + b \qquad (8)$$

Table I

	45°N			60°N		
	r	a	b	r	a	b
1500Z 22nd April, 1950	·28	·24	1·6	·59	·8	10·2
1500Z 12th June, 1950	·01	·01	2·5	·07	·1	1·2

r = correlation coefficient.

a and b (in feet per hour) = regression coefficients of equation (8).

Table II

Root mean squares in ft./hr.

(a) - of differences between computed and actual tendencies

(b) - of actual tendencies

	45°N		60°N	
	a	b	a	b
1500Z 22nd April, 1950	11·1	8·6	15·1	8·8
1500Z 12th June, 1950	9·9	7·5	9·5	7·6

From an inspection of Figures 6 and 7 and Tables I and II, it can be seen that there was poor agreement between the computed and actual 500 mb. tendencies for the case of June 12th, although there was a certain similarity between centres of maxima and minima. Although the correlation coefficients for the case of April 22nd (Figure 8) were greater than those for June 12th, the actual forecast for April 22nd (Figure 9) showed little agreement with actuality (Figure 10). This would be expected from the regression coefficients shown in Table I, and from Table II. In this case, the computed tendency was generally algebraically less than the actual tendency. The correlation coefficients of Table I only relate to 36 points in each case, and only refer to two parallels of latitude . From Table II it can be seen that a forecast of no change would have produced smaller R.M.S. errors in each case examined.

Conclusion.

Two cases only have been considered, and one cannot generalise from the results so far obtained. I think the main reason for the failure of the computations is that the model adopted is not sufficiently representative of the baroclinic atmosphere. Charney (2) suggests this is probably one of the reasons for the discrepancies. He suggests that the Sutcliffe (4) model with a thermal wind direction constant with height is synoptically more tenable than that of total wind direction constant with height. This conclusion had been arrived at independently at Dunstable. Some attempts have already been made in the Forecast Research Division to incorporate the effect of a thermal wind not parallel to the surface wind into Charney's (5) one-dimensional formula, but so far we have not attempted to include it in the two-dimensional method, although it is possible to do so.

In the work done at Dunstable, we have been comparing instantaneous computed tendencies with actual tendencies meaned over 24 hours. This may introduce apparent errors, but Charney (2) has carried out the time integration of equations (6) and (7) using 3-hour time intervals, and still has not succeeded in eliminating the major discrepancies which occur.

Both the work at Dunstable and that of Charney indicate that the "equivalent barotropic model" is an inadequate basis for forecasting by numerical integration of the equations of motion. However, the degree of agreement between charts 3 and 5, 6 and 7, and 9 and 10 does suggest that the idea of advection of absolute vorticity at the 500 mb. level is relevant to the problem of changes in the flow at that level. Moreover, Figure 4 shows that the effect of the variation of the Coriolis parameter is not negligible.

It therefore seems that it would be profitable to consider a model of the atmosphere which includes baroclinity. A suitable model is that used by Sutcliffe (4), and Charney (2) is contemplating work on these lines. It may be possible ultimately to combine the study of changes in the 500 mb. flow with evolution of development based on the work of Sutcliffe (4) as modified by Sumner (6).

References.

1. Charney, J.G. On a physical basis for numerical prediction of large-scale motion in the atmosphere. J. Met., Vol. 6, No. 6, 1949, p.371.

2. Charney, J.G., Fjörtoff, R. and Neumann J. Numerical integration of the Barotropic Vorticity Equation. Tellus, Vol. 2, No. 4, 1950, p.237.

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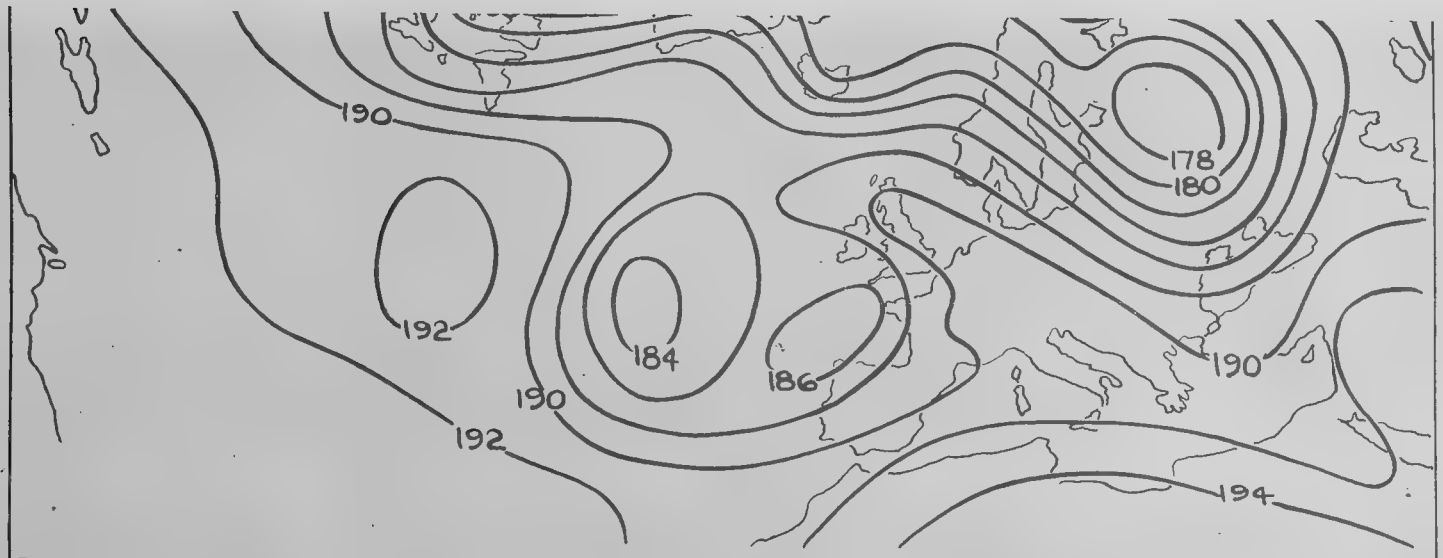

FIG I 500mb CONTOUR CHART (hundreds of feet.) 1500 G.M.T. 12th JUNE 1950.

References

3. Southwell, R.V. Relaxation methods in theoretical physics. Oxford, 1946.

4. Sutcliffe, R.C. A contribution to the problem of development. Q.J. Royal Met. Soc., Vol. 73, 1947, p.370.

5. Charney, J.G., and Eliassen A. A numerical method for predicting the perturbations of the middle latitude westerlies. Tellus, Vol. 1, No. 2, 1949, p.38.

6. Sumner, E.J. The significance of vertical stability in synoptic development. Q.J. Royal Met. Soc., Vol. 76, 1950, p.384.

tendencies for the case of June 12th, although there was a certain similarity between centres of maxima and minima. Although the correlation coefficients for the case of April 22nd (Figure 8) were greater than those for June 12th, the actual forecast for April 22nd (Figure 9) showed little agreement with actuality (Figure 10). This would be expected from the regression coefficients shown in Table I, and from Table II. In this case, the computed tendency was generally algebraically less than the actual tendency. The correlation coefficients of Table I only relate to 36 points in each case, and only refer to two parallels of latitude . From Table II it can be seen that a forecast of no change would have produced smaller R.M.S. errors in each case examined.

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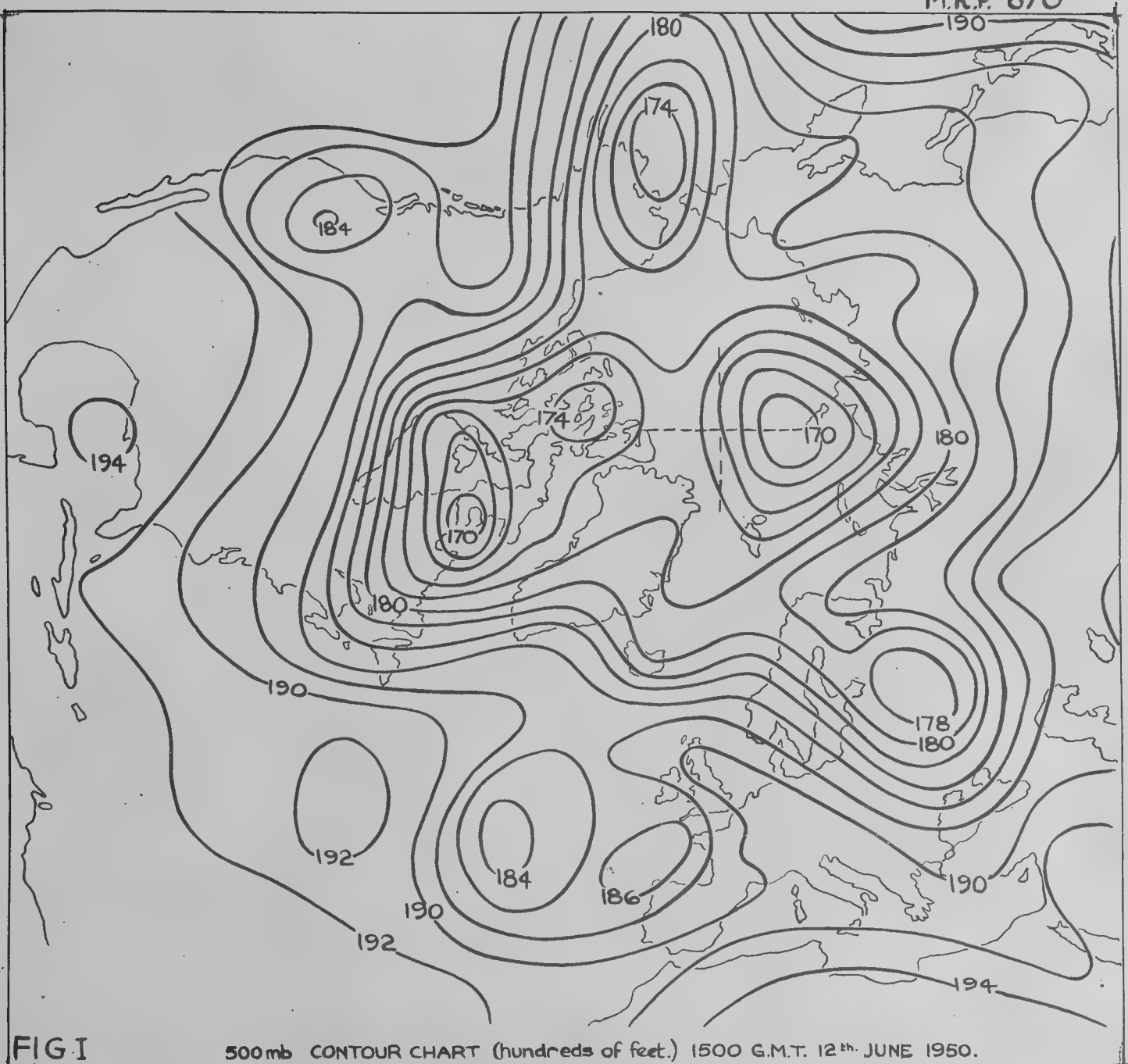

FIG I 500 mb CONTOUR CHART (hundreds of feet.) 1500 G.M.T. 12th JUNE 1950.

FIG 4. CORIOLIS' ADVECTION (10^{-4} hrs^{-2}) AT 1500 G.M.T. ON 12·6·50.

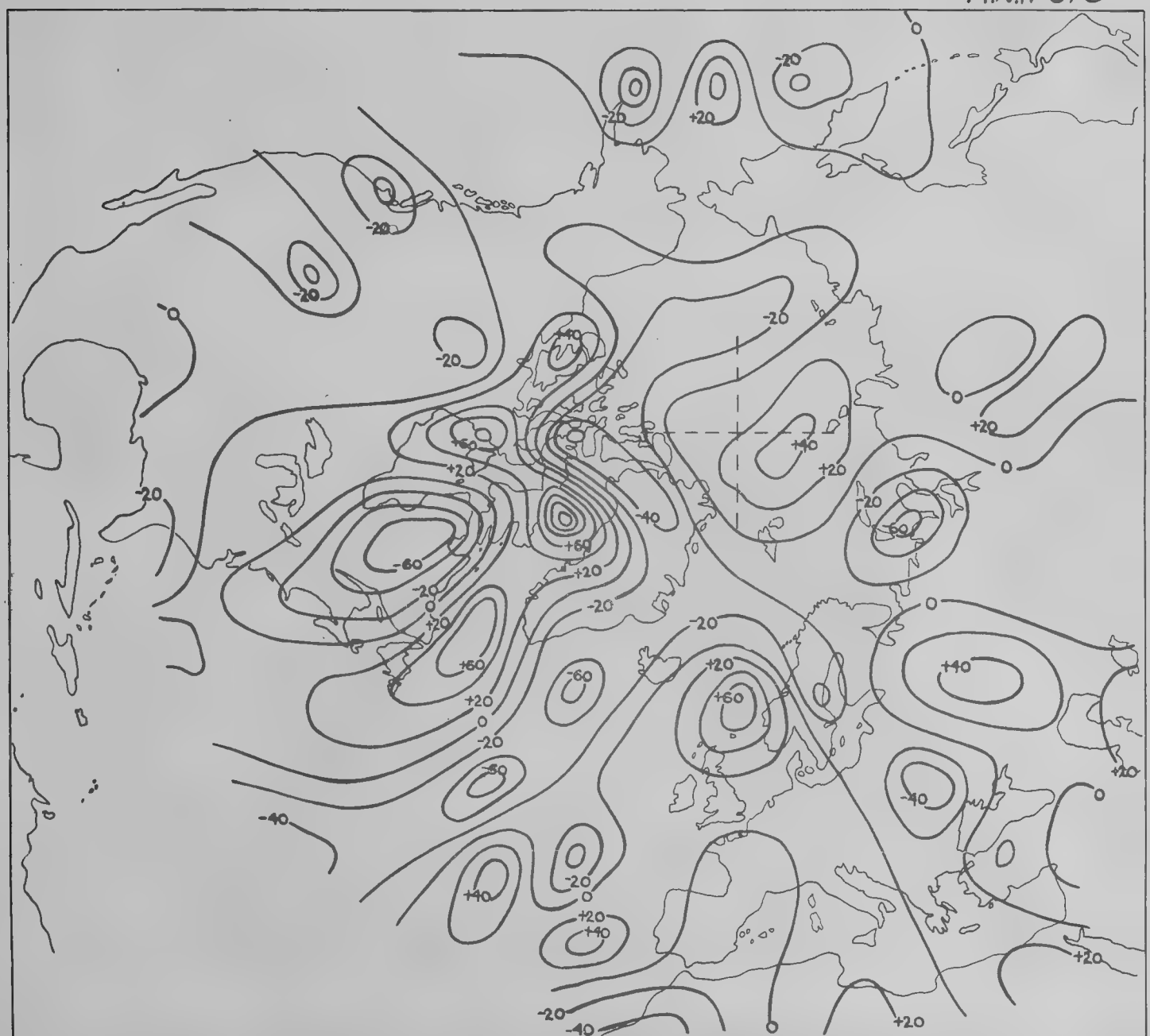

FIG. 5. LOCAL RATE OF CHANGE OF VORTICITY (10^{-4} hrs.$^{-2}$) AVERAGED FROM 0300 G.M.T. 12.6.50 TO 0300 G.M.T. 13.6.50

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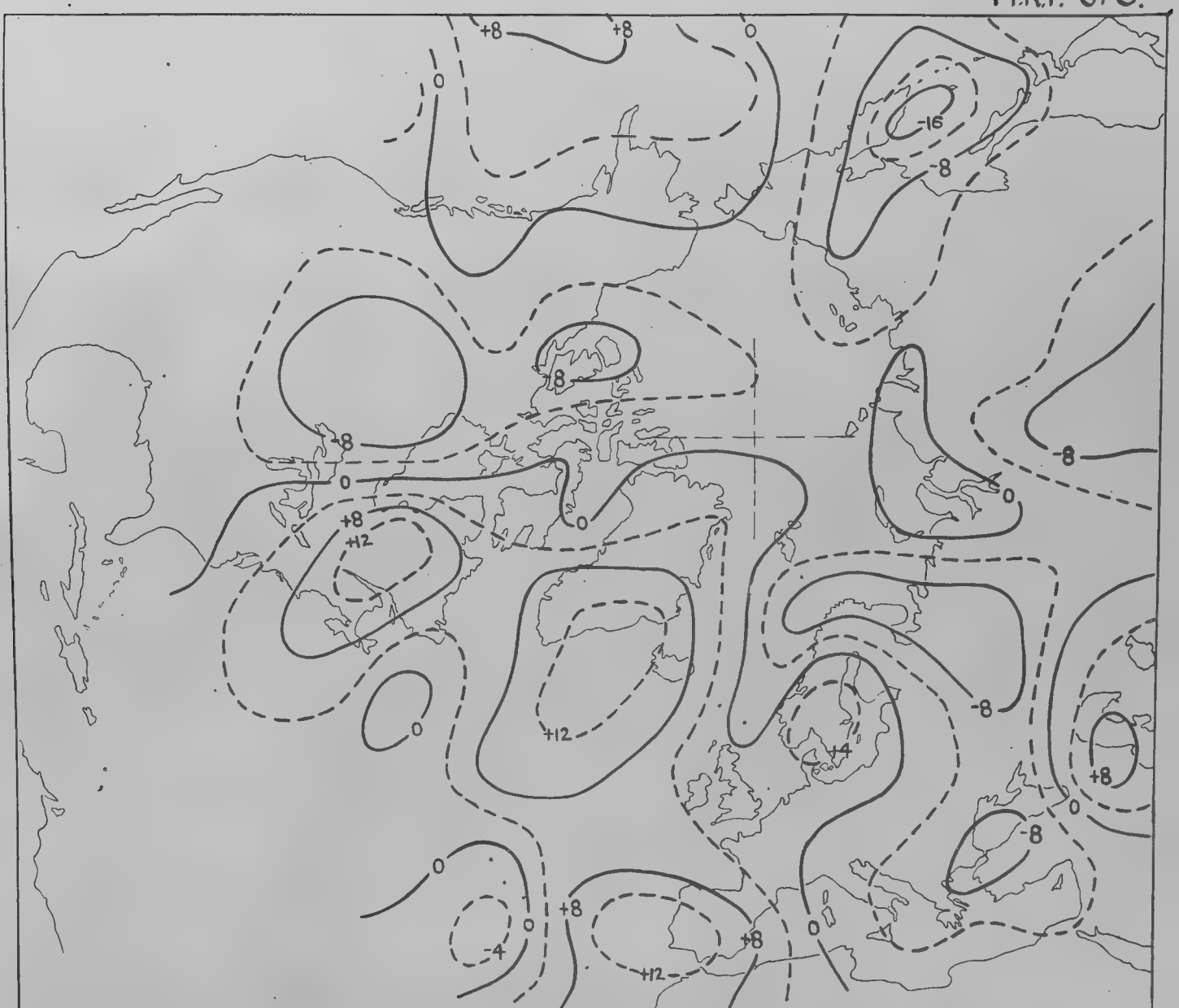

FIG 6. COMPUTED 500 mb. HEIGHT TENDENCY (ft/hr.) 1500GMT 12th. JUNE 1950.

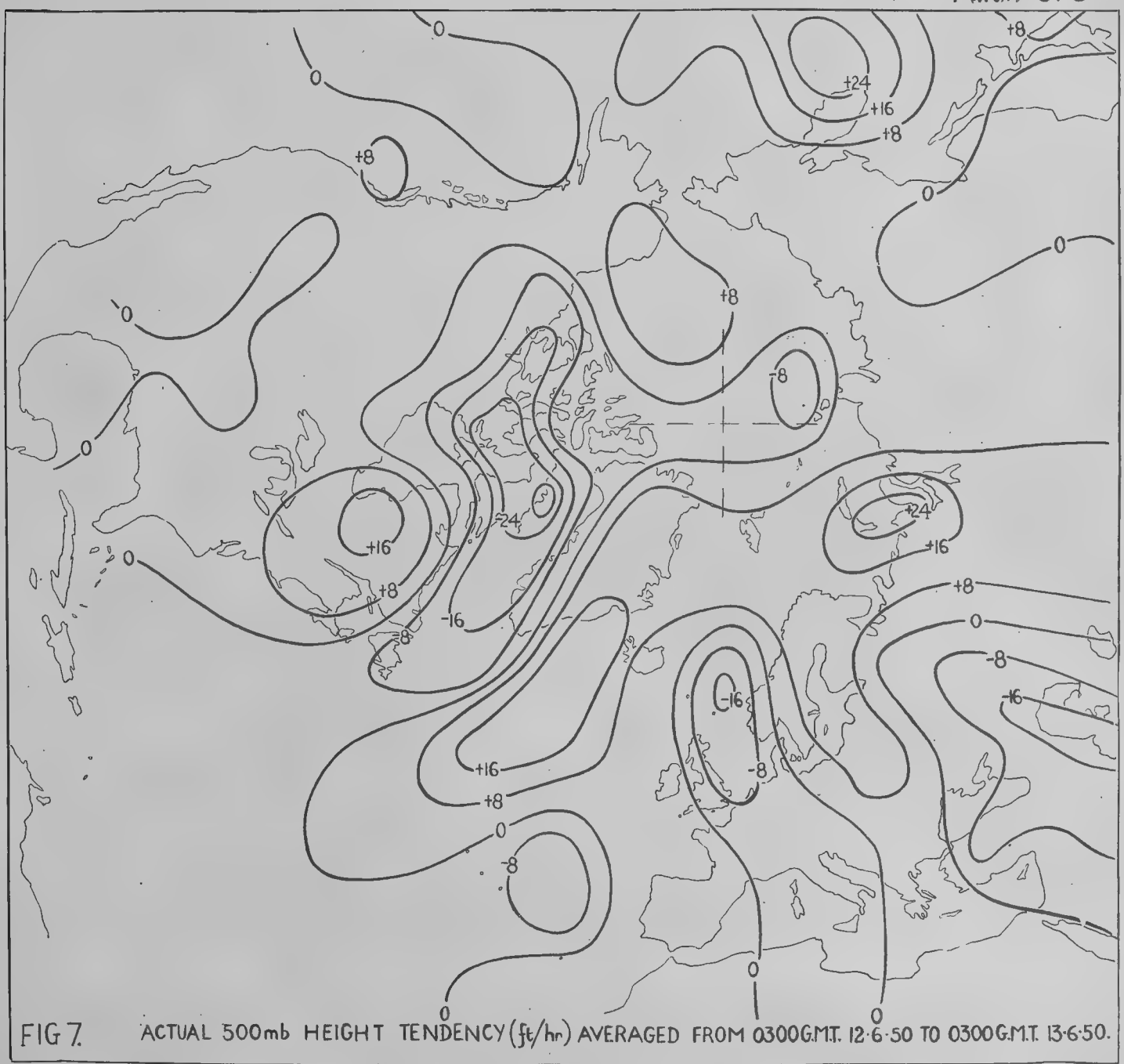

FIG 7. ACTUAL 500mb HEIGHT TENDENCY (ft/hr) AVERAGED FROM 0300 G.M.T. 12·6·50 TO 0300 G.M.T. 13·6·50.

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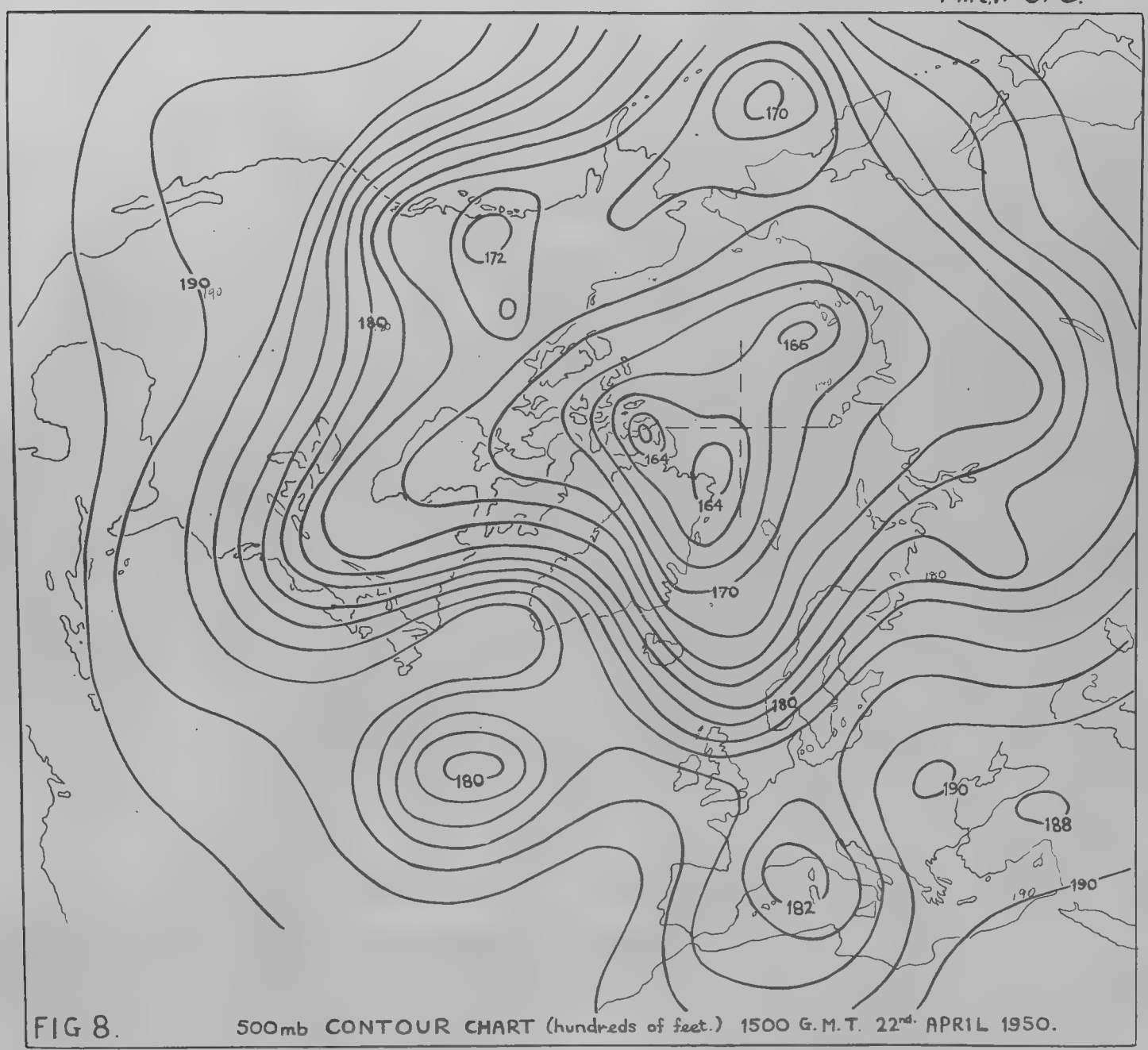

FIG 8. 500mb CONTOUR CHART (hundreds of feet.) 1500 G.M.T. 22nd. APRIL 1950.

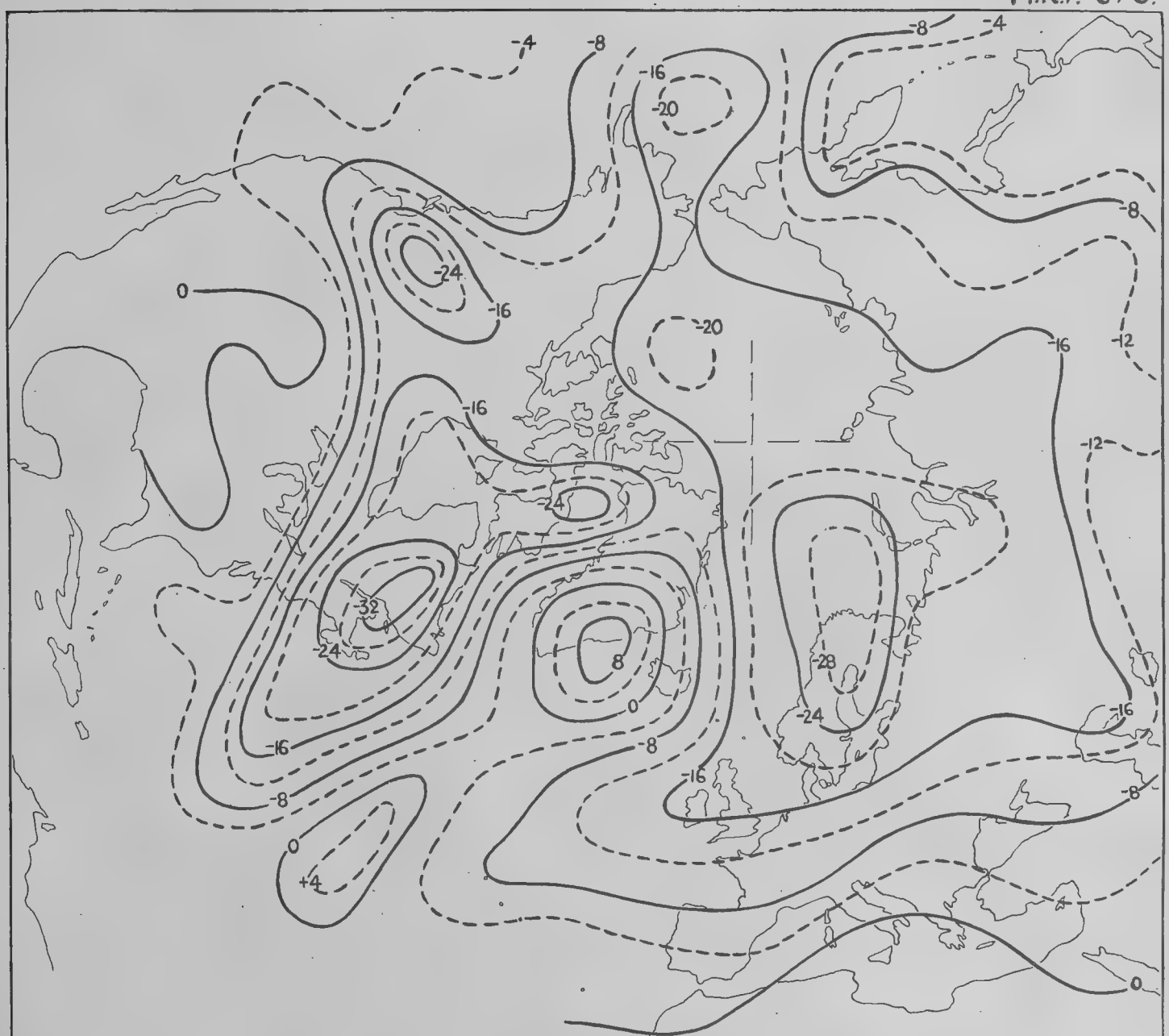

FIG.9. COMPUTED 500mb. HEIGHT TENDENCY (ft/hr.) 1500 G.M.T. 22nd APRIL 1950.

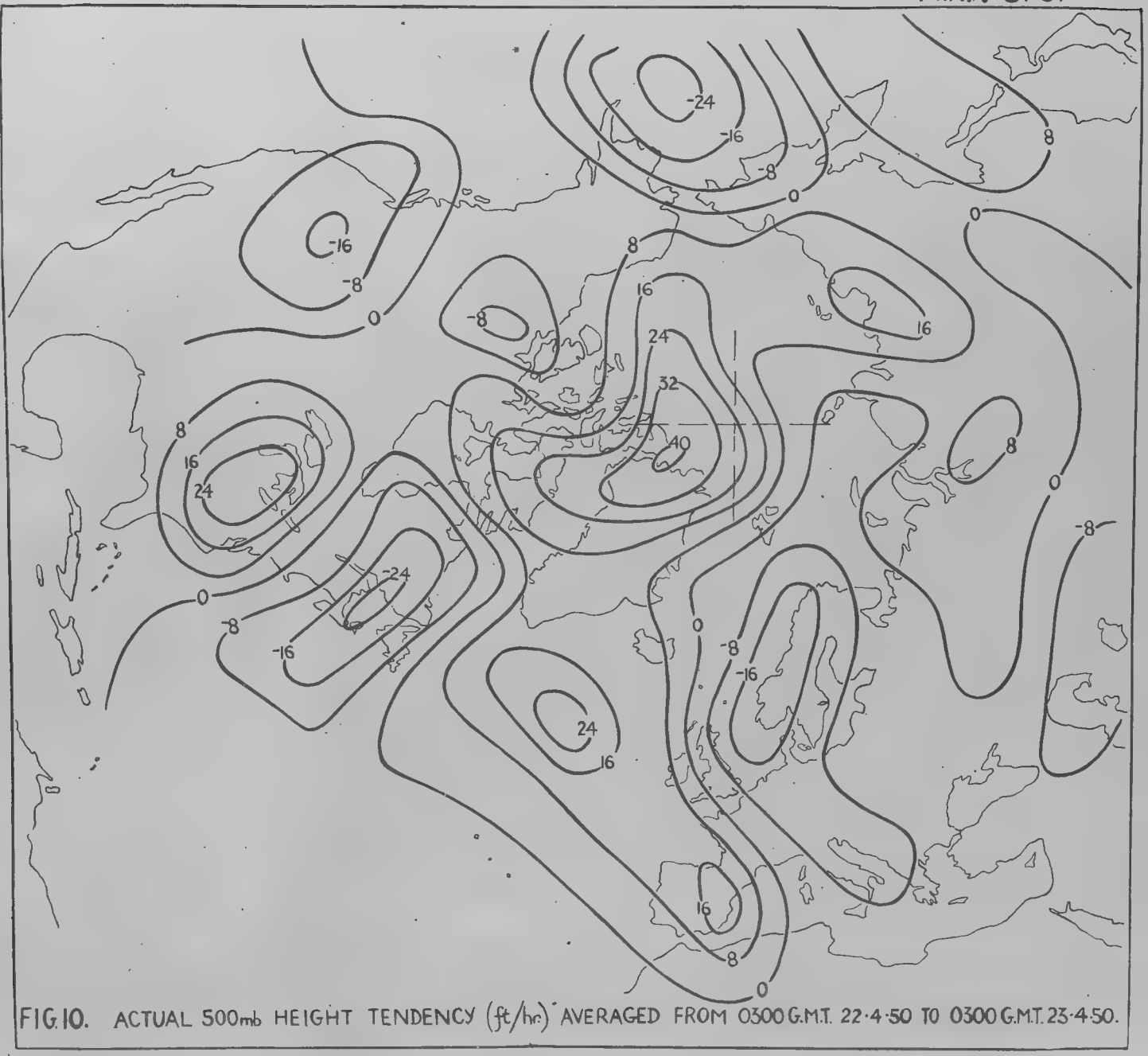

FIG.10. ACTUAL 500mb HEIGHT TENDENCY (ft/hr.) AVERAGED FROM 0300 G.M.T. 22·4·50 TO 0300 G.M.T. 23·4·50.

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METEOROLOGICAL RESEARCH COMMITTEE

The Diurnal and Seasonal Height of Pressure Contours

by

C.S. Durst (Meteorological Office)

1. Up to the present, no information has been published to throw light on the diurnal variation of pressure in the upper air. The data which are given below were first worked up in the hope that they might give an indication of the accuracy with which the diurnal variation of temperature can be calculated from radio sonde data, but the values of the diurnal variation of pressure are in themselves of considerable interest.

2. In Table I there are given the monthly heights of various pressure levels over Larkhill from the observations made in 1950.

TABLE I Mean monthly heights (tens of feet) of pressure levels over Larkhill, 1950, based on 4 observations a day

	Jan.	Feb.	Mar.	Apl.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Range
60 mb	6313	6287	6325	6366	6451	6493	6543	6542	6476	6390	6330	6249	294
100 mb	5261	5226	5274	5293	5353	5407	5436	5415	5391	5337	5260	5195	241
150 mb	4424	4376	4431	4428	4489	4543	4564	4545	4539	4499	4412	4353	211
200 mb	3834	3778	3844	3812	3879	3935	3945	3925	3931	3901	3810	3752	193
300 mb	2989	2925	3002	2946	3019	3066	3069	3048	3053	3037	2945	2902	167

These heights are based on the mean of four observations a day. The observations were fairly complete for the heights of 300, 200 and 150 mb and though at 100 mb there were some gaps, even in the worst month, November, they were based on 72 observations and in most months on between 90 and 100 observations. At 60 mb however the observations were scanty so the values in the table above were obtained by adding on the mean partial thickness 100 - 60 mb to the mean height of the 100 mb surface.

3. The diurnal variation of the heights of various pressure levels is given in Table II.

TABLE II Departure (in feet) from mean of day of the heights of certain pressure levels over Larkhill in 1950

	Hour	Winter	Spring	Summer	Autumn
60 mb	03	-83(10)	-307(8)	-255(6)	-275(11)
	09	-67(18)	+220(10)	+85(12)	+109(8)
	15	+190(31)	+215(28)	+236(23)	+168(24)
	21	+5(13)	-300(1)	-350(1)	-100(3)
Range		273(0.84mb)	522(1.61mb)	586(1.80mb)	443(1.36mb)
100 mb	03	-47	-187	-190	-117
	09	+27	+120	+107	+73
	15	+127	+160	+207	+123
	21	-107	-87	-130	-87
Range		234(1.17mb)	347(1.73mb)	397(1.99mb)	240(1.20mb)

Figures in brackets denote number of observations.

/150 mb

150 mb	Hour	Winter	Spring	Summer	Autumn
	03	-27	-130	-140	-70
	09	+10	+70	+107	+53
	15	+90	+140	+140	+87
	21	-80	-80	-100	-67
Range		170(1.34mb)	270(2.12mb)	280(2.20mb)	157(1.23mb)
200 mb	03	-20	-113	-110	-47
	09	0	+57	+50	+17
	15	+87	+97	+127	+63
	21	-70	-47	-73	-37
Range		157(1.50mb)	210(2.00mb)	237(2.25mb)	110(1.05mb)
300 mb	03	-17	-63	-50	-13
	09	-7	+13	+30	-10
	15	+50	+67	+60	+43
	21	-23	-13	-40	-7
Range		73(1.12mb)	130(2.00mb)	110(1.70mb)	56(0.86mb)

In calculating the 60 mb values weight has been given to the number of observations in each month of the quarter and in the result it looks as if the values give a fair representation of the diurnal variation at that level.

The ranges have been given in feet and these ranges have been converted into their equivalent value in millibars.

4. After the diurnal variation has been removed the random deviations for January and July are as shown in Table III.

TABLE III Standard deviation of height of pressure levels over Larkhill, 1950, after diurnal variation has been removed

HEIGHT	JANUARY		JULY	
	feet	mb ? 2.7 ?	feet	mb
100 mb	336	1.7	237	1.7
150 mb	310	3.1	253	2.0
200 mb	461	4.4	307	2.9
300 mb	478	7.4	319	4.9

5. Some further data which have been calculated for other regions are shown in Tables IV, V and VI.

TABLE IV Standard deviation of height of 100 mb pressure levels over Lagenhagen (near Hanover)

MONTH	JANUARY 1950	JULY 1949, 1950				
Hour	All hours	20, 21	02, 03, 04	08, 09	14, 15	Range
Height	52520 ft.	54260 ft.	54210 ft.	54590 ft.	54560 ft.	380 ft.
S.D.	300 ft.	160 ft.	200 ft.	210 ft.	260 ft.	
No. of Obs.	66	46	47	41	30	

TABLE V Standard deviation of height (feet) of 100 mb and 300 mb pressure levels over Nicosia (1949-50)

100 mb

Month	June	July	Aug.	June	July	Aug.	Dec.	Jan.	Feb.	Dec.	Jan.	Feb.
Hour	0300	0300	0300	1500	1500	1500	0300	0300	0300	1500	1500	1500
Height	54730	55165	54955	54900	55450	55170	52995	52915	53025	53085	53120	53135
S.D.	383	307	240	422	336	370	367	320	323	336	395	316
Number of Obs.	40	43	34	52	55	55	36	36	33	43	44	41

/300 mb

300 mb	June	July	Aug.	June	July	Aug.	Dec.	Jan.	Feb.	Dec.	Jan.	Feb.
Height	31305	31695	31680	31415	31830	31780	30030	30000	29860	30170	29990	29940
S.D.	307	221	202	305	240	268	299	458	508	380	482	486
Number of Obs.	44	46	39	57	59	56	44	46	42	60	58	54

TABLE VI Mean height (in feet) of 100 mb pressure level over Habbaniya in May to December, 1949

Month		May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Hour	0700								
	Height	53980	55060	55380	54950	54490	54050	53400	53090
	No. of Obs.	10	11	21	19	28	27	26	27
Hour	1400								
	Height	54230	55250	55420	55210	54670	54190	53390	53120
	No. of Obs.	27	23	25	19	27	20	27	23
Difference		250	190	40	260	180	140	-10	30
	(m.)	76	58	12	79	55	43	-3	9

6. These data can be summarised as follows:-

/(a)

T_s mean temperature of the platinum wire element.
$\overline{T}_a$ mean air temperature inside the pitot-head tube.
T_m mean temperature of the air in a channel.

(a) Over N.W. Europe the diurnal variation of pressure at constant level appears to increase with height up to about 40,000 feet but decreases above that height; near the normal height of the tropopause it averages about $2\frac{1}{4}$ mb in summer and about $1\frac{1}{2}$ mb in winter.

(b) Over Nicosia the diurnal range is about 0.70 mb (138 ft.) in winter at the 100 mb level and 1.30 mb (257 ft.) in summer; at the 300 mb level the range is almost negligible in winter and about 1.75 mb (115 feet) in summer.

(c) The standard deviation of pressure after removal of the diurnal variation decreases steadily with height above the 300 mb level over N.W. Europe both in winter and summer. It is greater in winter than in summer.

(d) The standard deviation of pressure after removal of the diurnal variation decreases over Nicosia in winter from about 7 mb at the 300 mb level to under 2 mb at the 100 mb level. In summer the decrease is from 4 to under 2 mb.

HEIGHT	JANUARY		JULY	
	feet	mb	feet	mb
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S.D.	300 ft.	160 ft.	200 ft.	210 ft.	260 ft.	
No. of Obs.	66	46	47	41	30	

TABLE V Standard deviation of height (feet) of 100 mb and 300 mb pressure levels over Nicosia (1949-50)

100 mb												
Month	June	July	Aug.	June	July	Aug.	Dec.	Jan.	Feb.	Dec.	Jan.	Feb.
Hour	0300	0300	0300	1500	1500	1500	0300	0300	0300	1500	1500	1500
Height	54730	55165	54955	54900	55450	55170	52995	52915	53025	53085	53120	53135
S.D.	383	307	240	422	336	370	367	320	323	336	395	316
Number of Obs.	40	43	34	52	55	55	36	36	33	43	44	41

/300 mb

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AIR MINISTRY

METEOROLOGICAL RESEARCH COMMITTEE

An assessment of the probable causes of variation of the speed correction coefficient of aircraft thermometers.

AIR WEATHER TECHNICAL RE...

by

D. D. Clark. (Meteorological Office)

Summary

The variations in the speed correction coefficient a of aircraft thermometers reported in M.R.P. 527 may arise from any of the following causes in greater or less degree as described in the following table.

Possible sources of variations in a	Form of Variation + increase - decrease o no change		Importance
	with altitude	with speed	
Variations in c_p, the specific heat of air (constant pressure)	+	o	Nil
Variations in Prandtl Number.	-	o	Nil
Variations in angle of attack (Thermometer on lower surface)	+	-	Important in bad sites
Ambient temperature error in indicator	+ or -	- or +	Important where it exists.
Variation in heat exchange in the thermometer (pitot-head only)	-	+	Most important
Transition from laminar to turbulent boundary layer	-	+	Most important

Notation.

u_o	air speed in cm/sec clear of aircraft.
V	air speed in knots.
u_1	air speed near the aerofoil where the thermometer is mounted, cm/sec.
u_2	air speed close to the thermometer itself, cm/sec.
u_m	mean speed inside the pitot-head thermometer = 1/5 U_o.
T_o	undisturbed air temperature.
T_T	total or stagnation temperature of arrested air in °C.
T	temperature at the surface of an aerofoil.
T_w	temperature of the outside wall of the pitot-head thermometer.
T_e	temperature at any point of the platinum wire temperature, element in the pitot-head thermometer.
T_a	temperature of the air at any distance x inside the pitot-head tube.
$\bar{T}_c$	mean temperature of the platinum wire element.
$\bar{T}_a$	mean air temperature inside the pitot-head tube.
T_m	mean temperature of the air in a channel.

ΔT	temperature rise of thermometer in °C.
ρ	air density in gm/cm³.
G	mass of air entering the pitot-head thermometer per second.
b	fore and aft width of a flat plate.
D	diameter of inside of pitot-head tube in cm.
d	diameter of circle of equivalent area to cross-section of temperature element and former, in cm.
a	half width of a channel.
l	effective length of inside of pitot-head tube = 3.7 cm.
α	speed correction coefficient for °F. and knots, $\Delta T = \alpha(V/100)^2$ °F.
β	speed correction coefficient for °C and cm/sec., $\beta = 0.2097 \times 10^{-7}\alpha$
λ	defined as $\lambda = 2 c_p \beta$
c_p	specific heat of air at constant pressure in units stated.
k	coefficient of thermal conductivity of air in units stated.
μ	coefficient of viscosity of air in c.g.s. units.
ν	kinematic viscosity = μ/ρ
σ	Prandtl number = $\mu c_p/k$
R	Reynolds number = xu/ν
Nu	Nusselt number.
A_w	rate of exchange of heat between air and wall inside pitot-head thermometer per unit length of tube.
A_e	rate of exchange of heat between air and platinum wire inside pitot-head thermometer per unit length of tube.
k_e	rate of loss of heat, per unit length of tube, from platinum wire by conduction through the frame.
C_f	skin friction coefficient.
Q_o	amount of heat transferred from unit area per second, ergs/sec.
k_H	heat transfer coefficient defined in the text.

1. Introduction.

In R.&.M. 527 (1) Shellard has summarised the evidence indicating that a variation takes place in the speed correction coefficient of aircraft thermometers both with altitude and, in one particular case, with air speed. He also mentions some possible reasons for the variations.

In what follows an attempt has been made to list all factors which could possibly affect the speed correction coefficient and to examine each thoroughly in turn.

Air which is arrested in the boundary layer of an aerofoil experiences a rise in temperature due to the conversion of kinetic energy into heat. Some of the acquired heat will be lost by conduction through the boundary layer to the main flow but, at the same time, more heat is being created in the boundary layer by friction which offsets the losses by conduction. The heat balance between the loss by conduction and the gain by friction depends on the non-dimensional parameter, the Prandtl number, σ, defined as

$$\sigma = \frac{\mu c_p}{k}$$

If the Prandtl number is unity the two effects cancel out and the innermost layer, which is stationary relative to the aerofoil, acquires the stagnation adiabatic temperature rise. When air is the medium, however, $\sigma < 1$, and there is a net loss of heat by conduction; so that, in this case, the temperature rise at the surface is somewhat less.

For laminar flow, Pohlhausen (2), assuming constant density, has shown that the temperature rise at a flat plate, where there is no heat exchange at the surface, is expressible as

$$\Delta T = f(\sigma) \frac{u_o^2}{2 c_p} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

where $f(\sigma)$ is a function of σ only, being equal to a close approximation to $\sigma^{1/2}$.

Applying equation (1) to the flow past an aerofoil, where T_1 is the temperature, and u_1 the air speed, near the surface, and T is the

temperature of the surface

$$T - T_1 = f(\sigma) \frac{u_1^2}{2 c_p}$$

whence, since

$$T_1 + \frac{u_1^2}{2 c_p} = T_o + \frac{u_o^2}{2 c_p},$$

$$T - T_o = \Delta T = \frac{u_o^2}{2 c_p} \left[1 - \left(\frac{u_1}{u_o}\right)^2 \left\{ 1 - f(\sigma) \right\} \right]$$

Substituting $f(\sigma) = \sigma^{1/2}$ this formula becomes

$$\Delta T = \frac{u_o^2}{2 c_p} \left[1 - \left(\frac{u_1}{u_o}\right)^2 \left\{ 1 - \sigma^{1/2} \right\} \right] \quad \ldots\ldots\ldots\ldots (2)$$

Crocco (3) has shown that this formula is valid also for laminar flow in a compressible medium and therefore can be applied at all air speeds.

Dryden (1936) (4) has shown that, in the boundary layer, on an aerofoil, three states of flow can exist, depending, for any given fluid, on the pressure gradient, the state of the surface, the Reynolds number and the degree of turbulence in the air stream. At low Reynolds numbers the flow is laminar, as the Reynolds number increases however, an eddy flow appears which finally develops into turbulent flow. The region of eddy flow, usually referred to as the transition zone, is of complex structure and varies erratically with time.

A more recent article by Emmons (11) describes in more detail the mechanism of transition as revealed by recent experiments. The turbulence according to Emmons starts in spots where the induced oscillations in the laminar layer become unstable; these spots then move separately downstream growing and increasing in number at the same time until all the boundary layer is turbulent.

In problems where both laminar and turbulent flow are considered it is customary to neglect the length of the transition zone and postulate an instantaneous transition. Dryden (4) found that the point of transition was very sensitive to small pressure gradients.

The equivalent expression for the temperature rise for a turbulent boundary layer has been given by Squire (5) in the form

$$\Delta T = \frac{u_o^2}{2 c_p} \left[1 - \left(\frac{u_1}{u_o}\right)^2 \left\{ 1 - \sigma^{1/3} \right\} \right] \quad \ldots\ldots\ldots (3)$$

There is every reason to believe that this formula also holds at all speeds provided no marked pressure gradients are present.

Formulae (2) and (3) can be applied to describe the flow over a flat plate thermometer. If u_1 refers to the flow past the thermometer and u_2 to the flow in the immediate vicinity of the thermometer surface then the speed correction factor β, defined as $\Delta T = \beta u_o^2$ is obtained from (2) by replacing u_1 by u_2, and is

$$\beta = \frac{1}{2c_p} \left[1 - \left(\frac{u_2}{u_o}\right)^2 \left\{ 1 - \sigma^{1/2} \right\} \right] \quad \ldots\ldots\ldots\ldots (4)$$

$$= \frac{1}{2c_p} \left[1 - \left(\frac{u_2}{u_1}\right)^2 \left(\frac{u_1}{u_o}\right)^2 \left\{ 1 - \sigma^{1/2} \right\} \right] \quad \ldots\ldots (5)$$

for a laminar boundary layer

and from (3)

$$\beta = \frac{1}{2c_p} \left[1 - \left(\frac{u_2}{u_o}\right)^2 \left\{ 1 - \sigma^{1/3} \right\} \right] \quad \ldots\ldots\ldots (6)$$

$$= \frac{1}{2c_p} \left[1 - \left(\frac{u_2}{u_1}\right)^2 \left(\frac{u_1}{u_o}\right)^2 \left\{ 1 - \sigma^{1/3} \right\} \right] \quad \ldots\ldots (7)$$

for a turbulent boundary layer.

Neither (2) nor (3) however are adequate to describe the conditions inside a pitot-head thermometer where the temperature changes due to heat losses are significant.

The effect on β of variations in all the quantities on the R.H.S. of (5) and (7) will now be considered, including the effect of the change in the exponent of σ with transition in the boundary layer.

2. Variations in c_p.

Various theoretical and empirical formulae for c_p have been given from time to time, all of which differ from each other. The following two selections quoted by Partington (6) show the pressure dependence of c_p

$$c_p = 0.23702 + 0.0015504\,(p - 1) \quad \text{(Lussana)}$$

$$c_p = 0.2414 + 0.000286\,p \quad \text{(Holborn and Jakob)}$$

where c_p is measured in calories per gram and p in atmospheres.

The changes in c_p with pressure indicated by these formulae are too small to affect our results. It will therefore be assumed that c_p for air does not vary with pressure changes in the atmosphere.

Partington (6) also gives a selection of formulae showing the temperature dependence of the molecular heat per mol at constant pressure for various gases, including oxygen and nitrogen. From these formulae an expression in terms of temperature for the specific heat of air at constant pressure has been calculated, assuming an oxygen to nitrogen ratio by weight of 1 : 3, and ignoring the presence of the rarer atmospheric gases.

The formulae for C_p, the molecular heat at constant pressure, for oxygen and nitrogen, are given in the following form

$$C_p = a + bT + cT^2$$

C_p being measured in calories per mol, and T in °K.

The following set of values for a, b and c, quoted by Partington, are due to Spencer and Justine (1934).

	a	$b \times 10^3$	$c \times 10^7$
O_2	6.0954	3.2533	- 10.171
N_2	6.4492	1.4125	- 0.807

and the following to Bryant and Taylor (1934)

	a	$b \times 10^3$	$c \times 10^7$
O_2	6.26	2.746	- 7.70
N_2	6.30	1.819	- 3.45

The corresponding values of a, b and c for the specific heat of air at constant pressure in calories per gram per °C. have been worked out from these formulae and are given below

Air	a	$b \times 10^3$	$c \times 10^7$
S and J	0.22037	0.06325	- 0.10108
B and T	0.21765	0.07018	- 0.15257
mean	0.2190	0.0667	- 0.1268

The values of c_p for air using the mean values deduced above for a, b and c have been calculated for four separate temperatures, three of which correspond in the I.C.A.N. scale to the pressure levels 900, 500 and 300 mb. and one in the tentative N.A.C.A. scale (7) to the isothermal stratosphere up to 32 km. These values of c_p are given in column 6 of Table I both in c.g.s. units and in calories. They show that the variation in c_p alone is

too small too affect β appreciably and that in any case its variation is opposed to that required to account for the observed α variation.

3. Variations in the Prandtl Number.

The Prandtl number is defined as

$$\sigma = \frac{\mu c_p}{k}$$

For air, $\sigma < 1$, and variations in σ could cause variations in the speed correction factor. We have just seen that small variations with temperature occur in c_p, it remains to be seen therefore what happens to the ratio μ/k at low temperatures.

On the assumption of equi-partition of energy in molecular collisions, μ/k can be expressed, to a second approximation, as a linear function of γ, - the ratio of the two specific heats - considered constant over the temperature range in question. In order to test the validity of the assumption of equi-partition of energy in molecular collisions for the lowest temperatures reached, the ratio μ/k has been calculated from independently determined experimental values of both μ and k. The values used for μ are those obtained by Johnston and McCloskey (1940) (8) in America, and for k, those by Taylor and Johnston (1946) (9), also in America. Both sets of values are claimed to be accurate to $\frac{1}{2}\%$. Only the variations with temperature are considered as the change with pressure in both μ and k is many times too small to have any effect.

The experimental values of μ and k, interpolated from the tables from the sources quoted, are given in columns 3 and 4 of Table I, and the calculated ratio μ/k is given in column 5. The values of σ, $\sigma^{1/2}$ and $\sigma^{1/3}$ formed from these values of μ/k together with the previously calculated values of c_p, appear in columns 7, 8 and 9. The values of $\sigma^{1/2}$ and $\sigma^{1/3}$ at the different altitudes scarcely change, but the differences between $\sigma^{1/2}$ and $\sigma^{1/3}$ at any one level are significant. This point will be discussed later.

4. Variations in (u_1/u_o) and the significance of u_2/u_1.

The ratio u_1/u_o at any part of an aerofoil does not change with the Reynolds number but does change with the inclination of the aerofoil to the airstream. At the distance from the surface at which the thermometer is situated, however, only a small part of the total variation in u_1 is felt. For an under-wing or under-fuselage mounting, u_1/u_o decreases as the angle of incidence of the aerofoil increases, which implies that any variation in β from this cause would consist of an increase with altitude and a decrease with speed, which is the opposite of that reported.

If a variation from this cause were present, then mounting the thermometer on an extension clear of the aircraft ought to remove it.

This may explain why the α measured for the under-nose mounting on aircraft ST 796 (Table III of M.R.P. 527, reproduced as Table II) remains constant at different altitudes whilst, on the same aircraft, a thermometer on an extension platform under the nose gives a decrease in α with altitude. It is difficult to see, however, why the same effect should not be present also on the other Halifax ST 817 for a similar under-nose mounting.

Since variations in u_1/u_o affect the flat-plate thermometer, but not the pitot-head thermometer, it is worth noting that the pitot-head thermometer on Halifax ST 796, mounted under the nose, indicates a decrease in α with altitude, whereas, as reported, the flat-plate thermometer, under the nose, has no α variation.

The ratio u_2/u_1 for any given thermometer should remain constant, on the average, and will depend on the thermometer thickness and the shape of the leading edge. Although the ratio u_2/u_1 is not, in itself, a cause of a variation in α, a high value of u_2/u_1 accentuates changes due to transition and augments any variation already present from this cause, as is shown in the increase in the disparity between the entries against

pressure for laminar and turbulent boundary layer conditions, of the value of β for $(u_2/u_1)^2 = 2$ as against $(u_2/u_1)^2 = 1$, (assuming $u_1 = u_o$) given in Table I.

5. Changes in boundary layer flow.

We can regard the flat plate thermometer as a thin section aerofoil with parallel sides. If u_2 is the air speed at any point near the surface of the thermometer and u_1 the air speed past the thermometer then u_2/u_1 is constant except at the edges. Since u_2 is small at the edges the rate of exchange of heat there, with the air, is also small, which means that the edges will contribute only slightly to the mean temperature of the thermometer surface.

Since the Reynolds number decreases with height, transition from laminar to turbulent flow, if present, should occur during descent. It could also occur at any level, with an increase of speed, provided the critical Reynolds number is passed.

Transition tends to occur behind regions of minimum pressure. On the flat-plate thermometer such regions would occur behind the leading edge and also behind any corrugation on the sides, since the sides could not be expected to be completely flat and smooth. Transition therefore will not be a gradual process but can be expected to proceed in stages as one favourable region after another becomes operative, until the whole surface behind the leading edge has a turbulent boundary layer.

Applying these theories to the data from the Mosquito flights plotted in curve 1 of Figure 1 (reproduced from M.R.P. 527) we obtain the following results.

Value of α at 300 kts = 1.82
corresponding value of λ at 300 mb. = 0.75
Value of α at 200 kts = 1.55
corresponding value of λ at 300 mb. = 0.64

If we assume that at the lowest speeds the boundary layer is wholly laminar, then, from equation 5, since

$$\lambda = 2 c_p \beta$$

$$\lambda = 1 - \frac{(u_2)^2}{(u_1)} (1 - \sigma^{1/3})$$

Putting $\lambda = 0.64$, $c_p = 0.9772 \times 10^7$ c.g.s. units, $\sigma^{1/3} = 0.8443$ (the two latter from Table I)

$$\therefore \frac{(u_2)^2}{(u_1)} = 2.308 \; ; \quad \frac{u_2}{u_1} = 1.519$$

If now the proportion of the total surface area at any time under a laminar boundary layer be 'a_1' and under a turbulent boundary layer be 'a_2' ($a_1 + a_2 = 1$) then for the whole thermometer

$$\lambda = a_1\lambda_1 + a_2\lambda_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(8)$$

where λ_1 and λ_2 refer to laminar and turbulent boundary layers respectively*.

From (5) and (7) remembering that $\lambda = \beta/2c_p$ and that $a_1 + a_2 = 1$

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*Equation (8) is only strictly true if the surface is a heat insulator but for the small temperature differences between surface and boundary layer which do arise from thermal conduction along the surface, the effects of the differential rates of heat exchange, as between laminar and turbulent portions, may be neglected.

$$\lambda = 1 + \left(\frac{u_2}{u_1}\right)^2 \left[a_1\sigma^{1/2} + (1 - a_1)\sigma^{1/3} - 1\right]$$

whence

$$a_1 = \frac{1 - \lambda - \left(\frac{u_2}{u_1}\right)^2 (1 - \sigma^{1/3})}{\left(\frac{u_2}{u_1}\right)^2 (\sigma^{1/3} - \sigma^{1/2})} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(9)$$

Putting $\lambda = 0.75$, and from Table I, $\sigma^{1/2} = 0.8486$ and $\sigma^{1/3} = 0.8962$ and using the value of $(u_2/u_1)^2$ just found, we obtain $a_1 = 0.028$.

In this case, therefore, the reported variation in α with air speed could be explained if almost complete transition to turbulence took place between the least and greatest air speeds. The corresponding Reynolds numbers $u_2 b/\nu$ where b = 2 cm. are

$$9.4 \times 10^4 \quad \text{and} \quad 1.4 \times 10^5.$$

It is interesting to note that the lowest critical Reynolds number at which Dryden (4) found transition to take place on a flat plate was 9×10^4. Curve 2 of Fig. 1 shows no variation with speed, although the corresponding values in Table II show a variation with altitude. Let us assume therefore, that, for this particular thermometer, the flow has remained laminar throughout the speed range at 500 mb.

The mean value of α taken from curve 2 is 1.64, corresponding to $\lambda = 0.68$ at 500 mb. At 900 mb, from Table II, $\alpha = 1.78$, i.e. $\lambda = 0.74$.

Assuming laminar flow at 500 mb, from equation (5) we obtain

$$\left(\frac{u_2}{u_1}\right)^2 = 2.084; \qquad \frac{u_2}{u_1} = 1.444$$

and from equation (7)

$$a_1 = 0.43$$

which means that, at 900 mb., transition would have to occur over a little more than half the thermometer surface to explain the observed change in α with height. The highest Reynolds number $u_2 b/\nu$ at 500 mb., corresponding to speed 236 knots, is 1.5×10^5, and the lowest Reynolds number at 900 mb. is 1.8×10^5. The conclusion is therefore that, either transition occurs later on this thermometer than on the one on the Mosquito, or else, that a counter-acting tendency is present, caused, say, by variations in u_1/u_0, as has been discussed above.

A similar treatment can be applied to the other observations in Table II which show variations with altitude of the meaned values with respect to airspeed of α at the different pressure levels.

In each case if we assume a laminar boundary layer at the lowest Reynolds numbers and transition or partial transition to a turbulent boundary layer at the highest Reynolds numbers, then all the observations in the tables can be accounted for except those for the under-nose mounting on Halifax No. ST 817 (Table II). In this case, proceeding on the same lines as before, but this time assuming turbulent flow at 900 mb, we get $\alpha = 2.05$, i.e. $\lambda = 0.85$, at 900 mb.

$$\therefore \left(\frac{u_2}{u_1}\right)^2 = 1.442$$

whence the corresponding value of α, assuming a purely laminar boundary layer at 500 mb, should be 1.93, whereas, in fact, it is 1.70. In this case therefore none of the reasons so far discussed can account for all of the variation. The possibility that instrumental inaccuracies may affect the answer will now be discussed.

6. Variations arising within the thermometer itself.

Apparent variations in the speed correction factor could have the following instrumental origins:-

(i) Changes in the thermometer temperature-indicator reading with changes in ambient temperature not allowed for in the calibration.

(ii) Changes of heat exchange efficiency inside the thermometer with changes in Reynolds number. Applicable only to the pitot-head thermometer.

Consider the first possibility, namely the existence of errors arising from ambient temperature effects. In the type of indicator used with the thermometers on which the α variations were detected, the specification allows a maximum variation of $\pm 0.4°F$. in reading for an ambient temperature range from $+120°F$ to $0°F$. Although it is unlikely that the ambient temperature varied by as much as $120°F$ during these flights, nevertheless, as a test case, we could assume that at the lowest temperature which the indicators experienced, the readings did suffer a + or - $0.4°F$ error, not allowed for in the calibration done at ground temperature. An examination of the laboratory test reports of this type of indicator showed that, whereas the majority acquire a positive error at the lowest ambient temperature, there are some which show a negative error: to explain a decrease in α with height a negative error would be necessary.

In fact if

ΔT_1 is the true temperature rise

ΔT_2 is the indicated temperature rise

then the corresponding speed correction factors α_1 and α_2 are given by

$$\Delta T_1 = \alpha_1 \left(\frac{V}{100}\right)^2$$

and

$$\Delta T_2 = \alpha_2 \left(\frac{V}{100}\right)^2$$

But if $\Delta T_1 = \Delta T_2 + 0.4$

then

$$\alpha_1 - \alpha_2 = 0.4 \left(\frac{100}{V}\right)^2$$

Since α_1 is the same at all levels $\alpha_1 - \alpha_2$ represents the variation in α. If V = 200 kts, $\alpha_1 - \alpha_2 = 0.1$.

Although it is unlikely that the permitted maximum error is ever present, except perhaps in a few cases, it is seen that, at speeds of 200 kts, its presence as a negative error would be sufficient to account for a large part of any reported decrease of α with altitude, but that, with increasing speeds, its influence would diminish.

The presence of an ambient temperature error can also cause an apparent variation of α with speed.

Let α_1, ΔT_1 and α_2, ΔT_2 be the values of α and ΔT at the two speeds V_1 and V_2 respectively, and assume again that the error is negative, then we have -

$$\left.\begin{aligned} \Delta T_1 &= \alpha_1 \left(\frac{V_1}{100}\right)^2 \\ \Delta T_2 &= \alpha_2 \left(\frac{V_2}{100}\right)^2 \end{aligned}\right\}$$

Let accents denote true values of ΔT_1 and ΔT_2

$$\therefore \quad \left.\begin{aligned} \Delta T_1' - 0.4 &= \alpha_1 \left(\frac{V_1}{100}\right)^2 \\ \Delta T_2' - 0.4 &= \alpha_2 \left(\frac{V_2}{100}\right)^2 \end{aligned}\right\}$$

i.e.

$$\left.\begin{aligned} \alpha\left(\frac{V_1}{100}\right)^2 - 0.4 &= \alpha_1\left(\frac{V_1}{100}\right)^2 \\ \alpha\left(\frac{V_2}{100}\right)^2 - 0.4 &= \alpha_2\left(\frac{V_2}{100}\right)^2 \end{aligned}\right\}$$

$$\therefore \quad \alpha_2 - \alpha_1 = 0.4\left[\left(\frac{V_2}{100}\right)^2 - \left(\frac{V_1}{100}\right)^2\right]\left(\frac{100}{V_1}\right)^2\left(\frac{100}{V_2}\right)^2$$

$$= 0.4\left[\left(\frac{100}{V_1}\right)^2 - \left(\frac{100}{V_2}\right)^2\right]$$

e.g. if V_1 = 200 kts, V_2 = 300 kts

$$\alpha_2 - \alpha_1 = 0.4\,(5)\,\frac{1}{9\times4}$$

$$= 0.06$$

The smaller V_1 the greater the variation in α will be. However, over the speed range of the Halifax flights the effect would be very small, but a significant variation would arise over the speed range of the Mosquito. The variation can only occur when the ambient temperature is least and can have any sign depending on the sign of the indicator error.

The second possibility, namely the variations in efficiency of thermal exchange, only applies to the pitot-head thermometer, because the air flow past the flat plate is sufficiently great to dominate the temperature indicated, whereas, in the interior of a pitot-head, the arrested air is surrounded by potential thermal exchangers, and the temperature which the sensitive element acquires depends on the balance of thermal exchange finally reached.

7. The Pitot-head thermometer.

The principle of the pitot-head thermometer is to measure the temperature of air which has been arrested in the interior of a pitot-tube or similar device. Because the arrested air is at a higher temperature than its surroundings great precautions have to be taken to prevent heat losses from the air before its temperature can be measured, and at the same time, an efficient thermal exchange with the sensitive temperature element must exist. Lack of attention to these details will result in inaccurate measurements.

The pitot-tube thermometer used in the test described in M.R.P. 527 was the resistance bulb, impact type IT 3-1 as shown in Fig. 2. In the prototype of this thermometer, the body of the tube was made of a thermally insulating material, but, in the production models such as were used, the material was brass. The temperature element was platinum wire (SWG 47) wound on a star-sectioned former of synthetic resin fabric, which was supported on a metal rod projecting from the base of the tube.

In a design of this character the following avenues of heat exchange will have to be considered:-

(i) Losses from the arrested air to the metal walls of the tube

(ii) Losses from the platinum wire by conduction through the synthetic bonded-resin former to the metal supporting rod.

Radiation losses are very small and have been neglected.

The equations of thermal balance are therefore

$$Gc_p(T_T - T_a) = \int_0^x A_c(T_a - T_c)dx + \int_0^x A_w(T_a - T_w)dx \quad \ldots\ldots(10)$$

$$A_c(T_a - T_c) = k_c(T_c - T_w) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(11)$$

The boundary conditions are $T_a = T_T$ at $x = 0$ and the solution to equations (10) and (11) is

$$T_T - T_a = (T_T - T_w)(1 - e^{-\eta x})$$

where

$$\eta = \frac{k_c A_e + k_c A_w + A_c A_w}{G c_p (k_c + A_c)}$$

If $\overline{T}_a$ represents the mean value of T_a over the whole length (l) of the inside of the tube then

$$T_T - \overline{T}_a = (T_T - T_w)\left[1 - \frac{1 - e^{-\eta l}}{\eta l}\right] \quad \ldots\ldots\ldots(12)$$

whence

$$T_T - \overline{T}_e = (T_T - T_w)\left[1 - \frac{A_c}{k_c + A_c}\left\{\frac{1 - e^{-\eta l}}{\eta l}\right\}\right] \quad \ldots(13)$$

where $\overline{T}_e$ is the average temperature of the platinum wire.
Now

$$T_T - \overline{T}_e = \frac{(1 - \lambda_e)}{2c_p} u^2 \quad \text{and} \quad T_T - T_w = \frac{(1 - \lambda_w)}{2c_p} u^2$$

$$\therefore \quad 1 - \lambda_e = (1 - \lambda_w)\left[1 - \frac{A_c}{k_c + A_c}\left\{\frac{1 - e^{-\eta l}}{\eta l}\right\}\right] \quad \ldots\ldots(14)$$

where λ_e refers to the thermometer as a whole and
λ_w refers only to the outside wall.

To apply this formula to the thermometer in question, values for A_w, A_c and k_c were calculated.

To find A_w we proceed as follows

Define $$k_H = \frac{Q_o}{\rho c_p u_m (T_m - T_w)}$$

whence $$A_w = k_H \rho c_p u_m \pi D$$

k_H is the heat transfer coefficient for flow in a channel. T_m is the mean temperature in the channel which in this case is $\overline{T}_a$.

In order to express k_H in terms of known quantities, Kármán's generalisation of the formulae based on Reynolds' analogy for heat transfer and skin friction was used, as given by Goldstein (10) p. 657, namely -

$$\frac{1}{k_H} = \frac{2}{c_f} + 5\sqrt{\frac{2}{c_f}}\left[\sqrt{\sigma - 1} + \log_e\left[1 + 0.83(\sigma - 1)\right]\right] \quad \ldots(15)$$

Putting $\sqrt{2/c_f} = 5.1\, R^{1/8}$, where $R = 2au_m/\nu$, which is the value which Prandtl found for flow in channels, (15) becomes

$$\frac{1}{k_H} = 5.1\, R^{1/8}\left[5.1\, R^{1/8} + 5\left[\sqrt{\sigma - 1} + \log_e(1 + 0.83\sqrt{\sigma - 1})\right]\right] \quad \ldots(16)$$

Now D = 1.2 and 'a' = 0.1

$\therefore$ <u>at 900 mb.</u> $R^{1/8} = 2.663$, $\frac{1}{k_H} = 183.25$

$$\underline{A_w = 4.54 \times 10^5}$$

$\therefore$ at 500 mb. $\quad R^{1/8} = 2.536, \quad \frac{1}{k_H} = 166.1$

$$\underline{A_w = 3.09 \times 10^5}$$

To evaluate A_e we use the relationship

$$A_e = Nu\, k\, \pi\, n\, r$$

where n is the number of turns of wire per centimetre length of tube and r is the length of one turn in cm. Given the fundamental interval of the resistance element is 40 ohm and the S.W.G. = 47

$$\therefore n = 19.2$$

Also $\quad r = 2\sqrt{2}$

Using Hilpert's values of Nusselt number against Reynolds number for a circular cylinder in transverse flow given on p.637 of ref. 10 we obtain,

at 900 mb. $\quad R = 64.3, \quad Nu = 0.50$

$$\therefore \underline{A_e = 2.066 \times 10^5}$$

at 500 mb. $\quad R = 43.5, \quad Nu = 0.18$

$$\therefore \underline{A_e = 0.682 \times 10^5}$$

For k_e only tentative values can be found based on plausible assumptions; for example making the following assumptions.

The wire is in contact with the support for 0.3 cm. of its length at each of the four points and that it is in contact for half its circumference. The synthetic resin former was assumed to conduct like a rectangular parallelepiped whose base is the projection of the area of contact with the wire and whose depth to the metal rod is 0.3 cm. The coefficient of conductivity of the material of the former was assumed to be 10^{-3} cal. per °C. across a cm^3. On the basis of these assumptions, and, remembering that there are four points of contact and 19.2 turns per cm.

$$k_e = 1.66 \times 10^4$$

Substituting these values for A_w, A_e and k_e and $l = 3.7$ in (14) we get

At 900 mb. $\quad 1 - \lambda_e = 0.1315(1 - \lambda_w)$

i.e. $\quad \lambda_e = 0.8685 + 0.1315\, \lambda_w \quad \ldots\ldots\ldots\ldots(17)$

and at 500 mb. $\quad 1 - \lambda_e = 0.2501(1 - \lambda_w)$

i.e. $\quad \lambda_e = 0.7499 + 0.2501\, \lambda_w \quad \ldots\ldots\ldots\ldots(18)$

Unfortunately the values of α found for the pitot-head thermometer as given in M.R.P. 527 are suspect because the greatest value, that of 2.52 for aircraft ST 796 at 900 mb., is greater than the maximum theoretical value, assuming no heat losses, which at 900 mb. has the value 2.407, corresponding with $\lambda = 1$. The same is also true of the value formed from the entries of $\alpha_{ph} - \alpha_{fp}$ and of α_{fp} for the same aircraft, ST 796.

e.g. at 900 mb. $\quad \alpha_{ph} - \alpha_{fp} = 0.71$

$$\alpha_{fp} = 1.78$$

$$\therefore \alpha_{ph} = 2.49$$

No values at all are given for ST 817 at 900 mb.

If it is assumed that no change takes place in λ_w then from (17) and (18)

$$\left(\frac{\lambda_c}{2c_p}\right)_{900} - \left(\frac{\lambda_c}{2c_p}\right)_{500} = 0.0571\ 10^{-7} - 0.0708\ 10^{-7}\ \lambda_w$$

putting $\lambda_w = \sigma^{1/2} = 0.85$

$$\therefore \quad \left(\frac{\lambda_c}{2c_p}\right)_{900} - \left(\frac{\lambda_c}{2c_p}\right)_{500} = 0.0054\ 10^{-7}$$

$$\text{i.e.} \quad \alpha_{e_{900}} - \alpha_{e_{500}} = 0.03$$

If on the other hand λ_w also suffers a decrease with altitude similar to the flat plate the variation of α would be correspondingly increased.

In fact if we assume that the maximum change corresponding to complete transition takes place, then, if $\lambda_{w_{900}}$ and $\lambda_{w_{500}}$ represent the values of λ_w at 900 and 500 mb. respectively,

$$\lambda_{w_{900}} = 1 - \left(\frac{u_2}{u_1}\right)^2 (1 - \sigma^{1/3})$$

$$= 1 - \left(\frac{u_2}{u_1}\right)^2 (0.1038)$$

$$\lambda_{w_{500}} = 1 - \left(\frac{u_2}{u_1}\right)^2 (1 - \sigma^{1/3})$$

$$= 1 - \left(\frac{u_2}{u_1}\right)^2 (0.1535)$$

$$\therefore \quad \lambda_{w_{900}} = 0.3238 + 0.6762\ \lambda_{w_{500}}$$

whence from (17) and (18) after converting the λs to αs

$$\alpha_{e_{900}} - \alpha_{e_{500}} = 1.551 - 0.646\ \alpha_{e_{500}}$$

If we put $\alpha_{e_{500}} = 2.22$ (the average value from Table II)

$$\alpha_{e_{900}} - \alpha_{e_{500}} = 0.12$$

which would make $\alpha_{e_{900}} = 2.34$.

The only value given for $\alpha_{e_{900}}$ is 2.52 which, as already stated, must be suspect.

To test the effect of other values of the conductivity loss k_e, $\alpha_{e_{900}} - \alpha_{e_{500}}$ has been calculated also for $k_e = 10^3$, 5×10^4, 10^5 for $\alpha_{e_{500}} = 2.22$ and the results given below.

When $k_e = 10^3$, $\quad \alpha_{e_{900}} - \alpha_{e_{500}} = 0.08$

" $k_e = 5 \times 10^4$ $\quad \alpha_{e_{900}} - \alpha_{e_{500}} = 0.11$

" $k_e = 10^5$ $\quad \alpha_{e_{900}} - \alpha_{e_{500}} = 0.12$

From these calculated values it is seen that only a small change in the variation is achieved by reducing the leak coefficient k_e from 10^5 to 10^3. Other points to note are that the loss to the frame, of heat, from the air, although contributing to the temperature deficiency of the thermometer, does not introduce any variation over and above that due to

transition on the outside of the tube. The two factors affecting the α variation are the balance between the rate of gain of heat by the platinmun wire element and the rate of loss from the platinum wire to the frame, and the variations in frame temperature due to transition in flow pattern on the outside. If the insulation of the element to frame were improved and if the tube were made of a non-thermal conductor, as in the prototype, then it ought to be possible to reduce the α variation for this type of thermometer to negligible quantities.

8. Conclusions and recommendations.

A pitot-head thermometer in which care has been taken to eliminate heat losses from the sensitive element to the frame ought not to suffer variations in α even if it does not record the full stagnation temperature rise owing to heat losses from the arrested air.

A flat-plate type of thermometer ought to be mounted on a platform clear of the aircraft to ensure an airstream whose speed relative to the thermometer is unaffected by the proximity of the aircraft.

Care should be taken in the design to maintain a laminar boundary layer over the sensitive surface area throughout the range of Reynolds numbers likely to be encountered. A completely laminar boundary layer is preferable to a completely turbulent one because, apart from questions of drag, it is not always possible to ensure complete turbulence right from the leading edge and, moreover, it is believed that there may be degrees of turbulence.

With these points in view it is considered that a conical or wedge shaped thermometer bulb should give good results.

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Table I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
									$\beta \times 10^7$							
P	T	$\mu \times 10^7$	k	$\mu/k \times 10^7$	$c_p \times 10^{-7}$	$\mu c_p/k = \sigma$	$\sigma^{1/2}$ (1)	$\sigma^{1/3}$ (2)	$(u_2/u_1)^2 = 0$ (1)	(2)	$(u_2/u_1)^2 = 1$ (1)	(2)	$(u_2/u_1)^2 = 2$ (1)	(2)	$\frac{\rho}{\rho_o}$	$\rho/\mu = 1/\nu$
900	281 (ICAN)	1760.8	2423 (5789 x 10^{-8} cal)	0.7268	0.9906 (0.2367 cal)	0.7199	0.8486	0.8952	0.505	0.505	0.428	0.452	0.352	0.400	0.909	6.330
500	252 (ICAN)	1617.8	2221 (5306 x 10^{-8} cal)	0.7285	0.9835 (0.2350 cal)	0.7164	0.8465	0.8948	0.508	0.508	0.430	0.455	0.352	0.401	0.565	4.282
300	228.5 (ICAN)	1433.9	(2047) (4893 x 10^{-8} cal)	0.7298	0.9772 (0.2335 cal)	0.7131	0.8443	0.8933	0.512	0.512	0.432	0.457	0.352	0.402	0.375	3.077
-	218 (NACA)	1436.8	1966 (4699.4 x 10^{-8} cal)	0.7307	0.9747 (0.2329 cal)	0.7122	0.8439	0.8922	0.513	0.513	0.433	0.458	0.353	0.402		

Table II

Mean values of α_{fp}, α_{ph}, $\alpha_{ph}-\alpha_{fp}$ from Halifax runs.

(from Table III of M.R.P. 527.)

1. Mean values of α_{fp}

Mounting and Aircraft.	Low Level 900 mb. approx.	Medium Level 700 mb. approx.	High Level 500 mb. approx.
Under nose ST 817	2.05 (1)	-	1.70 (1)
.. .. ST 796	1.77 (1)	-	1.77 (1)
On extension under nose ST 796	1.92 (1)	-	1.79 (3)
Under wing ST 796	1.78 (2)	-	1.64 (7)
.. .. ST 817	-	1.71 (1)	1.61 (1)
All mountings	1.86 (5)	1.71 (1)	1.69 (13)
All under wing mountings	1.78 (2)	1.71 (1)	1.64 (8)

2. Mean values of α_{ph}

Under nose ST 796	2.52 (1)	-	2.27 (2)
.. .. ST 817	-	2.32 (1)	2.11 (1)
All mountings	2.52 (1)	2.32 (1)	2.22 (3)

3. Mean values of $\alpha_{ph} - \alpha_{fp}$

ST 796 (fp under wing) (ph under nose)	0.71 (3)	0.64 (1)	0.59 (2)
ST 817 (.. .. ..)	0.66 (2)	0.63 (6)	0.59 (6)
All mountings	0.69 (5)	0.63 (7)	0.59 (8)

N.B. Bracketed figures indicate number of measurements.

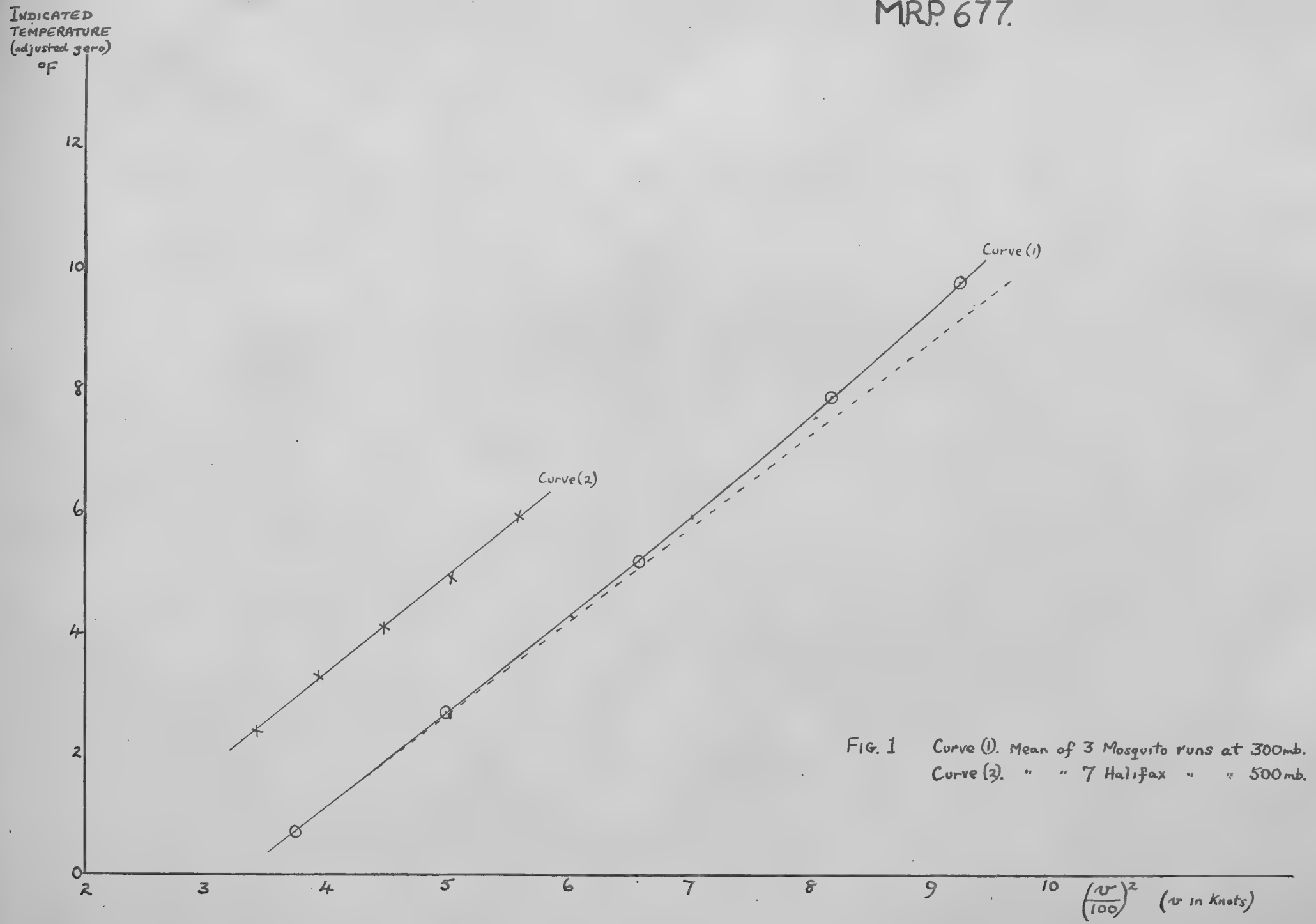
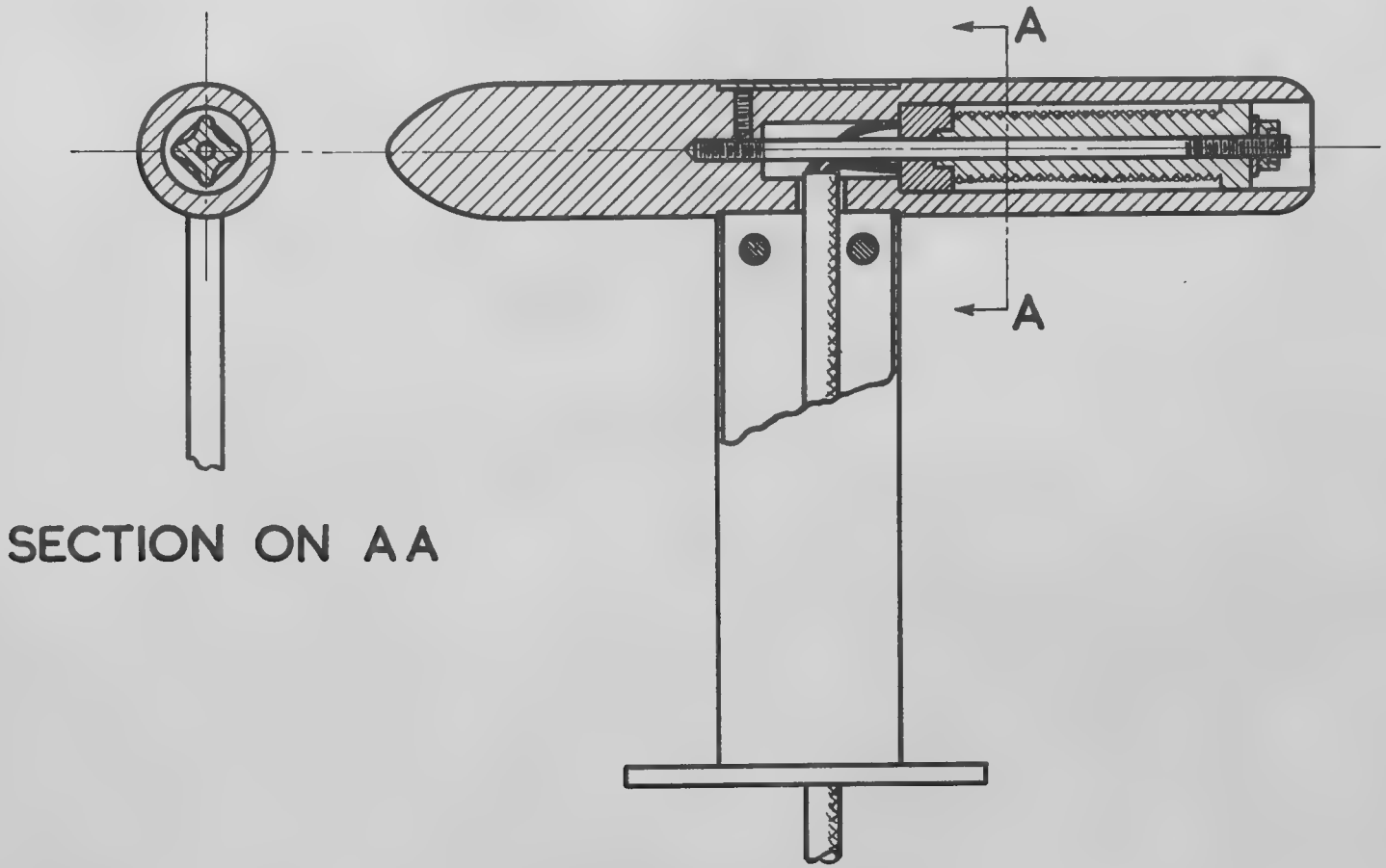


FIG. 1 Curve (1). Mean of 3 Mosquito runs at 300mb.
Curve (2). " " 7 Halifax " " 500mb.

FIG. 2

RESISTANCE BULB FOR AIR THERMOMETER – EXPERIMENTAL



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M.R.P.680
S.C.III/112
10th October, 1951

AIR MINISTRY

METEOROLOGICAL RESEARCH COMMITTEE

THE VISIBILITY CHARACTERISTICS OF NORTHOLT AIRPORT

by

G.A. Corby (Meteorological Office)

AIR MINISTRY TECHNICAL ... JAN 8 1952

1. INTRODUCTION

An analysis of the diurnal and annual variation of fog and thick fog at Northolt was recently undertaken by W.E. Saunders and W.D. Summersby (1). In the present investigation, the analysis has been extended to cover the whole visibility range, with particular reference to the variation of visibility with both wind speed and direction, as well as diurnally and annually. Hourly observations during the 5-year period 1st January, 1946, to 31st December, 1950, numbering over 40,000, were used for the purpose.

The aim has been to avoid lengthy tabulations, from which it is often difficult to recognise significant features. Instead, particular attention has been given to the devising of purely diagrammatic methods of illustrating the results, so as to make them of immediate practical value to forecasters and also to operational users of the Airport.

At the suggestion of Dr. Goldie, the results are presented also in terms of mean extinction coefficients for various combinations of wind speed and direction. This achieves greater conciseness and has advantages in that a definite physical meaning attaches to a mean extinction coefficient which is lacking in the arithmetic mean of a series of visibilities.

2. DIURNAL AND ANNUAL VARIATION OF MIST AND FOG

In Fig. 1 the diurnal and annual variation of mist and fog (visibility < 2,200 yards) at Northolt, based on the five years 1946 - 50, is illustrated. The representation takes the same form as that employed for fog and thick fog in reference (1).

The significant features which emerge, broadly follow the same pattern as those of the fog variation described in the latter article and generally speaking the explanatory notes of reference (1) apply equally. The variations from 0500 to 1200 during the period October - January, however, are particularly striking and further comment is not out of place. The high incidence of mist and fog (30 - 35%) from about 0500 - 1100 in October and November is no doubt the result of the predominance at this season of moist maritime airmasses having a water fog point in the region 35 - 45°F. The maximum can, thus, be ascribed to radiation mist and fog forming late in the night. In December and January the maximum, exceeding 40%, shifts to 0900 to 1100 and we have in fact a minimum at 0600 - 0700. This situation is attributable to the increasing frequency over this period of drier continental airmasses, of easterly origin, in which the liability to radiation fog is greatly decreased, whilst the liability to smoke pollution is increased at Northolt by virtue of the direction of airflow from London. The time of the 40% maximum in these months is clearly bound up with the time of lighting fires. The high incidence of dusk deteriorations in these months is also evident from Fig. 1, and can be explained on similar principles.

3. VARIATION OF VISIBILITY WITH WIND SPEED AND DIRECTION

Figs. 2 - 11 are intended to illustrate as exhaustively as possible the precise way in which wind speed and direction exercises control over the visibility at Northolt Airport. The diagrams are explained and commented upon in the following paragraphs.

/ Fig. 2

Fig. 2. Each curve represents for the winter months (Oct.-Mar.) the variation with wind direction of the % frequency of visibility less than a stated value. The boundary values used for the curves are 1,100 yards, 2,200 yards, 4,400 yards, 4 1/3 miles, 6½ miles and 12½ miles. Percentage frequencies rather than true frequencies were adopted to show more strikingly the far greater liability to poor visibility with some wind directions than others. The outer shaded area indicates the percentage frequency of winds from the various directions. The results presented on this diagram are irrespective of wind speed except insofar as calms have been excluded and dealt with separately.

Comments. The general pattern of the curves is approximately symmetrical about the West/East axis, but is markedly asymmetrical about the North/South axis. The effect of London smoke pollution with winds having an easterly component thus appears as the dominating feature. We see for example:-

that with easterly winds the probability of fog is 13 times greater than with westerlies,

that with easterlies the visibility was less than 2,200 yards for 46 out of every 100 hours, whilst with westerlies the figure was only 5 out of every 100,

that visibility was 4 1/3 miles or greater for 54 hours per 100 with westerlies, but for only 6 hours per 100 with easterlies.

Many other facts of this type may be recognised from the diagram.

Fig. 3. The central area (fog) of the previous figure is here enlarged to show the detailed variation with wind direction of the visibility within the fog range.

Comments. Proceeding down from the outer curve to that for visibility less than 220 yards we see that the direction for maximum frequency swings from E. to ENE. Also, the < 220 yards curve has relatively sharper peaks at certain directions than those for better visibilities. These points are interpreted as indicating that, in the thick fog range, more local smoke sources than the London area as a whole become important, as for example domestic smoke from Ruislip, Hillingdon and Uxbridge. The probability of thick fog is seen to vary enormously with wind direction, being 26 times greater with ENE'lies than westerlies.

Figs. 4, 5 and 6.

For these diagrams the winter observations were separated into three groups corresponding to ranges of wind speed of 1 - 6 knots, 7 - 10 knots, and ≥ 11 knots. The basis of the diagrams is the same as that adopted for Fig. 2.

Comments. The successive decrease in the incidence of bad visibility (and the corresponding increase in moderate and good visibility) with increasing wind speed is immediately apparent. It is seen also that increasing wind speed concentrates London smoke pollution within a successively narrower sector and intensifies still further the difference between the good and bad directions. For example, considering winds of 1 - 6 knots the liability to visibility less than 2½ miles is just over twice as great with easterlies as with westerlies, whereas for winds over 11 knots the factor becomes about 43 times. Many similar effects can be observed directly from the graphs.

/ Fig. 7

Fig. 7. We illustrate here the frequency distribution of visibility associated with calms in winter.

Comments. The degree and duration of bad visibility to be expected with calms in winter is all too clear. Under these conditions we see that Northolt has fog for nearly 1 hour out of 2 and thick fog ($<$ 220 yards) for about 1 hour per $4\frac{1}{2}$. Bearing in mind the well marked diurnal variation evident from Fig. 1, it is clear that the liability to fog at certain times of the day if the surface wind is calm in winter must be very great indeed. At the other end of the scale, the visibility with calms in winter is $6\frac{1}{4}$ miles or more for only 1 hour out of 55.

Fig. 8. This diagram shows for the summer months (April - Sept.) the variation of visibility with wind direction irrespective of speed on the same basis as that adopted for the winter. As before, the frequency distribution of wind direction is shown by the outer shaded area.

Comments. The overall improvement in visibility as compared with Fig. 2 (winter) is at once obvious. As regards smoke, industrial sources predominate in summer, domestic smoke being then negligible. These curves suggest, apart from London, another smoke haze source to the north and this is in all probability Watford. That this scarcely features on the winter curves is no doubt due to the swamping effect of widespread domestic smoke from numerous sources.

Fig. 9. This gives the frequency distribution of visibility associated with calms in summer.

No special comments are called for.

Figs. 10 and 11

The generalised extinction coefficient of the atmosphere expresses the degree of scattering and absorption of light and is, closely, a measure of the concentration of obscuring matter in the atmosphere. As defined (2), and subject to the usual assumptions involved in Koschmieder's work, it leads to the relation:

$$\sigma V = -\log_e E'$$

where σ = Extinction Coefficient
V = Meteorological Visibility
E' = Brightness contrast threshold for daylight visibility, normally taken as 0.02.

Mean extinction coefficients were computed for each of the 16 wind directions, firstly irrespective of wind speed and secondly for each of the three groups 1 - 6 knots, 7 - 10 knots and $\geqslant$ 11 knots. The curves were drawn from these values, Fig. 10 covering the winter months and Fig. 11 the summer. For convenience, as different scales have been used for winter and summer, the mean summer curve is indicated also on the winter diagram to facilitate comparison.

Comments. It will be appreciated that the arithmetic mean of a series of visibility values has no physical meaning. If we average the extinction coefficients corresponding to a number of occasions, however, we obtain the coefficient for an hypothetical airmass sample, such as would result from mixing equal volume samples of the airmass from each occasion until homogeneous distribution of the attenuating matter was attained. This, of course, is on the assumption that the nature and existence of the obscuring particles would be unaffected by the mixing.

/ The

The visibility corresponding to the mean extinction coefficient (i.e. the harmonic mean of the original visibilities) would, thus, be the visibility within the homogeneous mixture. This property makes the extinction coefficient a valuable quantity for expressing the visibility characteristics of a station when averaging is involved.

Figs. 10 and 11 show clearly the variation with wind speed and direction, and season, of the concentration of obscuring matter in the atmosphere at Northolt, and, in fact, constitute a concise summary of the contents of previous diagrams. This method of presentation would be eminently suitable for the comparison of the characteristics of different aerodromes. In particular, similar diagrams for a smoke free rural station or for an aerodrome such as Prestwick would be particularly interesting.

Some other mean values of the extinction coefficient, computed irrespective of the wind direction, are given in Table 1 below. These figures speak for themselves and would also be a useful basis for comparing different stations.

T A B L E 1.

MEAN VALUES OF EXTINCTION COEFFICIENT (σ)
IN MILES^{-1}, IRRESPECTIVE OF WIND DIRECTION
BASED ON 5 YEARS 1946 - 1950 AT NORTHOLT

Wind Speed	Winter	Summer	Whole Year
Calm	18 (390 yds.)	3.1 (2220 yds.)	11 (620 yds.)
1-6 kt.	4.5 (1530 yds.)	1.2 ($3\frac{1}{4}$ mls.)	2.7 (2500 yds.)
7-10 kt.	1.9 (2 mls.)	0.7 ($5\frac{1}{2}$ mls.)	1.3 (3 mls.)
$\geq$ 11 kt.	1.1 ($3\frac{1}{2}$ mls.)	0.5 (8 mls.)	0.8 (5 mls.)
All speeds inc. calms	5.0 (1370 yds.)	1.2 ($3\frac{1}{4}$ mls.)	3.1 (2220 yds.)

Note:- Figures in brackets are corresponding Harmonic Mean Visibilities.

In an analysis published in 1939 (3), Wright gives values of mean "opacity" in nebules per km. for a number of U.K. stations, but the variation with wind direction is not dealt with. The relation between extinction coefficient and the opacity is, of course, purely linear and Wright's values may be compared with the results of Table 1 by use of the relation:

$$\sigma = Kn$$

where σ = extinction coefficient in miles^{-1}
n = opacity in nebules per km.
K = 0.111

Fig. 12. This shows the location of Northolt relative to the built-up areas of Greater London. No attempt has been made to distinguish between industrial and domestic development.

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3. Wright, H.L.: Atmospheric Opacity: a study of visibility observations in the British Isles. Q.J. Roy. Met.Soc. 65, 1939.

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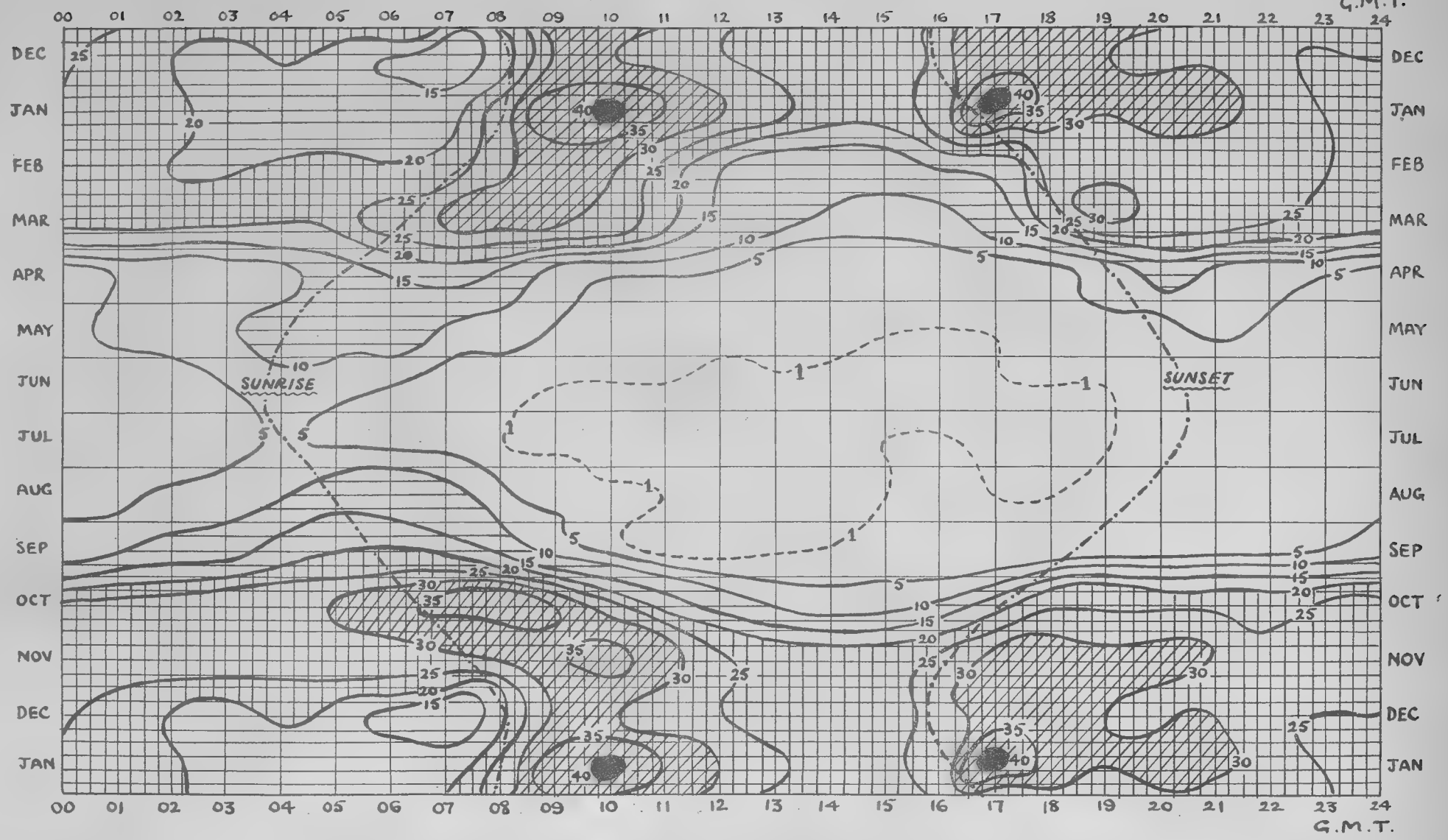

FIG. 1. % INCIDENCE OF MIST AND FOG (VISIBILITY < 2200 YDS.) AT NORTHOLT (BASED ON 5 YRS. 1946-50)

DIURNAL AND ANNUAL VARIATION

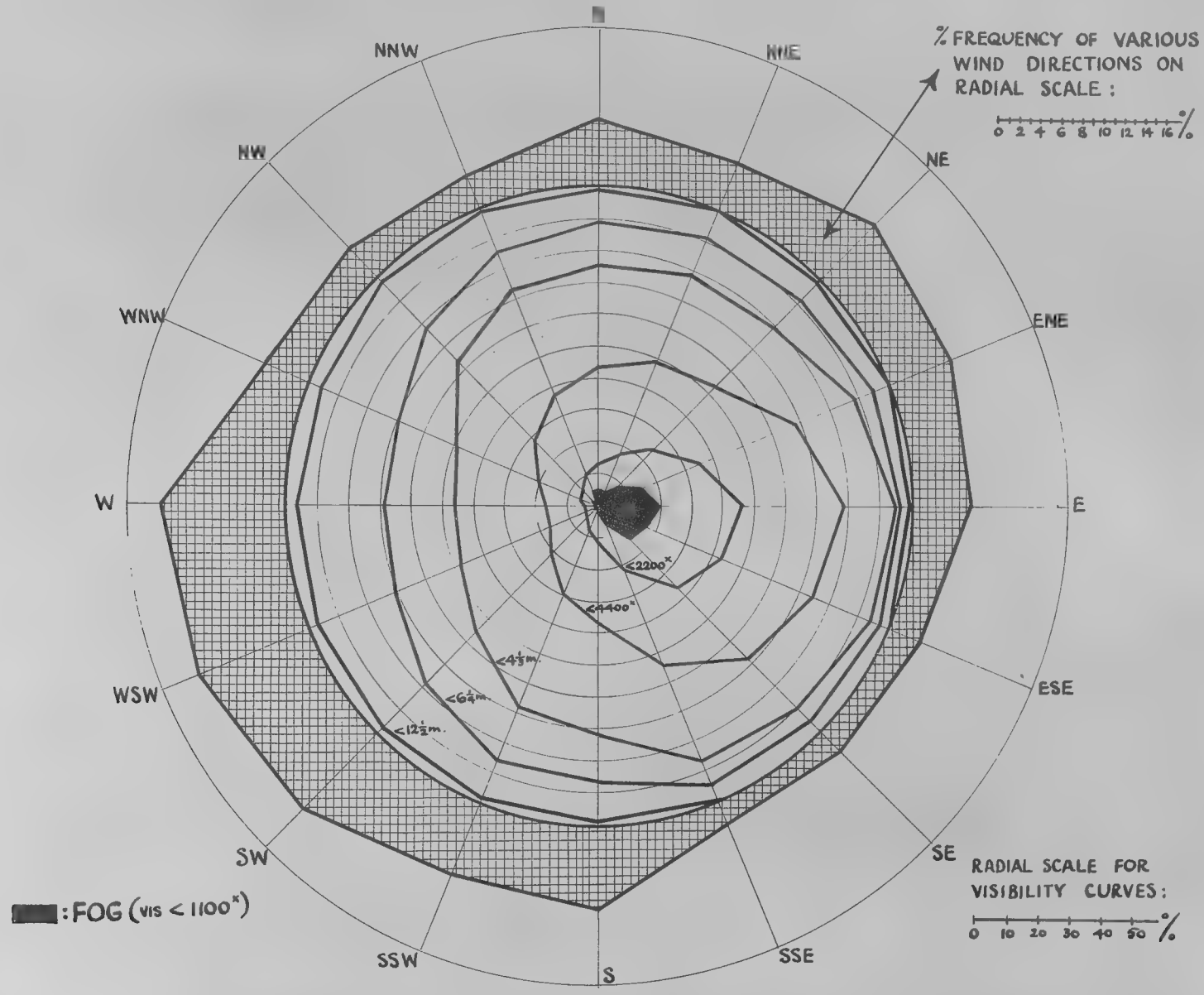

FIG. 2. NORTHOLT AIRPORT - VARIATION WITH WIND DIRECTION OF THE % FREQUENCY OF VARIOUS VISIBILITIES (BASED ON 5 YRS. 1946–50)

WINTER - IRRESPECTIVE OF WIND SPEED

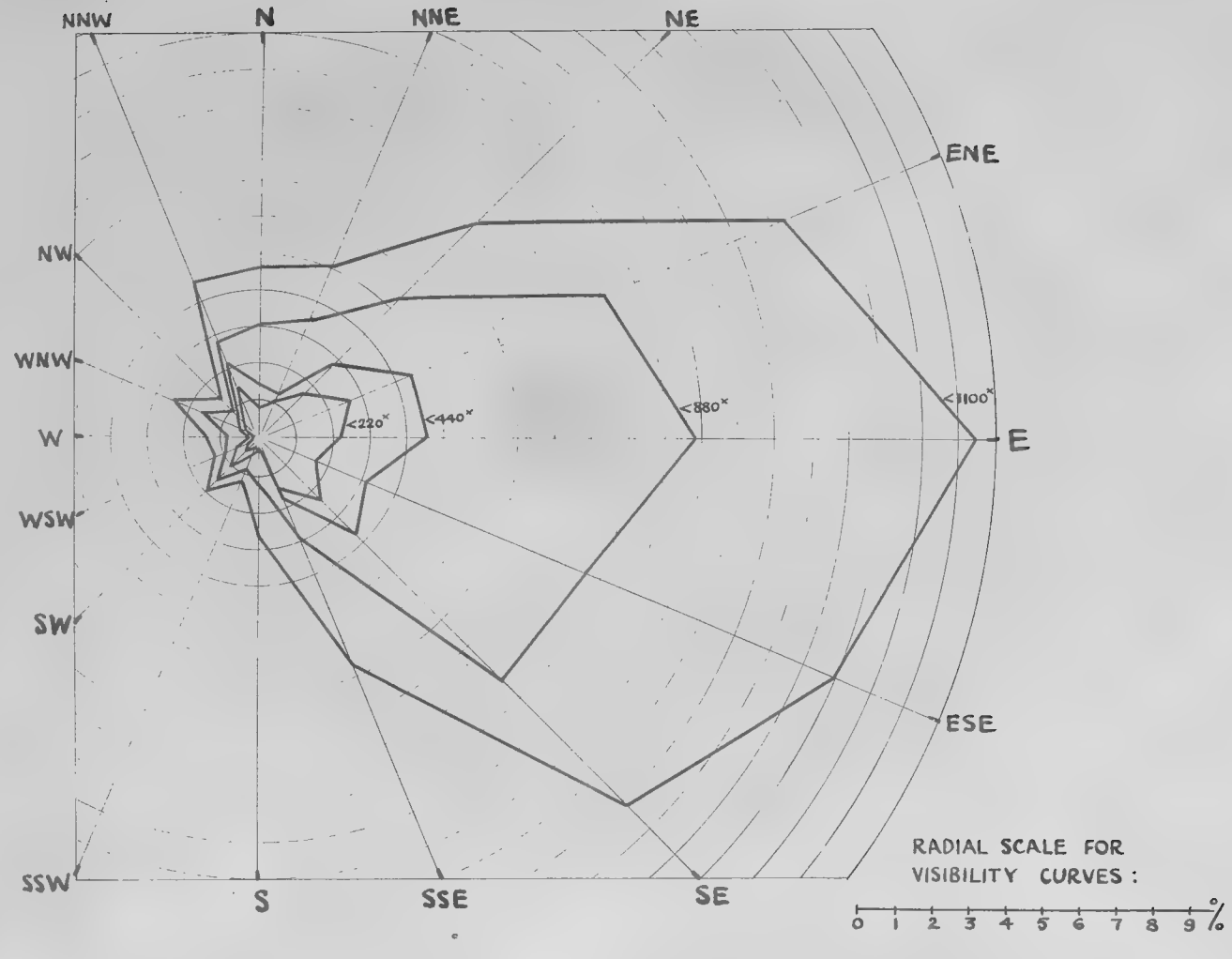

FIG. 3. NORTHOLT AIRPORT – ENLARGEMENT OF CENTRAL AREA OF FIG. 2.

WINTER – IRRESPECTIVE OF WIND SPEED

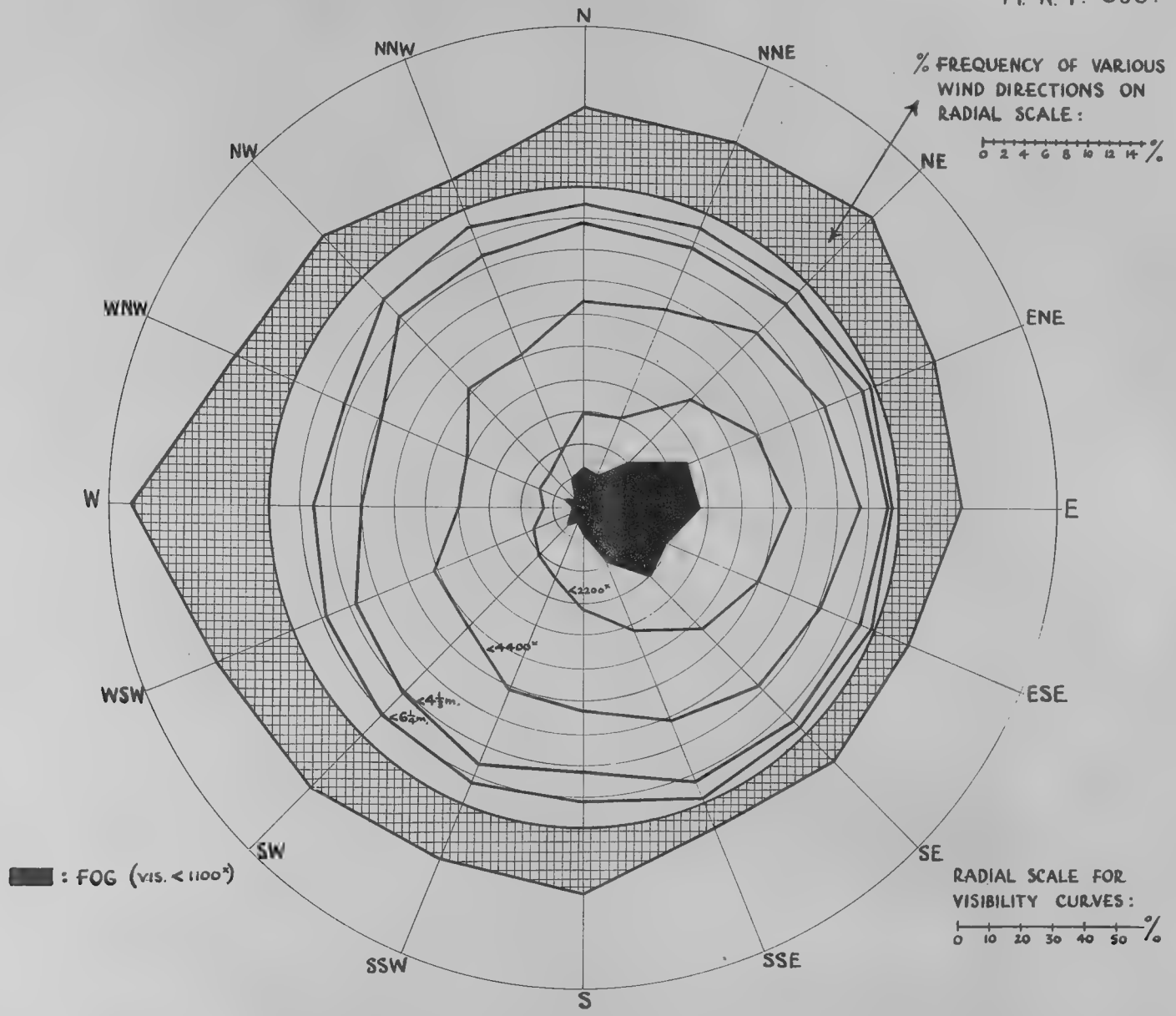

FIG. 4. NORTHOLT AIRPORT - VARIATION WITH WIND DIRECTION OF THE % FREQUENCY OF VARIOUS VISIBILITIES (BASED ON 5 YRS. 1946-50)

WINTER — WINDS 1-6 KNOTS ONLY

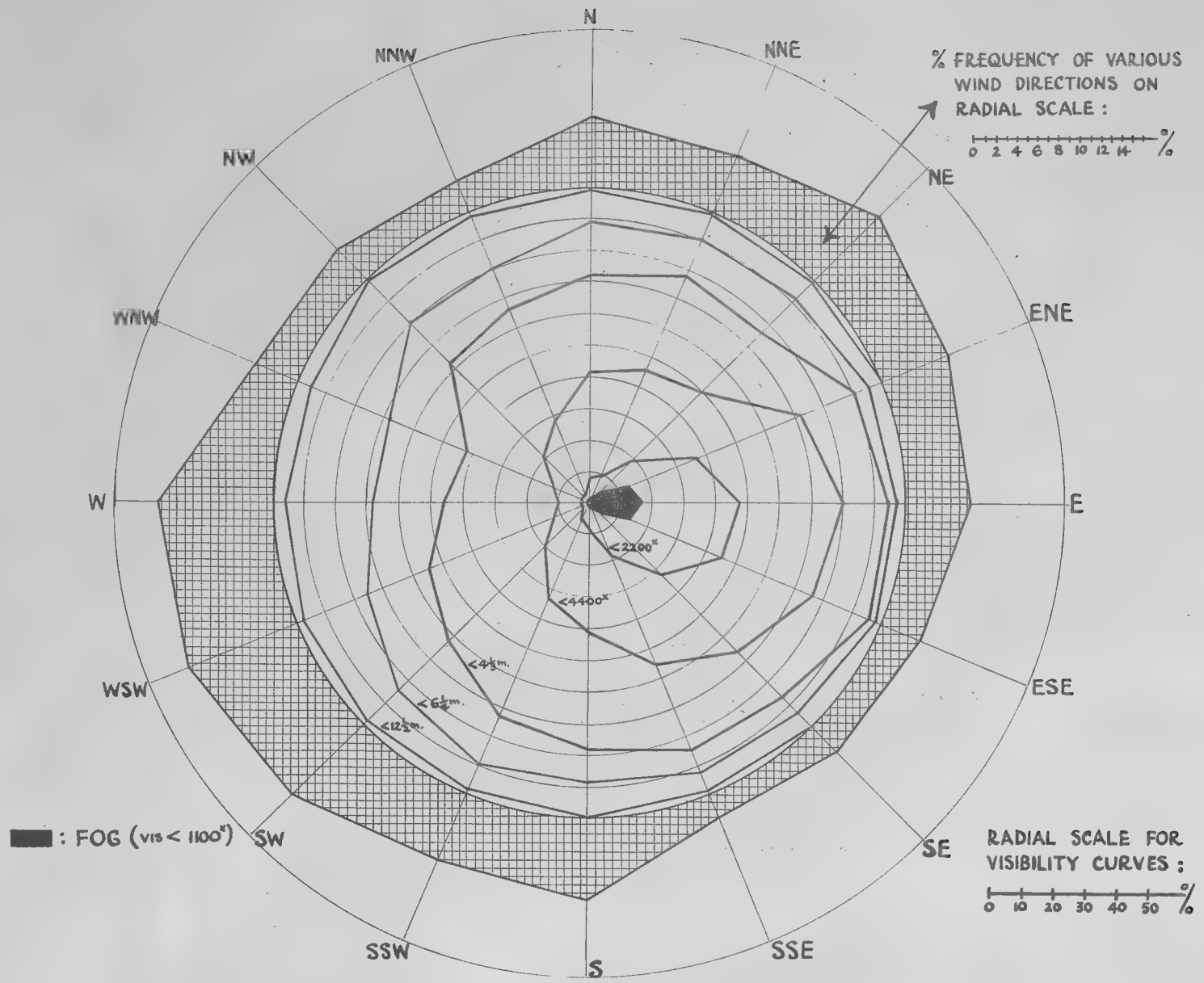

FIG. 5. NORTHOLT AIRPORT - VARIATION WITH WIND DIRECTION OF THE % FREQUENCY OF VARIOUS VISIBILITIES (BASED ON 5 YRS. 1946-50)

WINTER - WINDS 7-10 KNOTS ONLY

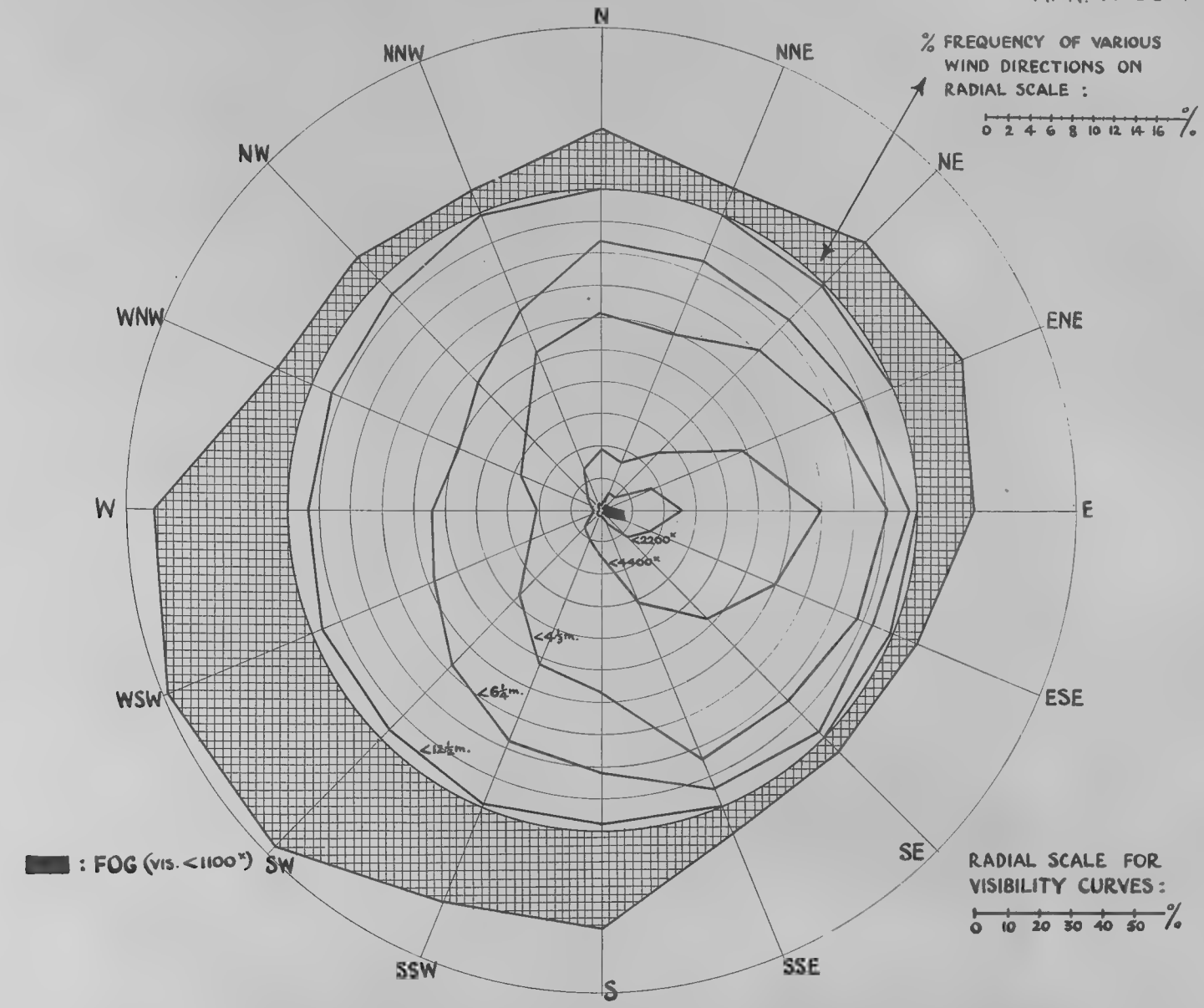

FIG.6. NORTHOLT AIRPORT - VARIATION WITH WIND DIRECTION OF THE % FREQUENCY OF VARIOUS VISIBILITIES (BASED ON 5 YRS. 1946-50)

WINTER - WINDS ≥ 11 KNOTS ONLY

M. R. P. 680

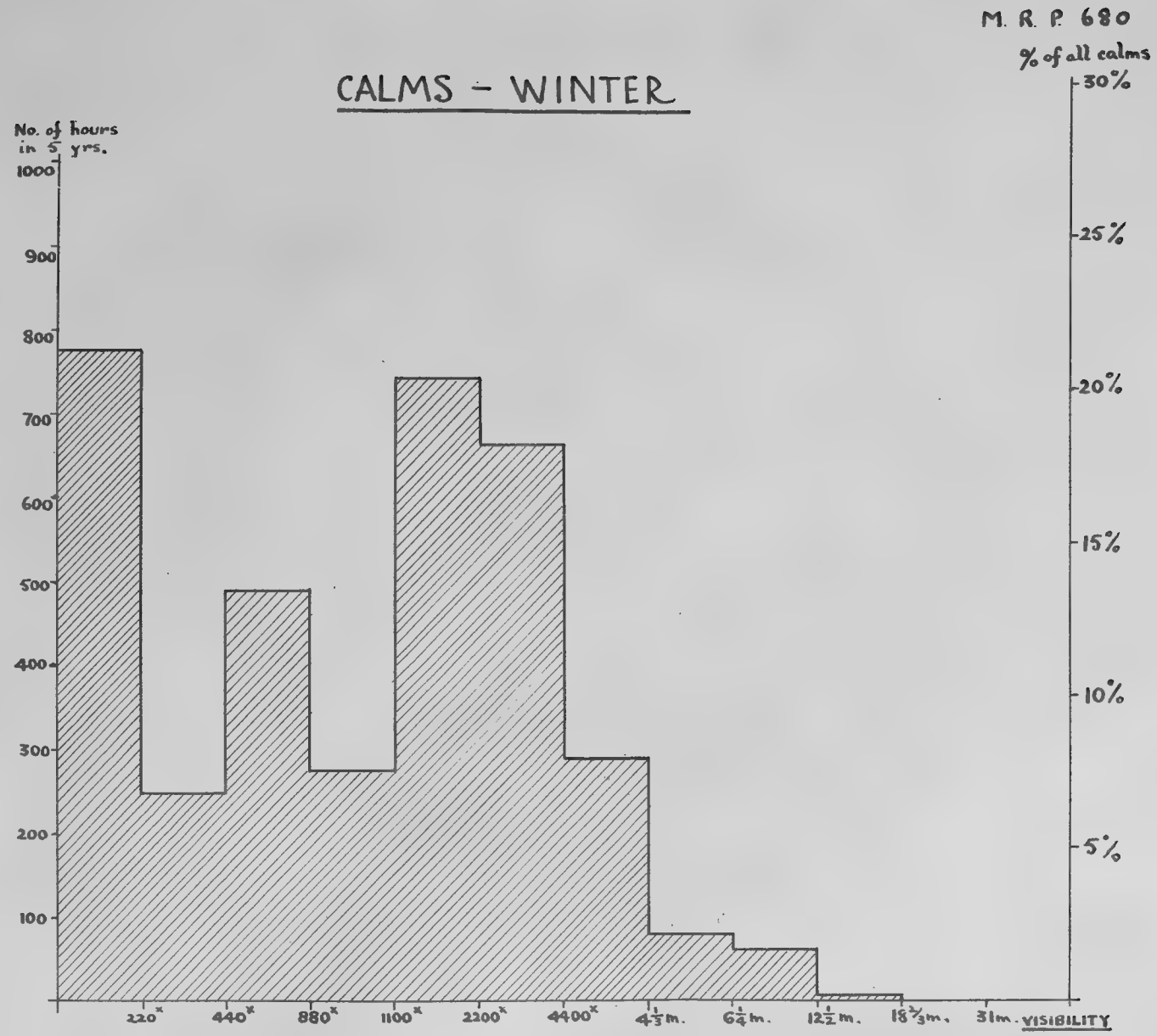

FIG. 7. NORTHOLT AIRPORT - FREQUENCY DISTRIBUTION OF VISIBILITY WITH CALMS IN WINTER (BASED ON 5 YRS. 1946 - 1950)

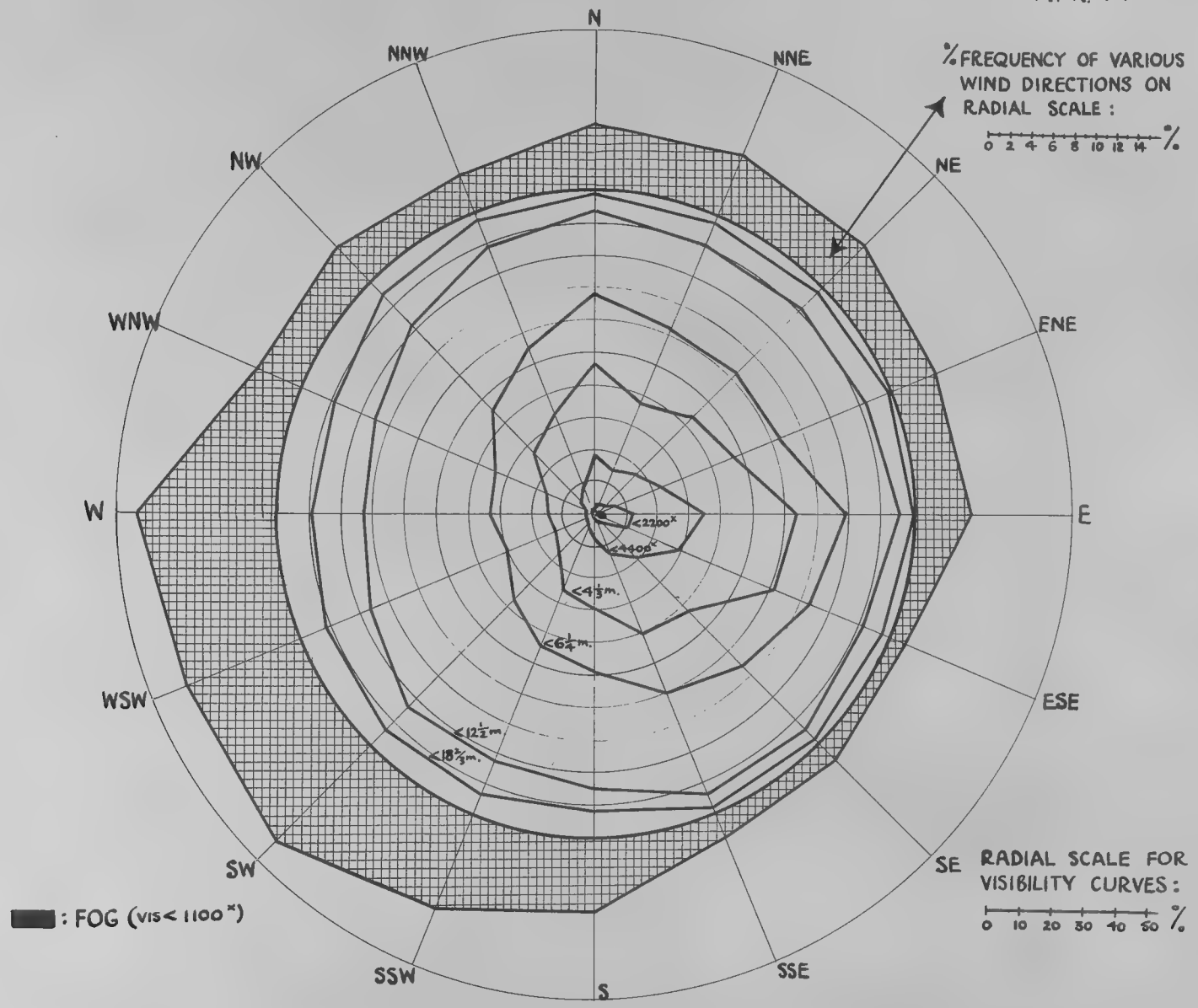

FIG. 8. NORTHOLT AIRPORT - VARIATION WIND DIRECTION OF THE % FREQUENCY OF VARIOUS VISIBILITIES (BASED ON 5 YRS 1946-50)

SUMMER - IRRESPECTIVE OF WIND SPEED

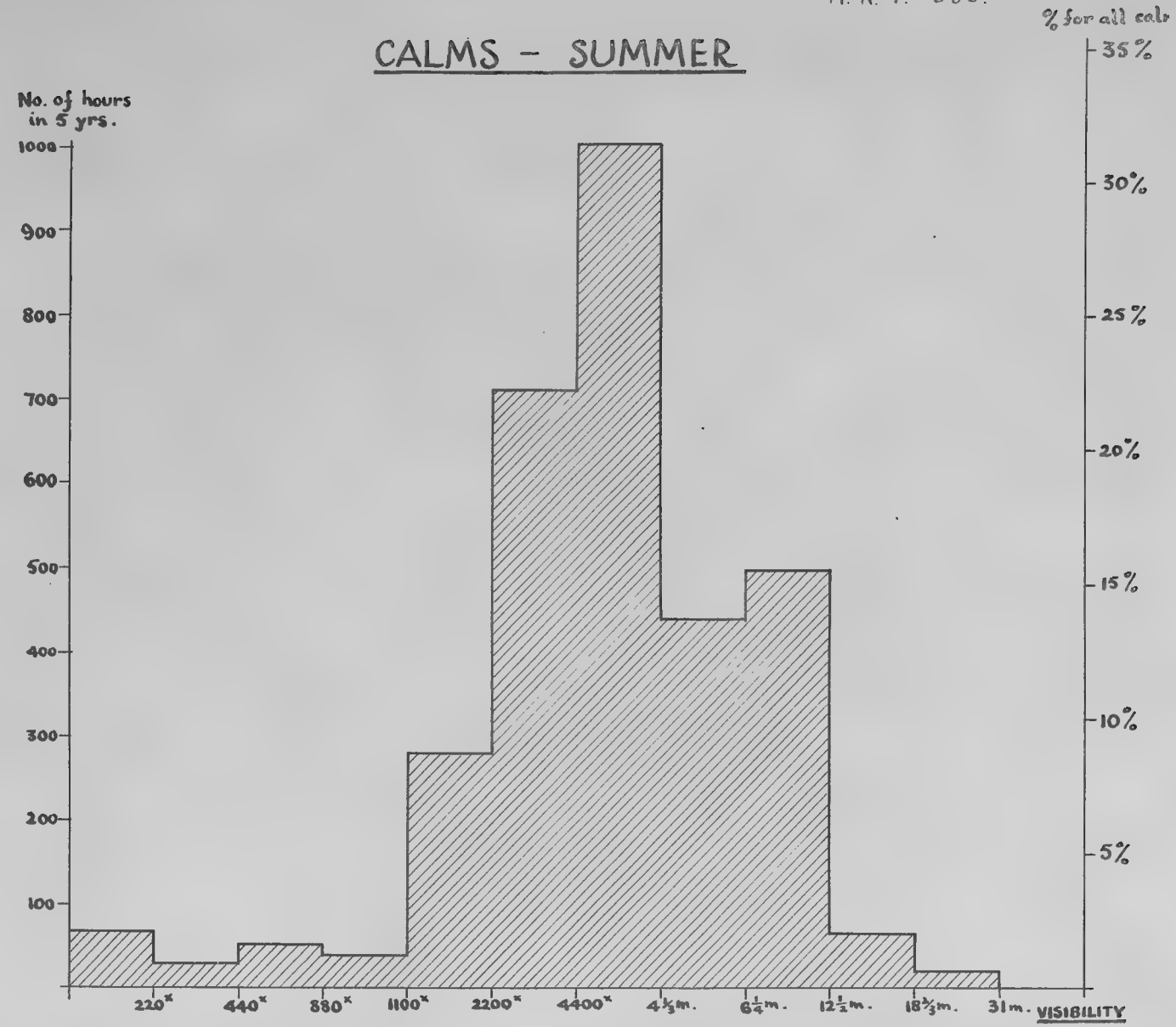

FIG. 9. NORTHOLT AIRPORT - FREQUENCY DISTRIBUTION OF VISIBILITY WITH CALMS IN SUMMER (BASED ON 5 YRS. 1946-1950)

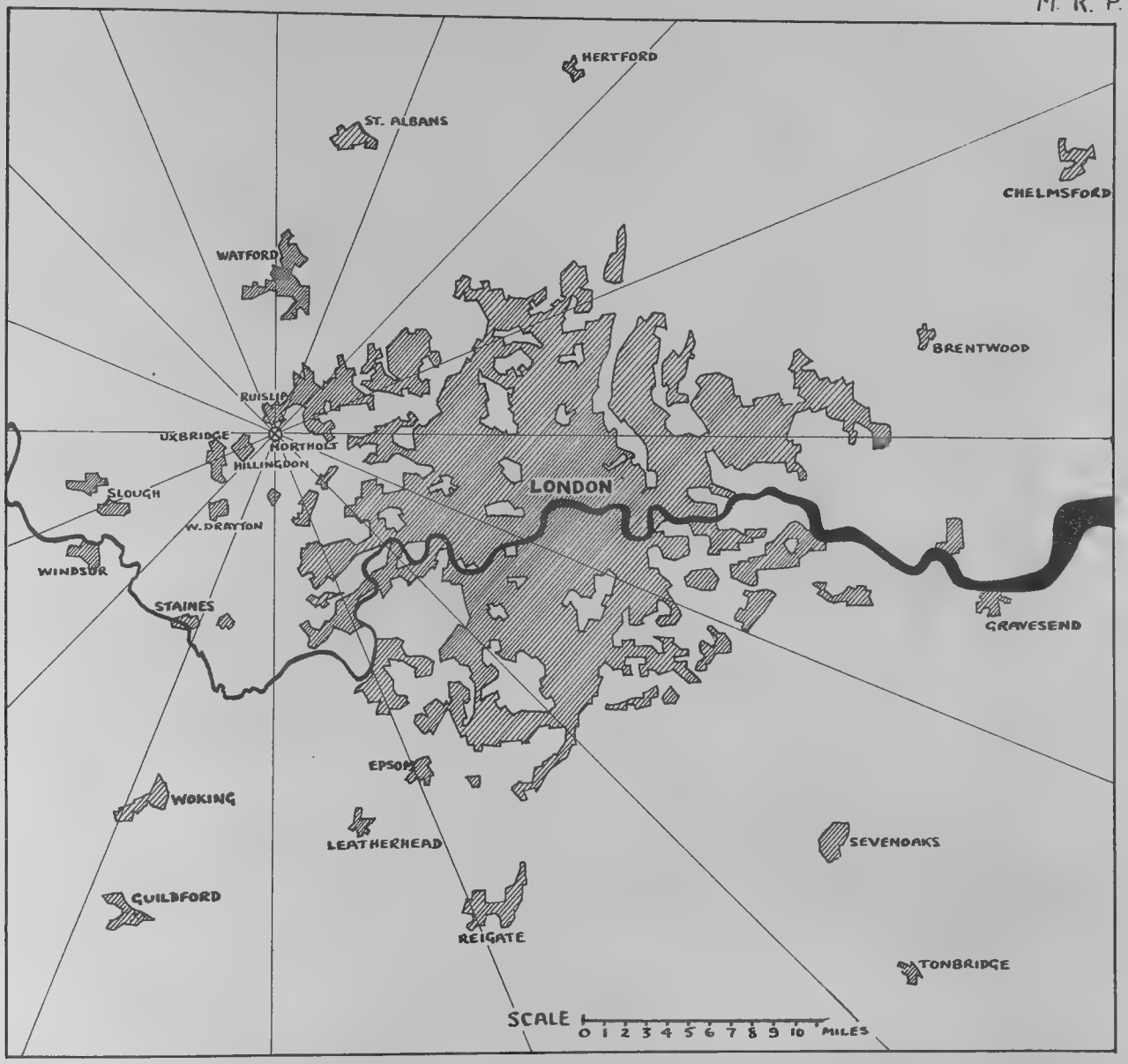

FIG. 12. NORTHOLT AIRPORT – LOCATION RELATIVE TO THE BUILT-UP AREAS OF GREATER LONDON.

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A copy is available in the Library of the Meteorological Office."

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S.C.II/97
26th September, 1951

AIR MINISTRY

METEOROLOGICAL RESEARCH COMMITTEE

Computation of the field of mean vertical velocity in the 1000-500 mb. layer of the atmosphere and its effect on the thickness of the layer

by

NOV 14 1952

F.H.Bushby (Meteorological Office)

AIR WEATHER SERVICE
TECHNICAL REFERENCE FILES

1. Introduction.

In recent years dynamical principles have been gradually introduced into weather forecasting. The equations governing atmospheric motion are much too complicated for a general solution to them to be obtained, but various research workers have applied the fundamental equations of meteorology to special fields of motion in the hope that the behaviour of these restricted fields of motion will correspond sufficiently to the behaviour of the real atmosphere to be of use to the forecaster. In America much work has been done by Rossby, Charney and others on theories relating to motion in a barotropic atmosphere, and in this country Sutcliffe (1) has shown how in quasi-geostrophic motion cyclonic and anticyclonic development can be interpreted in terms of rates of change of vorticity. The forecaster is, of course, mainly concerned with forecasting regions of maximum development, i.e. regions of maximum vertical motion, and the purpose of the present paper is to show how, by making certain approximations, the field of vertical velocity can be evaluated.

Following the treatment of Sutcliffe (1) as modified by Sumner (2), the field of vertical velocity averaged logarithmically with respect to pressure for the 1000 mb. - 500 mb. layer of the atmosphere has been computed for six synoptic situations. The effect of vertical stability on vertical motion is examined and the relationship between the mean vertical velocity of a layer of the atmosphere and the thickness tendency of that layer is also investigated. A brief reference is also made to the relationship between vertical velocity and rates of rainfall.

2. The equations.

Sumner (2) has shown that the "development equation" of Sutcliffe (1) which related the difference of horizontal divergence at two pressure levels to the thermal wind of the layer and the vertical component of vorticity at the two pressure levels, can be expressed in the form

$$l(\mathrm{div}\,\underline{V}_1 - \mathrm{div}\,\underline{V}_0) = -\underline{V}'\,\mathrm{grad}\,(l + \zeta_0 + \zeta_1) - \frac{R}{I}\log\frac{p_0}{p_1}\nabla^2(\overline{\Gamma w'}) \quad \ldots (1)$$

where $\underline{V}'$ represents the thermal wind between the two pressure levels p_0 and p_1; $\underline{V}_0$, $\underline{V}_1$ are the horizontal wind vectors and ζ_0, ζ_1 the corresponding vertical components of vorticity at the levels p_0 and p_1 respectively; Γ represents the departure of the actual lapse of temperature $(-\partial T/\partial z)$ from the wet or dry adiabatic lapse rate γ; l is the Coriolis parameter; w' is the vertical velocity relative to a moving isobaric surface and the bar represents the average over the layer in question with respect to a logarithmic pressure scale.

The main assumptions made in deducing equation (1) are that the motion in the atmosphere is quasi-geostrophic, that the direction of wind shear does not vary with height and that the vertical advection of vorticity can be neglected. The variation of wind with pressure is shown by the following equation:

$$\underline{V} = \underline{V}_0 + A(p)\,\underline{V}' \quad \ldots\ldots\ldots\ldots (2)$$

/where

where $\underline{V}$ is horizontal wind vector at pressure level p. Before equation (1) can be solved for the mean vertical velocity, it is necessary to make some assumption about the variation of divergence with pressure. The simplest tenable assumption is probably a linear variation of divergence with pressure, passing through zero at some pressure level p_2 in mid-troposphere. This gives

$$\text{div } \underline{V} = \frac{p - p_2}{p_o - p_2} \text{ div } \underline{V}_o \qquad \ldots\ldots\ldots \quad (3)$$

Since p_o corresponds approximately to the pressure at the earth's surface, we can generally neglect dp_o/dt in comparison with dp/dt. Thus we get from the equation of continuity for a hydrostatic field of pressure

$$\frac{dp}{dt} = -\int_{p_o}^{p} \text{div } \underline{V} \; dp \qquad \ldots\ldots\ldots \quad (4)$$

Also

$$w' = -\frac{1}{g\rho}\frac{dp}{dt} \qquad \ldots\ldots\ldots \quad (5)$$

and with usual notation

$$p = \rho RT \qquad \ldots\ldots\ldots \quad (6)$$

Substituting from equations (3), (4) and (6) in equation (5) and using the approximation that

$$\overline{T\, f(p)} = \overline{T} \cdot \overline{f(p)} \qquad \ldots\ldots\ldots \quad (7)$$

since variations in T are small compared with T, it can easily be shown that

$$\overline{w'} = \frac{R\overline{T} \text{ div } \underline{V}_o \left\{(p_o - p_1)(2p_2 + p_1 - p_o) - 2p_1p_2\log_e(p_o/p_1)\right\}}{2\, gp_1(p_o - p_2)\log_e(p_o/p_1)} \quad \ldots\ldots \quad (8)$$

Hence from equations (3) and (8)

$$\text{div } \underline{V}_1 - \text{div } \underline{V}_o = \frac{2g\,(p_1 - p_o)p_1 \log_e(p_o/p_1)\,\overline{w'}}{R\overline{T}\left\{(p_o - p_1)(2p_2 + p_1 - p_o) - 2p_1p_2\log_e(p_o/p_1)\right\}} \qquad \ldots\ldots\ldots\ldots \quad (9)$$

Assuming that Γ is constant with respect to pressure, and that the first order space derivatives of the Coriolis parameter are negligible compared with those of ζ_o and ζ_1 equations (1) and (9) give

$$\nabla^2(\Gamma\overline{w'}) + \alpha(\overline{w'}) + \beta\, \underline{V}' \cdot \text{grad}\,(\zeta_o + \zeta_1) = 0 \qquad \ldots\ldots\ldots \quad (10)$$

where

$$\alpha = \frac{2l^2 g(p_1 - p_o)\, p_1}{R^2\overline{T}\left\{(p_o - p_1)(2p_2 + p_1 - p_o) - 2p_1p_2\log_e(p_o/p_1)\right\}}$$

$$\beta = \frac{1}{R\log_e(p_o/p_1)}$$

Since horizontal variations of l and $\overline{T}$ are small compared with the absolute values, α and β have been treated as constants with $l = 11.2 \times 10^{-5}$ sec^{-1} and $\overline{T} = 271°A$. Equation (10) can be solved numerically for $\overline{w'}$ by the relaxation technique (Southwell (3)), but if $\Gamma \neq 0$ it is easier to re-write equation (10) as follows

$$\nabla^2(\Gamma\overline{w'}) + \frac{\alpha}{\Gamma}(\Gamma\overline{w'}) + \beta\underline{V}' \cdot \text{grad}\,(\zeta_0 + \zeta_1) = 0 \quad \ldots\ldots\ldots \quad (11)$$

and solve for $\Gamma\overline{w'}$.

If h represents the thickness of the layer between pressure levels p_0 and p_1 then by neglecting non-adiabatic processes and following the analysis of Sutcliffe and Godart (4) we get

$$\left(\frac{\partial h}{\partial t}\right)_p = -\overline{\underline{V}} \cdot \text{grad}\, h + \frac{R}{g}\int_{p_1}^{p_0} \Gamma w'\, d(\log p) \quad \ldots\ldots\ldots \quad (12)$$

$$= -\overline{\underline{V}} \cdot \text{grad}\, h + \frac{R\Gamma\,\overline{w'}}{g} \log \frac{p_0}{p_1} \quad \ldots\ldots\ldots \quad (13)$$

The first term on the right-hand side of equation (12) represents the advective component of the thickness tendency and the second term represents the component due to vertical velocity.

/3

3. Numerical solutions of the equations.

The vertical velocity and its effect on the thickness tendency has been evaluated in the Forecast Research Division at Dunstable for six particular synoptic situations. One of these cases is described in detail in section 4, and the stages of the computations can best be illustrated by reference to that situation.

The layer of the atmosphere chosen for the experiment was from 1000-500 mbs. The area chosen included the British Isles, Eastern North Atlantic, and the Atlantic sea-board of Europe. There is a good network of upper air stations over all of this area except the extreme south-west.

The level of non-divergence p_2 was taken to be 600 mb. Thus we have a parabolic relationship between w' and p, w' being zero at the 1000 and 200 mb. pressure levels, and passing through a maximum at 600 mbs.

To simplify the finite difference approximations a square grid on a conical orthomorphic projection of the earth's surface (standard parallels 30°N and 50°N) was used. The mesh length of the grid was 140 nautical miles in latitude 50°N. As a relatively small area was considered, the variations of mesh length with latitude were negligible, but if a more extensive area were considered, variations in mesh length would have to be accounted for in the analysis.

The first stage in the computation was the evaluation of $\underline{V}'.\ \mathrm{grad}(\zeta_0 + \zeta_1)$ at the grid points. This term was evaluated by using the technique and scale suggested by Sawyer and Matthewman (5). If there were no damping due to vertical stability,

$$\text{i.e. } \nabla^2 \Gamma \overline{w'} \equiv 0$$

then the mean vertical velocity would be directly proportional to the term $\underline{V}'.\mathrm{grad}(\zeta_0 + \zeta_1)$ (see equation 10). Isopleths of this "undamped" vertical velocity are shown in Fig. 3 relating to the synoptic situation shown in Fig. 1.

In the analysis of section 2, Γ has been assumed constant with respect to pressure. Therefore in estimating Γ it is consistent to measure a value averaged vertically throughout a column of air from the 1000 mb. to 500 mb. pressure level.

To obtain a value of the appropriate wet or dry adiabatic lapse rate between the 1000 and 500 mb. pressure levels, the following procedure was adopted. From the tephigram it was estimated that dry air would be cooled 53°C when raised adiabatically from the 1000 to 500 mb. pressure level. This value varies only slightly for different mean potential temperatures. A table was made of similar values, $y(\overline{\theta})$, for saturated air at various mean potential temperatures ($\overline{\theta}$). Each ascent was examined and the pressure interval for which the air was saturated was measured, say p_s. Then the appropriate adiabatic rate of cooling for air represented by the ascent was taken as

$$\frac{500 - p_s}{500} \cdot 53 + \frac{p_s}{500}\, y(\theta) \quad °C \text{ per } 500 \text{ mb.}$$

The actual temperature difference between 1000 and 500 mbs. was measured from the tephigram. Taking the average thickness of the 1000-500 mb. level to be 18,000 ft. we can thus get a crude approximation to Γ. To increase the number of observations for each particular case, Γ was evaluated from ascents 6 and 12 hours before the time used for the computations, these values being advected forward with the geostrophic wind. The field of Γ for 2100 G.M.T. on 14th February, 1950 is shown in Fig. 2.

Taking the grid length used to be a and describing values of the variable by using as suffixes the letters shown in Fig. 6, we get at any point A the

/following

following finite difference equation to replace equation (10).

$$\Gamma_B\overline{w'_B} + \Gamma_C\overline{w'_C} + \Gamma_D\overline{w'_D} + \Gamma_E\overline{w'_E} - \left(4 - \frac{\alpha a^2}{\Gamma_A}\right)\Gamma_A\overline{w'_A} + a^2\beta\left[\underline{V}'.\text{grad}(\zeta_0 + \zeta_1)\right]_A = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (14)$$

At all grid points equations such as (14) can be easily solved for $\Gamma\,\overline{w'}$ by the relaxation technique, if boundary values of $\Gamma\,\overline{w'}$ are known. This particular type of equation can be solved by a slight extension of the method described by Southwell (1946) for obtaining numerical solutions to Poisson's equation. In this case, however, the relaxation pattern will be different at each grid point.

The boundary values of $\overline{w'}$ are not known, and therefore have to be fixed arbitrarily. However, it would seem probable that the boundary conditions only affect the solution for a limited area. Therefore by taking a boundary outside the systems we are interested in, the effect of the boundary conditions on the part of the solution we are most interested in is minimised. In preliminary experiments, the boundary condition $\overline{w'} = 0$ was used. As a result of these experiments it was found that vertical stability usually reduced the "undamped" value of the vertical velocity to about a quarter of its value. Therefore it seemed that better results would be obtained by assuming that the boundary values of $\overline{w'}$ were given by

$$\overline{w'} = -\frac{1}{4}\frac{\beta}{\alpha}\underline{V}'.\text{ grad }(\zeta_0 + \zeta_1) \quad \ldots\ldots\ldots (15)$$

The field of $\overline{w'}$ can easily be obtained from the field of $\Gamma\,\overline{w'}$. Isopleths of $\overline{w'}$ for 2100 G.M.T. on 14th February, 1950 are shown in Fig. 4.

From equation (13) it can be seen that the component of thickness tendency due to vertical motion is proportional to $\Gamma\,\overline{w'}$. This component of the thickness tendency evaluated in this way has been compared with the difference between the actual thickness tendency and the advective component (see Fig. 5). The actual thickness tendency was estimated by plotting a curve of thickness against time at 6-hourly intervals for a period of 24 hours centred at the time used for the computations. The slope of this curve was taken as the thickness tendency. The advective component of the thickness tendency was measured by evaluating $\underline{V}_0.\text{ grad h}$, since from equation (2)

$$\overline{\underline{V}}.\text{ grad h} = \underline{V}_0.\text{ grad h}$$

The geostrophic approximation to $\underline{V}_0$ was used in evaluating this advective component.

4. Discussion of Results.

The various stages in the computations relating to the synoptic situation at 2100 G.M.T. on 14th February, 1950, are shown in Figs. 1-5, although the area for which computations were made was somewhat larger than that shown. This case is quite typical of the other situations that were examined. It has been chosen for decription in preference to the other cases because Sawyer (6) has made a full investigation of the rainfall associated with the warm front as it crossed Great Britain.

By reference to Fig. 2 it can be seen that in this case $-\Gamma$ varied between .06 and .52 of the dry adiabatic lapse rate. In all the ascents that were examined, Γ was always negative and it was thus possible to use equation (11) rather than the more cumbersome equation (10) to obtain the field of $\overline{w'}$.

/It

It does appear that in middle latitudes it is unusual for the whole of a column of air from 1000-500 mbs. to be in unstable or neutral equilibrium, although certain intermediate layers may be unstable. However, should a case arise when the field of Γ does contain a zero isopleth, the field of $\overline{w'}$ can be obtained by solving equation (10).

By comparing Figs. 3 and 4 the importance of including the vertical stability term when solving equation (10) or (11) for $\overline{w'}$ can be seen. The inclusion of the vertical stability term has the effect of reducing the maxima and minima of the computed mean vertical velocity to about one quarter of the value it would otherwise have had, although a minimum area of undamped vertical velocity of small extent may be almost completely swamped by an adjacent larger and more extensive maximum. In this particular case the minimum area of undamped vertical velocity over South Wales has become a much less significant minimum farther to the south-west in the field of damped vertical velocity. Maximum and minimum values of the damped velocity are, on the average taken over all the cases, 0.27 of the corresponding maximum and minimum values in the field of undamped vertical velocity.

The inclusion of the vertical stability term has only slightly altered the positions of the main maxima and minima, and in this respect this case is also typical of the other situations that were examined. So far, most use of the Sutcliffe (1) development ideas in forecasting has been made by a qualitative assessment of the areas of maximum vertical motion. This has in general been more by estimating the areas in which the term $\underline{V}'.\text{grad}(\zeta_0 + \zeta_1)$ has possessed maximum or minimum values. From the work done in the present experiment, it would appear that although the aforementioned technique neglects the effects of vertical stability, it should in general be adequate enough to predict important regions of maximum vertical motion.

Sawyer (6) has made a full investigation of the rainfall associated with this synoptic situation. In this experiment we have only evaluated the field of vertical velocity at one particular time, but it is interesting to note that the maximum rainfall occurred over Scotland and whilst generally only a slight precipitation occurred south of a line from the Bristol Channel to East Anglia, there were some indications of heavier precipitation along the SE coast of England. The reports of rainfall over France from 1800 G.M.T. 14.2.50 to 0600 G.M.T. 15.2.50 showed a maximum of 3-4 mms. over extreme NE of France and Belgium, elsewhere in NW and central France only 1-2 mms. were reported, thus supporting the maximum area of vertical velocity in that region.

If one assumed that the field of vertical velocity remained stationary relative to the warm front as it moved ENE over the British Isles, one would broadly expect a distribution of rainfall similar to that which actually occurred. An upward vertical velocity of 5 cms/sec. is approximately equivalent to 1.5 mms. of rain per hour. (Bannon (7)). Some stations in Scotland did report twice this rate of rainfall, but undoubtedly some of this was due to orographic effects.

In each of the six cases examined the component of thickness tendency due to the computed field of vertical motion has been computed. To test the degree of success of the forecasting of this component of the thickness tendency it was compared with the difference between the actual thickness tendency and its advective component (as in Fig. 5). The two should be identical if non-adiabatic processes are negligible.

By comparing computed and measured values of the component of thickness tendency due to vertical motion at the eight radio-sonde stations in Great Britain for each of the six cases examined, a root mean square difference of 24 ft./hr. was found. A forecast of zero thickness tendency due to vertical motion everywhere gave a root mean square error of 36 ft/hr.

A correlation coefficient of .61 was found between the computed and measured values, and the regression equation for the best fitting straight

/line

line giving the measured vertical component of the thickness tendency (Yft/hr) in terms of the computed change (X ft./hr) was

$$Y = 1{\cdot}44\ X + 1$$

From these results, it does seem that this method gives a fairly representative picture of the effect of vertical velocity on the thickness tendency, although the regression equation does show that the effect of vertical motion has been underestimated. The physical significance of this is not immediately obvious, but it may be that in neglecting variations of vertical stability with respect to pressure, the degree of damping is over-estimated.

5. Conclusion.

In order to obtain numerical solutions to the governing equation of motion, state and continuity by the method described in this paper, it has been necessary to assume that the complex motion of the atmosphere can be approximated to by the motion of a simple atmospheric model, viz. one in which the thermal wind per unit layer is constant in direction and magnitude with respect to pressure height and also that the divergence of the horizontal wind vector is a linear function of pressure passing through zero in mid-troposphere. This model has many limitations, but it is encouraging that by using this model one does obtain a reasonable degree of correspondence between actual and forecast component of thickness tendency due to vertical motion. By extending the present model it may be possible to increase the accuracy of the results, but it would seem that the present method does give a good first approximation to the field of vertical velocity.

Although it is impossible to generalise from only six situations, there are two inferences that can be made which are probably generally true. It seems extremely probable that important areas of maximum vertical velocity can be deduced by assessing the areas of maximum $\underline{V}'.\ \mathrm{grad}(\zeta_0 + \zeta_1)$, the Sutcliffe development term, and that an approximate value for the magnitude of the vertical velocity can be calculated by taking one quarter of the value obtained when damping due to vertical stability is not allowed for.

It has not so far been possible to compare the degree of success obtained from the computed component of thickness tendency due to vertical motion with that obtained by an experienced forecaster, but the degree of success achieved by this objective method is certainly appreciable. It might be possible to improve it by some empirical modifications. Some of the apparent discrepancies may be due to the neglect of non-adiabatic processes in assessing the measured vertical velocity, although generally these will be a second order effect.

At the moment it is not possible to integrate equations(13) and (9) with respect to time as there are three variables dependent on time. However, by introducing a third equation similar to a modified form of Charney's (8) equation for the contour height of the "equivalent barotropic" level of the atmosphere, it is possible to obtain a closed set of simultaneous differential equations which could possibly be solved by modern electronic devices.

6. References.

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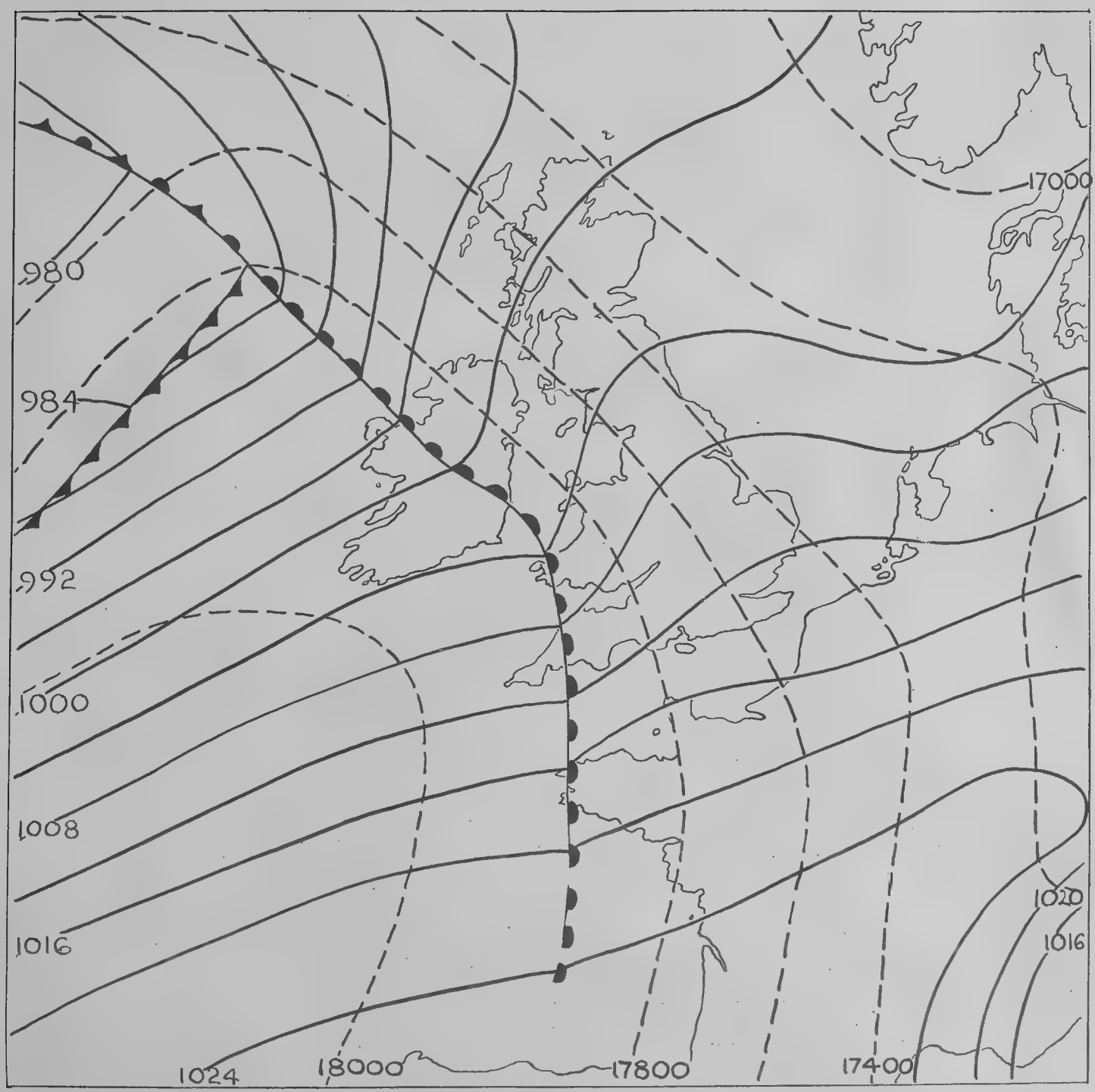

FIG.1. SYNOPTIC SITUATION. — ISOBARS. -- 1000-500mb. THICKNESS LINES. 2100 G.M.T. 14.2.50.

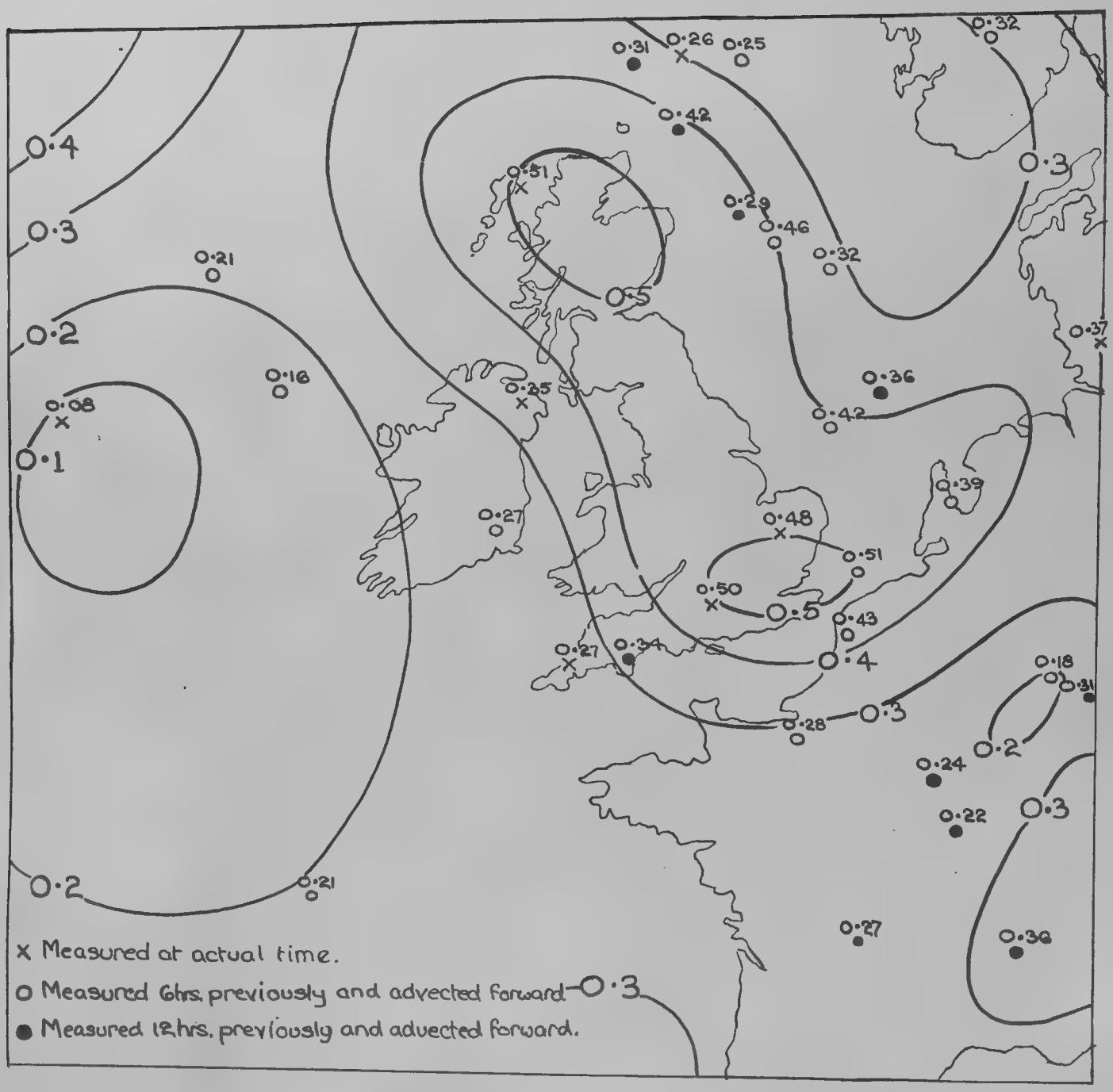

FIG. 2. FIELD OF Γ (AS FRACTION OF DRY ADIABATIC LAPSE RATE) 2100 G.M.T. 14.2.50

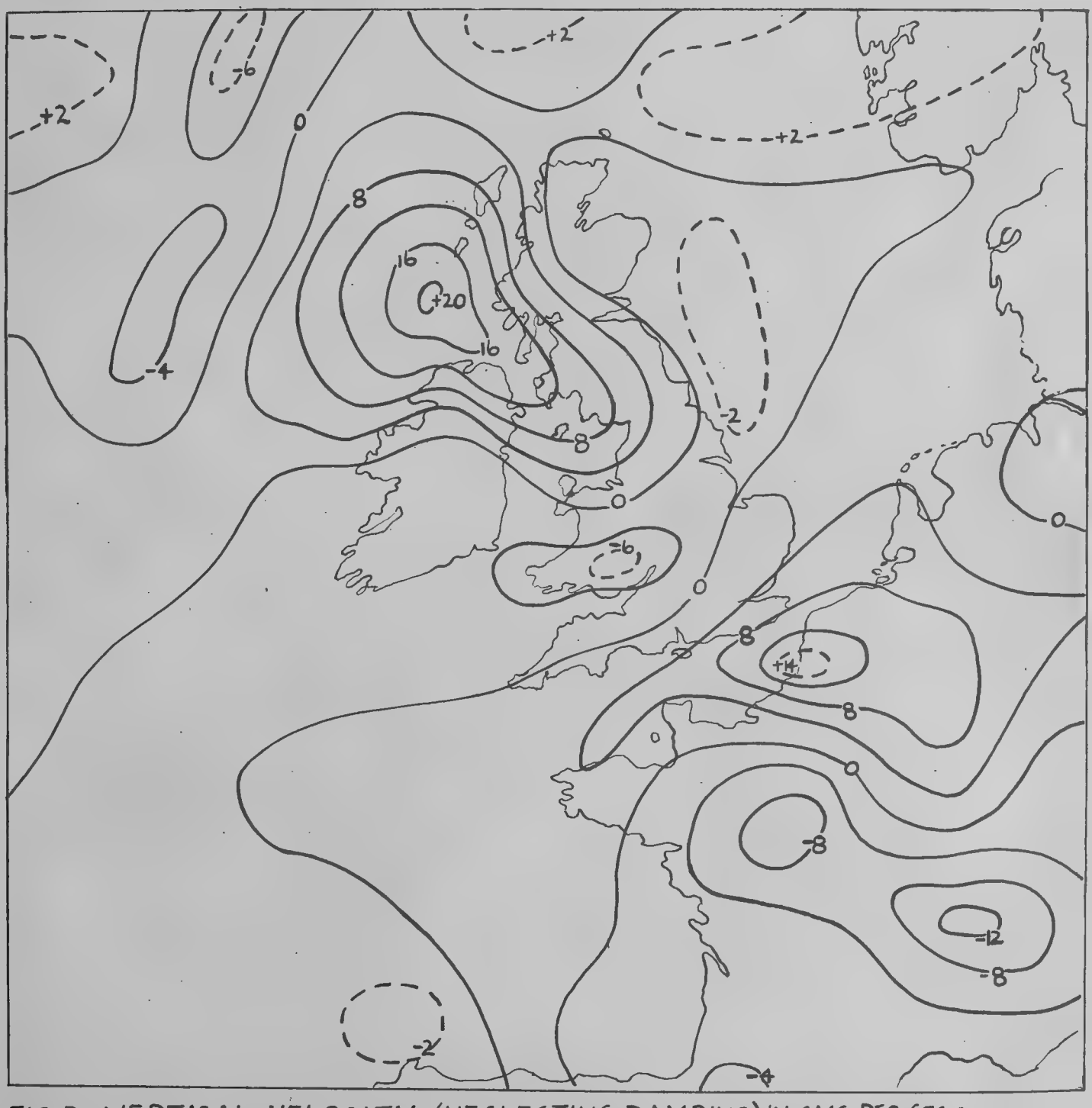

FIG.3. VERTICAL VELOCITY (NEGLECTING DAMPING) IN CMS. PER SEC. 2100 G.M.T. 14-2-50

FIG. 4. VERTICAL VELOCITY DAMPED IN CMS/SEC. 2100. G.M.T. 14.2.50.

M. R. P. 682.

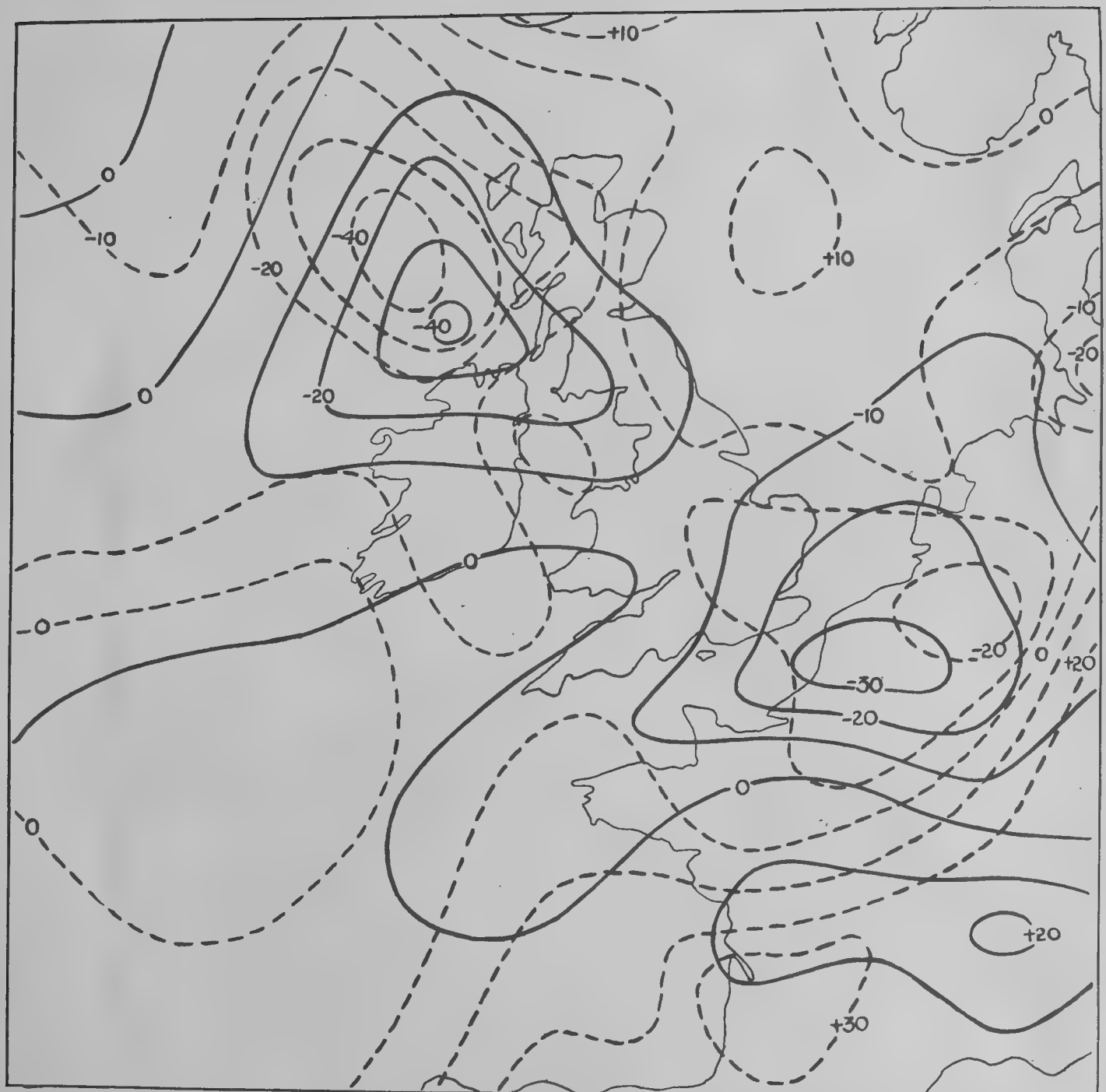

FIG 5. – – – ACTUAL MINUS ADVECTIVE THICKNESS TENDENCY IN FT./HR. 2100. G.M.T. 14·2·50.
——— COMPONENT OF THICKNESS TENDENCY DUE TO VERTICAL MOTION IN FT./HR.

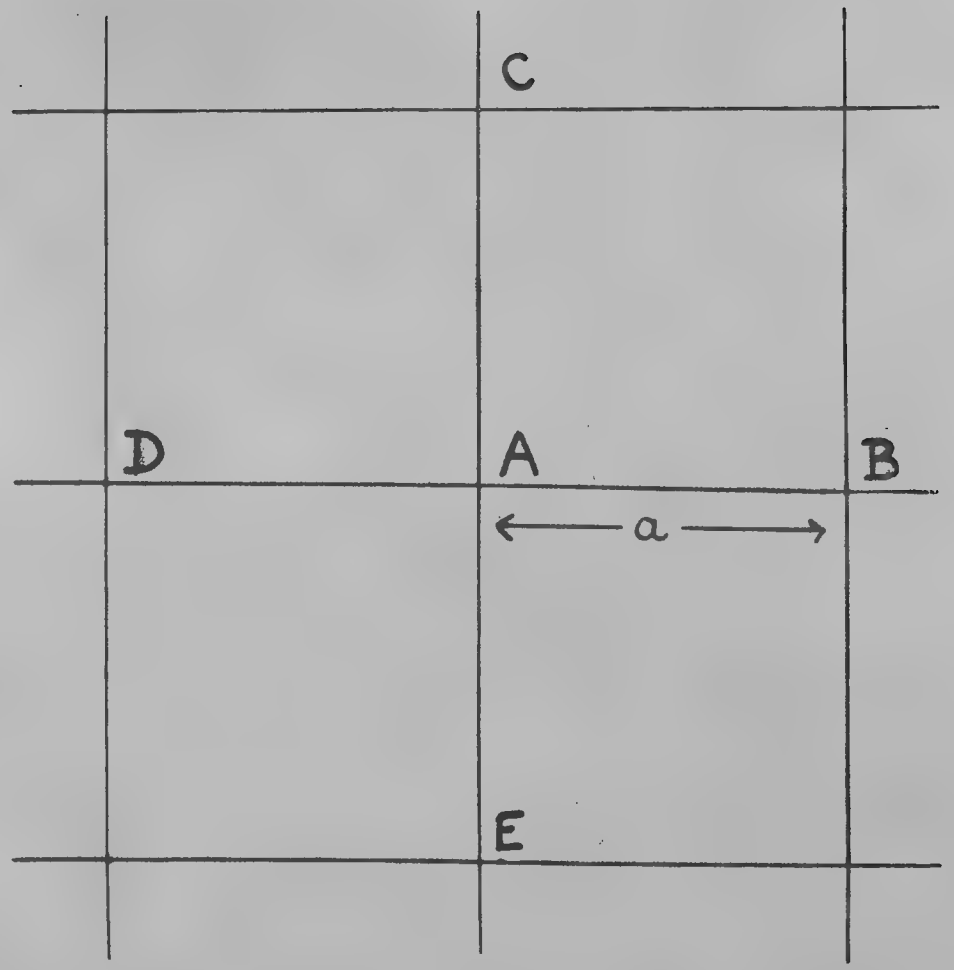

FIG. 6. GRID USED IN FINITE DIFFERENCE APPROXIMATIONS.

been published. Any author desiring to quote this paper should refer to it as "A paper of the Meteorological Research Committee (London). M.R.P. No..... by Mr..... A copy is available in the Library of the Meteorological Office."

M.R.P. 683

S.C.III/113

16th November, 1951.

AD 223031

AIR MINISTRY

METEOROLOGICAL RESEARCH COMMITTEE.

Aircraft observations of radar reflecting particles above the freezing level.

by

R.F. JONES (Meteorological Office).

AIR WEATHER SERVICE TECHNICAL REFERENCE FILE

Introduction

It is now well known that echoes are received on 10 cm and 3 cm radar equipment from precipitation. When this precipitation is undoubtedly in the form of water drops the relation between the intensity of the echo received and the size and number of the water drops has been well established both in theory(1) and in practice(2,3,4,). When, however, the radar echoes are received from the region of the freezing level and above, the explanation of the radar echoes and their intensities is on less secure foundations, because of lack of knowledge of the physical state of the reflecting particles.

An explanation of the various types of radar echo received from above the freezing level has been attempted by Bowen(10) and by the writer(5), while the explanation of the enhanced echo sometimes observed in the vicinity of the freezing level - the bright band - has been given by Ryde(1), Hooper(6) and others(e.g.7). There remains, however, an obvious necessity for direct and detailed observation of the reflecting particles responsible for the weather echo above the freezing level. If their size, character (e.g. ice crystals, agglomerates of ice crystals, hail, liquid water) and concentration can be reasonably well established, the interpretation of weather echoes received will be taken out of the realm of speculation and the value of radar in investigating precipitation processes greatly enhanced.

For this reason a number of flights have been made by the Meteorological Research Flight in cooperation with the ground radar station at East Hill to investigate clouds from which weather echoes extending above the freezing level were received and these flights form the subject of Part I of this report. The type of echo received was principally that associated with extensive cloud systems but some information is available, from another source, of the echo characteristics associated with isolated convection clouds and this is discussed in Part II of this report.

PART I - Investigations by the Meteorological Research Flight

1. The Observations

The difficulties of making quantitative measurements of precipitation from an aircraft are well known but, in these experiments, a qualitative estimate was made of the numbers and sizes of ice crystals in precipitation by observing the impact of the crystals on a black rod protruding from the side of the aircraft. Due to lack of suitable instrumentation, however, it was not possible to assess the sizes and numbers of any water droplets which may also have been present, although some indication of the presence of water drops large enough to hit the rod could be obtained from the amount of ice deposited on the rod (i.e. ice distinguishable from the individual ice crystals which were hitting the rod at the same time).

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The echo was therefore probably received from large numbers of ice crystals which were large enough to be visible individually as they struck the windscreen of the aircraft.

7. Flight on 1st February, 1950

This flight was made in the cloud and precipitation ahead of an active warm front moving east. The area covered by, and intensity of, the radar echoes received increased appreciably during the flight.

The vertical cross-section of the echo showed it to be of a very uniform character and free from any sudden discontinuities or marked variations in intensity, except at the bright band which was centred at about 3,000 ft. This bright band was not so clearly defined as in the flight of Section 5 above, particularly with respect to the top of the bright band layer. There was considerable echo extending well above the bright band on many occasions and occasionally a second layer of echo was visible.

Throughout the flight the aircraft was in cloud and precipitation could be seen streaming past the aircraft.

It was clear from the observations that the size and number of ice crystals was frequently the governing factor as to the intensity of the echo received from above the bright band, when allowance was made for the differing ranges from the radar set at which observations were made. Thus crystals up to 0.3 mm diameter of moderate intensity (on an arbitrary scale governed by the estimated number of crystals hitting the rod in a given time) gave a diffuse echo at 10 - 20 miles range and crystals up to 1mm in diameter of similar intensity gave a similar echo at 20 - 25 miles range. An increase of crystal size at approximately constant range gave a stronger echo and decrease in size led to disappearance of the echo.

There was however also the probability that a further factor influenced the echo intensity. Entry into the top of the bright band layer of high echo intensity was accompanied by the sticking of the ice crystals to the rod and at a lower height the rod became wet, while in the upper band of echo, in which the intensity was greater than anywhere else except the bright band, although the rate of precipitation and size of crystals was not appreciably greater than elsewhere, there was heavy icing at a temperature of about 19°F. Indeed this was the only report of noticeable icing. It seems therefore that an increase of echo intensity can also occur when considerable supercooled water is present, which is evidenced by the occurrence of icing. If this supercooled water was in the form of large numbers of droplets and if the ice crystal surfaces became wet on falling through these droplets, the observed increase in echo intensity would be explained.

8. Flight on 27th April, 1950

Clouds giving rise to a few column type echoes were investigated. The echoes were not of the isolated type but rather associated with stronger columns within a more diffuse mass of echo. From the aircraft reports it was clear that the columns were associated with cumulus tops reaching above the general cloud level to a height of about 11,000 ft.

Flight through one of these cumulus tops at 11,000 ft., 4°F, coincided with passage through - or possibly very slightly above - the top of a strong echo column. In cloud, visibility was very poor, the wing tip of the Halifax was only just visible, and there was rapid icing with a few large ice crystals hitting the rod. The ice crystals were about 2 mm in diameter. Similar conditions were encountered on returning through the cloud top at 10,600 ft.

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A third run through the cloud while descending from 8,500 to 7,500 ft. was very bumpy, large numbers of ice crystals were observed and there was severe icing.

It is clear from the report of rapid icing observed on a very short flight through the top of the cumulus that considerable supercooled water must have been present. It is impossible to say whether this water was present in the form of large or small drops but, with only a few large ice crystals present, it seems possible that the radar echo may have been received from large water drops rather than from the ice crystals. On the other hand it is possible that the aircraft was just above the echo top as very many more ice crystals were observed within the echo at about 8,000 ft., still with severe icing.

The strong echo column in this case may therefore be due to either (a) large supercooled water drops or (b) large numbers of ice crystals in the presence of considerable supercooled water - not necessarily in the form of large drops.

9. Flight on 19th May, 1950

On this occasion, associated with a thundery trough, some patches of echo were received. In the vertical cross-section the echoes showed as a mixture of column type and bright band echoes, the columns being quite pronounced. The aircraft investigated the clouds giving rise to these columns of echo and it was clear from the reports that the column type echoes were associated with cumuliform tops to the cloud although layers of cloud were also present at various heights.

Flights just in the echo column tops were associated with reports of (i) icing and a few ice crystals, temperature 11°F, (ii) icing, probably no ice crystals, temperature 11°F, (iii) rime ice, probably no ice crystals, temperature 9°F and (iv) ice crystals, very bumpy. On no penetration was the aircraft in cloud for as long as one minute.

Flight well within an echo column for $1\frac{3}{4}$ minutes encountered large ice crystals or snowflakes and heavy icing with $\frac{1}{4}$ in. of ice deposited on the rod.

On a flight through cloud 2,000 ft. above the top of a fairly strong echo column the cloud was reported as a water cloud, temperature 3°F; but above a weak echo exhibiting a bright band icing and ice crystals were reported.

It appears therefore that the echoing volume of the cloud was generally associated with large amounts of supercooled water and on some occasions, when few or no ice crystals were reported, it seems likely that this supercooled water was in drops large enough to give a radar echo. Well within a strong column of echo, however, there were snowflakes but with $\frac{1}{4}$ in. of ice forming in $1\frac{3}{4}$ mins there must also have been considerable free water.

10. Flight on 2nd June, 1950

Areas of rain occurring in an east to southeast airstream, but without any marked front, were investigated. The clouds were reported as very patchy and not confined to clearly defined layers. The vertical cross-sections of the radar echoes showed them to be bright band echoes but with some variations in intensity and, at times, a tendency towards column structure. Echoes extended on some occasions 8,000 ft. above the bright band.

An extensive flight was made investigating the echoes above the bright band. When in the diffuse echo about 5,000 ft. above the bright band snow was reported with ice crystals up to 3 mm in diameter, accompanied by icing

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which was usually slight but occasionally severe. When 3,000 ft. above the bright band and in an echo which showed some tendency towards column structure, particularly on the 3 cm radar presentation, there were many large crystals and some turbulence accompanied by very rough clear ice on the rod.

When clear of echo the ice crystals were usually small but some slight icing was reported. One one occasion "lots of ice crystals" when not in cloud did not give an echo even though the range of the aircraft at the time was less than 15 miles.

It would appear that the size (and probably the number) of the ice crystals was often a factor in the occurrence of an echo; but with icing, which was occasionally severe, reported there must also have been considerable supercooled water. It is noteworthy that when many ice crystals were reported without giving an echo the aircraft was not in cloud and therefore not in a region where supercooled water droplets would be encountered in large numbers.

11. Flight on 14th June, 1950

The echoes observed on this occasion were associated with widespread rain from an occlusion, and were very extensive and strong. In the vertical cross-section the echo consisted of a strong bright band centred at 5,000 - 6,000 ft. with considerable echo above extending to a level top at 15,000 - 16,000 ft. over a considerable range. This height of echo top was never exceeded, although very frequently attained, throughout the observations.

The fact that the echo top over a considerable range was level at 15,000 - 16,000 ft. implies that the increase of radar reflectivity at this height was quite sudden since the effect of increased range did not lead to any decrease in height of the echo top until beyond 24 miles (cf. Section 5 last para.). At 16,000 ft., temperature 6°F, the aircraft reported that it was at the top of the cloud with only Ci above and a little later "still just in top of this ice crystal cloud with thin layer of Ci above". A halo was visible. Later in the flight at 16,500 to 17,000 ft., temperature 3°F to 0°F, the aircraft was just in the top of the cloud. The radar echo top therefore was very close - probably within 1,500 ft. at the most - to the top of the principal cloud layer with only thin Ci cloud above. In the top of this cloud layer very small ice crystals were reported but it is clear that the radar reflectivity of the crystals, or other particles, increased considerably a few hundred feet lower in the cloud. From the aircraft reports it appeared that this increase in radar reflectivity occurred frequently with an increase in size of the ice crystals but there were one or two striking exceptions. Thus in one part of the flight a radar echo was received from large numbers of small ice crystals (less than $\frac{1}{2}$ mm in diameter) while later in the flight, at the same ranges, bigger crystals (of diameter ranging from $\frac{1}{2}$ mm to 2 mm) also in large numbers gave no echo.

It appears therefore that on some occasions the size and number of the ice crystals were not the only factors affecting the intensity of the radar echoes received. The observations made do not give positive evidence of any other factor but it may be significant that, while very slight icing was often reported in the weak echoes associated with small ice crystals, no icing was reported when the crystals were large but not giving a detectable echo. This suggests that the presence or absence of supercooled water drops is a factor of importance.

It seems probable that the top of the main cloud layer as seen from the aircraft was the top of the cloud containing supercooled water droplets and this cloud was being seeded from above by ice crystal precipitation from the Ci layer.

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12. Conclusions based on Meteorological Research Flight data

(i) If an echo is received from above the freezing level in conditions leading to the formation of a well-marked bright band in the vicinity of the freezing level, then the volume from which the echo is received contains large numbers of ice crystals of sizes of the order of $\frac{1}{2}$ mm diameter or greater. (Sections 7,10 and 11). It is probable that these crystals must also be associated with the presence of supercooled water but the size of the water droplets could not be obtained with the techniques used.

(ii) The co-existence of supercooled water can be inferred from the frequent reports of icing, although often only slight, when the aircraft was within the echo and also from the fact that the aircraft was always in cloud when in the echo. Icing, however, was encountered on some occasions in parts of the cloud from which no echo was received but the intensity was always slight.

(iii) The size and number of ice crystals frequently appear to be the governing factors in the occurrence or not of a radar echo above a bright band. Some observations, however, suggest that there may be other factors and increased reflectivity may, for example, be obtained when the ice crystal surface becomes wet by collision with supercooled water droplets. (Sections 5,7,10 and 11).

(iv) The bright band is associated with the change of state of the ice crystals or snowflakes to raindrops. (Section 5)

(v) When the radar echo takes the form of a column without obvious bright band characteristics, considerable supercooled water must be present since, even on very short flights through the echo, rapid icing is experienced. It appears possible that the echo on some occasions is received from large supercooled water drops but the observing technique used was unable to verify this conclusion. On other occasions large numbers of ice crystals or small hail must have contributed to the echo but the amount of free water must still have been large. (Sections 4,8,9).

(vi) One exploration of the precipitation associated with an occlusion suggested the possibility that the precipitation originated in the seeding of a lower layer of cloud, containing supercooled water droplets, by ice crystal precipitation from an upper layer of Ci cloud. (Section 11)

11. Recommendations for future flights

It is clear from the foregoing that, before conclusive evidence can be obtained on the nature of the radar-reflecting particles above the freezing level, a technique is necessary whereby the amount of supercooled water present in a cloud can be measured. Further, it is important to decide whether the supercooled water is in the form of cloud particles or larger drops. A rate of icing meter may lead to an assessment of the amount of liquid water present but the measurement of the size of the supercooled water drops has so far not been achieved.

Until satisfactory instruments have been devised for these purposes it is doubtful whether further flights will greatly increase the knowledge already gained from the flights described above.

/PART II

PART II - Investigations of cumulus and cumulonimbus clouds by Spitfire aircraft from the Royal Aircraft Establishment

1. Introduction

Flights by Spitfire aircraft from the Aero Flight, Royal Aircraft Establishment, have been made to investigate the turbulence within cumulus and cumulonimbus clouds, most of which gave a weather echo, and the procedure adopted and results obtained have been described elsewhere(8,9).

On most of these flights some observations were made of the principal weather features on traversing the clouds within the limitations imposed by (i) the aircraft was a single-seater and the pilot had many other duties to perform than recording the weather and (ii) the aircraft was usually in cloud on each traverse for a period of less than three minutes. The reports are therefore qualitative only and are presumably confined to the outstanding features of each traverse, so that absence of mention of a particular phenomenon e.g. precipitation, icing, does not necessarily imply that it was totally absent.

2. Data available

111 traverses, above the environment freezing level, of cumulus and cumulonimbus clouds are available for analysis.

The flights have been arranged according to the temperature of the environment at the height at which the traverse was made, as given by the relevant radiosonde ascent, and the pilots' reports have been tabulated and are given in Table II.

3. Conclusions based on Spitfire flights in cumulus and cumulonimbus cloud.

(i) The column type of radar echo received from parts of cumulus and cumulonimbus clouds is frequently associated with reports of precipitation, observable from an aircraft traversing the cloud. At heights well above the environment freezing level this precipitation is often reported as rain or rain and snow (sleet) mixed. On only few occasions was snow only or hail reported while on a large majority of the other flights, on which precipitation was not reported, icing was experienced.

It appears therefore that the column type radar echo may frequently be received from waterdrops of raindrop size at heights well above the environment freezing level.

(ii) Towards the top of the echo the size and number of reflecting particles presumably decreases since visible precipitation is reported less frequently, although it must be remembered that the time the aircraft takes to cross the echo, already small, is reduced with a correspondingly less time for observations to be made.

(iii) When no precipitation was reported within the echo there are two possibilities; (a) although precipitation was present, the pilot did not report it, due, for example, to restricted vision with the windscreen iced up or because the degree of icing was considered more important; (b) the cloud particles did not appear to be numerous or, perhaps, large enough to warrant classification as precipitation. The fact that icing was usually reported, even though the traverses were of very short duration, implies that considerable supercooled water must have been present and it seems reasonable to suppose that the reflecting particles were again water drops.

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(iv) Above the echo and in clouds which failed to give an echo icing was again frequently reported, with the inference that liquid water was present, but precipitation was not reported. In this case it can be assumed that the reflecting particles, although almost certainly liquid water drops or in the presence of liquid water, were too small to give a detectable radar echo, or to be classed as precipitation by the pilot.

4. Recommendations for future work

It is clear that more detailed investigation of the precipitation within cumulus and cumulonimbus clouds is desirable, but this is unlikely to be achieved with single-seater aircraft. The turbulence hazards make investigation by much larger aircraft unwise.

It is suggested therefore that any further investigations should be made in a twin-seater fighter aircraft with the spare seat occupied by an observer qualified to make the necessary reports.

The assessment of the amount of liquid water present and the size of the water drops is necessary and is the same problem as that already referred to in Part I, Section 11 above.

Summary

Aircraft investigations of clouds from which radar weather echoes extending above the freezing level were received shows that, for those echoes which contain a bright band about the height of the freezing level, the echo from above the bright band is received from ice crystals or snow flakes of diameter of the order of $\frac{1}{2}$ mm or greater. The size of crystals necessary to give a detectable echo will, however, depend on the type and wavelength of the radar equipment used and on the range at which the crystals are viewed. The intensity of the echo received frequently increases as the size and number of the crystals increase but would appear also to depend on the amount of supercooled water present. The inference is made that on some occasions the echo intensity is increased by the effect of the ice crystal surface becoming wet on collision with numerous supercooled water droplets. It has not been possible to measure the amount of supercooled water nor the size of the drops in which it occurs but its presence is inferred from the frequent reports of icing encountered by the aircraft.

The bright band is certainly associated with the region of the cloud in which the melting of ice crystals to form raindrops is taking place.

The stronger column type of echo, not exhibiting a marked bright band, is usually associated with cumulus and cumulonimbus clouds, although these clouds may on occasions be embedded in layer or frontal clouds. The amount of supercooled water present in the echo producing region above the freezing level is much greater than for bright band echoes and it is probable that on many occasions the echo is received from large water drops, even at heights well above the freezing level. Even when ice crystals are present the amount of free water is large and may exist in the form of large drops. This latter possibility cannot be verified however with existing observational techniques.

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TABLE I

Flights made by Meteorological Research Flight aircraft investigating radar reflecting particles above the freezing level

No.	Date	Time (GMT)	Type of echo examined	Synoptic situation
1.	22. 6.48	1100 - 1140	Bright band with column type echoes mixed.	Trough in polar air, depression NE Scotland.
2.	9.11.49	1100 - 1200	Bright band with very weak echo above for limited distance.	Warm front moving east.
3.	16.11.49	1450 - 1520	Diffuse echo of limited horizontal extent above weak bright band. Visible on 3 cm radar only - not on 10 cm.	Stationary front to west of area.
4.	1. 2.50	1120 - 1240	Bright band with considerable echo above.	Active warm front moving northeast to east.
5.	27. 4.50	1440 - 1520	Column type echoes.	West to northwest airstream with some limited instability.
6.	19. 5.50	1015 - 1140	Mixed column and bright band.	Thundery trough extending across British Isles from depression west of Portugal.
7.	2. 6.50	1000 - 1055	Bright band with echoes above - sometimes like columns.	Anticyclone Southern Baltic, east to southeast airstream. Limited areas of rain.
8.	14. 6.50	0940 - 1100	Strong bright band with considerable echo above.	Occlusion and cold front moving southeast, depression Southern Scandinavia. Widespread rain.

TABLE II

Flights by Spitfire aircraft through cumulus and cumulonimbus clouds above the environment freezing level

Pilots' reports of weather encountered

Environment Temperature (°F)	Rain without mention of icing.			Rain with icing (i.e. freezing on impact)			Sleet			Snow. No icing reported			Hail. No icing reported			Icing with no precipitation or no mention of pptn.			No icing. No mention of pptn.			Number of flights			Total
	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	
30 - 21	7	2	0	4	0	0	1	2	0	1	0	0	2	0	0	4	3	1	2	1	5	21	8	6	35
20 - 11	3	0	0	3	0	0	4	0	0	2	1	0	1	0	0	9	7	0	0	1	0	22	9	0	31
10 - 01	1	0	0	0	0	0	1	0	0	2	0	0	0	0	0	6	2	3	0	3	0	10	5	3	18
00 - -09	0	0	1	1	0	0	1	0	0	0	2	0	0	0	0	3	1	7	0	2	0	5	5	8	18
-10 - -19	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	1	1	2	1	1	4
-20 - -29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	1	1	1	1	3
-30 - -39	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	2	0	0	2
Totals	11	2	1	8	0	0	7	2	0	6	3	0	3	0	0	26	14	11	2	8	7	63	29	19	111

Notes Column A Flights through the radar echo received from the cloud.
Column B Flights through the tops of the radar echoes.
Column C Flights above the echo or in clouds which did not give an echo.
On 10 of 12 occasions on which rain intensity was reported, the intensity was moderate or heavy.
On four of the seven occasions of sleet in Column A, icing was also reported, 1 at 30 - 21°F, 2 at 20 - 11°F and 1 at 00 - -09°F.
On the occasions of icing with no pptn., the icing was reported as severe on 5 occasions in Col.A, 2 occasions in Col.B. and on no occasion in Col.C.
Of the 19 flights in Column C, 7 were above the echo and 12 through clouds which gave no echo. The 12 occasions of cloud with no echo were 5 with no icing at 30 - 21°F, 6 with icing but no pptn. at 00 - -09°F and 1 with icing but no pptn. at 30 - 21°F.

pptn. = precipitation.

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AIR MINISTRY

METEOROLOGICAL RESEARCH COMMITTEE

Diurnal Changes at High Levels

by
A.H.R. Goldie (Meteorological Office)

1. Introduction

A year or two ago the writer, in an elementary discussion of the dynamics of thermal circulation in vertical planes in the atmosphere, concluded that in the "warm" region of the globe the convection of heat upwards from the surface should be checked at about the 17.5 km. level because this was the level of maximum adiabatic effect. The calculations indicated that the adiabatic effects of the circulation in reducing temperature at the 400 mb. level, the 85 mb. level (about 17.5 km.) and the 40 mb. level should be in the ratio 1.00: 2.14: 1.08. It was concluded that the 17.5 km. level in the Tropics should become the coldest part of the planetary system. Up to this level in the Tropics convection would determine temperature. Above this level radiation in the long run would mainly determine temperature, (unless the sense of the circulation were reversed above 17.5 km., in which case subsidence and radiation would be the determining factors).(1)

In the "cold" region, on the other hand, of a major thermal circulation these effects would be reversed. Temperature rise, owing to the circulation, would be a maximum at 17.5 km. and radiation would have a chance to become effective in the levels from 17.5 km. downward.

An attempt was made at the same time to estimate what sort of diurnally varying field should prevail over the globe at high levels. This attempt was not published because no data regarding the diurnal changes of temperature in the upper air were available for the Tropics. Since then, however, it has been discovered that the regime in the upper air at Habbaniya (lat. 33°N.)(2) is characteristically tropical, (i.e. there is the high tropopause and very cold stratosphere) from about mid-May to mid-September. Incidentally at the same season the Geographical Equator, as represented by Nairobi, loses its position as the region of highest tropopause. This is a matter of great interest, since the thermal equator - as determined by the surface isotherms - lies on or to the south of the geographical equator in January, whilst in July it stretches across Asia in about lat. 30°-35°N, thence into Arabia and thence across Africa in about lat. 20° to 25°N. Thus it happens that Nairobi lies near the January thermal equator whilst Habbaniya lies in the July thermal equator. Observations from Habbaniya from 14 h. and 2 h. are available and are used in para. 5 of this paper.

2. Preliminary Estimation of diurnal pressure ranges in the Tropics up to 24 km.

First an estimate is made of the expansion of the atmosphere which might be expected by daytime heating if no circulation took place between the heated region and other regions. For the lower layers we use the pressure data of mountain observatories. The expansion of the lower atmosphere by daytime heating lifts a greater amount of air above the mountain, so that at the mountain top* there is a maximum of pressure about 16h., a minimum about 8 h. The diurnal variation of pressure difference between Madras (7 metres) and Dodabetta (2631 metres) answers to a vertical expansion and

/contraction

* if the height is sufficiently great

contraction of the intervening atmosphere of some 22 metres in range, corresponding to about 2.5°C in diurnal range of temperature. Other mountain data lead to estimates of the same order.

To estimate the diurnal wave at higher levels in the absence of circulation we assume for the Tropics:-

(a) that the air in the first 3 km. (roughly the first 300 mb.) has a daily temperature range of 2.5°C. Strictly speaking this is a minimum, being derived from actual values which include the effect of circulation

(b) that the air from 3 km. to the tropopause at 17 km. (roughly a 620 mb. interval) has a daily temperature range averaging 1.5°C.

(c) that the air from 17 km. to 24 km. (a 63 mb. interval) has a daily temperature range averaging 3/4°C. (Only about 3% of the atmosphere lies above 24 km.)

The evidence for (a) is, in the first place, the mountain observations of pressure. And the observed diurnal variations of temperature on tropical islands indicate that the value here assumed should not be an over-estimate for the Tropics generally. The justification for the tentative assumption (b) is that the air from 3 km. to 17 km. in the Tropics has approximately the adiabatic lapse rate, so that a mean actual daily heating of 2.5°C below 3 km. seems scarcely possible at first sight without something nearly as great all the way up to the tropopause. The assumption (c) we leave for the present.

3. Estimate from Solar Radiation

We can estimate independently whether the solar radiation justifies the assumptions, in the preceding paragraph, as a whole. The heat required to achieve (a), (b) and (c) together amounts to 410 gm. cal. per sq. cm. The total solar heat available at the equator at the equinox from sunrise to sunset (on a solar constant of 1.94 gm. cal. per sq. cm. per minute and neglecting emission) is about 890 gm. cal. per sq. cm. The gross amount received from sunrise up to 4 p.m. should be of the order of 800. Thus, on the whole, the required figure of 410 is perhaps not unreasonable. It represents the absorption (by 97% of the atmosphere) of about 50% of the total insolation, but even if the assumed heating above 3 km. had to be scaled down by 20 or 30% it would not substantially alter the conclusions drawn below.

4. Computed Pressure Wave and Diurnal Wind Pattern

The columns under A of Table I give the computed expansion of the atmosphere up to various pressure levels and the corresponding rises of pressure which would be produced at fixed heights with the temperature increases postulated in para. 2 provided there were no circulation to other regions.

It thus seems that the rises of temperature estimated in para. 2 lead to a total expansion of the 24 km. of air of about 140 metres or a diurnal pressure wave at the fixed level of 24 km. in the atmosphere of about 0.6 mb.

The world pattern deduced therefrom is that in the Tropics, where the time is 16 h., the isobaric surfaces at the 24 km. level would be bulged up by 140 metres as compared with their position where the time is 8 h. To a lesser extent all the lower isobaric surfaces would be bulged up. Thus at the equinoxes, for example, the world pattern in the diurnally varying pressure field at high levels is that there is a centre of high pressure on the equator where the time is 16h., a low where the time is 8h. There

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should be a high level off-flow of air from the tropical "blister" in all directions. This argument so far, however, assumes in effect that the complete heating takes place before the circulation starts and that then there is only one circulation in vertical planes, namely expansion in the Tropics where the local time is afternoon, off-flow at high levels to other regions, namely to temperate latitudes and/or to the Tropics where the local time is night or early morning, thence subsidence of air and flow in the reverse sense in low levels. The step by step computation shows that the pressure gradient increases up to about 9 km. and then falls off. The complication, however, is that the primary circulation must all the time be tending to modify the temperature structure.

5. Computation of actual conditions as based on Habbaniya data

The temperature differences 14h.- 2h.for Habbaniya for the various months of the years 1948-50 are given in Table II. Too much importance cannot be attached to individual values in this table, but the compact group of negative (or at most negligible positive) values in the summer months from the 100 mb. level down to 200 or 300 mb. is probably significant. Such an effect could only arise if the adiabatic cooling of air in the higher troposphere in the daytime is greater than the heating by insolation (plus any artificial instrumental effect). Thus we seem justified in assuming that the tropical regime there is strong off-flow of air by day, at high levels, - so strong that by 14h. it has actually brought decrease of temperature in the region from about 14 km. up to 17 km. (the tropopause).

A revised estimate has been made, using the actual values of the heights of isobaric surfaces for summer for Habbaniya. The results are shown in columns B of Table I. The expansions in levels above the tropopause have been computed on the assumption that the 14h. - 2h. temperature difference remains at its 19 km. value of 1.8°F. The total heat required for all this, however, is only about 185 gm. cal. per sq. cm. so apparently over a half of the heat which might have been gained from the local insolation has been carried away elsewhere or used up as work.

In columns C are given the differences in terms of height and pressure of columns A and B. These differences may be regarded as representing the amount of off-flow of air (or at least the order of this off-flow) at each level which has taken place whilst daytime heating of the atmosphere was going on. There is another way of checking that the off-flow of air, thus estimated, is of about the right order. The pressure at surface level at Habbaniya at 15h. has fallen on the average by 1.6 mb. as compared with the value at 3h. This 1.6 mb. is equivalent to a layer of air about 15 metres thick at ground level or 40 metres at the 9 km. level or 60 metres at the 12 km. level.

6. Adiabatic effects of off-flow of air

The temperature reduction which would be caused at the various levels by reducing the pressure by "unloading" the amounts given in columns C is shown in column D. It goes on increasing up to the 18 km. level, after which it seems (though the observational data become sparse after 19 km.) that it must decrease.

Thus, so far as the observations go, the consequences of thermal circulation seem to be in line with the theoretical discussion quoted at para. 1 and it seems that the expansion actually attained by the atmosphere by daytime heating in the Tropics is only about a third to a half of that which might be expected in the absence of a thermal circulation. This seems to be a reasonable result, since the effect of circulation should be to level

/down

down the effects of unequal heating in different parts of the globe and therefore to reduce substantially the expansion which would have obtained in the hottest region if there had been no circulation. Above the tropopause the rate of linear expansion, according to the Habbaniya data, may increase again substantially but in terms of heat involved this does not amount to much, having regard to the small amount of air above this level. (Anyway we cannot yet determine how much of the apparent daytime rise of temperature in the stratosphere is real).

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TABLE I

Daytime expansion of the atmosphere in the Tropics

Height in Km.	A Estimate (no thermal circulation)		B Actual (derived from Habbaniya Heights of Isobaric Surfaces)		C Difference A-B		D Adiabatic effect of pressure difference (A-B) °F
	Expansion in metres	Equivalent Pressure Range (mb.)	Expansion in metres	Equivalent Pressure Range (mb.)	Expansion in metres	Equivalent Pressure Range (mb.)	
24	140	.61	(68)	0.29	72	0.32	1.3
22	134	.85	(58)	0.37	76	0.48	1.3
20	126	1.15	(48)	0.44	78	0.71	1.4
18	118	1.59	(38)	0.51	80	1.08	1.5
16	110	2.09	40	0.76	70	1.33	1.3
14	96	2.38	35	0.87	61	1.51	1.0
12	80	2.65	35	1.16	45	1.49	0.9
11	73	2.70	34	1.26	39	1.44	0.8
10	67	2.79	32	1.33	35	1.46	0.6
9	61	2.85	30	1.40	31	1.45	0.6
8	55	2.84	29	1.50	26	1.34	0.5
7	49	2.83	27	1.55	22	1.28	0.3
5	37	2.61	25	1.83	12	0.78	0.2
3	26	2.22	17	1.45	8	0.77	0.1

/TABLE II

TABLE II

Habbaniya - Temperature difference 14h. - 2h.

Means of 1948-50 (°F)

Pressure Mb.	Jan.	Feb.	Mar.	Apl.	May	Jne.	July	Aug.	Sept.	Oct.	Nov.	Dec.
60	-1.6	1.7	1.6	3.3	2.3	1.1	2.1	2.2	1.8	2.3	0.5	-1.6
100	0.4	2.3	1.8	1.5	-0.5	-0.2	-2.2	-0.7	1.4	2.1	0.9	1.0
150	-0.1	0.7	0.9	0.6	0.0	-0.1	-1.7	1.6	1.5	1.3	-0.1	-0.1
200	0.4	2.2	1.5	0.9	0.0	0.2	-1.1	2.0	1.1	-0.9	0.1	1.3
300	1.0	0.1	1.9	2.0	1.5	0.5	-1.1	1.3	1.3	-1.9	0.7	0.0
400	0.5	1.5	0.6	1.6	1.9	1.1	0.4	0.6	0.7	0.3	0.3	0.2
500	0.9	0.8	0.6	1.5	2.1	0.7	0.5	0.8	0.7	0.0	0.4	0.4
600	2.0	0.1	0.5	1.8	2.5	1.6	1.6	0.9	0.8	0.2	0.5	0.9
700	0.4	1.3	1.2	1.2	2.4	2.1	1.8	1.1	1.1	1.2	0.6	0.8
800	-0.1	0.2	0.2	1.5	2.1	3.1	1.4	1.8	1.3	0.9	0.0	0.7
900	1.2	2.5	2.1	3.4	4.7	5.0	4.6	4.1	3.7	2.1	0.0	0.5
Surface	11.8	17.2	16.8	20.6	22.8	27.4	27.6	28.2	27.4	22.6	19.0	13.0

TABLE III

Habbaniya - Seasonal Means of Temperature

Difference 14h. - 2h. (°F)

Ht/Km.	Mb.	June July Aug.	October to March	April & Sept.
19	60	+1.8	+0.5	+2.5
16	100	-1.0	+1.4	+1.4
14	150	-0.1	+0.4	+1.0
12	200	+0.4	+0.8	+1.0
9.5	300	+0.2	+0.3	+1.6
7.5	400	+0.7	+0.6	+1.1
6	500	+0.7	+0.5	+1.1
4	600	+1.4	+0.7	+1.3
3	700	+1.7	+0.9	+1.2
2	800	+2.1	+0.3	+1.4
1	900	+4.6	+1.4	+3.5
	Surface	+27.7	+16.7	+24.0

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SOME FURTHER ASPECTS OF NIGHT COOLING UNDER CLEAR SKIES

by

W.E. SAUNDERS (Meteorological Office)

Summary

An attempt is made to forecast the cooling curve for a radiation night by first forecasting the time and temperature of the evening discontinuity in rate of cooling and then estimating the further cooling to the night minimum.

Evidence is given that the discontinuity in cooling is the result of the beginning of dew formation.

1. Introduction

In aviation forecasting it is a routine requirement to estimate not only whether radiation fog will form but the time when it will form, and it is the practice at many stations to do this by first estimating a likely "fog point", the temperature at which fog would form if the air at screen level were sufficiently cooled, and then to estimate the time when this temperature will be reached, if at all. This method is in regular use at Northolt, and is known to have been practised for at least 10 years at some stations, though little has been written on the subject. Detailed forecasting of night cooling is also increasingly a requirement of agriculture and of industry. It must then be emphatically stated that this paper is not concerned with a hypothetical requirement, or with a new problem, but is an attempt to systematise an established method.

It is a fact of observation that the cooling curve is typically not a smooth curve but is characterised by a very steep decline in the evening followed by a much more gradual decline, the two phases being separated by a marked discontinuity. The fact is probably familiar to many forecasters, and was certainly taken into account at some stations during the war years. It was discussed recently by the writer (1).

The present approach therefore is first to forecast the time and temperature of the discontinuity and then to estimate the night minimum as the further depression from this value. In this way a fair fix on the whole curve is obtained and the temperature at any time can be interpolated.

2. Time and temperature of the discontinuity

As further evidence of the reality of the discontinuity in cooling attention is drawn to a recent American paper (2). Half-hourly readings of temperature are given for various levels above and below the ground from which it appears that the discontinuity is pronounced very near the ground, less so at screen level, delayed by about an hour at a depth of 2 in., and difficult to recognise above and below. An example is given in Fig.1. It seems then that the phenomenon is primarily a feature of the boundary layer, and it has been suggested to the author, independently by S.F. Whitcombe and J.M. Craddock, that the discontinuity marks the time of first deposit of dew, the latent heat effect presumably retarding the temperature fall. This interpretation is supported by a few simple experiments made by the writer on seven clear-sky, light wind evenings in June, 1950. Over a closely mown lawn wet and dry bulb

thermometers were set up at a height of 1 in. and frequent readings were made. In every case, somewhere between 2000 and 2010, the time of saturation and reduced rate of cooling coincided with an obvious and sudden formation of heavy dew. The thermometers were screened from direct radiation with aluminium foil. There are some unresolved difficulties. On three occasions the deposit coincided with a sudden slight rise of temperature, and this is known to occur from time to time at screen level. Also the sudden discontinuity regularly appears on more windy nights when there is no visible dew deposit on the grass. It may be suggested that the phenomenon is due to a sudden increase in turbulence in the boundary layer, but this could not bring moisture downwards unless there were an inversion of the hydrolapse and it is not easy to see how this could be set up without the previous deposit of dew. On the whole the evidence supports the view that the deposit of dew is the cause and not an effect of the discontinuity of cooling.

The phenomenon was further studied from screen temperatures at Northolt taken at half-hourly intervals through the period August, 1948 - April, 1950. Having eliminated cases where the discontinuity might have been caused by passing cloud, and cases where it appeared there was advective change, the time and temperature of discontinuity were noted for each clear evening. The times are represented by crosses in Fig.2 and, as some indiction of the soil moisture, the daily rainfall is shown by ordinate lines.

In both years there is a sudden change in the time during the first week in October, and in spite of the fact that this precedes the main rainfall it is believed that it marks the transition to the winter period when the top soil remains permanently moist, when relative humidity in the near-surface layer is always high, and when dew forms very readily. During the winter, rainfall does not appear to affect the time, but in summer there is an indication, e.g. in May-June, 1949, that the time is earlier after rain. This effect is more pronounced in spring and early summer than in late summer. The monthly average times are in fair agreement with those given previously for Abingdon. They agree well with the times for Bovingdon. The times for London Airport and Croydon are systematically later, no doubt due to soil differences.

Following the method of Craddock (3) we have obtained regression equations for forecasting T_r (screen temperature at the change of cooling rate) in terms of T_{max} (maximum temperature) and T_d (dew point at the time of T_{max}). It was found by inspection that T_r also depends upon the afternoon temperature lapse rate, being significantly lower in cases of stability, higher with a steeper lapse. By separating the cases into two rather arbitrary groups, as there was or was not an afternoon inversion with base at or below 900 mb., quite small scatter was obtained about the regression lines. It should be noted that we refer to an inversion in the air mass, not to the normal evening ground inversion. The formulae are:-

with no inversion $T_r = \frac{1}{2}(T_{max} + T_d) - 0.6^\circ F$ (78 occasions, s.d.1.04°F)

with inversion $T_r = \frac{1}{2}(T_{max} + T_d) - 4.0^\circ F$ (27 " " 0.88°F)

On evenings when the discontinuity could not be discerned at screen level the temperature T_r given by these equations was reached at about the appropriate time.

3. Subsequent Cooling

On general theoretical grounds it must be expected that subsequent cooling will be affected by many factors, of which the nature and state of soil (conductivity, diffusivity, specific heat), characteristics of the air mass (temperature and humidity profiles, probably haze content), nature of condensation (dew or hoar frost), and wind speed, are certainly important variables. There is little prospect of being able to deal with all these factors in theory, and the choice of forecasting parameters must

be based on consideration of their relative importance and on general inspection of the data.

Without labouring the arguments or describing the trials which led to this choice of classification, the night minimum (T_{min}) for clear nights (mean cloud amount $\leq 1/8$) was derived in the following ways:-

(i) With calm and light winds T_{min} was expressed as a function of the calculated value of T_r, separately for the summer and winter periods (i.e. temperature regarded as the main variable during periods in which the state of top soil is unchanged, except after rain). An allowance was made for the decreased length of time available for subsequent cooling during the midsummer period.

(ii) With stronger winds the fall of temperature was expressed as a function of the gradient wind speed.

The results are presented in graphical form (Figs.3, 4, 5 and 6). Fig.3 gives the relation for calm and light winds for spring and late summer, taken as 25/3 - 4/5 and 11/8 - 7/10. Fig.4 gives the corresponding relations for winter.

The transitional dates were chosen independently to give the best fit, but are clearly related with the change of season indicated in Fig.2, and so, as we have surmised, with the state of ground.

The separation into days of inversion and no inversion introduced in para.2 was carried forward into this analysis, the observations being represented by circles and crosses. In summer afternoons, stability has no obvious effect on subsequent cooling, but in winter there is a tendency for cooling to be greater in the stabler air mass (Fig.4, AB compared with CD). A separation by gradient wind speed was also adopted.

It was found necessary to apply a time correction during the midsummer part of the summer half-year, when the period of subsequent cooling falls below about 8 hrs. The amount of cooling indicated by Fig.3 is reduced to the extent indicated by Fig.5, provided the top soil is dry. This shows that subsequent cooling on calm nights is reduced from about 15°F in April and September to about 12°F in late June. A difference of this order is readily predictable on theoretical grounds. Brunt's equation (4) shows that, with other factors materially unchanged, the number of degrees cooling is proportional to $\sqrt{t}$ where t is the cooling period. The subsequent cooling is 15°F in April and again in mid-August to late September, when the period averages 8 hours. In late June when the period is 6 hours, the subsequent cooling is reduced to $15 \times \sqrt{\frac{6}{8}} = 12°F$ approx. The time correction clearly applies to the length of period of subsequent cooling, not to the total length of night. After rain T_r is reached early and subsequent cooling tends towards the full 15°F. There is no indication of a corresponding time correction during winter, when probably cooling becomes so slow towards the end of the night that variations in time do not affect the result.

Fig.6 gives the relation for stronger winds, suggesting that the correlation increases with the wind speed.

4. Application to forecasting

The forecasting procedure for Northolt is:-

(i) Take the observed T_{max} and T_d for the afternoon. Consider if they are representative of the air likely to be over the locality during the night. If not, make a judgement from observations upwind.

(ii) Estimate synoptically, allowing for advection, whether or not there is an inversion at or below 900 mb.

(iii) Calculate T_r from the appropriate regression equation.

(iv) Estimate the time of T_r from Fig.2, making a correction of 1 hr. early in early summer and $\frac{1}{2}$ hr. early in late summer for wet top soil if necessary.

(v) From the forecast T_r and forecast mean gradient wind speed use the following table to obtain T_{min}:-

	Forecast mean gradient wind mi/hr.	Method for T_{min}
Summer late March-early Oct.	0 - 14	Fig.3 curve AB) Add to T_{min} a time
	15 - 21	" " " CD) correction from Fig.5 during 5 May - 10 Aug., except after rain.
Summer (contd.)	> 21	By $T_{min} = T_r$ (calculated) - Δ°F where Δ given by Fig.6
Winter early Oct. - late March	0 - 19	Fig.4, AB or CD according to afternoon lapse rate.
	19 - 24	Fig.4, EF.
	> 24	As in summer

This approach to the practical problem provides no simple all-purpose formula but requires care and judgement. But forecasters do not expect to see simple results to complex problems, and will not object to the procedure if it provides useful results.

Northolt is on clay soil, and the method should give comparable results on clay soils elsewhere. But on other soils corrections will certainly be necessary. For example a comparison of T_{min} between Northolt and London Airport on 25 clear sky/calm nights showed that T_{min} was always higher at L.A.P. where there is no clay near the surface. The mean difference was as much as 4.0°F (s.d. 1.5°F). The mean T_{max} on the previous afternoon was only 0.5°F higher at L.A.P., so that the difference in T_{min} must be attributed almost solely to differences in soil and exposure. The difference in T_{min} decreases as the wind speed increases. Clearly, therefore, in applying this method at other stations, separate corrections must be deduced from a sufficient number of cases for each section of the above table.

Cooling near snow covered surface has not been included, owing to lack of snow during the period.

Occasions of fog where "sky obscured" was reported were regarded as cloudy nights and omitted. Other fog or thick haze cases were included.

5. Comparison with earlier methods

The method outlined in the previous section was tested against Northolt data for April-July, 1951. Of 34 clear nights there were 4 in which the behaviour of the dew point gave clear indications of advective change. On the remaining nights the mean square error from the forecast T_{min} was 1.28°F.

The present work suggests that the partial success of earlier methods which used $\frac{1}{2}(T_{max} + T_d)$ has been largely due to the fact that this is in many cases almost equal to T_r, the initial temperature of the subsequent cooling. For example, McKenzie (5), in a formula which has been widely applied, gives for clear skies and calm:-

$$T_{min} = \tfrac{1}{2}(T_{max} + T_d) - 15°F$$

Curve AB (Fig.3) is practically the straight line $T_{min} = T_r - 15$, and T_r differs from $\frac{1}{2}(T_{max} + T_d)$ by only 0.6°F in the non-inversion case. Hence, in non-inversion cases in spring and late summer McKenzie's formula would be very accurate. In mid-summer it would give too low an estimate. In winter (Fig.4) it would give T_{min} too high when the temperature is high, too low when it is low. Sometimes the error would be quite appreciable - in an inversion case where $T_r = 55°F$, $\frac{1}{2}(T_{max} + T_d)$ would be 59°F, and McKenzie's formula would give $T_{min} = 44°F$, whereas Fig.4, curve AB, would give 35°F.

6. The grass minimum temperature

The relation between the screen minimum (T_{min}) and grass minimum (T_g) temperatures was investigated for the same period with results as given in Tables 1 and 2.

Table 1.
Variation of difference between screen and grass minimum temperatures with wind speed

Season	Gradient wind speed mi/hr.	Mean $T_{min} - T_g$ °F	s.d. °F
Summer	0 - 14	8.0	1.5
Winter	0 - 19	(varying with temperature in accord with Table 2 below)	
Summer	15 - 28	11.1	2.5
Winter	20 - 28		
Whole year	over 28	7.4	1.3

Table 2.
Variation of difference between screen and grass minimum temperatures with temperature for occasions of calm and light wind during the winter.

T_{min} °F	Mean $T_{min} - T_g$ °F
31 - 35	4
26 - 30	7
21 - 25	9
16 - 20	11

Table 1 shows that the first effect of increasing wind speed is to increase $T_{min} - T_g$, corresponding with the stage at which turbulent mixing with warmer air extends down to screen level but not so effectively to the grass. When the gradient wind exceeds 28 mi/hr. mixing evidently does reach the grass level. The rapid increase in $T_{min} - T_g$ on calm winter nights is no doubt due to the decreased amount of moisture available for forming hoar frost and the decreased latent heat released. If T_{min} is, say, 34°F $T_{min} - T_g$ will be 8°F in summer with dry soil, but only 4°F in winter when the soil is permanently moist.

7. Conclusion

The results show that increased accuracy in forecasting night cooling follows from consideration of more parameters.

The Northolt results require verification on other clay soils, and corrections for other soils could be deduced. Corrections for use on overcast and partly cloudy nights could be tabulated. Experimental work is needed to determine, for various soils, what changes in moisture content govern variations in the time of the discontinuity, and in the transitional dates for summer and winter curves.

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FIG. 1

DRY BULB TEMPERATURES - MANOR, TEXAS

27th/28th SEPT. 1948

M.R.P. 687.

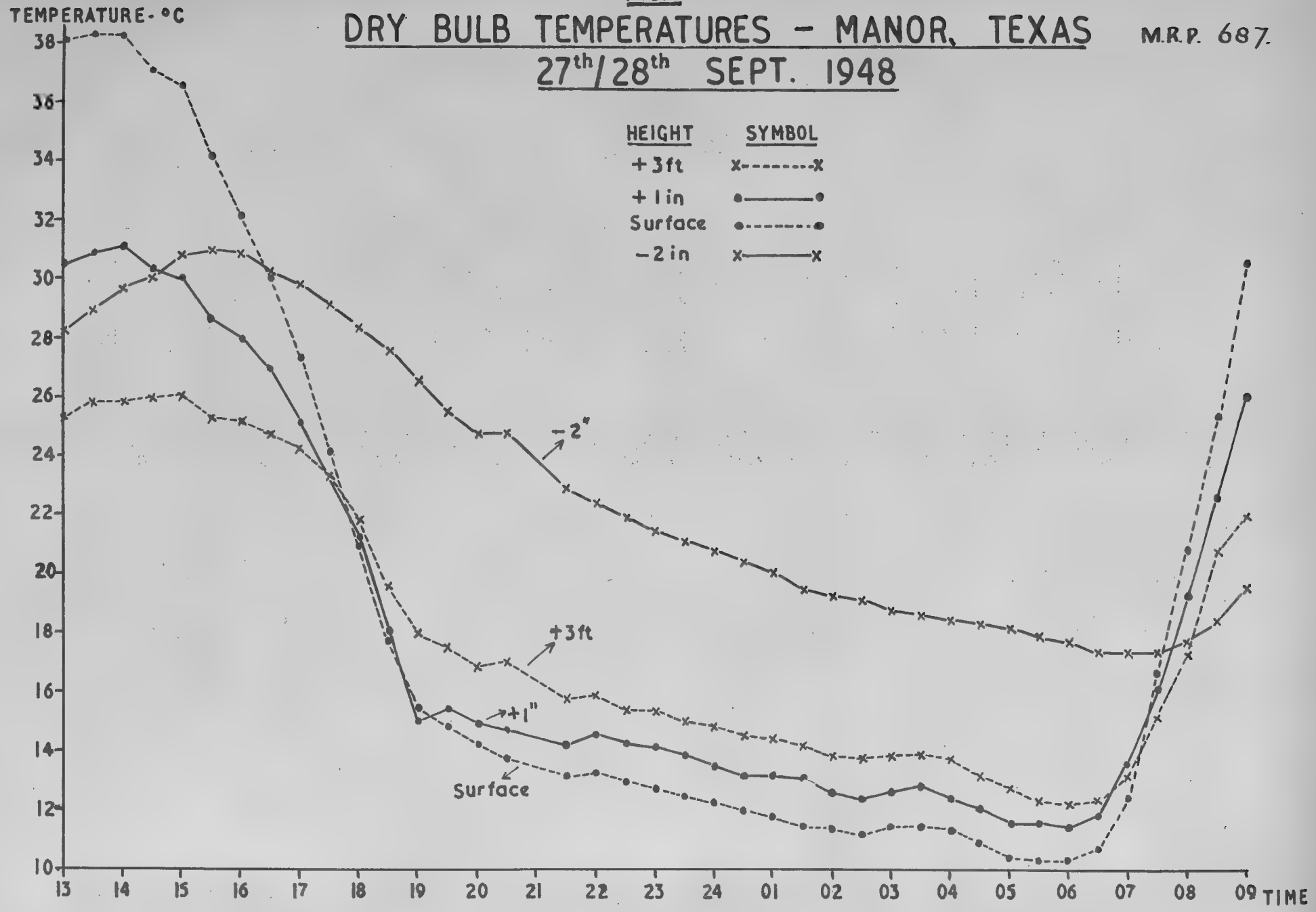
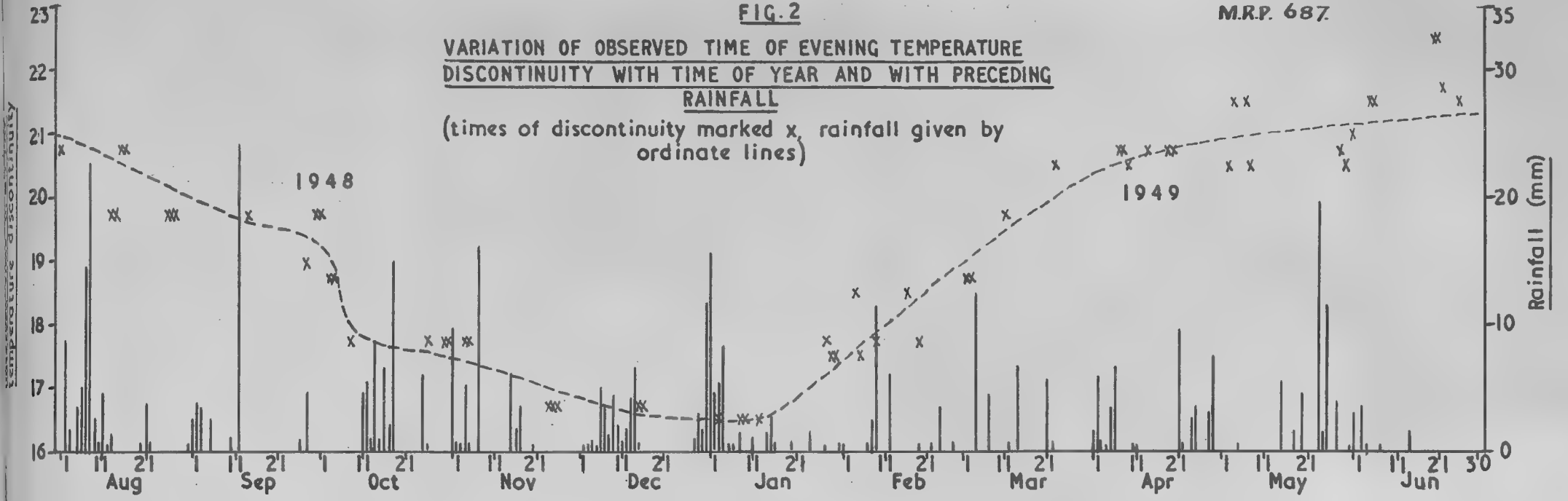


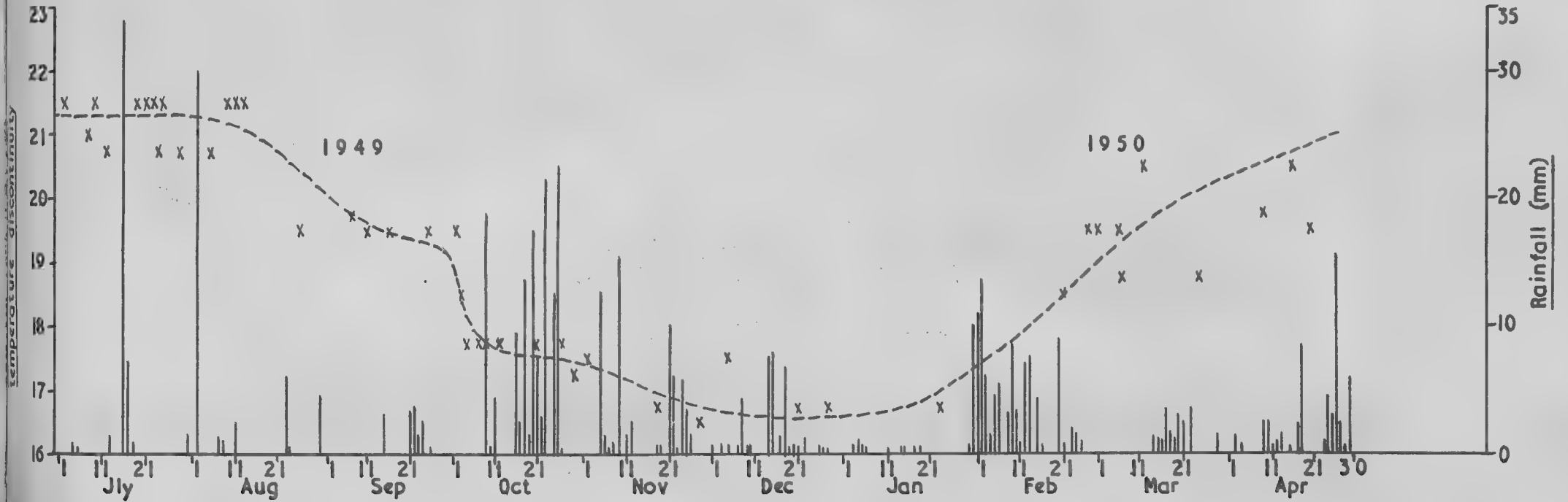
FIG. 2

M.R.P. 687.

VARIATION OF OBSERVED TIME OF EVENING TEMPERATURE DISCONTINUITY WITH TIME OF YEAR AND WITH PRECEDING RAINFALL

(times of discontinuity marked x, rainfall given by ordinate lines)





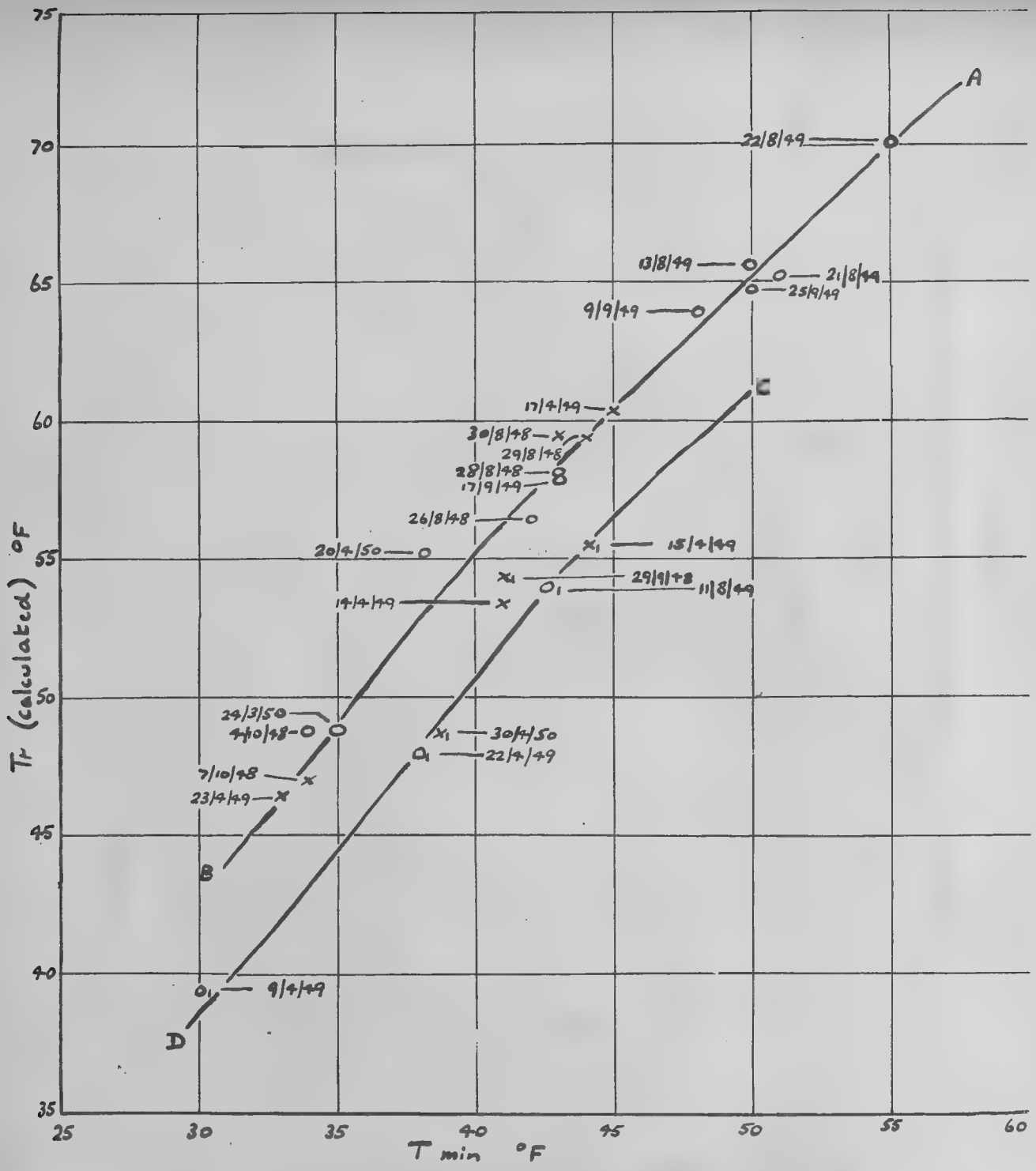


FIG. 3

THE RELATION BETWEEN INITIAL AND FINAL TEMPERATURES OF THE PERIOD OF SUBSEQUENT COOLING DURING THE SPRING AND LATE SUMMER FOR OCCASIONS OF CALM (MEAN GRADIENT WIND 0-14 MI/HR) AND LIGHT WIND (MEAN GRADIENT WIND 15-21 MI/HR).

Cases which were "non-inversion" for calaulation of Tr are marked o if the mean gradient wind was 0-14 mi/hr., and o_1 if the mean gradient wind was 15-21 mi/hr.
Cases which were"inversion" for calculation of Tr are marked x if the gradient was 0-14 mi/hr., and x_1 if the gradient was 15-21 mi/hr.
AB is the mean of cases ox and CD the mean of cases $o_1 x_1$.

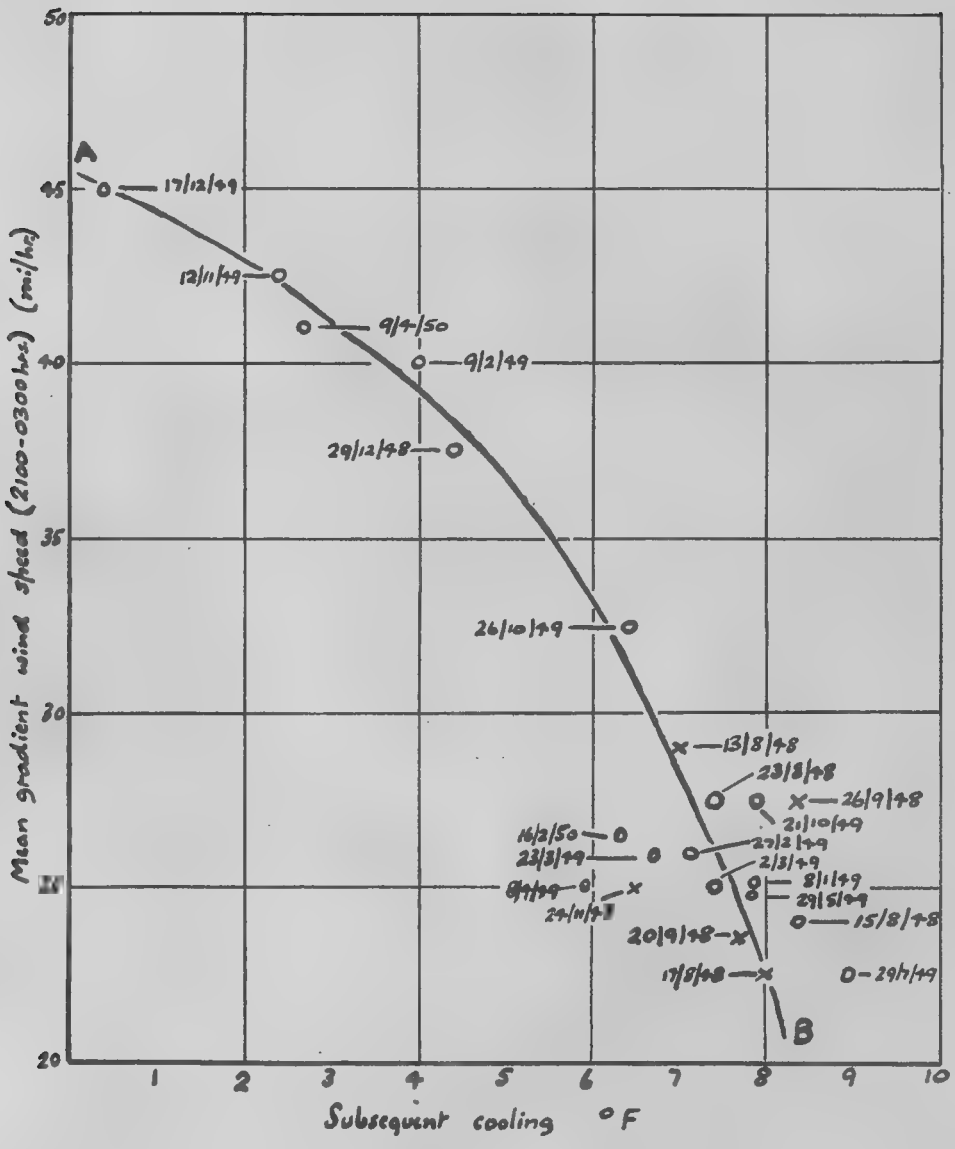


Fig. 6.

THE RELATION BETWEEN THE NUMBER OF DEGREES SUBSEQUENT COOLING AND THE MEAN GRADIENT WIND SPEED FOR OCCASIONS OF STRONGER WIND.

(Lower limit of gradient wind speed 22 mi/hr. in summer and 25 mi/hr. in winter).

Cases which were "non-inversion" for calculation of Tr are marked o and "inversion" cases x.

AB is the mean of cases plotted.

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Transverse flow at entrance and exit to jet streams

By

R. Murray and Sylvia Daniels (Meteorological Office)

1. Introduction

Theory indicates that an ageostrophic flow across the isobars occurs in regions of isobaric (or contour) confluence and difluence (e.g. Sutcliffe 1938): looking downwind, the cross-flow should be to the left in the confluence and to the right in the difluence. The effect should be most pronounced in the entrance and exit regions of jet streams. Indeed Namias and Clapp (1949) specifically refer to the cross-isobar flow in the region of confluence as an essential element in the jet stream mechanism. Sawyer (1951) also invoked the existence of cross-flow in the confluence region to explain the formation of a "folded" tropopause. It was therefore considered desirable to examine wind observations in the neighbourhood of jet streams to see whether the cross-jet wind components could be detected and, if so, how the components varied immediately below and above the jet axis.

The variation of the height of the jet axis along its length has also been examined.

2. Method

By inspection of the 1500 G.M.T. 300 mb. contour charts the positions of jet streams in the vicinity of the British Isles were estimated. For this purpose a jet stream was regarded as defined by more or less "straight" and parallel contours with maximum wind speed greater than 70 kt. and with the contour gradient decreasing on both sides of the maximum. Wind observations within 200 miles of the axis of such jet streams were extracted. All observations were placed into one of six groups; namely, observations occurring near the right and left entrance, the right and left centre and the right and left exit of the jet stream. The entrance (exit) was taken as the region where the 300 mb. contours showed noticeable confluence (difluence). Furthermore, the observations in the entrance and exit regions were only used provided they were not farther away than about 200 miles from the zone with sensibly parallel contours. The six sectors are shown schematically in Fig. 1.

The level of the jet stream was taken as defined by the maximum wind reported on each sounding (winds were reported only at 50 mb. levels). The axis of the jet stream was defined by the approximately straight and paralled contours on the 300 mb. chart in the region of the jet. The wind components perpendicular to this axis were evaluated on each sounding at 50 mb. intervals below and above the level of the jet. The wind component was regarded as positive for cross-flow from right to left, looking downstream, and as negative for flow from left to right.

The method of analysis has obvious shortcomings. In the first place, the jet axis at 300 mb. cannot be placed accurately, although it is most unlikely to be more than 200 miles from the estimated position, whilst the direction of the jet is probably little different from the estimated direction. Secondly, the height of the jet axis need not necessarily be the same as the height of the wind maximum observed at a point near the jet, but the difference is generally small - normally this procedure under-estimates the height of the axis to the left and over-estimates it to the right of the jet stream. Thirdly, the classification of observations into the six categories by visual inspection of the 300 mb. charts must be subjective to some extent. However, a statistical analysis of the data should enable the main features to be thrown up, even though they may be rather obscure or uncertain in any particular case.

3. The mean cross-flow

The mean values of the transverse components in the various sectors are presented in Table I. Only those wind reports which reached at least 100 mb. above the level of the jet axis were used.

Table I

Mean cross-jet flow (kt.) at levels above and below jet axis

Region	Above jet		At jet	Below jet				No. of obs.
	100 mb	50 mb	0	50 mb	100 mb	150 mb	200 mb	
Entrance left	3.4	2.8	6.2	3.7	1.0	-2.4	-3.2	22
right	6.2	6.5	10.1	9.2	6.9	4.9	2.0	21
combined	4.8	4.6	8.1	6.4	3.8	1.1	-0.7	43
Centre left	-1.0	-1.0	-1.9	-4.8	-3.2	-2.4	-2.3	30
right	-2.5	-5.2	-5.9	-5.0	-4.7	-5.1	-3.7	24
combined	-1.7	-2.9	-3.7	-4.9	-3.8	-3.6	-2.9	54
Exit left	-4.5	-5.5	-7.3	-3.1	-2.1	2.4	4.2	29
right	-5.9	-10.2	-12.0	-5.6	-3.1	-1.7	0.9	30
combined	-5.2	-7.9	-9.7	-4.4	-2.6	0.4	2.5	59

In the initial stages of the work observations for the year 1949 only were used. Although giving results broadly similar to those in Table I, the number of wind soundings in the entrance and exit regions were insufficient to be conclusive. The data were therefore extended to include data from other years to bring the number of soundings in each sector to more than 20. Thus the values for the entrance region are based on the 3-year period 1948-50; those for the central region are based on 1949 only; and those for the exit on the years 1949-50.

The mean components in the entrance and exit regions are represented graphically in Figs. 2 and 3 respectively.

Fig. 2 shows that the maximum cross-flow in the entrance region is at the level of the jet, and is from right to left, looking downwind. The profiles for the right, left and combined sectors are essentially similar. The magnitudes of the mean wind components in the right and left sectors differ by some 4 kt. at each level, but this effect is probably due to the different direction of the contour lines (and geostrophic wind) in the right compared with the left sector. Observations in the right sector must usually have a positive geostrophic wind component perpendicular to the jet axis, that is, in the same direction as the cross-component indicated in Fig. 2; whereas in the left sector the streamlines are such that a negative geostrophic wind component must normally occur, in the opposite direction to that indicated in Fig. 2. However, combining the right and left sector observations produces the mean profile of Fig. 2, which should represent the cross-circulation at the jet axis fairly accurately. The cross-components are probably ageostrophic winds across the contours resulting from the acceleration of the air downstream.

Supporting evidence for the reality of the mean profile of Fig. 2 is given by examination of the statistical data employed. The mean of the difference between the cross-components at the level of the jet axis and 200 mb. below is 8.8 kt. and the standard error of our estimate of the mean is 2.1 kt. (observations in right and left sectors combined); and this mean difference is highly significant, according to Student's 't' test. Also, the mean of the difference between the cross-components at the jet level and 100 mb. above is 3.4 kt. and the standard error of our estimate of the mean is 1.7 kt.; and this mean is nearly significant at the 5% level of significance. Thus it is almost certain that the difference between the transverse components at the jet level and 200 mb. below is real; whereas there is a 5% probability that

/the

the difference between the components at the jet level and 100 mb. above is merely a chance one. In other words the general shape of the mean profile is probably substantially correct. However, the magnitudes of the mean cross-components are in doubt to some extent, although an idea of the degree of inaccuracy involved may be obtained by considering the mean component at the level of the jet axis. The mean value at the jet axis equals 8.1 kt. and the standard error of our estimate of the mean is 3.5 kt. implying a 95% probability that the mean value (8.1 kt.) is correct to within 7 kt.; thus it is highly probable that the mean value is at least a positive quantity or, in other words, that there is a small cross-component in the direction from right to left, looking downstream.

Fig. 3 demonstrates that, in contrast to the entrance region, the maximum cross-flow in the exit region is at the level of the jet axis and from left to right, looking downwind; it is across the jet axis in a direction opposite to that occurring in the entrance region. The differences between the profiles of the right and left sectors can be explained by the direction of the isobaric contours (geostrophic winds) in each of these sectors. The combined mean profile eliminates to a great extent any bias due to a predominance of observations in one particular side of the jet axis.

That the mean profile of Fig. 3 is likely to be substantially correct is indicated by the following considerations. The mean difference between the transverse components at the level of the jet axis and 200 mb. below is highly significant (mean difference is 12.2 kt. and the standard error of our estimate of the mean is 2.1 kt.). The mean difference between the cross-flow at the jet level and 100 mb. above is significant at the 2% level of significance (mean difference is 4.5 kt. and the standard error of our estimate of the mean is 1.9 kt.). Thus there is at least a 98% probability that the differences discussed are real, so that the general shape of the mean profile is well established. The maximum mean cross-component, which occurs at the level of the jet axis, is equal to -9.7 kt. and the standard error of our estimate of this mean is 2.4 kt. Hence there is approximately a 95% chance that the true mean cross-component lies between -4.9 and -14.5 kt.; it is at least reasonably certain that the mean component is negative, i.e. from left to right, looking downstream.

Regarding the small negative mean components in the centre region of the jet (See Table I), it is not thought that these are significant. They can be accounted for by the fact that the majority of wind observations in the central part tended to be nearer the exit than the entrance region - which is to be expected bearing in mind the greater frequency of occurrence of jet exits compared with jet entrances near the British Isles.

The transverse components given in Table I (or Figs. 2 and 3) would be consistent with the existence of circulations in opposite directions above and below the jet stream, as indicated in the schematic cross-sections of Fig. 4. The circulation would be right-handed above and left-handed below at the entrance and the reverse at the exit. However, there is no direct evidence for the vertical branches of the circulations and the observations could almost equally well be explained by the upper or lower circulation alone. In view of the recognised temperature distribution both circulations would be 'direct' thermal circulations at the entrance and 'indirect' at the exit. *

4. The mean pressure of the jet axis in the entrance and exit regions

The pressure level of the jet was taken as the pressure level of maximum observed wind speed in the vicinity of the jet. Some additional wind observations were employed in this section since it was not necessary that they reached 100 mb. above the jet axis, as in section 3. The results are summarised in Table II.

/Table II

Footnote: * A 'direct circulation' is one in which the vertical motion is upwards in the warm air and downwards in the cold air. The reverse type of circulation in which the vertical motion is downwards in the warm air and upwards in the cold air is called 'indirect'. A direct circulation is in the same sense as natural convection and converts potential into kinetic energy: an indirect circulation is opposed to convective overturning and reduces kinetic energy.

Table II

Mean pressure (mb.) of wind maxima together with the standard errors of the estimates of the mean values

Region		Mean pressure (mb.)	S.E.	No. of obs.
Entrance	left	298	9	26
	right	259	8	23
	combined	279	6	49
Centre	left	295	6	48
	right	284	7	51
	combined	289	5	99
Exit	left	314	11	31
	right	298	10	32
	combined	306	7	63

Table II shows that the mean pressures of the wind maxima observed in the left sectors are generally greater (i.e. mean height lower) than those of the corresponding right sectors. This is not unexpected since the tropopause, where the wind speed is normally a maximum, is generally at a lower elevation to the left and at a higher elevation to the right of jet streams. However, combining the observations in the left and corresponding right sectors (e.g. combining left and right entrance observations) should give reasonably accurate values for the mean pressures of the jet axes in the entrance, centre and exit regions. The mean pressures of the jet axes are 279, 289 and 306 mb. in the entrance, centre and exit regions respectively. The mean pressure (height) of the jet axis in the entrance region is less (greater) than in the exit region by 27 mb. (about 2,000 ft.) - a small difference, yet significant at the 2% level.

In none of the cases used in the analysis was it possible to obtain height observations of the entrance and exit regions of the same jet over the British Isles, so that the mean pressures found are based on different jets. Thus the obvious interpretation of the mean pressure difference - namely that jets tend to descend slightly along their length downstream - is not necessarily correct.

Berggren et al (1949) state that the height of the jet centre is lowest at the bottom of an upper trough and tends to rise upstream on the rear side and downstream on the foreward side of the trough. They quote as evidence for this the height variation of the wind maxima at Larkhill during a period of several days when marked troughs and ridges moved eastwards; but the evidence cannot be regarded as entirely satisfactory for Larkhill was not situated in the same position relative to the strongest flow throughout the period. A similar idea is implicit in a paper by Palmen and Nagler (1949) who present cross-sections normal to the northwesterly upper flow in the rear of a trough (their Fig. 3) and normal to the westerly upper flow at the bottom of the trough (their Fig. 11), which show the jet centre at a higher elevation in the first section. However, only one example is shown and also there is a time interval of 36 hours between the two sections.

Nevertheless, on the assumption that there is in fact a tendency for the height of a jet to fall along the jet from north to south (on rear side of upper trough) and to rise from south to north (on foreward side of trough) then it was thought that in our case the difference between the mean pressures at the jet entrance and exit regions might be due to a predominance of jets with a north-south component. The orientations (whether north-south or south-north) of the individual jets used in compiling the statistics were examined, but north-south and south-north jets were found to be almost equal in number. The mean latitude of the stations used for specifying the positions of the entrance and exit regions was also practically the same in each case. Thus the mean pressure differences found do not appear to be merely the result of a predominance of jets with a north-south component.

/Another

Another possible explanation of the mean pressure difference between the entrances and exit regions might be that the jet axis tends to rise with time on moving east across our area; for the jet entrance statistics are likely to refer to later stages in the life history of the individual jets than do the exit region data.

It is clear from the above discussion that a conclusive and definite interpretation of the mean pressure difference between entrance and exit regions cannot be given, although it is tentatively suggested that there might be some tendency for the jet axis to descend slightly downwind along its length. The question will require further investigation.

5. Conclusion

(a) In the entrance to jet streams there is a component of wind towards the left (looking downstream) and this reaches a maximum at the level of the jet and falls off above and below. It is highly probable that this arises from ageostrophic flow across the contours as a consequence of the acceleration of the air downstream. It would be consistent with the existence of a "direct" thermal circulation either above or below the jet stream, or both.

(b) In the exit region the cross-stream wind component is towards the right (looking downstream); it reaches a maximum at the level of the jet and falls off above and below. This type of cross-stream flow probably arises from ageostrophic motion across the contours as a result of the deceleration of the air downstream. It would be consistent with an "indirect" thermal circulation either above or below the jet stream, or both.

/(c)

(c) In the mean, over the British Isles, the jet centre in the entrance regions is some 2,000 ft. (27 mb.) higher than in the exit regions. However, it can only very tentatively be put forward that the jet axis tends to be slightly tilted since the observations employed in computing the mean entrance and exit region pressures (heights) did not refer to the same jet streams.

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FIG 1.

Sectors of the jet stream used in classification of wind observations.

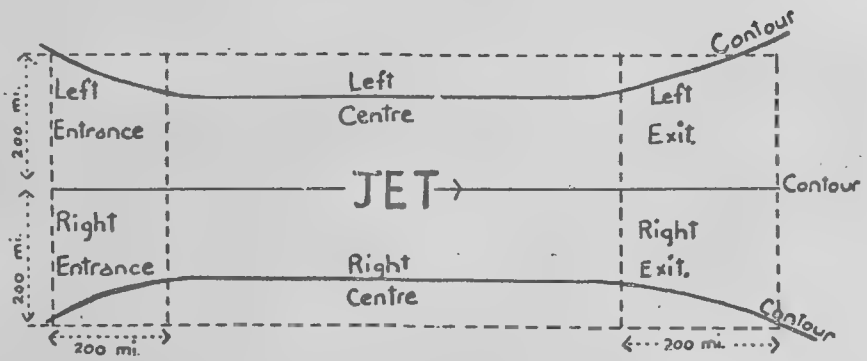

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FIG 4.

The probable circulations shown schematically in sections through entrance and exit to jet (Looking downstream.)

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22nd October, 1951

AIR MINISTRY

METEOROLOGICAL RESEARCH COMMITTEE

Visibility Statistics - The Variation of Extinction with place and season

by
A.H.R. Goldie (Meteorological Office)

The physical feature determining visibility is the turbidity of the air. Visibility is inversely proportional to the "extinction coefficient" (E). Abundant statistics of visibility are available for numerous stations and the purpose of this note is to examine how the extinction coefficients vary on the average for season and locality.

Given tables of the percentage frequency of various visibility distances (as in M.O.521) we can calculate the mean extinction coefficients. This has been done for Nairn as representing the very pure air of the north; Felixstowe as pure air for the south-east of England; Renfrew, Croydon and Greenwich as representing relatively polluted suburban and city air. (A glance over M.O.521 suggests that these stations are fairly representative of their various types). Values of mean E per km. are given in Table I below with, in brackets in each case, the visibility distance (in km.) which would correspond. The differences between the three types of station are enormous.

At Nairn, even in Autumn-Winter, the air is so clear that its average extinction corresponds in round figures to visibility of about 21-30 km., going up to about 35 km. at midday in Summer.

At Felixstowe the corresponding values are $2\frac{1}{2}$ to 7 km. in Winter and about 25 km. at midday Summer.

At Croydon they come down to about $1\frac{1}{2}$ to 3 km. and about 20 km; at Greenwich down to 1 to $2\frac{1}{2}$ km. and again about 20 km. Renfrew approximates to Croydon and Greenwich.

Nairn, Felixstowe and Greenwich are all not much above sea level and are near the sea or an estuary. In the main the differences between them must be a matter of smoke pollution. Croydon is more in the nature of a true inland station; its height is not great, 217 ft.

Table II, in which everything is referred to a normal optimum visibility of 35 km., corresponding to summer conditions at Nairn (extinction .11 per km.), brings out the differences even more forcibly. Greenwich at 9h. in Autumn - Winter is seen to have over 30 times as high an extinction coefficient, Croydon at 7h. 20 times as high, even Felixstowe at 7h. 13 times as high.

Table I Mean values of Extinction
(E per km.)

Season & Hour		Nairn		Felixstowe		Croydon		Greenwich 9,15 & 21h.		Renfrew	
Autumn-Winter	7	.14	(27.4)	1.50	(2.6)	2.30	(1.7)	3.55	(1.1)	1.96	(2.0)
	13	.13	(29.5)	.59	(6.6)	1.40	(2.8)	1.45	(2.7)	1.70	(2.3)
	18	.19	(20.9)	.77	(5.1)	2.06	(1.9)	1.78	(2.2)	1.45	(2.7)
Summer	7	.14	(27.4)	.37	(10.5)	.64	(6.1)	.46	(8.5)	.66	(5.9)
	13	.11	(34.8)	.17	(23.6)	.21	(18.7)	.19	(21.0)	.24	(16.2)
	18	.11	(34.3)	.15	(26.4)	.18	(20.9)	.28	(14.0)	.23	(17.1)

/Season

Table II - Ratio of E to a normal optimum (35 km.) value
(as attained at Nairn on the average at 13h. and 18h. in Summer)

Season and Hour		Nairn	Felixstowe	Croydon	Greenwich	Renfrew
Autumn)	7	1.3	13.2	20.4	31.4 (9h)	17.7
Winter)	13	1.2	5.3	12.3	13.1 (15h.)	14.9
	18	1.7	6.8	18.2	15.8 (21h.)	13.4
Summer	7	1.2	3.4	5.8	4.2 (9h.)	5.9
	13	1.0	1.5	1.9	1.7 (15h.)	2.1
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AIR MINISTRY

METEOROLOGICAL RESEARCH COMMITTEE

Condensation nuclei and the development of radiation fog.

by

A. C. Best.

(Meteorological Office)

Summary

The development of radiation fog upon condensation nuclei is considered in two stages separated by the instant at which the air becomes just saturated. In the first stage it is assumed that the droplets which form upon the condensation nuclei are always in instantaneous equilibrium with the cooling environment. Equations are developed to give the opacity and liquid water content of the atmosphere at various humidities (less than 100 per cent) as a result of condensation of water upon an assumed size distribution of nuclei. Numerical computation then shows that at relative humidities less than 100 per cent the shape of the size distribution curve of nuclei has little influence on visibility or liquid water content. Opacity and liquid water content are both directly proportional to the total salt content of the air; the former is not much affected by the size of the nuclei but the liquid water content varies as the square root of the mass of the individual nuclei. The values obtained for visibility and liquid water content at 100 per cent relative humidity and with reasonable assumptions about the size and number of nuclei are in accord with experience. A formula for drop size distribution in fog at saturation is also developed and found to agree with measurements.

In the second stage - when the air is supersaturated - it is assumed that all the nuclei are of the same size. Allowance is now made for the fact that condensation is essentially a time process and it is no longer assumed that the droplets are in instantaneous equilibrium with the supersaturated ambient air. Equations are developed to represent the growth of the droplets as the air cools and, after making some approximations, solutions are obtained to these equations. These solutions are used to compute the deterioration in visibility and increase in liquid water content in fog 10, 30 and 90 minutes after saturation is reached and for various combinations of values of nucleus size, total salt content and rate of cooling of the air. The results of these computations show that the extent to which the visibility will deteriorate after supersaturation sets in can not be assessed solely on the basis of the visibility and total salt content of the air at saturation. A knowledge of the nucleus size is also necessary.

1. Introduction. Fog results when a sufficient number of particles, liquid or solid, are suspended in the air. It is generally accepted that the liquid particles occur owing to the condensation of water vapour upon hygroscopic nuclei. Such nuclei may derive either from the oceans or from local sources. The solid particles on the other hand are likely to be of local origin only. Condensation on the hygroscopic nuclei starts when the relative humidity is substantially below one hundred per cent. The purpose of this paper is to trace the effect of this condensation, as the relative humidity increases, upon the visibility. Condensation is essentially a time process governed by the rate of diffusion (molecular) of water vapour to the nucleus. The equations governing this condensation process in an atmosphere in which the temperature is steadily falling do not seem to be susceptible to exact solution and it will be necessary to consider the process in two distinct phases. In connection with a similar problem it has been shown elsewhere (Best, 1951 a) that the rate of growth of a droplet upon a hygroscopic nucleus is extremely rapid at low relative humidities (and small droplet sizes) but becomes very much slower as the relative humidity approaches a critical value which exceeds 100 per cent. It will, therefore, be assumed that as the ambient temperature falls and the relative humidity rises the droplets always remain in instantaneous equilibrium with the ambient conditions while the relative humidity remains below 100 per cent. After supersaturation sets in the fact that condensation is a time process becomes increasingly important and for this phase an approximate solution to the equations will be obtained.

It is unlikely that condensation nuclei are all of the same size and in the first part of the work it will be assumed that there is a distribution of nucleus sizes. It will appear, however, that although the mean nucleus size is a factor of some importance the shape of the curve of size distribution of nuclei is comparatively unimportant and accordingly in the second part of the paper - the development of fog in supersaturated conditions - it will be assumed that the nuclei are of uniform size.

It seems to be generally accepted that condensation nuclei consist of particles of sea salt. Accordingly it will be assumed that this is true; it will thus be possible to assign numerical values to various parameters. The quantitative results so obtained will thus be dependent upon this assumption but the qualitative results will not depend upon this assumption.

2. The extinction coefficient as a function of drop size. The first step is to relate the visibility to the number and size of the water droplets suspended in the atmosphere. It can be shown (see e.g. Wright, 1939) that the visibility V is related to the extinction coefficient σ by the equation

$$V = \frac{3.91}{\sigma} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

and that σ may be regarded as composed of three additive parts, one part resulting from the air molecules, one part from the solid particles and one part from the liquid droplets suspended in the air. The precise value of the constant in the right hand side of equation 1 depends upon assuming a value 0.02 for the minimum perceptible contrast. The primary purpose of the present paper is to compute σ (and hence V from equation 1) for various conditions. Wright quotes a value 1.6×10^{-7} cm^{-1} for that part of σ resulting from the air molecules. We shall see that the computed values of σ resulting from water droplets are much greater than this and we shall accordingly ignore the contribution from the air molecules. We shall also ignore the contribution from the solid particles in the air but for the reason that they are likely to be of local significance only. The problem thus reduces to the evaluation of the contribution to σ owing to the suspended water droplets. Houghton and Chalker (1949) have published some results of computing the scattering coefficient K of

water drops for visible light. K is related to σ by the equation

$$\sigma = \pi \int K f r^2 \, dr \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

where f dr is the number of drops per unit volume with radius between r and r+dr and the integration extends over all the drops present. The evaluation of σ (and hence V) thus depends upon computing the integral in equation 2. Houghton and Chalker tabulate K as a function of $2\pi r/\lambda$ where λ is the wave length of light. The variation of K with r/λ is essentially similar to that indicated by Wright (1939) in his figure 1, showing a rapid increase as r increases from zero to λ, after which there are marked oscillations in the value of K until r reaches a value of about 5λ, the oscillations diminishing as r increases and K having a value of about 2.0 for large values of r. Since it was intended to compute σ for relative humidities below 100 per cent, when the drops are very small, it was necessary to allow for the oscillatory nature of K in the first part of the work. In practice light is not monochromatic and this introduces a degree of approximation into the work. A further consideration was the desire to fit an explicit formula to K in order to simplify the calculations. Ultimately the following procedure was adopted. From the tabulated values of K given by Houghton and Chalker values were computed for each of the four wave lengths 4, 5, 6 and 7×10^{-5} cm. and for a selection of values of r. The four values of K were then averaged for each value of r. These values are shown plotted in figure 1. The next step was to fit a curve of the form

$$K = c + dr^v \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

to the plotted points for various ranges of r. The values found for the constants c, d and v are shown in table 1 and the corresponding curves are shown in figure 1.

Table 1.

The scattering coefficient as a function of drop size.

Values of constants in the formula

$K = c + dr^v$

Range of r cm x 10^{-5}	c	d	v
<1	0	4.9×10^{15}	3.28
1 to 4.2	- 0.83	1.03×10^{5}	1.0
4.2 .. 5.5	3.5	0	0
5.5 .. 9.9	5.04	-2.8×10^{4}	1.0
9.9 ..16.8	0	5.73	0.1
>16.8	2.0	2.89×10^{-9}	-2.16

With this explicit form for K it is now possible to integrate equation 2 if f is known as a function of r.

3. Drop size as a function of the nucleus in an unvarying atmosphere.

The next step in the evaluation of the integral in equation 2 is to express f, the number of drops of radius r, as a function of r. It is clear that f will be dependent upon the mass of the nucleus and the relative humidity and accordingly we first consider the relation between drop size, nucleus mass and relative humidity. Wright (1936) has given the equation for vapour pressure over a drop of salt solution in the following form

$$\frac{p'}{p} = \exp\left(\frac{2TM_w}{r\rho_1 R\theta}\right) - \frac{750\,mG}{\rho_1 \pi \beta_1 r^3} \quad \ldots\ldots\ldots\ldots\ldots\ldots(4)$$

where

p' = vapour pressure over the drop (dynes cm^{-2}).
p = saturation vapour pressure over a plane water surface at the same temperature as the drop (dynes cm^{-2}).
r = drop radius (cm.).
ρ_1 = drop density (g cm^{-3}).
T = surface tension (74 dynes cm^{-1} for water).
M_w = molecular weight of water (18).
R = gas constant per mol. (8.314×10^7 erg deg^{-1} mol^{-1}).
θ = absolute temperature.
m = mass of solute in drop (g).
β_1 = molecular weight of solute (58 for NaCl).
G = the proportionality factor in the relation between the fractional lowering of vapour pressure and the number of mols of solute per 1000 g of water and has a value of 0.04 for NaCl.

It is convenient to rewrite equation 4 in the form (after substituting numerical values for the constants and putting $\rho_1 = 1$)

$$H = \frac{p'}{p} = \exp(P/2r) - \frac{m}{6r^3} \quad \ldots\ldots\ldots\ldots\ldots\ldots(5)$$

where $P = 6.408 \times 10^{-5}/\theta$ and 100H is the ambient relative humidity if there is no significant temperature difference between the drop and the surrounding air. As Wright mentions, it is easily seen that there is a value of r which makes p', and hence H, a maximum. This critical value of r is given by

$$P \exp(P/2r_c) = \frac{m}{r_c^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots(6)$$

Since P is much smaller than r we may expand the exponential and we then get

$$r_c^2 = \frac{m}{P} \quad \ldots\ldots\ldots\ldots\ldots\ldots(7)$$

$$H_c = 1 + \frac{P}{3r_c} \quad \ldots\ldots\ldots\ldots\ldots\ldots(8)$$

where H_c corresponds to the maximum value of p' and r_c is the corresponding drop radius. This critical radius r_c is the boundary between two different phases in the life of the drop. While the radius is less than r_c the drop will grow to a size which is in equilibrium with the ambient relative humidity but once r exceeds r_c the drop will grow indefinitely until it falls out or the ambient conditions change.

Equation 5 gives the relation between drop size, nucleus mass and humidity but the form of the equation is awkward for computation. If the exponential term is expanded, second and higher powers of P/2r neglected and h written for 1-H then

$$m = 6r^3h + 3r^2P \quad \ldots\ldots\ldots\ldots\ldots\ldots(9)$$

If now log r is plotted against log m for specific values of h and using equation 9 and a value of P appropriate to a particular temperature, the resulting curves will be almost straight lines. Thus to a very close approximation m can be expressed as a function of r, over limited ranges of r, by an equation of the form

$$m = a\, r^b \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

where a and b are constants over limited ranges of r with values which depend upon h and, to a small extent, upon the temperature θ. This procedure was adopted, the end points for the ranges of r being chosen to agree with those in table 1. The temperature θ was assumed to be 283°A giving $P = 2.264 \times 10^{-7}$. The values found for a and b are shown in table 2.

Table 2.

Nucleus mass as a function of drop radius.
Values of constants in the formula

$$m = a\, r^b$$

h	0.2	0.1	0.05	0.01	0.001	0.0
Range of r cm x 10^{-5}			Value of a			
0.1 to 1.0	1.52×10^{-1}	2.04×10^{-2}	3.02×10^{-3}	1.70×10^{-5}	1.35×10^{-6}	6.79×10^{-7}
1.0 ..16.8	1.014	4.27×10^{-1}	1.62×10^{-1}	6.92×10^{-3}	2.14×10^{-5}	6.79×10^{-7}
>16.8	1.20	6.03×10^{-1}	3.02×10^{-1}	4.57×10^{-2}	1.58×10^{-3}	6.79×10^{-7}
			Value of b			
0.1 to 1.0	2.82	2.70	2.59	2.23	2.05	2.00
1.0 ..16.8	2.98	2.96	2.93	2.75	2.30	2.00
>16.8	3.00	3.00	3.00	2.96	2.80	2.00

The fact that it was not necessary to subdivide the range of r 1 to 16.8×10^{-5} cm results from the closeness of the curve connecting log r and log m to a straight line. The values of a and b for h = 0 can, of course, be deduced from equation 9.

4. The size distribution of nuclei and of the resulting drops. Having related drop size to nucleus size and humidity the next step in the solution of equation 2 is to evaluate the drop size distribution f in terms of the nucleus size distribution. There appear to be no data available about the size distribution of salt nuclei in the atmosphere over land and so some suitable assumption must be made. The type of formula adopted has been found convenient in other connections and gives a distribution curve which has an intuitively reasonable shape. Let the total mass of salt per unit volume of air be S and let the nucleus sizes be distributed according to the formula

$$1 - F = \exp\left[-(m/m_o)^q\right] \qquad \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

where F is the fraction of salt in the atmosphere comprised by nuclei with mass less than m and m_o and q are constants. Then it is easily seen that the number of nuclei per unit volume of air with mass between m and m+dm is jdm where

$$j = \frac{S\,q}{m_o^q}\, m^{q-2} \exp\left[-(m/m_o)^q\right] \qquad \ldots\ldots\ldots\ldots\ldots (12)$$

(a) Variation in q, i.e. in the shape of the nucleus distribution has very little effect. This is particularly true at high humidities.
(b) For a fixed value of S a large value for the mean nucleus mass (and hence a small number of nuclei) implies better visibility (i.e. smaller values for σ). The effect may be very marked in near saturation conditions (e.g. 99 per cent relative humidity) but is not so important when the atmosphere is saturated. At low humidities (99 per cent or less) the amount of condensed water is influenced only slightly by the mean nucleus size (still assuming S is constant) but at saturation the amount of condensed water varies as the square root of the nucleus mass.
(c) With a large number of small nuclei the change in σ and W as the relative humidity increases from 99.9 per cent to 100 per cent is not very great but with a small number of large nuclei the corresponding change is very great.
(d) Both σ and W are directly proportional to S.

It appears (see for example Wright, 1940 a and b) that experimental measurements of S vary over a wide range of values but that a reasonable value is about 5×10^{-12} g cm^{-3}. Now suppose we put $\sigma/6\pi S = 8\times10^{3}$ and 5×10^{5} at 80 and 100 per cent relative humidities respectively. Then the values of σ are 7.54×10^{-7} cm^{-1} and 4.71×10^{-5} cm^{-1}. In order to obtain the visibility from equation 1 we must add the component of σ owing to the air molecules viz 1.6×10^{-7} cm^{-1}. This has a negligible effect at saturation of course. We thus obtain 43 km and 830 m for the visibility at 80 per cent and 100 per cent relative humidity respectively. These are reasonable values. The liquid water content is of interest only at saturation. We may take $W/8\pi S = 28$ or 280 depending upon whether the mean nucleus mass is 10^{-14} g or 10^{-12} g. These values give liquid water concentrations of 0.0035 g m^{-3} and 0.035 g m^{-3} for the small and large nuclei respectively.

7. Drop size distribution at 100 per cent relative humidity. At saturation we see from equation 9 that $a = 3P$ and $b = 2$. Then from equation 17 the fraction F_1 of liquid water in the atmosphere comprised by drops with radius less than r is given by

$$F_1 = \frac{\int_0^{u(1+1/2q)^{1/2}} t^{1/2q} \exp(-t)\, dt}{\Gamma(1 + 1/2q)} \qquad \ldots\ldots\ldots(18)$$

where

$$u = \frac{(3Pr^2/m_o)^q}{(1 + 1/2q)^{1/2}}$$

Elsewhere (Best 1951b) an expression for F_1 somewhat similar to that given by equation 11 has been deduced from published measurements for drop size distribution in fog. Although these measurements were not made at exactly 100 per cent relative humidity it is of interest to compare the size distribution of fog droplets given by equation 18 with that found from these measurements. Now as a result of comparing numerical values of the two expressions it is found that, numerically, there is little difference between F_1 as given by equation 18 and F_1 as given by equation 19

$$F_1 = 1 - \exp\left[-(u/b)^{1+1/4q}\right] \qquad \ldots\ldots\ldots\ldots\ldots(19)$$

provided b_1 is chosen to satisfy the relation

$$\log_{10} b_1 = \frac{0.6}{4q+1}$$

If q is infinitely large the two expressions for F_1 (equations 18 and 19) are identical. For three other selected values of q table 4 shows a comparison between the two expressions for a few values of u.

Table 4.

Comparison of values of F_1 computed from equations 18 and 19.

q	5		1		0.5	
u	Eqtn 18	Eqtn 19	Eqtn 18	Eqtn 19	Eqtn 18	Eqtn 19
0.1	0.076	0.080	0.030	0.039	0.009	0.016
0.2	0.154	0.158	0.079	0.090	0.033	0.044
0.5	0.361	0.363	0.253	0.258	0.158	0.163
3.0	0.948	0.948	0.938	0.939	0.925	0.926

We see from table 4 that provided F_1 exceeds about 0.1, i.e. provided we are not concerned with the 10 per cent of water comprised by the smallest drops, the two formulae for F_1 give very similar answers. The computations were repeated for five other values of u between 0.5 and 3.0 and the maximum difference between corresponding values of F_1 was about 4 per cent of F_1. This occurred for $q = 0.5$. For the other two values of q the maximum difference was about 2 per cent, the agreement being closest for the largest value of q. We may conclude therefore that, provided F_1 exceeds 0.1, equation 19 is a reasonable approximation to equation 18. Now after a little rearrangement equation 19 can be written

$$F_1 = 1 - \exp\left[-(r/a_1)^n\right] \quad \ldots\ldots\ldots\ldots\ldots(20)$$

$$n = 2q + 0.5$$

$$a_1 = b_1^{1/2q}(1+1/2q)^{1/4q}(m_o/3P)^{1/2}$$

When q takes the values 1.1 and 3.0 the values of n are 2.7 and 6.5 respectively. The values of n found experimentally (Best 1951b) vary from 2.82 to 4.23 so that our arbitrary choice of these two values for q seems to have been fortunate. Similarly if we put $\theta = 283°A$, $P = 2.264 \times 10^{-7}$ we find $a_1 = 1.48\mu$ and 1.25μ for $q = 1.1$ and 3.0 respectively when $m_o = 10^{-14}$g. When $m_o = 10^{-12}$ g these values of a_1 must be multiplied by 10 of course. The values appropriate to the larger value of m_o correspond very closely to the values based upon experimental measurements (Best 1951b) but the comparison must not be pursued too closely since the theoretical results of this section apply to conditions at exactly 100 per cent relative humidity and the measurements may have been made in slightly different conditions.

8. Equations of growth in supersaturated conditions. So far we have considered the growth of droplets when the nuclei are not all the same size and when the relative humidity does not exceed 100 per cent. We have also assumed that the droplets are in equilibrium with the instantaneous ambient conditions. We wish now to consider what happens when the air becomes slightly supersaturated as a result of a further fall in temperature. Mathematical analysis seems somewhat intractable if a distribution of nucleus sizes is postulated so we shall now assume that all the nuclei have the same mass m. On the other hand the rate of growth of the drops is now smaller and we shall no longer assume that the drops are in instantaneous equilibrium with the ambient conditions.

From Best (1951a) we see that the equations governing the growth of a single drop can be written

$$\frac{p'}{p_{\theta+\Delta\theta}} = \exp(P/2r) - \frac{m}{\sigma r^3} = \psi \text{ (say)} \quad \ldots\ldots(21)$$

$$\Delta\theta = \frac{\lambda D}{R\theta k}(Hp_\theta - p') \quad \ldots\ldots(22)$$

$$r\frac{dr}{dt} = \frac{MD}{R\theta}(Hp_\theta - p') \quad \ldots\ldots(23)$$

where

$\Delta\theta$ = excess of droplet temperature over ambient temperature (°A)
λ = latent heat of evaporation (cal. mol^{-1})
D = coefficient of diffusion of water vapour in air (cm^2 sec^{-1})
k = thermal conductivity of air (cal. cm^{-1} sec^{-1} deg^{-1})
p_θ = saturation vapour pressure at temperature θ (dynes cm^{-2})

and the other symbols have the same meanings as before. Equation 21 is, of course, a somewhat more exact form of equation 4. Provided $\Delta\theta$ is small we then easily get

$$\Delta\theta = \frac{p_\theta \lambda D}{R\theta k} \cdot \frac{H - \psi}{1 + \frac{\partial p_\theta}{\partial \theta}\frac{\lambda D \psi}{R\theta k}}$$

Now computation shows that $(\partial p_\theta/\partial\theta)(\lambda D/R\theta k)$ varies from 0.8 to 2.6 as θ varies from 273°A to 293°A. Since ψ differs from unity only by a small amount, of the order 10^{-4}, we may accordingly replace ψ by unity in the denominator of this last expression for $\Delta\theta$. Combining this modified expression for $\Delta\theta$ with equations 22 and 23 we then obtain

$$r\frac{dr}{dt} = A(H - \psi)$$

where

$$A = \frac{MDkp_\theta}{(R\theta k + \lambda D \partial p_\theta/\partial\theta)}$$

and computation shows that A varies only from 5.9×10^{-7} to 12.0×10^{-7} as θ varies from 273°A to 293°A. Next we expand the exponential term in the expression for ψ (equation 21) neglect second and higher powers of P/2r, write h = H-1 (this has the sign changed from the h used earlier) and so obtain

$$r\frac{dr}{dt} = A\left(h - \frac{P}{2r} + \frac{m}{\sigma r^3}\right) \quad \ldots\ldots(24)$$

This equation describes the rate of growth of a single drop in terms of the supersaturation h.

9. Equation for growth of a cloud of drops in a varying atmosphere.

We now consider the interaction between the cloud of fog droplets and the ambient water vapour. If ρ and ρ_θ denote the ambient and saturation vapour densities

$$\rho = H \rho_\theta$$

Differentiating with respect to time, writing β for $-\partial\theta/\partial t$ (the rate of fall of temperature), and replacing H by unity in the resulting equation (since H differs very little from unity) we get

$$\frac{\partial \rho}{\partial t} = \rho_\theta \frac{\partial H}{\partial t} - \beta \frac{\partial \rho_\theta}{\partial \theta}$$

Now the rate of decrease of ρ must be equal to the rate of increase of condensed water and so we have

$$\frac{S}{m} 4\pi r^2 \frac{\partial r}{\partial t} = \beta \frac{\partial \rho_\theta}{\partial \theta} - \rho_\theta \frac{\partial h}{\partial t} \quad \text{.....(25)}$$

since the number of drops per unit volume is S/m and $\partial H/\partial t = \partial h/\partial t$.

Equations 24 and 25 together define the rate of growth of the fog droplets after allowance has been made for the water condensing onto them and so tending to diminish the humidity. It is convenient however to change the dependent variables r and h to dimensionless quantities. From equations 7 and 8 we have

$$r_c = (m/P)^{1/2} \qquad h_c = H_c - 1 = P/3r_c$$

If now we write

$$h = x h_c \qquad r = y r_c \quad \text{.....(26)}$$

then equations 24 and 25 reduce to

$$\frac{dy}{dt} = \frac{L}{y}\left(\frac{x}{3} - \frac{1}{2y} + \frac{1}{6y^3}\right) \quad \text{.....(27)}$$

$$\frac{dx}{dt} = M - Ny\left(\frac{x}{3} - \frac{1}{2y} + \frac{1}{6y^3}\right) \quad \text{.....(28)}$$

where

$$L = \frac{AP}{r_c^3}$$

$$M = \frac{\beta}{h_c \rho_\theta} \frac{\partial \rho_\theta}{\partial \theta}$$

$$N = \frac{4\pi S}{m\rho_\theta} \frac{AP}{h_c}$$

Since θ is a function of time the coefficients L, M and N will vary with time. The variation however is slow. For example as θ decreases from 293°A to 273°A L decreases in the ratio 1.9 to 1.0 while M and N increase in the ratio 1.18 to 1.0 and 1.86 to 1.0 respectively. Since the ambient temperature is unlikely to fall by more than 5°C after fog has formed we shall accordingly treat L, M and N as constants. Eliminating x between equations 27 and 28 a second order differential equation in y is obtained. This can be integrated once by the normal methods but of course the result involves the values of the various quantities at time t = 0. In order to define the starting conditions it will be assumed that at time t = 0 the fog droplets are instantaneously

in equilibrium with 100 per cent relative humidity, i.e. $dy/dt = 0$ and $x = 0$ at $t = 0$. Then from equation 24 we get $y = 1/3^{1/2}$ when $t = 0$. If we also write z for y^2 we find that equations 27 and 28 reduce to

$$\frac{dz}{dt} = \frac{2LM}{3} t + \frac{2N}{9}\left\{\frac{1}{3^{3/2}} - z^{3/2}\right\} + L\left\{\frac{1}{6z^{3/2}} - \frac{1}{2z^{1/2}}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots(29)$$

and we note that $z = 1/3$ when $t = 0$. Equation 29 is not susceptible to an exact solution and approximations are necessary. Now it is easily seen that if in equation 21 we put $\psi = 1$, that is if we neglect the influence of salt content and curvature on the vapour pressure over the drop after the relative humidity has reached 100 per cent, then equation 29 will be changed only by the omission of the terms in the last pair of brackets on the right hand side. It seems a reasonable approximation to neglect the salt and curvature effects after the droplets reach the size corresponding to saturation but having obtained the full form of equation 29 we can check this approximation. It is clear that the values of $2N/9$ and of L will be largely governed by the values of m and S. If we let m take the values 10^{-14} g, 10^{-13} g and 10^{-12} g and S take the values 10^{-11} g, 10^{-10} g and 10^{-9} g then we find that with $\theta = 283°A$ the ratio of L to $2N/9$ is generally less than 10^{-2}, exceeding this value only when there are a small number of small nuclei. On the other hand computation shows that for values of $z > 1/3$ (the value of z when $t = 0$) the terms $(1/3^{3/2} - z^{3/2})$ and $(1/6z^{3/2} - 1/2z^{1/2})$ are of the same order and sign. Thus we can justifiably approximate by writing

$$\frac{dz}{dt} = \frac{2LM}{3} t + \frac{2N}{9}\left(\frac{1}{3^{3/2}} - z^{3/2}\right) \quad \ldots\ldots\ldots\ldots\ldots(30)$$

Equation 30 is still not in a form capable of explicit solution but we can obtain an approximate explicit solution for large values of t. If we write

$$T = t + \frac{N\, 3^{1/2}}{27\, LM}$$

$$z = \left(\frac{3LM}{N}\right)^{2/3} T^{2/3} + \epsilon$$

we find that provided

$$\epsilon \ll \left(\frac{3LM}{N}\right)^{2/3} T^{2/3} \quad \ldots\ldots\ldots\ldots\ldots\ldots(31)$$

equation 30 reduces to

$$\frac{d\epsilon}{dT} = -\frac{2}{3}\left(\frac{3LM}{N}\right)^{2/3} \frac{1}{T^{1/3}} - \frac{N}{3}\left(\frac{3LM}{N}\right)^{1/3} T^{1/3}\epsilon \quad \ldots\ldots(32)$$

An exact solution of equation 32 is possible but is not necessary. Provided the first term on the right hand side is small compared with the second term we can neglect the first term. The solution to the equation so obtained is

$$\epsilon = C \exp\left[-\frac{N}{4}\left(\frac{3LM}{N}\right)^{1/3} T^{4/3}\right] \quad \ldots\ldots\ldots\ldots(33)$$

where C is a constant and the condition for neglecting the first term on the right hand side of equation 32 is

$$\frac{2}{NT^{2/3}\epsilon}\left(\frac{3LM}{N}\right)^{1/3} \ll 1 \qquad \ldots\ldots\ldots\ldots\ldots\ldots(34)$$

Thus we see from equation 33 that provided the two inequalities 31 and 34 are satisfied ϵ rapidly approaches zero as T increases and we then have

$$z\left(=\frac{r^2}{r_c^2}\right) = \left(\frac{3LM}{N}\right)^{2/3}\left(t + \frac{N3^{1/2}}{27LM}\right)^{2/3} \qquad \ldots\ldots\ldots\ldots(35)$$

The two inequalities 31 and 34 do not give a sufficient condition for equation 35 to hold since we do not know the value of ϵ but if corresponding sides of the two inequalities are multiplied we get a necessary condition which is independent of ϵ and which reduces to

$$T \gg \left(\frac{8}{3LMN^2}\right)^{1/4} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(36)$$

10 Numerical check of equation 35. It is of interest to check the validity of equations 35 and 36 numerically. If in equation 30 we write $z = z_1 + u$, where z_1 is the value of z at the beginning of a small interval of integration, and if we make the interval of integration so small that we can neglect second and higher powers of u/z_1 then equation 30 reduces to a simple differential equation for u which can be solved explicitly. This solution can be used to obtain the value of z a short time after z has the value z_1. This new value of z can then be used as the initial value for another small interval of integration and so on. Thus z can be computed numerically at a succession of time intervals. This procedure was adopted for the three values 10^{-6}, 3×10^{-5} and 10^{-3} for 2LM/3 and the three values 2×10^{-3}, 6×10^{-2} and 2 for 2N/9. In each of the nine cases the initial value of z was 1/3 at t = 0 (i.e. at the beginning of the first interval of integration) and in each step the increment in t was chosen so that $u/z_1 < 0.1$. The numerical integration in each case was carried to the stage where $z > 2.25$. With the corresponding pairs of values of z and t so obtained $\log_{10}(t + N3^{1/2}/27LM)$ was plotted against $\log_{10}z$ for values of z greater than unity. In all cases the points for the larger values of z approximated closely to a straight line. From equation 35 the slope of this line should be 2/3 and from the intercept on the axis we should be able to evaluate $(3LM/N)^{2/3}$ provided t is large enough. The results of these computations showed that the value of the slope found graphically was outside the range 0.67 to 0.71 in only two cases. In the same two cases only $(3LM/N)^{2/3}$ differed from the nominal value in a ratio exceeding 1.5. Further examination showed that in these two cases the ratio of T to $(8/3LMN^2)^{1/4}$ was 2.7 and 1.0 respectively when z had the value 2.25 so that the inequality 36 was not satisfied.

It should perhaps be mentioned that the values chosen for 2LM/3 and 2N/9 were selected after consideration of the values likely to arise in practice.

11 Opacity and liquid water content in conditions of supersaturation. Since we are now dealing with droplets of uniform size we may write

$$\sigma = \pi K \frac{S}{m} r^2 \qquad\qquad W = \frac{4}{3}\pi \frac{S}{m} r^3$$

and, in view of the fact that the droplets are now of substantial size and the fact that various approximations have already been adopted we may put K = 2 independently of the size of the droplets. Now for droplets which are in equilibrium with the ambient conditions at 100 per cent relative humidity we have (from equation 24)

$r = (m/3P)^{1/2} = r_c /3^{1/2}$ and so

$$\sigma = 3\,\sigma_{100}\, z \qquad W = 3^{3/2}\, W_{100}\, z^{3/2}$$

where σ_{100} and W_{100} are the values of σ and W when the droplets are in equilibrium with a saturated atmosphere. Then combining these two expressions with equations 35 and 36 we get

$$\sigma = \sigma_{100}\, X_t^{2/3} \qquad W = W_{100}\, X_t \quad \text{.....(37)}$$

where $$X_t = \frac{27\,LM}{N3^{1/2}}\, t + 1 \quad \text{.....(38)}$$

and provided $$X_t \gg 27 \left(\frac{2LM}{3N^2}\right)^{3/4} \quad \text{.....(39)}$$

and by substituting for L, M and N in the right hand side of inequality 39 we find that this criterion is independent of m. Table 5 has been compiled to show the development of radiation fog in several different sets of natural conditions. The assumed values of m, S and β are believed to cover many of the values which arise in practice. The critical quantities r_c and h_c have been computed from equations 7 and 8 for a temperature of 283°A. V_{100} and W_{100}, the visibility and liquid water content at saturation, have been assessed from table 3 (and equation 1) assuming a value 5×10^5 for $\sigma_{100}/6\pi S$ and values 28 and 280 for $W_{100}/8\pi S$ when m is 10^{-14} g and 10^{-12} g respectively. The values of X for 10, 30 and 90 minutes were computed from equation 38. The visibility and liquid water content can be found by using equations 1 and 37.

We note first, from a comparison of X with the last column of table 5, that the inequality 39 is satisfied for all cases except numbers 25, 26 and 27 and the 10 minute values in cases 13, 14 and 15. Table 5 demonstrates a number of points, all of which could, of course, be deduced from equation 38.

(a) With a small number of small nuclei the thickening of the fog after saturation is very considerable. An increase in either the size or number of nuclei diminishes this effect. Since V_{100} is independent of m it follows that on two occasions when the total salt concentration in the air is identical the visibility might be the same at saturation but the subsequent rate of thickening of the fog might be very different. Compare for example cases 4 and 16.

(b) It may be an appreciable time after saturation is reached before the fog is really thick. In case 14 for example the visibility is greater than 4 km at saturation and 90 minutes later is 186 m.

(c) On those occasions when the visibility deteriorates substantially after saturation (i.e. when $X \gg 1$) the value of X is roughly proportional both to t and β, i.e. the visibility is inversely proportional to (fall of temperature after saturation)$^{2/3}$.

(d) It is easily seen that the drop size is given by $r = r_c\, X_t^{1/3} /3^{1/2}$. For the three values of m considered in table 5 the greatest values of r at 90 minutes after saturation are achieved in cases 3 and 15 and are 12.1μ and 26.1μ respectively (case 27 does not satisfy the inequality 39). These are of the order of magnitude indicated by measurements but it must be remembered that the present work takes no account of sedimentation and a droplet of radius 25μ has a terminal velocity of about 7 cm sec^{-1}.

(e) Considering now the liquid water content it is rather remarkable how little the size of the nuclei or the value of S affect W. If we compute W for 90 minutes after saturation and a rate of fall of temperature of 1.5°F/hour we find that it lies between 0.7 g/m^3

and 0.9 g/m³ for cases 3, 6, 9, 15, 18, 21 and 27. This value of W is concordant with measured values. With a value 10^{-9} g for S values of 1.4, 2.93 and 7.7 g/m³ are obtained for W in cases 12, 24 and 33. These are rather high but 10^{-9} g is an unusually large value for S, though such a value has been found experimentally.

Table 5.

Development of radiation fog.

Case	m g	S g cm^{-3}	β °F/hr	r_c μ	h_c	V_{100} m	W_{100} g m^{-3}	X_{600}	X_{1800}	X_{5400}	$27\left\{\frac{2LM}{3N^2}\right\}^{3/4}$
1	10^{-14}	10^{-12}	0.2	2.1	3.60	4150	7.03	16	45	134	1.10
2			0.5		x10^{-4}		x10^{-4}	38	112	333	2.20
3			1.5					112	333	998	5.00
4		10^{-11}	0.2			415	7.03	2.48	5.43	14.3	3.5x10^{-2}
5			0.5				x10^{-3}	4.69	12.1	34.2	7.0x10^{-2}
6			1.5					12.1	34.2	101	1.6x10^{-2}
7		10^{-10}	0.2			41.5	7.03	1.15	1.44	2.33	1.1x10^{-3}
8			0.5				x10^{-2}	1.37	2.11	4.32	2.2x10^{-3}
9			1.5					2.11	4.32	11.0	5.0x10^{-3}
10		10^{-9}	0.2			4.15	7.03	1.01	1.04	1.13	3.5x10^{-5}
11			0.5				x10^{-1}	1.04	1.11	1.33	7.0x10^{-5}
12			1.5					1.11	1.33	2.00	1.6x10^{-4}
13	10^{-18}	10^{-12}	0.2	6.65	1.14	4150	2.22	5.67	15.0	43.0	
14			0.5		x10^{-4}		x10^{-3}	12.7	36.0	106.0	
15			1.5					36.0	106	316	
16		10^{-11}	0.2			415	2.22	1.47	2.40	5.20	
17			0.5				x10^{-2}	2.17	4.50	11.5	
18			1.5					4.50	11.5	32.5	
19		10^{-10}	0.2			41.5	2.22	1.05	1.14	1.42	
20			0.5				x10^{-1}	1.12	1.35	2.05	
21			1.5					1.35	2.05	4.15	
22		10^{-9}	0.2			4.15	2.22	1.00	1.01	1.04	
23			0.5					1.01	1.03	1.11	
24			1.5					1.03	1.11	1.32	
25	10^{-12}	10^{-11}	0.2	21.0	3.60	415	7.03	1.15	1.44	2.33	
26			0.5		x10^{-5}		x10^{-2}	1.37	2.11	4.32	
27			1.5					2.11	4.32	11.0	
28		10^{-10}	0.2			41.5	7.03	1.01	1.04	1.13	
29			0.5				x10^{-1}	1.04	1.11	1.33	
30			1.5					1.11	1.33	2.00	
31		10^{-9}	0.2			4.15	7.03	1.00	1.00	1.01	
32			0.5					1.00	1.01	1.03	
33			1.5					1.01	1.03	1.10	

(f) Table 5 has been computed for a temperature of 283°A. During development of the fog after supersaturation sets in the influence of temperature enters only through the coefficient of t, $27LM/N^{3/2}$, in equation 38 and substituting the values of L, M and N we find that this coefficient is

$$\frac{27\ \beta\ P^{3/2}\ \frac{\partial \rho_\theta}{\partial \theta}}{4\ 3^{1/2}\ S\ m^{1/2}}$$

The only parts of this expression which depend upon temperature are $P^{3/2}$ and $\partial\rho_\theta/\partial\theta$. For the usual variations in θ the variations in $P^{3/2}$ will

be small. The variation in $\partial\rho_\theta/\partial\theta$ as θ varies from 273°A to 293°A is in the ratio of about three to one but we see that results for temperatures other than 283°A can be deduced from table 5 by suitable adjustment of the value of β.

It remains only to consider how the humidity should be computed during the period of supersaturation. We could, of course, return to the original differential equations but a more direct approach is given by the formula

$$\text{Relative humidity} = \frac{W_{100} + \rho_\theta - W_{100}\, X_t}{\rho_\theta - \beta t}$$

where ρ_θ is the saturation vapour density at 283°A and $\rho_\theta - \beta t$ is the corresponding value at the temperature prevailing at time t. If we apply this formula to case 16 at time 30 minutes after saturation for example we obtain a value 100.0117 but there seems little reason in tabulating such values which are not susceptible to verification by measurement.

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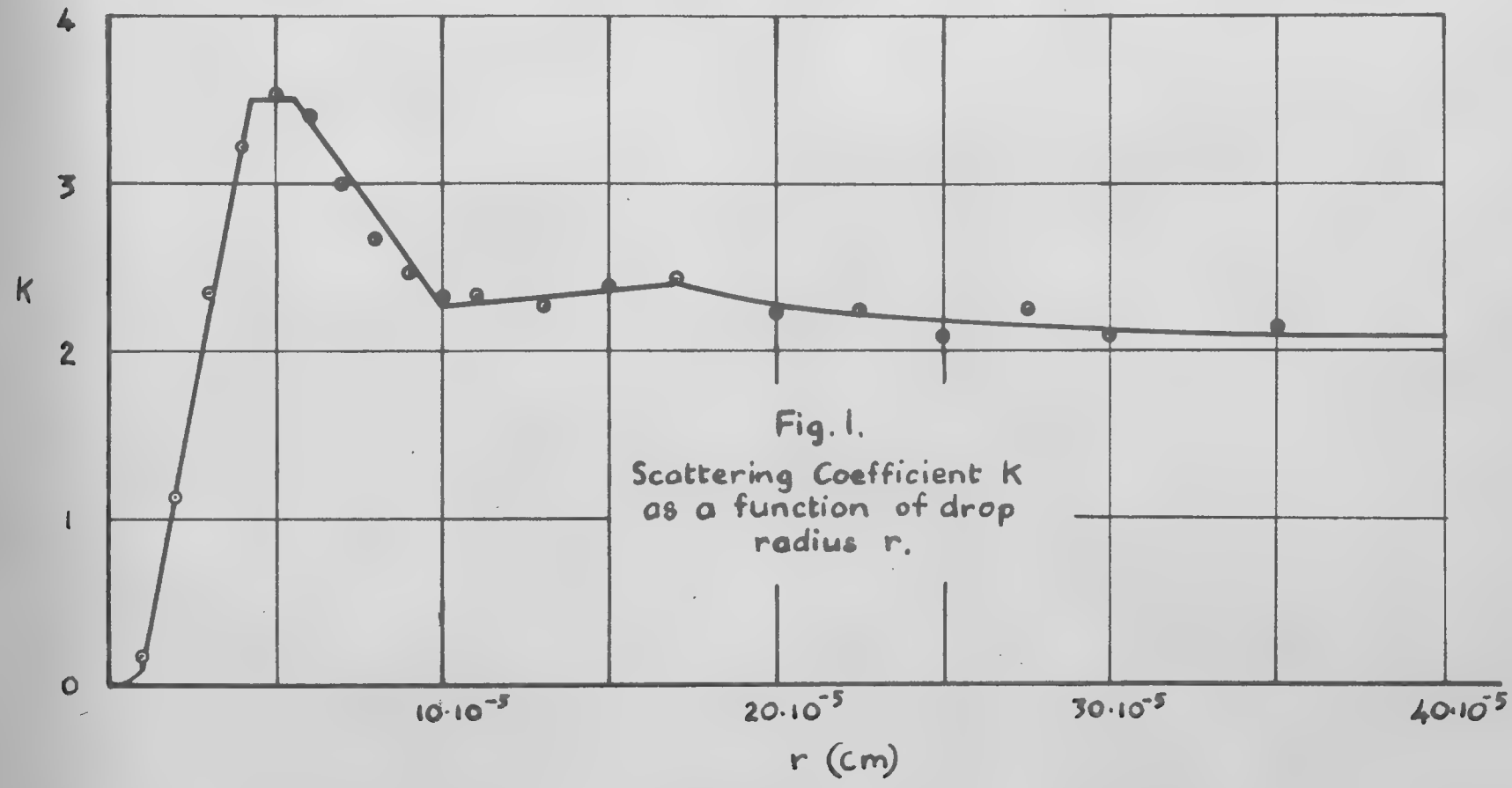

Fig. 1.
Scattering Coefficient K as a function of drop radius r.

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AIR MINISTRY

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The Reproduction of Ice Crystals by 'Splintering'.

by

H. P. Palmer

I. Introduction

It seems probable that much of the rain which falls in these latitudes is formed by the Bergeron process, but the origin of the ice particles which take part in this process is still obscure. In the atmosphere there are relatively few ice-forming nuclei which are active in the temperature range 0°C- -15°C, but clouds are observed to give rain although the cloud top is no colder. It has been suggested (Brewer and Palmer 1949) that ice crystals probably multiply by splintering, so that the ice crystals which grow on these few nuclei could initiate a type of chain reaction which gave sufficient ice particles for precipitation.

MRP 493 drew attention to some work by W. Findeisen (1940, 1942) on the electrical effects when splinters of ice were released from ice-covered surfaces. The author carried out some preliminary experiments which confirmed that splinters were released when ice grew by sublimation or by riming, and that work was also reported by MRP 493. The present paper reports some of the further laboratory work suggested in MRP 493. The results suggest that splinters are released by all ice particles which fall through deep supercooled water clouds, whether the ice particles grow by sublimation or by riming. The rate at which splinters are released in natural clouds is quite unknown.

II Splinters released by ice crystals during growth by sublimation.

(a) Apparatus.

The layer of hoar frost which released splinters was formed on the cooled surface S of a cylindrical aluminium block (Figs. 1 & 3). This was cooled by conduction down the copper rod C from the bath of solid CO_2 and petrol. An electric heater was wound on the block just above the surface S, and a simple thermostat controlled the current in this heater to keep the temperature of the surface within .25°C of any selected temperature between 0°C and -25°C. The hoar frost was observed with a low-power microscope, and the surface S was anodised and dyed black in order to give a dark background.

The cooled surface was installed in the large low-temperature cloud-chamber described in MRP 493. This consists of a large tank (30 litres) (Fig. 2) cooled by paraffin, which is pumped round the tank and through a heat exchanger immersed in solid CO_2 and alcohol. The paraffin could be cooled to -30°C, and its temperature could be kept constant by adding CO_2 to the alcohol at an appropriate rate.

The conditions under which the hoar frost grew were made as similar as possible to those in which ice crystals grow in the atmosphere. An airstream was passed over the surface at a speed which was of the same order as the terminal velocity of snowflakes. (About 40 cms/sec. Nakaya 1939). The airstream was as nearly as possible saturated with respect to ice at the temperature of the paraffin bath. This was achieved by passing air from the compressed air line through a flowmeter, and then drying it in a spiral and filter immersed in the paraffin bath. The spiral was connected by a short tube to a tufnol tunnel in which about 6 cm^2 of the surface were exposed to the airflow.. The tunnel was carefully designed so that the air reached the cooled surface while still at the temperature of the paraffin bath. In check experiments the air temperature was measured with a fine wire thermo-couple, and the cooled surface was used as a frost-point hygrometer. Both air temperature and frost-point were found to be within 1°C of the temperature of the paraffin. The rate of growth of the

/hoar

hoar frost deposit on the surface was controlled by the difference of temperature between the surface and the paraffin bath.

The splinters were not seen leaving the surface, but they were visible when they fell into the beam of light L from a mercury arc. The beam passed across the bottom of the tank, and in it water cloud droplets and ice particles were easily seen, and easily distinguished, against a background of black velvet. When the hoar frost was releasing splinters a few were immediately visible as they fell into the beam, but many more were seen when a water cloud was formed in the tank, presumably because, though too small to be seen initially, they had grown to visible size by sublimation as they fell through the water cloud. Expansions which formed cloud for about 30 sec. were therefore made at intervals of about three minutes, and it was found convenient to make this process automatic. The airstream over the surface was continuous, and when the air pressure in the tank rose 5 cm. Hg. above atmospheric the mercury in the manometer made contact with a platinum point, and a pump was turned on until the pressure fell 5 cm. below atmospheric. This is an advance on Findeisen's technique, for he observed only those splinters which were visible before they had grown, and there are many more which are smaller than that.

(b) Results.

There were two types of growth of the hoar frost; a tabular growth of solid hexagonal plates, and a dendritic growth of many-branched crystals giving a deposit with a very open structure. Splinters were released only when the hoar frost was growing dendritically, and this occurred only when growth was very rapid. At lower rates of growth the ice deposit was tabular, and splinters were not released.

Dendritic growth occurs very frequently in the atmosphere, but it only occurred on the cooled surface at very high rates of growth. Because of the large latent heat of ice, there is an important difference between the conditions during growth by sublimation on a freely falling crystal, and on an ice deposit on a cooled surface. In the case of a crystal falling freely, the heat of sublimation is conducted away from the surface of the crystal by the air through which the crystal is falling, and the crystal is rather warmer than the air. The corners will be better ventilated than the body of the crystal, and they will therefore be cooler then the rest of the crystal, though still warmer than the air. The corners will also have a better supply of vapour. These two effects together will make the corners tend to grow faster than the rest of the crystal, and the branched dendritic form, which is so well known in snowflakes, will result with all but the slowest rates of growth. In the case of hoar frost growing on a cooled surface, the heat of sublimation is conducted away through the body of the crystal, and through the cooled surface. The body of the crystal is then cooler than the corners. Dendritic growth still occurs at very high rates of growth because the more plentiful supply of vapour to the better ventilated corners still increases their rate of growth relative to the body of the crystal. On the cooled surface, dendritic growth did not commence until the surface was about 6°C colder than the airstream, but once this form of growth commenced, it developed cumulatively. The longer the dendritic needles grew, the less important became the conduction of latent heat down the needles and through the cold surface, and the more the conditions of growth approximated to those of a freely falling crystal.

When the surface was cooled at the beginning of an experiment a dew deposit appeared first, which did not freeze until it was cooled to about -20°C., and initial cooling to this temperature was necessary before experiments with hoar frost layers at higher temperatures. It had been expected that many splinters would be released as the dew deposit froze, but in fact none were seen at this time. This point is discussed in § 3.

Dendritic growth.

With the airstream at -5°C and the surface 6°C colder, fast dendritic growth occurred, and splinters were released at a rate of about 10/min/cm^2 of cooled surface. With the airstream at about -15°C, and the surface 6°C colder than that, the growth was slower but still dendritic. The rate

of release of splinters was then considerably slower, about 2/min/cm^2 of cooled surface. A few ice particles were visible as they fell into the beam at the bottom of the chamber when no cloud was present, but many more became visible when a water cloud was formed, presumably because the small splinters grew by sublimation while falling through it. The rate at which splinters appeared did not change immediately when a change was made in the conditions under which the hoar frost was growing, but many splinters were released when the airstream was restarted after an interruption. The rates of release reported above were observed when conditions were steady, and a water cloud was present.

Sudden movements could be seen on the surface when it was examined under the microscope. A dendrite was sometimes seen to fall over in the direction of the airstream, but none were seen to fall away, although it seems likely that the few large splinters seen when no cloud was present were whole dendrites. The rate at which the splinters appeared was not less than the rate at which dendrites fell over. Sometimes it was about the same, (one fall - one splinter) sometimes considerably greater, (one fall - several splinters). This suggests that the small splinters released from dendritic hoar frost layers were true splinters of ice formed as dendrites fractured.

A rough estimate of the size of the small splinters may be made as follows. The individual droplets of a water cloud were visible when they had grown to about 10μ diameter, and as the small splinters were invisible until they had grown, they were certainly smaller than this. On the other hand, the airstream was unsaturated w.r.t. ice in some of the earlier experiments, but splinters appeared at about the same rate. Few can have evaporated during the 1/2 second or so that they were in unsaturated air, and few can have been smaller than about 1μ diameter.

Tabular growth

When the surface was less than 6°C. colder than the airstream the ice crystals grew as solid hexagonal plates. A few comparatively large ice particles appeared in the beam, but no movements were visible on the surface. The rate at which these ice particles appeared was approximately constant whether cloud was present or not, and no small splinters seem to have been formed under these conditions. The few larger splinters were probably whole ice crystals which had fallen off the surface, not true splinters formed by the fracture of an ice crystal.

(c) Discussion

The rate at which splinters are released by ice particles in the atmosphere is still quite unknown. It had been hoped to use the results from the present work to make estimates of the rate at which splintering occurs in natural clouds. But this has not been possible because the rate of release of splinters, during sublimation at any rate, was found to depend very much on the type of growth of the hoar frost.

Findeisen reported that splinters were released at a rate of about 25/min/cm^2 of cooled surface when an ice deposit was growing by sublimations, and his work was done with apparatus which is similar to that described above. At first sight there is reassuring agreement between this figure and the 10/min/cm^2 reported above. However, Findeisen's work is to be criticised at several points; there was no airflow over the cooled surface, the ice was growing very fast, and he does not appear to have considered the effect which the cooled surface had on the form of growth of the hoar frost; there was no provision for making the splinters grow after release, and all those which he reported were large enough to be visible to the naked eye. It is probable that a large number of smaller, and invisible, ones were released at the same time.

A rate of 50/min/cm^2 of cooled surface was reported by the author in MRP 493, but in that case also the ice was growing very fast, and the airstream was only passing intermittently. This gives a high rate of release each time the airstream is restarted. The rate reported in this paper, 10/min/cm^2 of cooled surface, is that found when the conditions are as near as possible to those in natural clouds. But it was also found that splintering only occurs during

/dendritic

dendritic growth, and the conditions of growth on a cooled surface and on a freely falling crystal are very different. It is not reasonable to follow Findeisen and use results from laboratory work of this sort in order to make quantitative estimates of the importance of the process in natural clouds.

III. The release of splinters of ice as water freezes.

In the course of an investigation into condensation processes at low temperatures (MRP 492) it was observed that many splinters of ice were released during the freezing of the water on the walls of the small cloud chamber used in that work. M. J. Neiburger attempted to repeat the experiments during a visit to the Clarendon Laboratory. He used the same apparatus which had not been used for some months, but he observed no splinters during the freezing of a heavy deposit of water which had been freshly condensed on to the walls of the cloud chamber. He then contaminated this deposit with some rainwater, and after this it produced a great many splinters whenever it was frozen. There is an error on this point in MRP 492. The experiments described in I.c.2. were in fact done with deposits of laboratory distilled water, not freshly condensed water. It therefore appears that both rainwater and laboratory distilled water produce splinters during freezing, while freshly condensed water does not. The contamination, whatever it is, which is introduced by the impure waters cannot be removed by normal methods, but seems to become inactive when the chamber is left undisturbed for some months.

It was expected that many splinters would be released during the freezing of the dew deposit on the cooled surface S which was used for the experiments on hoar frost. Careful observations were therefore made during the freezing, but no splinters were seen. The deposit was of course 'freshly condensed' but the surface had not been cleaned particularly carefully.

Findeisen investigated the electrical effects associated with the release of splinters during riming. He reported that these effects were 10^3 times greater than those which occurred when an ice deposit was growing by sublimation, though his technique did not show whether this was because individual splinters were more highly charged, or because moderately charged splinters were released in greater numbers during riming. Findeisen's work on riming was done by exposing a cold body to natural supercooled water cloud on the Nebelhorn, or to a spray in the laboratory. Both the natural cloud droplets and the water which was sprayed must have been impure by these standards, for both gave splinters on freezing.

The release of splinters during freezing is clearly a very variable phenomenon, and the purity of the water an important factor. Some rainwater (Oxford) and some natural cloud droplets (Nebelhorn) are known to give splinters when they freeze. The process may be important in shower clouds, particularly those forming hail, for the air and cloud droplets in these clouds must be relatively dirty. In the other clouds the ice particles will grow mainly by sublimation, so that the release of splinters during riming is not likely to be an important process, and in any case the air and droplets will be cleaner, so that riming may not always lead to the release of splinters.

IV. Splintering in the atmosphere.

(a) Present knowledge.

It is now possible to make very tentative statements of the conditions under which the reproduction of ice crystals by splintering is likely to be important in the atmosphere. It will be seen that it may be an important step in the formation of rain by the Bergeron process.

(1) Splintering occurs when all but the very purest water freezes. Splinters were released when cloud particles froze in Findeisen's apparatus, and are probably released in natural cloud when ice particles grow by riming.

(2) Splinters are released when ice crystals grow by sublimation, if the form of growth is dendritic. This process is probably most important in mixed clouds between -5 and -15°C, for at lower temperatures the slower growth reduces the rate at which splinters are released.

/(3)

(3) Splinters are probably released when dendritic ice particles evaporate. This process is not likely to be very important in the atmosphere, and has not been investigated in the present work.

The size of the splinters released when ice crystals grow dendritically varies over a large range, and a large proportion are not visible to the naked eye until they have grown. The size of the splinters released during riming is not known. This process is very variable and is not clearly understood, but it seems important both as a mechanism by which ice crystals are reproduced, and possibly because of the associated electrical effects.

(b) Future work.

It is not possible to make numerical estimates of the rate at which any of the splintering processes occurs in the atmosphere. None of the experimental techniques used so far has simulated the conditions in natural cloud sufficiently closely. A much closer approach to natural conditions is possible, and the methods might yield some numerical estimates, so that it should then be possible to decide whether or not splintering is an important process in the atmosphere.

The release of splinters during sublimation could be observed under conditions much closer to those in natural cloud if the cooled surface S in the large cloud chamber were replaced by a fine wire. Hoar frost could be grown on the wire by passing over it an airstream which was super-saturated w.r.t. ice. The necessary supersaturation could be produced by mixing two saturated air streams which were at different temperatures to obtain an airstream which was super-saturated though not sufficiently supersaturated to contain water drops. Ice crystals could then be grown at rates, and at temperatures, comparable to those at which the process may be important in the atmosphere. The heat of sublimation would be removed primarily by the airstream, and only a small fraction down the wire, so that the major objection to the present apparatus would be overcome, and the ice crystals would be growing under conditions which were not very different to those in natural clouds.

The release of splinters during riming would be more difficult to investigate, for the process is extremely sensitive to the purity of the supercooled water, and may depend on the presence of minute quantities of impurities which have not been identified. It must be remembered, however, that practically no work has been done on this process since Findeisen's primarily electrical measurements. A supercooled water cloud can also be produced by the mixing method, and the riming process could therefore be investigated with the same apparatus. This should at least show whether the large electrical effects are due to the release of large numbers of moderately charged splinters, or to the release of a smaller number of splinters each having a large charge, and it would probably give some information about the size of the splinters produced.

MRP 493 V discussed the detection of meteorologically important concentrations of ice-forming nuclei with an airborne apparatus. It was not certain whether an apparatus which an aircraft could carry would be large enough to detect the very low concentrations of these nuclei which may be important. It was suggested that the subject should be reconsidered when further results had been obtained about the reproduction of ice crystals by splintering. The results reported in this paper suggest that this is probably an important process in the atmosphere, and that low concentrations of ice-forming nuclei may have significant effects. An airborne cloud chamber would have to be very large before all significant concentrations of nuclei could be detected, and it would therefore appear to be more advantageous to concentrate for the present on other work, primarily on splintering.

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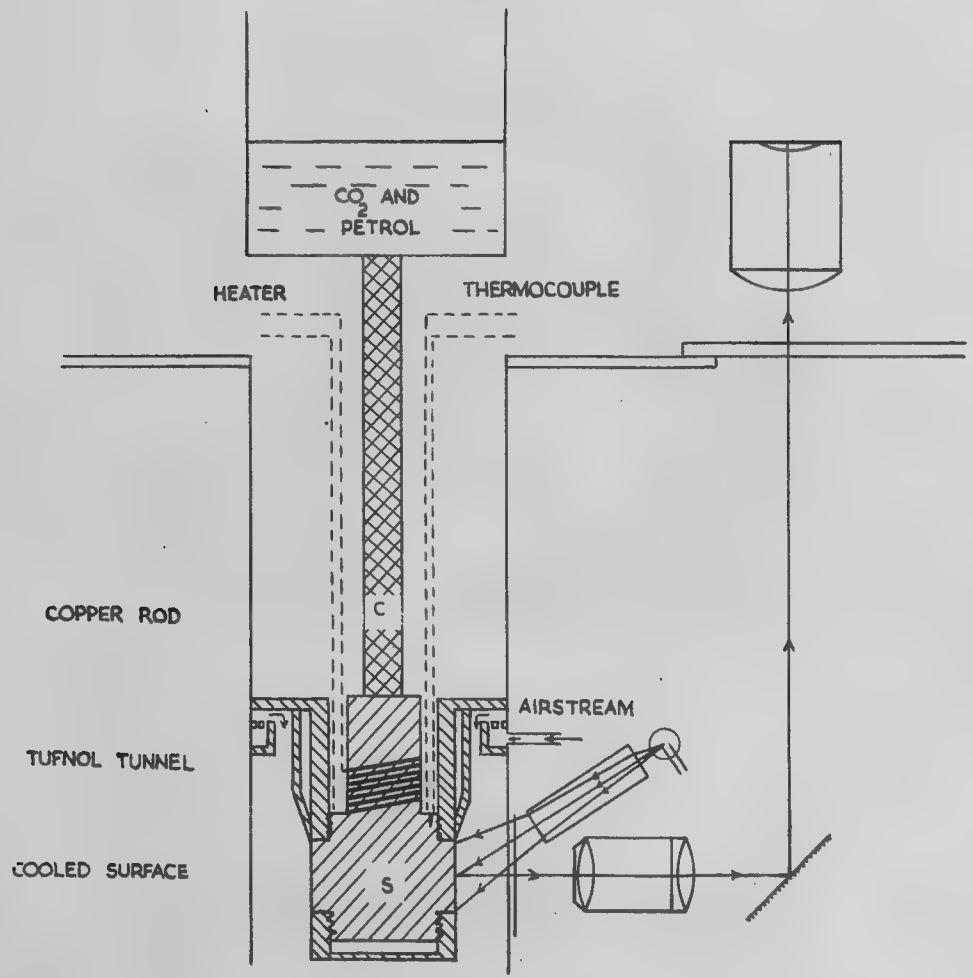

FIG.I.

THE COOLED SURFACE

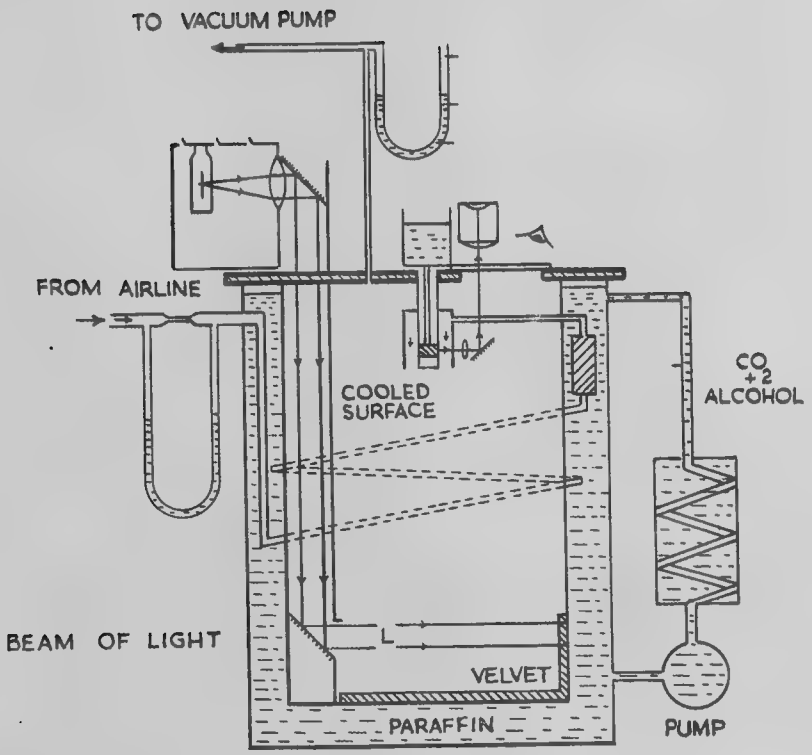

FIG.2

THE LARGE CLOUD CHAMBER

FIG 3 THE COOLED SURFACE WITH TUNNEL REMOVED

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METEOROLOGICAL RESEARCH COMMITTEE

The Upper Air Circulation in Low Latitudes and its relation to certain Climatological Discontinuities.

by

R. Frost (Meteorological Office)

Introduction

In a recent note in the Meteorological Magazine D. Dewar (1) discussed the variation of the tropopause pressures and temperatures at Habbaniyah and shewed that during the winter months the tropopause occurred at a pressure level of approximately 220 mb. and during the summer months at a pressure level of about 80 mb. The most striking features however were the abrupt changes in the pressure levels which occurred in late May and to a lesser extent in October, cf. Fig. 1 which shews the tropopause pressures for 1950.

Climatological data for Habbaniyah shew that surface discontinuities of wind and temperature also occur in these months. Table I which exhibits the monthly frequency of winds at Habbaniyah shews that during the winter months the surface winds are variable whilst during the summer months from June to October there is a steady NW wind. The onset of this NW wind, known as the Shamal, which is the northerly extension of the Arabian branch of the Indian monsoon, occurs in late May or early June and is frequently accompanied by dust storms, and whilst not as spectacular in its arrival as the 'burst of the monsoon' over India is almost equally abrupt. Table II, which gives the mean monthly west-east and south-north components of the winds at Habbaniyah at standard pressure levels, shews that during the summer months there is a northerly component of the wind at all levels from the surface to 700 mb., whereas during the winter months, with the exception of January, there is a southerly component at these levels.

The discontinuous nature of the temperature at Habbaniyah is readily seen from the mean monthly maximum temperatures shewn in Fig. 2.

In the course of an investigation into these climatological discontinuities seasonal charts of tropopause pressure levels over the Mediterranean, Africa, Middle East and India and seasonal meridional cross-sections for latitude 40°E. were prepared. These are reproduced as Figs. 3 - 10 and are discussed in the following sections.

It is suggested that these climatic discontinuities and also the onset of the Indian Monsoon are a direct consequence of this abrupt change in the tropopause regime.

Observational Material

The main observational material used in this discussion are the twice daily radiosonde and radarwind observations at 0300 and 1500 G.M.T. from January, 1948, which are available for the following stations, Aden (12°50'N 45°01'E) Bahrein (26°14'N 50°25'E) Habbaniyah (33°22'N 43°34'E) Nicosia (35°09'N 33°17'E) Benina (32°05'N 20°16'E) Malta (35°50'N 14°27'E) Gibraltar (36°09'N 05°21'W) Larkhill (51°11'N 01°48'W) and Lerwick (60°11'N 01°11'W) and the once daily observations at 0600 G.M.T. from Nairobi (01°17'S 36°50'E). Radiosonde observations for shorter periods from Marshall Field Liberia, from Salisbury (17°50'S 31°01'E) Rhodesia, from Pretoria (25°46'S 28°14'E)

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Union of South Africa, from Nandi (17°45'S 177°17'E) and Canton Island (02°49'S 171°43'W) New Zealand, from published data of Australian Stations, Loewe and Radok (2), and from published data for Manchuria and Japan, Tokio Central Meteorological Observatory publications, have also been utilised. These have been supplemented by observations from aircraft ascents and from sounding balloons made over many years, which have been extracted from various publications, for stations in Europe, India and Batavia.

Use of the Observational Material

A difficulty which was encountered at the outset in discussing the spatial and seasonal variations of the mean tropopause was the fact that the existing definition of the tropopause is not sufficiently precise for regions in which multiple tropopauses occur. In current Meteorological practice, the type and position of the tropopause are specified by the following conventions which are based on those given by W.H. Dines (3) in 1919.

Type I. When the stratosphere commences with an abrupt change in the lapse rate to an inversion the tropopause is at the base of the inversion.

Type II. When the stratosphere commences with an abrupt change to a lapse rate of less than 2°A. per kilometre, or to isothermal conditions, the tropopause is at the level of that change.

Type III. When there is no abrupt change of lapse rate, the tropopause is taken as the level of the point at which the lapse rate becomes less than 2°A. per kilometre, providing that it does not subsequently exceed this value for any subsequent kilometre.

It will be noted that with the exception of a Type III tropopause the above definitions imply that the stratosphere is clearly recognisable. In the subtropics where multiple tropopauses are common there is a tendency on the part of some observers to call the inversion at the highest level a Type I tropopause and to neglect any inversions or abrupt changes of lapse rates at lower levels. Other observers on the other hand report as tropopauses levels of abrupt changes which comply with the above definitions, but which are difficult to reconcile with any idea of a boundary surface between troposphere and stratosphere, whilst to complicate the picture, other observers appear to exercise their initiative and judgement and exclude all inversions below some arbitrary limit. As a result tropopauses in the subtropics especially at certain seasons of the year apparently range haphazardly over pressure levels ranging from 500 mb. to 100 mb. in the course of a few days, and means of such values are very unsatisfactory to say the least.

The difficulty of attempting to frame a satisfactory definition can be seen from Fig. 11 which depicts upper air soundings at Habbaniyah on 6th, 7th and 9th February, 1950, after a burst southwards of unusually cold air on the 5th. The tropopauses reported on those days were 500 mb., 121 mb. and 130 mb. respectively.

This drawback in the 'mean' level of the tropopause in regions where temperate and tropical tropopauses overlap is well known, and various attempts to overcome this have been proposed. One method which at first sight appears attractive and which has been adopted by many writers including, inter alia, Hess (4), Hutchings (5) and Clarkson (6) is to use the 'minimum' tropopause which is less subjective. They make use of the fact that when the mean vertical distribution of temperature above a station is plotted, there is one (or in rare cases two) level at which $\frac{\partial T}{\partial z} = 0$. With certain reservations they call this level the 'minimum' tropopause, or if there are two such levels they designate the higher as the tropical tropopause and the lower as the polar tropopause. The drawback about this definition is the unfortunate fact that the tropopause as thus defined cannot be reconciled with the average of individual tropopause levels and, as Hutchings has shown, may exceed the latter by as much as five kilometres.

/For

For individual soundings this 'minimum' definition of the tropopause has little, if any, advantage over the Meteorological Office conventions, for frequently the ascent is ill-defined and is easily masked by instrumental errors so that it is impossible to define a strict level of minimum temperature.

J. Bjerknes (7) has stated that at first sight one would place the tropopause simply where the lowest temperature occurs whilst the true tropopause, which is lower, would be passed over, as an insignificant feature in the upper troposphere, and he considers that to arrive at a coherent system it is necessary to admit the diminution in the vertical lapse rate of temperature as a sign of the tropopause.

In a later paper, J. Bjerknes and Palmen (8) regard all sudden decreases of lapse rate in the upper tropopause and lower stratosphere as a sign of the true tropopause, and more recently Flohn and Penndorf (9) have suggested that no attempt should be made to define a single tropopause level, but that a tropopause layer should be substituted extending downwards a conventional distance from the level at which isothermal conditions are reached.

Despite the imperfections of the 'mean' tropopause obtained by the averaging of individual tropopauses, this 'mean' tropopause has been used throughout the present paper. In some cases the tropopauses as reported have been rejected, and discontinuities at lower levels selected.

In constructing the meridional cross-sections stations were selected to lie as far as possible along the meridian of 40°E, which has the advantage that all the stations, with the possible exception of Aden, are situated in large continental land masses, and are not subject to the serious difficulties of interpretation which arise when some stations are subject to oceanic and others to continental influences.

Mean E/W and N/S components of the wind at Nairobi, Aden and Bahrein for January, April, July and October are given in Table III, and mean E/W components for Nairobi, Aden, Bahrein and Habbaniyah are depicted in Figs.3-6.

Mean temperatures at standard pressure levels of the same seasons for Nairobi, Aden, Bahrein, Habbaniyah and Rostov are given in Table IV, and mean temperature differences at the levels for the same stations and seasons derived from Table IV are given in Table V.

In the present paper actual wind components, up to in some cases 40 mb., have been used for stations south of latitude 35°N, and components of the mean geostrophic wind computed from G.M. No.85 (10) have been used north of 35°N.

The Mean Zonal Wind Field. The most significant features of the mean zonal wind field are the appearance of a pronounced high level westerly jet stream of over 100 kt. in January at a pressure level of approximately 200 mb. at about 26°N. This jet moves very slowly to a position about 29°N by April, weakens slightly to about 90 kt. and then moves more rapidly to about 40°N by July when it decreases to about 35 kt. By October it has moved southwards to about 33°N and has increased in speed to about 65 kt. A most striking feature of the mean zonal wind field in July is the appearance of an easterly jet of over 70 kt. at a pressure level of about 100 mb. at approximately 15°N, with an almost vertical boundary between the easterlies and westerlies at 30°N.

This easterly jet weakens rapidly by October and is non-existent in both January and April, its place being taken by westerlies in both the upper troposphere and lower stratosphere.

It will be noted that in drawing the easterly jet the centre has been placed at or above the 100 mb. level. The reason for this is that, owing to the strong upper winds at these levels, a large proportion of the balloons failed to reach the levels at 100 mb. and above, and hence the mean components at levels at which the winds were still increasing are too low. An estimate of the true value at 100 mb., from a comparison of the wind speeds at 150 mb. and 100 mb. for all winds which reached at least 100 mb., suggests that the mean wind speed at this level should be increased to about 75 kt. No attempt was made to correct the winds at higher levels.

/Comparison

Comparison with cross-sections in the subtropics by other writers show some important differences but it is important to note that, in the absence of adequate observations of wind at high levels, all these writers have based their cross-sections on pressure and temperature data with theoretical computations of winds based on geostrophic assumptions, and the accuracy of such winds near the equator in regions where observations at ground level are scanty, and at upper levels non-existent, is open to question. Hess (11), from mean cross-sections at 80°W longitude for the combined winter months of January and February and for the combined summer months of July and August for the years 1941 to 1945 inclusive, found in winter a high level westerly jet of about 100 m.p.h. at about latitude 35°N and at a height of about 12 km., whilst in summer it occurred at latitude 55°N and its speed had decreased by a half. Chaudury (12) who prepared a cross-section for January and February over India and Pakistan along the meridian of 76°E, from one year's data only, found three westerly jets, one between 15°N and 20°N, another to the south of the Himalayas at about 31°N and another to the north of the Himalayas.

Cross-sections prepared by Loewe and Radok for the SW Pacific in longitude 150°E, a mainly continental regime, show a winter jet of 120 m.p.h. at about 25°S at a height of about 12 km., and a summer jet of 50 m.p.h. at 40°S at a height of about 10 km. The winds used in their cross-sections were components of the mean geostrophic wind, and a comparison which they carried out between such computed winds and winds observed by radar at three stations between latitudes 19°S and 34°S in Australia, showed that the observed winds were only 55% of the computed winds. In a later note Loewe and Radok (13), from new Tasmania data, suggest that the summer maximum should be twice as high as that given in their earlier paper.

Hutchings, who prepared a meridional cross-section for a purely oceanic region in the SW Pacific at 170°E, found from the computed geostrophic wind components a single maximum of 100 kt. at 30°S in winter, and in summer a maximum of 60 kt. at 46°S, and a secondary maximum of 50 kt. at 31°S.

The Mean Zonal Temperature Field and Tropopause

The most important features of the mean zonal temperature field are the concentration of latitudinal temperature gradients, the baroclinic zone, in the troposphere beneath the westerly jet at all seasons of the year, and in July a second baroclinic zone south of 30°N. It can be seen that the latitudinal gradients of temperature vanish at a pressure level of about 200 mb., whilst above this level the gradients of temperature are reversed, and temperatures in the lower stratosphere above the cold air to the north are warmer than temperatures in either the lower stratosphere or upper troposphere above the warm tropical air. The lowest temperatures occur in the lower stratosphere above the thermal equator, which in January is located at or near the geographical equator, and in July is located at approximately 30°N.

The tropopause over the 'warm' regions occurs between pressure levels of 80 mb. and 100 mb. whilst over the 'cold' regions to the north the tropopause occurs between pressure levels of 200 and 250 mb., c.f. Table VI which shows the monthly values of the mean tropopause pressures and temperatures. It can be seen that at Nairobi the tropopause is highest in the winter and spring at a pressure level of about 80 mb. and lowest in the summer and autumn at a pressure level of about 90 mb. At Aden the tropopause is highest in March and April at a pressure level of about 85 mb., and lowest in September and October at a pressure level of about 95 mb. At Bahrein the tropopause is at its highest in June at a pressure level of less than 80 mb., and lowest in December with a pressure level of about 100 mb., whilst at Habbaniyah the tropopause occurs at a pressure level of 83 mb. in July and at a pressure level of about 220 mb. in January.

These figures shew that as the sun moves northward there is a close relationship between the height of the tropopause and the position of the thermal equator, and that the highest tropopauses occur at the thermal equator which is not unexpected. What is unexpected, however, is the fact that as the thermal equator moves south the height of the tropopause apparently falls with it. This, if real, suggests that the simple pattern of meridional circulation

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in which air ascends at the thermal equator south of the jet and subsides north of the jet requires modification when the sun is moving southwards. In support of this suggestion, the differences between the N/S components of the wind at Nairobi and Aden from Table III shew that in April there is little or no horizontal divergence or convergence at Aden at levels between 100 and 700 mb., whereas in October there is marked horizontal divergence at the 100 mb., 150 mb. and 200 mb. levels.

The nature of the transition between the high level tropopause and the low level tropopause is also obscure and it is uncertain whether in the baroclinic zone the tropopause should be regarded as a continuous surface or as a series of discontinuous surfaces. K.R. Ramanathan and K.R. Ramakrishnan (14) who first recognised the discontinuous nature of the tropopause surface over Northern India in 1933 attributed this to the large scale folding of the tropopause resulting from meridional circulations in the vertical plane. A study of the day to day soundings at Habbaniyah shews that in January there is one tropopause at about 220 mb., and on about one day in two, there is also a second tropopause at about 90 mb. In February and March conditions are somewhat similar although in February the proportion of upper tropopauses appears to be rather less. In April and May there appear to be at least three tropopauses at pressure levels of approximately 80 mb., 160 mb. and 220 mb., whilst on many occasions there are up to five. In June, July, August, September and most of October, observations shew that there is only one tropopause at a pressure level between 80 and 100 mb. In late October and November the same multiplicity of tropopauses as in April and May occurs, whilst in December there appear to be two tropopauses with the lower predominating at about 220 mb. At Bahrein observations shew that there is always a high tropopause between 80 mb. and 100 mb., but in December and January there is also evidence of the low temperate tropopause. In January therefore the tropical tropopause at 80/100 mb. and the temperate tropopause at 200/220 mb. overlap by at least 500 miles.

Observations from stations in the Mediterranean shew that there is a marked longitudinal variation of the tropopause. At Benina and Nicosia the variation of the tropopause is similar to that at Habbaniyah, although the transition from temperate to tropical tropopause occurs slightly later and is less abrupt than at Habbaniyah. At Gibraltar the temperate tropopause persists throughout the whole year, and from February to mid-June there is little evidence of an upper tropopause. From July to January there appear to be two tropopauses, but in July and August the tropical tropopause is the most marked. The observations from Malta are intermediate between those of Gibraltar and Nicosia. These general conclusions may be compared with those of Nagai and Matuoka (15) who investigated the tropopause from radiosonde ascents at Tateno (36°03'N, 140°08'E) from January, 1948, to April, 1949, and found that in the winter and early spring double tropopauses occurred at heights of about 10 km. and $15\frac{1}{2}$ km., whilst from March to November there was only a single tropopause which rose rapidly from under 12 km. in April to over 14 km. in June, and then more gradually to over 16 km. in September, October and November - the highest value occurring in September.

Reference to Fig. 12, which depicts the mean monthly temperatures at Habbaniyah at standard pressure levels and the mean monthly tropopause pressures and temperatures during the first seven months of the year, shews that during the period from January to May the temperature increase due to the surface heating is distributed evenly throughout the troposphere between the surface and about 200 mb., whilst in the stratosphere above this level the thermal structure is very stable and shews very little change. Between May and June however the continued rise in temperature in the troposphere is accompanied by a marked cooling in the lower stratosphere at about 100 mb., so that the low tropopause, which originally formed a complete barrier to penetrative convection from below, is easily broken through with the result that the surface heating is distributed through a deeper layer of the atmosphere, thereby producing the discontinuity in the rise of the maximum temperatures shewn in Fig. 2. H. Arctowski (16) who first noted temperature discontinuities of this kind at Melbourne and elsewhere, described them by stating that the natural course of the temperature due to local conditions appeared to be

/interrupted

interrupted just as if the station had been lifted into another climate similar to but colder than it might be during the summer. An analysis of the break-down in May, 1949, with the aid of Fig. 13, shews that from the 1st to the 23rd the temperatures increased at all pressure levels from the surface to approximately 200 mb., whilst above 200 mb. there was little change apart from a slight cooling of perhaps 7°F. at 100 mb. Between the 23rd and 24th however there was very rapid cooling in the lower stratosphere, the temperature at the 90 mb. level now being some 20°F. lower than on the 1st. By the 25th this cooling had extended downwards to the 150 mb. level, accompanied by very rapid warming between 260 mb. and 200 mb. which appeared to be due to subsidence. The warming of the lower layers continued between the 25th and 26th and by the latter date the inversion at 260 mb. appeared only as an insignificant feature of the tropopause, which, even if real, no longer constituted any barrier to penetrative convection from below, leaving only an upper tropopause at the level of maximum cooling.

It has been noted by several writers, e.g. Bjerknes and Palmen (17) and Kraus (18) that in Europe folded tropopauses occur with all major warm frontal surfaces, and that the first formation of the folded tropopause appears to take place about 500/1000 miles ahead of the surface warm front. As the thermal equator moves north, the baroclinic zone is a warm front transitional zone, and hence it might be anticipated that a similar characteristic folding of the tropopause would occur. On the other hand instead of cooling by advection it is possible that the cooling at high levels might be the result of horizontal divergence as suggested by Goldie (19) in which case the tropopause surfaces would be discontinuous.

It does not appear to be possible either from the observations at Habbaniyah or the rather scanty observational network in this area to decide between these alternatives. Although of theoretical importance however it does not appear to be of very great practical importance which theory is correct, and in view of the very large number of multiple tropopauses encountered at Habbaniyah in the transitional months, it was considered best, at least on aesthetic grounds, to depict the tropopause on the seasonal cross-sections as a continuous folded surface.

The spatial and seasonal variations of the mean tropopause, together with the position of the jet stream taken from Geophysical **Memoir** No.85, are shewn in Figs. 7-10. These charts which are based upon all the data available are believed to be reasonably reliable and indicate that the main features of the meridional cross-section at 40°E apply with minor modifications to the whole of the Afro-Asian continents. No attempt was made on these charts to depict either a folded or discontinuous tropopause, and the tropopause is shewn as a steeply sloping continuous surface to the north of the jet stream.

The Jet stream and the Latitudinal Gradient

There is at present no generally acepted theory of the formation of the jet stream. Rossby and collaborators (20) argue that the transfer of absolute vorticity by large scale lateral mixing could account for some of the more important features of the jet stream, and that the latitudinal variation of temperature is dynamically produced by the kinetic energy of the jet stream. Palmen and Newton (21) consider that the jet stream is always closely associated with the polar front. Namias and Clapp (22) have advanced their confluence theory (kinematic frontogenesis), according to which the jet stream is caused by the gradual flowing together of a current of warm midtropospheric air curved anticyclonically from the south and of cold midtropospheric air curving cyclonically from the north. Bleeker (23), however, whilst recognising the importance of confluence, considers that physical frontogenesis along a line of differential heating is the main cause of the jet stream.

Of these theories, Bleeker's appears to afford the most likely explanation of the Afro-Indian high level jet stream.

/The

The intimate relationship between the mean jet stream and the integrated mean latitudinal temperature gradient can be readily demonstrated. In the case of steady geostrophic flow we have

$$2\omega u \sin\varphi = -\frac{1}{\rho}\frac{\partial p}{\partial y} \qquad (1)$$

$$g = -\frac{1}{\rho}\frac{\partial p}{\partial z} \qquad (2)$$

$$p = R\rho T \qquad (3)$$

where U and V are the velocities along the xy axes which are directed to the east and north respectively, z is measured vertically upwards and p, R, ρ, T, ω & $\cos\varphi$ have their usual significance. From (2) and (3) we have

$$\frac{g}{RT^2}\frac{\partial T}{\partial y} = \frac{\partial}{\partial y}\left(\frac{1}{\rho}\frac{\partial p}{\partial z}\right) = \frac{\partial}{\partial z}\left(\frac{1}{\rho}\frac{\partial p}{\partial y}\right) \qquad (4)$$

$$\therefore \quad \frac{g}{RT^2}\left\{\left(\frac{\partial T}{\partial y}\right)_p + \frac{\partial T}{\partial p}\frac{\partial p}{\partial y}\right\} = -g\rho\frac{\partial}{\partial p}\left(\frac{1}{R\rho T}\frac{\partial p}{\partial y}\right) \qquad (5)$$

where $\left(\frac{\partial T}{\partial y}\right)_p$ is the latitudinal variation of temperature along a pressure surface.

$$= -\frac{\partial p}{RT}\frac{\partial}{\partial p}\left(\frac{1}{\rho}\frac{\partial p}{\partial y}\right) + \frac{g}{RT^2}\frac{\partial p}{\partial y}\frac{\partial T}{\partial p} \qquad (6)$$

whence

$$R\left(\frac{\partial T}{\partial y}\right)_p = -p\frac{\partial}{\partial p}\left(\frac{1}{\rho}\frac{\partial p}{\partial y}\right) \qquad (7)$$

and hence from (1) & (7)

$$2\omega u \sin\varphi = R\int_p^{p_0}\left(\frac{\partial T}{\partial y}\right)_p \frac{\partial p}{p} \qquad (8)$$

and hence the maximum wind occurs at the pressure level at which $(\partial T/\partial y)_p = 0$, which level, from the mean meridional cross-sections, is about 200 mb.

The winds at 200 mb. calculated from equation 8 for latitudes 26°N and 40°N, using the temperature differences between Bahrein and Habbaniyah for January and between Habbaniyah and Rostov for July respectively from Table V, are 105 kt. and 40 kt. This very close agreement is probably accidental, but the general agreement suggests that the thermal gradient could be the cause of the jet stream. The fact that the jet stream appears on the cross-sections from January to July as a weakening core of fast moving air with large shears on either side suggests that it is subject to decay by the action of large scale horizontal eddies which mix the fast moving thermally induced stream with stationary air to the north and south. It is not however clear what property is transferred by the horizontal eddies, and as Sheppard (24), who discusses Rossby's theory, points out, the assumption that absolute vertical vorticity is transferred implies serious limiting assumptions about the motion.

/Wind

Wind tunnel experiments on the turbulent spreading of symmetrical jets shew that calculations made by Howarth (25) on the vorticity transfer theory do not agree with the experimental results, whereas calculations by Tollmien (26) on the momentum transfer theory shewed good agreement.

The consequences of the assumption that in the atmosphere momentum is transferred by lateral mixing will be examined in a later paper.

Relation with the Onset of the Indian Monsoon

As early as 1686, Halley (27) had stated that the cause of the monsoon was due to differential heating, but until recently no very satisfactory explanation had been given for the retardation of its onset over India as compared with Burma, and why it subsequently advances over India in a spectacular manner.

Maung Tun Yin (28), who carried out a study of the onset of the monsoon, suggested that its retardation over India as compared with Burma could be explained by the fact that during the winter and spring months the seasonal flow patterns at high levels shewed that there was an upper trough line, presumably orographically determined, orientated N/S at about 85°E, which would accelerate the monsoon over Burma, which lies to the east of this upper trough, whilst retarding it over India, which lies to the west of the trough. In summer the mean flow patterns at high levels over India shew that the upper trough is now located at 75°E. Maung Tun Yin found that the displacement of this upper trough, which occurred relatively abruptly, appeared to be related to the advance of the monsoon over India, and pointed out that the displacement of this trough westward could explain the advance of the monsoon, or alternatively that the advance of the monsoon could explain the displacement of the trough westwards. As the latter precluded an explanation of the onset of the monsoon, he investigated the possibility of explaining the shift of the upper trough westwards by some other mechanism. Using values of the geostrophic wind at 500 mb. for five day intervals averaged over the whole of Eurasia from 0°E to 130°E, he found that the region of strongest westerlies, about 24 kt. at the beginning of May, was situated at about 30°N and that this region moved to about 40°N in the last week of May and disintegrated rapidly during the first week of June. From this he argued that the westerly stream, which in the premonsoon season had circled the Himalayas along its southern border, was displaced to the north of the Himalayas during this period, and that this resulted in a rapid orographically determined displacement westwards of the upper trough.

Whilst this explanation is undoubtedly ingenious there is little evidence to shew that the upper level trough in summer is orographically determined, and his deductions from geostrophic winds averaged from 0°E to 130°E over a region which embraces the Mediterranean, the Iraq and Persian deserts, the Himalayan plateau and Indo-China, where pressure regimes vary considerably, and where, especially in the area in question, observations are scanty, are not altogether convincing.

According to the Habbaniyah data, the westerly component of the 200 mb. wind falls rapidly from a maximum of 84 kt. in March to 56 kt. in April, then more slowly to 49 kt. in May, then increases to 51 kt. in June and falls off rapidly to 12 kt. in July. This pattern is repeated at the 300 mb. level, whilst at the 500 mb. level there is a slow decrease from 34 kt. in March to 27 kt. in April to 23½ kt. in May to 23 kt. in June and then a rapid fall to 6 kt. in July, and this pattern shews little variation from year to year.

As the jet is related to the latitudinal temperature gradient, any weakening in the thermal field should be reflected in the strength of the jet. Since the jet must be near but still south of Habbaniyah when the abrupt change of tropopause occurs, there should be a slight intensification of the temperature gradient southwards and hence a slight increase in the strength of the jet. It would thus appear from the Habbaniyah data that the jet moves through Habbaniyah late in June and that the collapse of the jet does not take place until some time after the 'burst of the monsoon' which usually occurs at the end of May.

/In

In a discussion of the tropopause observations in May, 1949, it was shewn that the change in tropopause level occurred on May 27th. In 1948 it occurred on June 9th, and in 1950 on May 26th. (The date of change in 1950 given by Dewar from the reported tropopauses was May 19th, but an examination of the original data shews that although the upper tropopause was well developed by this date, the lower tropopause still constituted an effective barrier to a break through from below until May 26th).

The dates of the 'burst of the monsoon' along the Malabar coast during these years, extracted from the Indian Daily Weather Reports, are as follows:-

1948, June 10th; 1949, May 23rd; 1950, May 27th.

It is suggested therefore that the sudden onset of the monsoon over India results from the breakdown of the lower tropopause barrier due to the combination of the rapid heating of the layers between the surface and 200 mb. and the cooling, either by advection or dynamical effects, of the layers above, and that the onset of the monsoon precedes the movement of the jet to a position north of the Himalayas.

Summary

The most striking features of the Upper Air Circulation over Arabia are the westerly jet stream of over 100 kt. at a pressure level of about 200 mb. in latitudes 25°/30°N during the winter months and an easterly jet stream of over 70 kt. at a pressure level of 100 mb. in about latitude 15°N during July.

The westerly jet stream reaches its maximum intensity in midwinter, moves slowly northwards and decreases slightly during the Spring and then moves to a position about 40°N and weakens rapidly during mid-summer. The easterly jet weakens rapidly during the Autumn and is non-existent during the Spring and Winter.

As the westerly jet moves northwards over Iraq, there is an abrupt change in the level of the tropopause and it is suggested that the breakdown in this tropopause barrier helps to explain the surface discontinuities of temperature and wind which occur during these months and the spectacular onset of the monsoon over India.

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hrein and Habba..i

abbaniyah	Ros
214.6	
212.1	
212.5	(218
217.2	(218
217.4	(218
229.8	227
243.7	
255.6	254
265.1	
273.5	269
281.5	
285.3	276
289.3	
294.3	

perature at stan

B-H	H-R
7.4	-
7.5	-
8.3	-(5.5)
4.0	-(0.8)
3.2	-(0.6)
5.7	+ 2.8
7.4	-
5.6	+ 1.6
5.0	-
5.7	+ 4.5
7.2	-
5.7	+9.3
5.7	-
5.0	-

Monthly Tropop

											mp.	June Pressure	
Nairobi	83.5	191°	84.8	192°	84.4	191°	82.5	191°	81.5	191°		85.3	1
Aden	90.3	191°	90.9	192°	85.1	192°	85.9	191°	87.2	191°		92.6	1
Bahrein	89.7	200°	92.0	201°	87.9	201°	81.7	200°	80.6	197°		76.7	
Habbaniyah	224	216°	231	216°	235	218°	212	216°	200	215°		88	2
Benina	217	218°	238	218°	243	219°	213	216°	197	216°		162	2
Nicosia	234	218°	253	219°	260	219°	240	218°	218	218°		160	2
Malta	235	217°	237	216°	251	217°	224	216°	196	214°		190	2
Gibraltar	221	215°	217	215°	221	214°	210	217°	209	219°		184	2

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Table I. Monthly Frequency of Wind Directions at Habbaniyah at 0600 G.M.T. (0900 Local Time)

Period 10 years 1937 - 1946

	NNW	N	NNE	NE	ENE	E	ESE	SE	SSE	S	SSW	SW	WSW	W	WNW	NW	CALM
Jan.	24	7	3	4	-	12	11	23	8	15	7	9	12	19	25	77	54
Feb.	31	8	4	6	3	10	11	29	10	10	5	10	5	20	17	67	36
March.	41	25	4	8	7	12	7	17	14	15	12	15	6	14	17	77	19
April.	37	28	12	12	6	20	11	18	10	14	7	6	5	11	12	69	22
May.	49	38	12	13	3	12	2	4	2	5	4	6	5	14	36	91	12
June.	74	29	4	8	2	3	-	-	2	3	1	2	2	6	21	134	9
July.	73	14	1	1	1	1	-	1	-	1	2	-	-	6	29	176	4
August.	83	21	1	7	-	1	-	-	-	1	-	1	-	3	16	169	7
Sept.	73	26	6	6	-	1	1	-	-	3	-	3	1	4	16	135	25
Oct.	76	26	10	18	9	16	3	3	-	11	2	9	2	2	11	82	30
Nov.	37	20	7	21	9	15	6	11	3	10	2	7	5	11	7	81	48
Dec.	42	10	9	8	9	13	12	22	5	8	5	8	4	16	14	72	53

Table III. Seasonal components of wind speed at standard pressure levels. Period 1948 - 1950 inc.
(Components of the wind from west and south are reckoned positive)

mb.	JANUARY Nairobi W/E	JANUARY Nairobi S/N	JANUARY Aden W/E	JANUARY Aden S/N	JANUARY Bahrein W/E	JANUARY Bahrein S/N	APRIL Nairobi W/E	APRIL Nairobi S/N	APRIL Aden W/E	APRIL Aden S/N	APRIL Bahrein W/E	APRIL Bahrein S/N	JULY Nairobi W/E	JULY Nairobi S/N	JULY Aden W/E	JULY Aden S/N	JULY Bahrein W/E	JULY Bahrein S/N	OCTOBER Nairobi W/E	OCTOBER Nairobi S/N	OCTOBER Aden W/E	OCTOBER Aden S/N	OCTOBER Bahrein W/E	OCTOBER Bahrein S/N
40																	-40	+3						
60	-24	-6			+ 26	+12	- 1	0			+ 5	- 1	+12	+1	-45	0	-40	-1	+ 4	-1			- 6	-2
100	-11	0	+ 5	0	+ 57	- 1	- 5	0	+ 4	+2	+38	+ 8	-11	-1	-66	+5	-35	+4	- 3	+1	-17	+6	+ 2	+4
150	-12	-2	+14	-6	+ 89	-19	+ 1	+1	+18	-1	+66	+17	-31	-3	-71	+4	-23	+2	+ 5	+4	-12	+8	+17	+5
200	-23	+7	+14	-3	+102	-18	-1	-1	+17	-1	+88	+21	-14	+1	-48	0	-16	+1	+ 6	+3	0	+7	+23	+3
300	-12	-4	+14	-5	+ 89	- 9	+ 8	+1	+16	-1	+59	+12	+ 2	+3	-24	-4	-11	-3	- 3	+4	- 7	+3	+29	+1
500	-13	-1	- 3	-1	+ 54	0	-13	-1	- 1	0	+28	+ 5	- 2	-4	- 9	-9	- 9	-4	-12	-1	- 5	-1	+11	-5
700	- 6	-9	- 2	+2	+ 23	+ 1	- 7	-3	- 7	-4	+17	+ 1	- 2	+1	- 1	-3	- 6	-5	- 9	+1	- 8	+12	- 2	-8

Table II. Monthly components of the wind speed at standard pressure levels. Period 1948 - 1950
(Components of the wind from west and south are reckoned positive)

mb.	Jan. W/E	Jan. S/N	Feb. W/E	Feb. S/N	March W/E	March S/N	April W/E	April S/N	May W/E	May S/N	June W/E	June S/N	July W/E	July S/N	August W/E	August S/N	Sept. W/E	Sept. S/N	Oct. W/E	Oct. S/N	Nov. W/E	Nov. S/N	Dec. W/E	Dec. S/N
40													-24.3	+3.3	-19.6	+ 3.0	- 4.5	+ 0.2						
60	+31.4	+2.0			+24.5	+ 1.6	+ 9.8	+ 1.4	+ 0.1	+ 4.8	- 7.1	+ 2.7	-15.1	+4.8	-12.7	+ 4.6	+ 0.6	+ 1.3	+ 6.5	- 0.4	+22.8	-0.3	+25.0	+ 2.4
100	+45.8	-4.1	+43.4	+4.9	+47.3	+ 3.2	+33.3	+ 4.2	+23.2	+15.3	+20.2	+13.1	+ 1.5	+10.0	+ 5.0	+ 8.4	+22.7	+12.1	+34.2	+ 7.2	+35.8	+1.8	+40.9	+ 1.2
150	+65.2	-4.7	+67.2	+4.4	+71.7	+ 5.7	+50.9	+ 7.8	+43.8	+20.4	+41.8	+18.6	+10.0	+9.6	+15.8	+ 9.1	+39.0	+15.4	+57.0	+15.9	+47.7	+4.8	+56.1	+ 2.2
200	+72.3	-4.0	+77.2	+6.3	+84.3	+12.7	+56.0	+13.1	+49.0	+22.1	+50.7	+19.4	+12.2	+7.7	+20.1	+ 6.9	+43.7	+15.9	+61.6	+18.1	+49.7	+5.6	+67.6	+ 3.7
300	+62.6	+3.4	+68.1	+7.7	+68.4	+16.3	+46.8	+11.9	+40.2	+18.2	+45.7	+14.0	+ 5.8	+3.2	+19.1	+ 4.3	+40.0	+11.8	+46.1	+11.9	+38.7	+6.5	+58.7	+ 4.0
500	+33.0	+1.8	+34.5	+6.1	+33.8	+ 8.7	+26.7	+ 7.2	+23.5	+10.0	+22.8	+ 3.2	+ 5.8	+5.2	+10.3	+ 1.8	+18.7	+ 4.2	+23.9	+ 5.7	+17.7	+4.1	+31.1	+ 3.1
700	+14.0	+1.3	+16.9	+3.3	+17.3	+ 5.3	+13.5	+ 3.2	+16.0	+ 3.7	+14.7	- 3.5	+ 7.4	-0.6	+ 9.2	- 2.4	+15.0	- 1.6	+17.7	+ 2.5	+ 9.5	+3.1	+13.8	+ 2.5
800	+ 9.0	0	+11.0	+2.3	+10.6	+ 2.7	+ 7.6	+ 1.6	+ 8.2	+ 0.2	+11.3	- 8.2	- 9.2	-7.1	+ 8.6	- 6.0	+ 9.4	- 5.8	+10.9	+ 0.7	+ 5.6	+1.8	+ 7.3	+ 0.4
850	+ 6.8	-0.5	+ 7.8	+1.8	+ 6.8	+ 1.4	+ 4.1	+ 1.3	+ 5.7	- 2.1	+10.3	- 9.4	+10.9	-9.5	+ 9.3	- 7.4	+ 6.8	- 7.1	+ 6.8	- 0.9	+ 3.0	+1.3	+ 4.3	+ 0.9
900	+ 5.0	-1.2	+ 5.5	+0.9	+ 4.3	+ 0.8	+ 2.4	+ 0.9	+ 4.0	- 3.5	+ 9.2	-11.4	+11.6	-11.7	+ 8.3	- 9.3	+ 6.2	- 8.3	+ 3.4	- 2.3	+ 1.3	+0.7	+ 2.3	+ 0.3
Surface	+ 0.5	-0.3	+ 1.1	+0.4	+ 1.5	+ 0.5	+ 0.1	- 0.2	+ 1.5	- 1.8	+ 3.7	- 2.9	+ 4.0	-2.2	+ 2.3	- 1.4	- 0.6	- 1.0	+ 0.4	0.0	- 0.5	+0.3	0.0	0.0

Temperatures for Nairobi, Aden, Bahrein and Habbani

...at Standard Pressure Levels.

...ah 1948/1950. Temperatures for Rostov 1937/1941.

mb.	JANUARY Nairobi	Aden	Bahrein	Habbaniyah	Rostov	APRIL Nairobi	Aden	Bahrein	Habbaniyah	Rostov	JULY Nairobi	Aden	Bahrein	Habbaniyah	Rostov	OCTOBER Nairobi	Aden	Bahrein	Habbaniyah	Rostov
40	208.4		(202.3)	210.1		207.3		207.2	214.6		211.0		(210.2)	209.7		213.6	216.1	219.1	215.4	
60	197.5	195.8	203.9	210.2		198.1	206.1	204.6	212.1		206.3	204.9	202.0	204.2		206.4	205.7	204.7	209.7	
100	196.1	194.8	203.7	210.4	(219.0)	196.3	194.8	204.2	212.5	(218.0)	199.7	195.7	199.7	202.3	(222.0)	196.2	194.6	199.2	204.7	(214.0)
150	208.6	206.6	211.7	215.9	(218.0)	209.4	208.2	213.2	217.2	(218.0)	208.2	207.3	213.6	215.1	(224.0)	207.2	206.0	209.1	210.8	(215.0)
200	220.9	219.8	219.7	218.6	(218.0)	222.4	221.3	220.6	217.4	(218.0)	219.3	221.1	226.6	228.1	(226.0)	220.6	219.6	220.7	218.7	(218.0)
300	241.9	241.5	233.2	227.1	222.0	243.7	242.4	236.5	229.8	227.0	241.1	243.1	248.6	248.3	239.0	242.4	241.1	240.1	235.3	235.0
400	257.7	256.7	247.4	240.1		258.2	257.2	251.1	243.7		256.9	258.1	262.5	262.5		257.5	256.9	254.6	249.6	
500	267.1	267.8	258.6	251.2	245.0	267.6	267.8	262.2	255.6	254.0	266.6	266.7	270.2	271.6	263.0	266.8	267.8	265.9	261.2	258.0
600	274.7	276.3	267.7	260.5		275.7	276.2	271.1	265.1		274.2	277.4	279.8	278.7		274.4	275.0	274.7	270.5	
700	283.0	283.4	274.6	267.7	261.0	282.3	284.1	280.2	273.5	269.0	280.8	288.2	290.2	287.8	278.0	281.5	281.8	282.2	279.9	275.0
800	286.0	288.1	280.5	273.8		288.3	291.6	288.7	281.5		284.6	296.5	298.8	296.3		286.3	289.9	290.8	288.0	
850		288.8	282.9	276.3	267.0		294.2	292.0	285.3	276.0		300.1	303.1	300.5	290.0		293.4	294.6	291.7	278.0
900		290.8	284.8	278.6			297.1	295.0	289.3			303.2	306.8	304.8			296.4	298.2	295.2	
Surface		298.3	289.8	284.2			301.1	297.3	294.3			305.9	306.9	306.9			301.3	301.1	296.4	

Table V. Mean differences of temperature at standard pressure levels constructed from Table IV.

mb.	JANUARY N-A	A-B	B-H	H-R	APRIL N-A	A-B	B-H	H-R	JULY N-A	A-B	B-H	H-R	OCTOBER N-A	A-B	B-H	H-R
40	-	-	-	-	-	-	-7.4	-	-	-	+0.5	-	-2.5	-3.0	+3.7	-
60	1.7	-8.1	-6.3	-	-3.5	-3.0	-7.5	-	+1.4	+2.9	-2.2	-	+0.7	+1.0	-5.0	-
100	1.3	-8.9	-6.7	-(8.6)	+1.5	-5.4	-8.3	-(5.5)	+4.0	-4.0	-2.6	-(19.7)	+1.6	-4.6	-5.5	-(9.3)
150	2.0	-5.1	-4.2	-(2.1)	+1.2	-5.0	-4.0	-(0.8)	+0.9	-6.3	-1.5	-(8.9)	+1.2	-3.1	-1.7	-(4.2)
200	1.1	+0.1	+1.1	+(0.4)	+1.1	+0.7	+3.2	-(0.6)	-1.8	-5.5	-1.5	+(2.1)	+1.0	-1.1	+2.0	+(0.7)
300	0.4	+8.3	+6.1	+5.1	+1.3	+5.9	+6.7	+ 2.8	-2.0	-5.5	+0.3	+ 9.3	+1.3	+1.0	+4.8	+ 0.3
400	1.0	+9.3	+7.3	-	+1.0	+6.1	+7.4	-	-1.2	-4.4	+0.0	-	+0.6	+2.3	+5.0	-
500	-0.7	+9.2	+7.4	+6.2	-0.2	+5.6	+6.6	+ 1.6	-0.1	-3.5	-1.4	+ 8.6	-1.0	+1.9	+4.7	+ 3.2
600	-1.6	+8.6	+7.2	-	-0.5	+5.1	+6.0	-	-3.2	-2.4	+1.1	-	-0.6	+0.3	+4.2	-
700	-0.4	+8.8	+6.9	+6.7	-1.8	+3.9	+6.7	+ 4.5	-7.4	-2.0	+2.4	+ 9.8	-0.3	-0.4	+2.3	+ 4.9
800	-2.1	+7.6	+6.7	-	-3.3	+2.9	+7.2	-	-11.9	-2.3	+2.5	-	-3.6	-0.9	+2.8	-
850	-	+5.9	+6.6	+9.3	-	+2.2	+6.7	+9.3	-	-3.0	+2.6	+10.5	-	-1.2	+2.9	+13.7
900	-	+6.0	+6.2	-	-	+2.1	+5.7	-	-	-3.6	+2.0	-	-	-1.8	+3.0	-
Surface	-	+8.5	+5.6	-	-	+3.8	+5.0	-	-	-1.0	+0.0	-	-	+0.2	+4.7	-

Table VI. Mean Monthly Tropopause Pressures and Temperatures (° A.)

	January Pressure	Temp.	February Pressure	Temp.	March Pressure	Temp.	April Pressure	Temp.	May Pressure	Temp.	June Pressure	Temp.	July Pressure	Temp.	August Pressure	Temp.	September Pressure	Temp.	October Pressure	Temp.	November Pressure	Temp.	December Pressure	Temp.
Nairobi	83.5	191°	84.8	192°	84.4	191°	82.5	191°	81.5	191°	85.3	194°	95.0	195°	94.0	197°	93.8	196°	91.7	193°	83.8	191°	78.0	191°
Aden	90.3	191°	90.9	192°	85.1	192°	85.9	191°	87.2	191°	92.6	193°	96.3	194°	96.2	195°	96.5	194°	97.8	193°	92.9	192°	92.5	192°
Bahrein	89.7	200°	92.0	201°	87.9	201°	81.7	200°	80.6	197°	76.7	194°	78.2	194°	85.5	194°	84.6	194°	90.0	196°	91.0	200°	94.0	201°
Habbaniyah	224	216°	231	216°	235	218°	212	216°	200	215°	88	201°	83	198°	90	199°	97	199°	150	205°	184	210°	202	211°
Benina	217	218°	238	218°	243	219°	213	216°	197	216°	162	213°	93	201°	97	202°	107	206°	151	210°	163	210°	206	214°
Nicosia	234	218°	253	219°	260	219°	240	218°	218	218°	160	215°	97	203°	98	203°	99	204°	125	209°	164	211°	221	215°
Malta	235	217°	237	216°	251	217°	224	216°	196	214°	190	216°	123	210°	119	210°	142	211°	177	212°	200	212°	228	216°
Gibraltar	221	215°	217	215°	221	214°	210	217°	209	219°	184	215°	137	211°	137	211°	169	212°	189	213°	174	209°	224	217°

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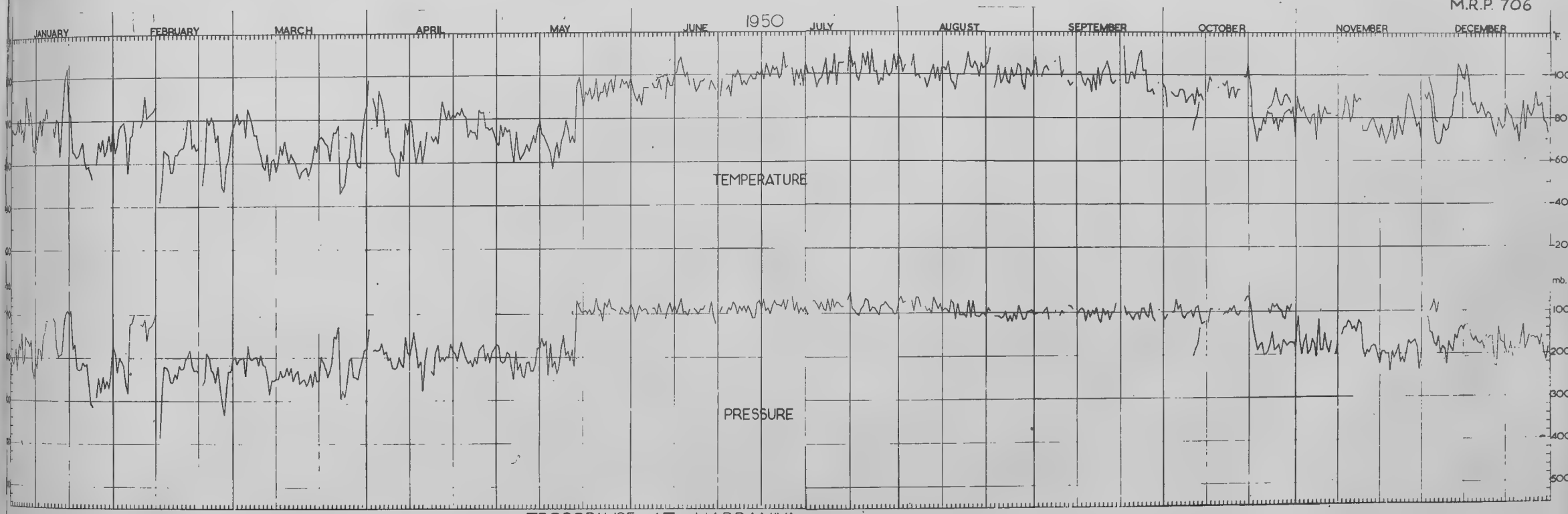
M.R.P. 706



TROPOPAUSE AT HABBANIYA 33°22'N 43°34'E 144ft.

Fig. 1. Hours of observation 0200 and 1400 G.M.T. (0500 and 1700 Zone Time)

Standard Pressure Levels.

M.R.P. 706.

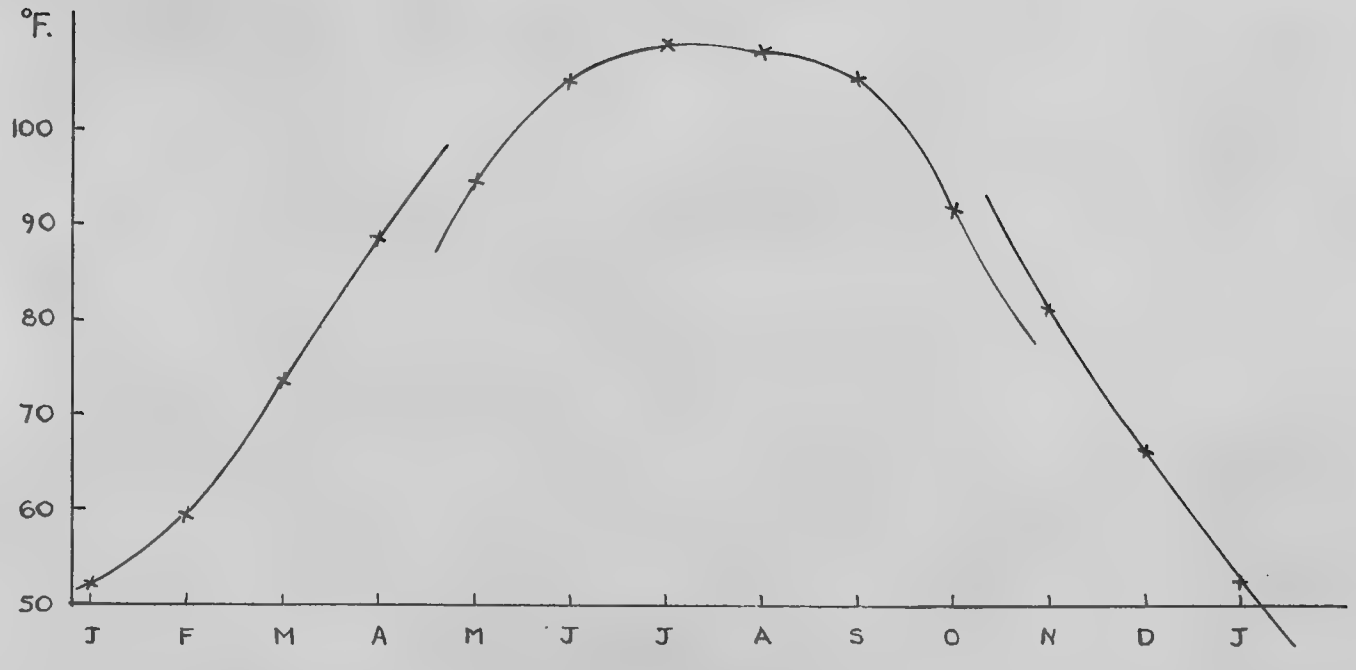
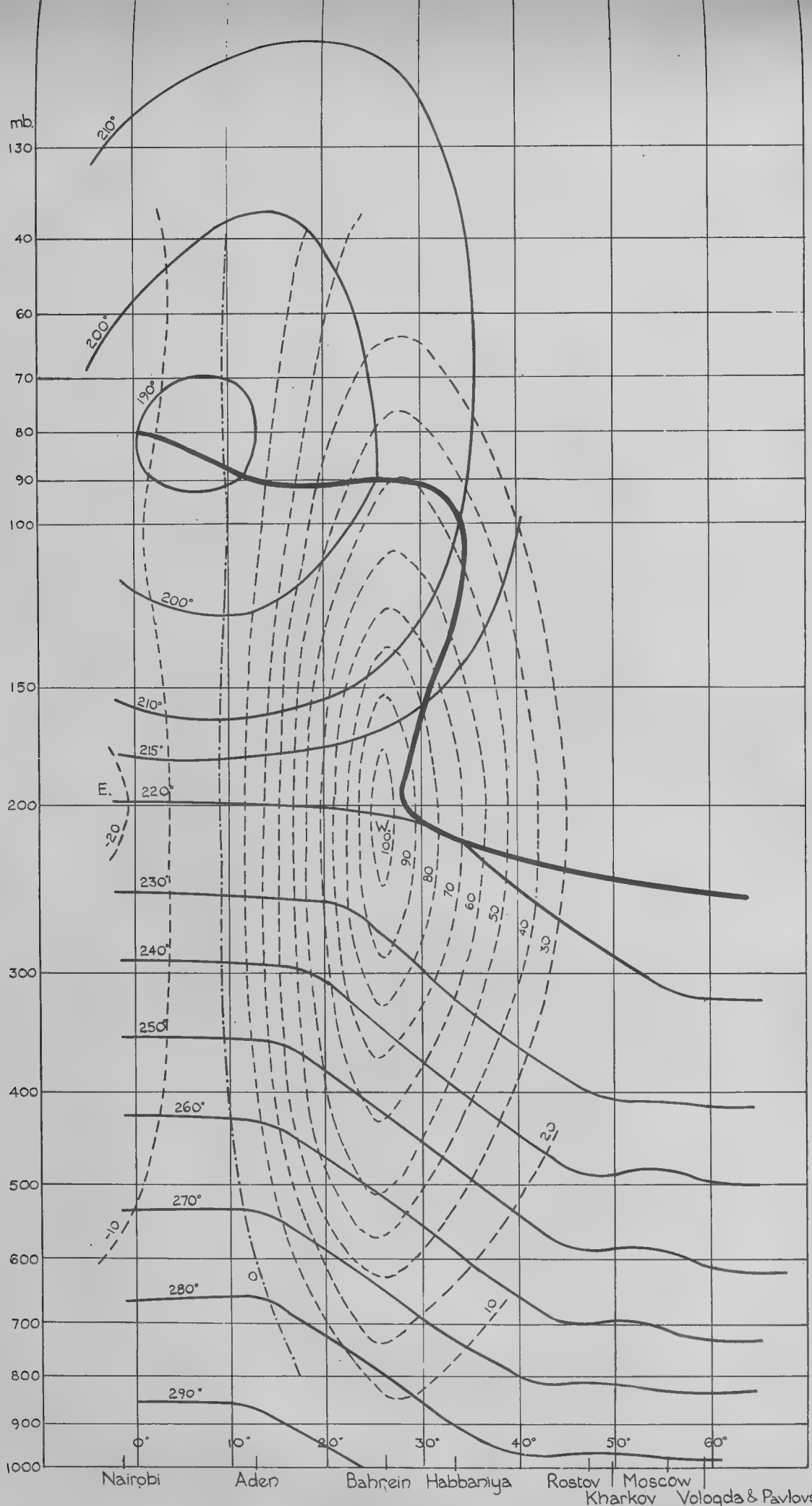


Fig. 2 Monthly mean maximum surface temperatures at Habbaniya in 1950.



Atmospheric Cross section at 40° E. Longitude showing mean temperature & mean East-West wind component. (Isotherms are shown by continuous lines, isokinetics by broken lines, the zero isokinetic by a chain-dotted line & the tropopause is shown by a heavy continuous line.)

JANUARY

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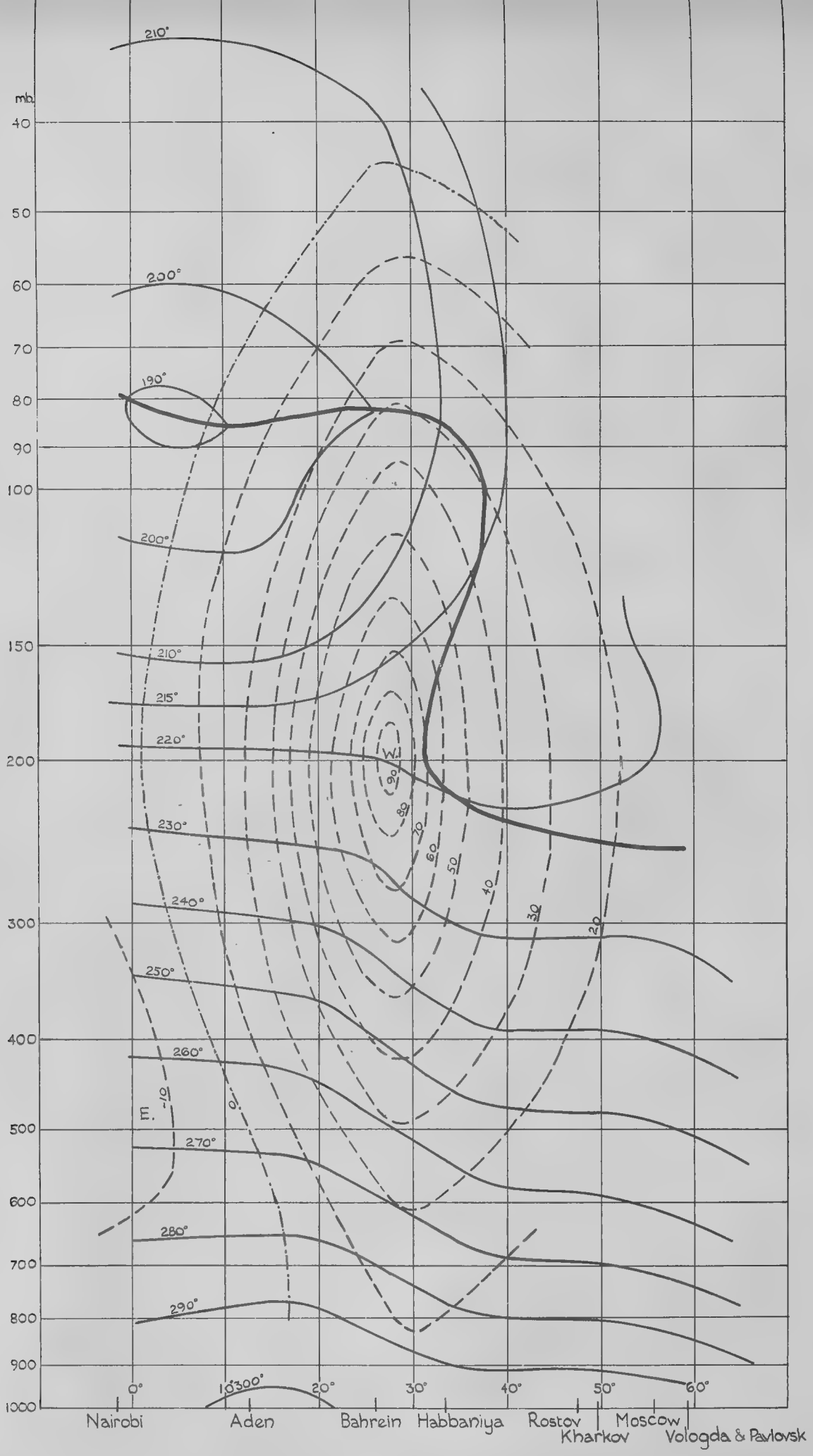
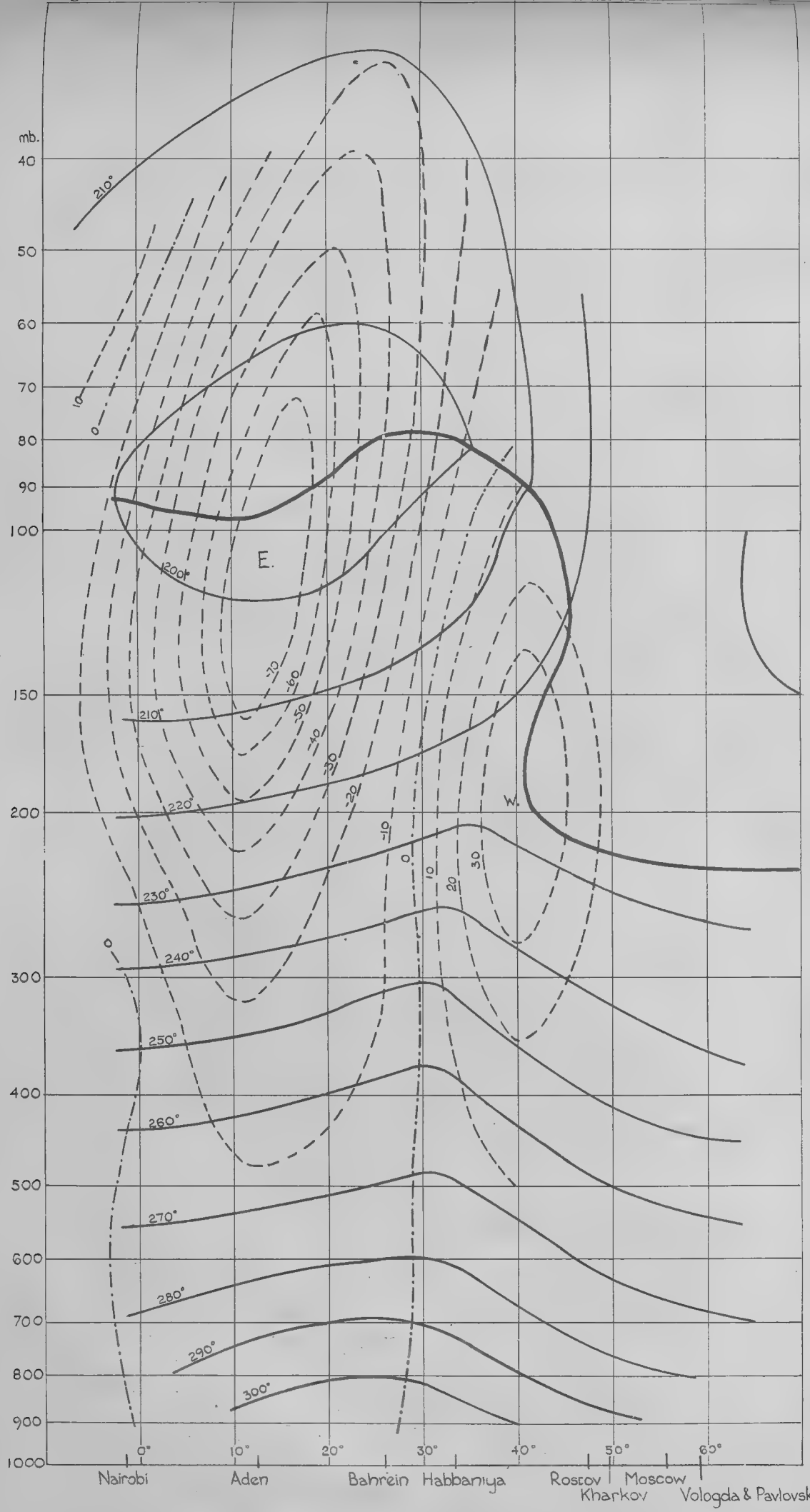


Atmospheric Cross section at 40°E. Longitude showing mean temperature & mean East-West wind component. (Isotherms are shown by continuous lines, isokinetics by broken lines, the zero isokinetic by a chain-dotted line & the tropopause is shown by a heavy continuous line.)

APRIL



Atmospheric Cross section at 40° E. Longitude showing mean temperature & mean East-West wind component. (Isotherms are shown by continuous lines, isokinetics by broken lines, the zero isokinetic by a chain-dotted line & the tropopause is shown by a heavy continuous line.)

JULY

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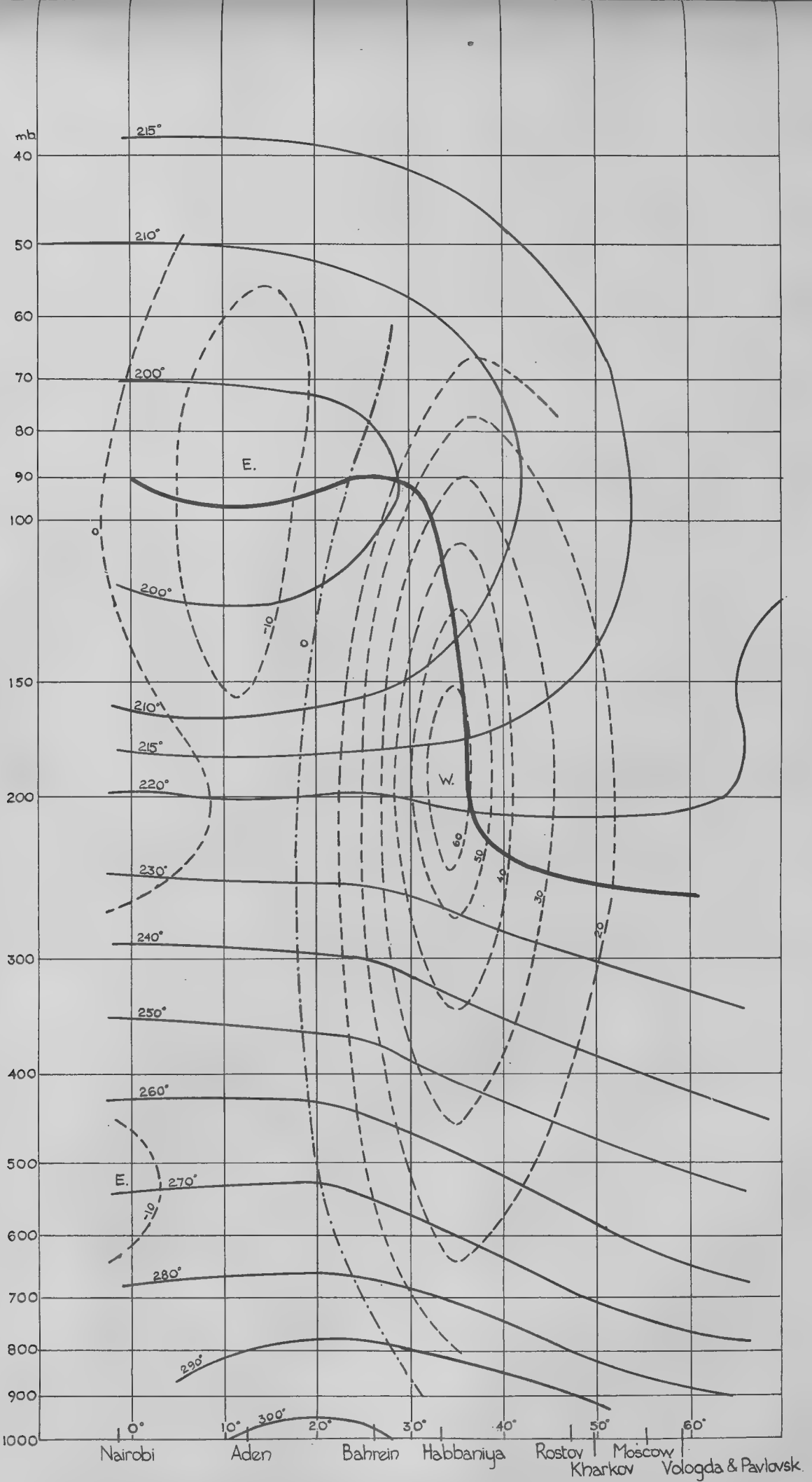

Atmospheric Cross section at 40° E. Longitude showing mean temperature & mean East-West wind component. (Isotherms are shown by continuous lines, isokinetics by broken lines, the zero isokinetic by a chain-dotted line & the tropopause is shown by a heavy continuous line.)

OCTOBER

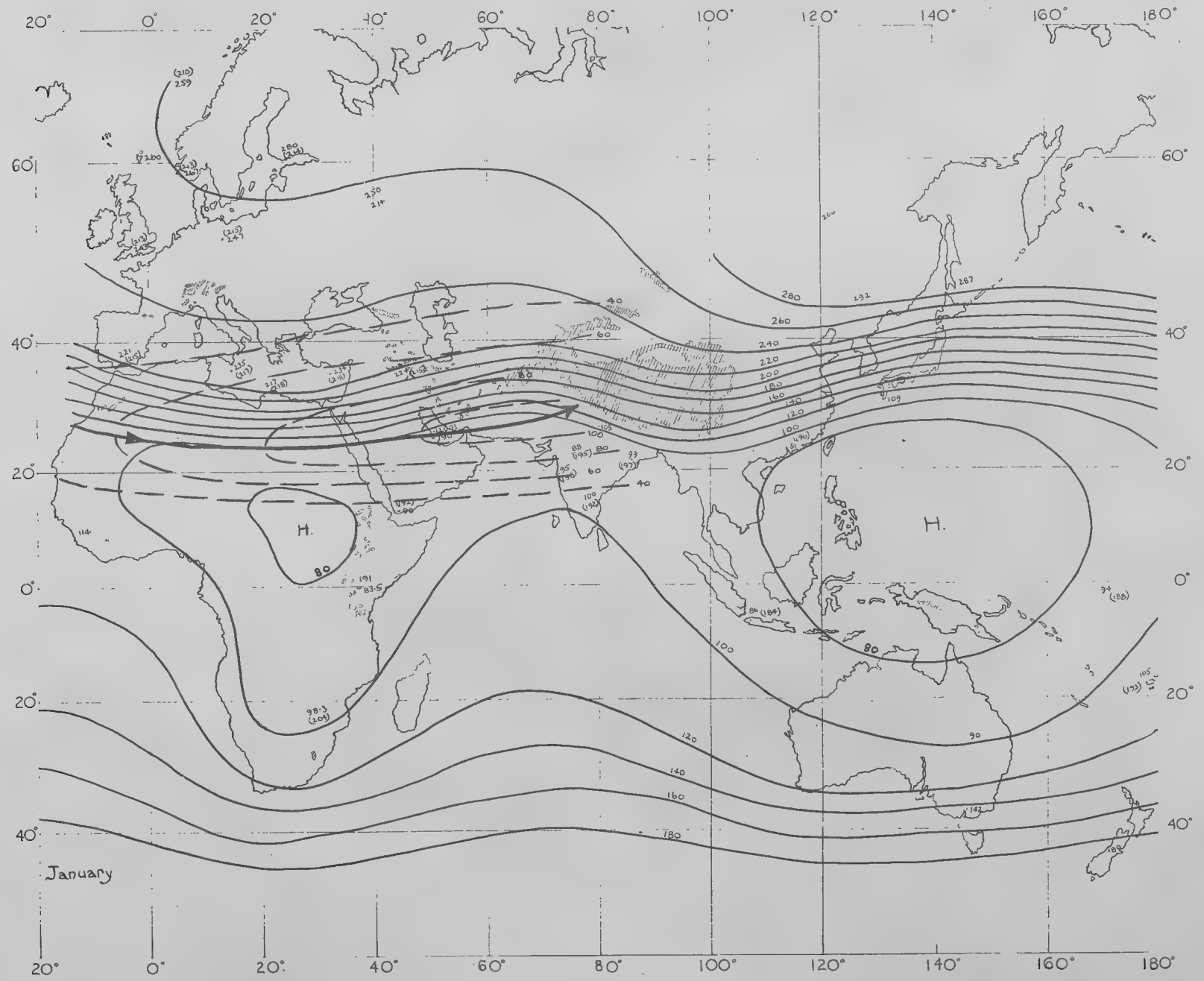

Fig. 7. Mean pressure chart of the tropopause with isopleths of vector mean wind at 200 mb. (Isobars are indicated by continuous lines, isopleths of vector mean wind by broken lines and the axis of the jet over India and Africa by a heavy line with arrows). Temperatures at the tropopause are in brackets.

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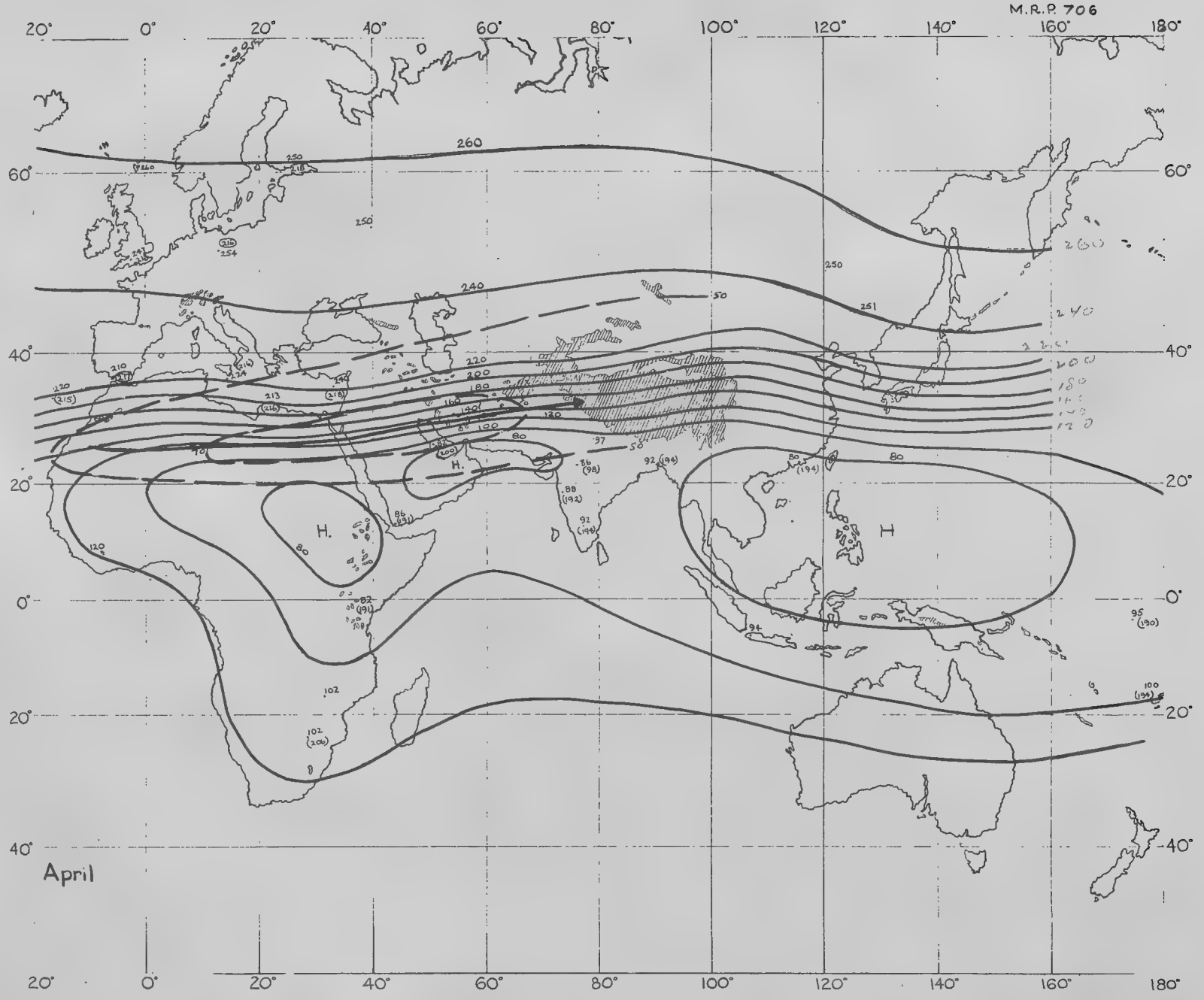

Fig.8 Mean pressure chart of the tropopause with isopleths of mean vector wind at 200 mb. (Isobars are indicated by continuous heavy lines, isopleths of vector mean wind by broken lines and the axis of the jet over India and Africa by a heavy line with arrows). Temperatures at the tropopause are in brackets.

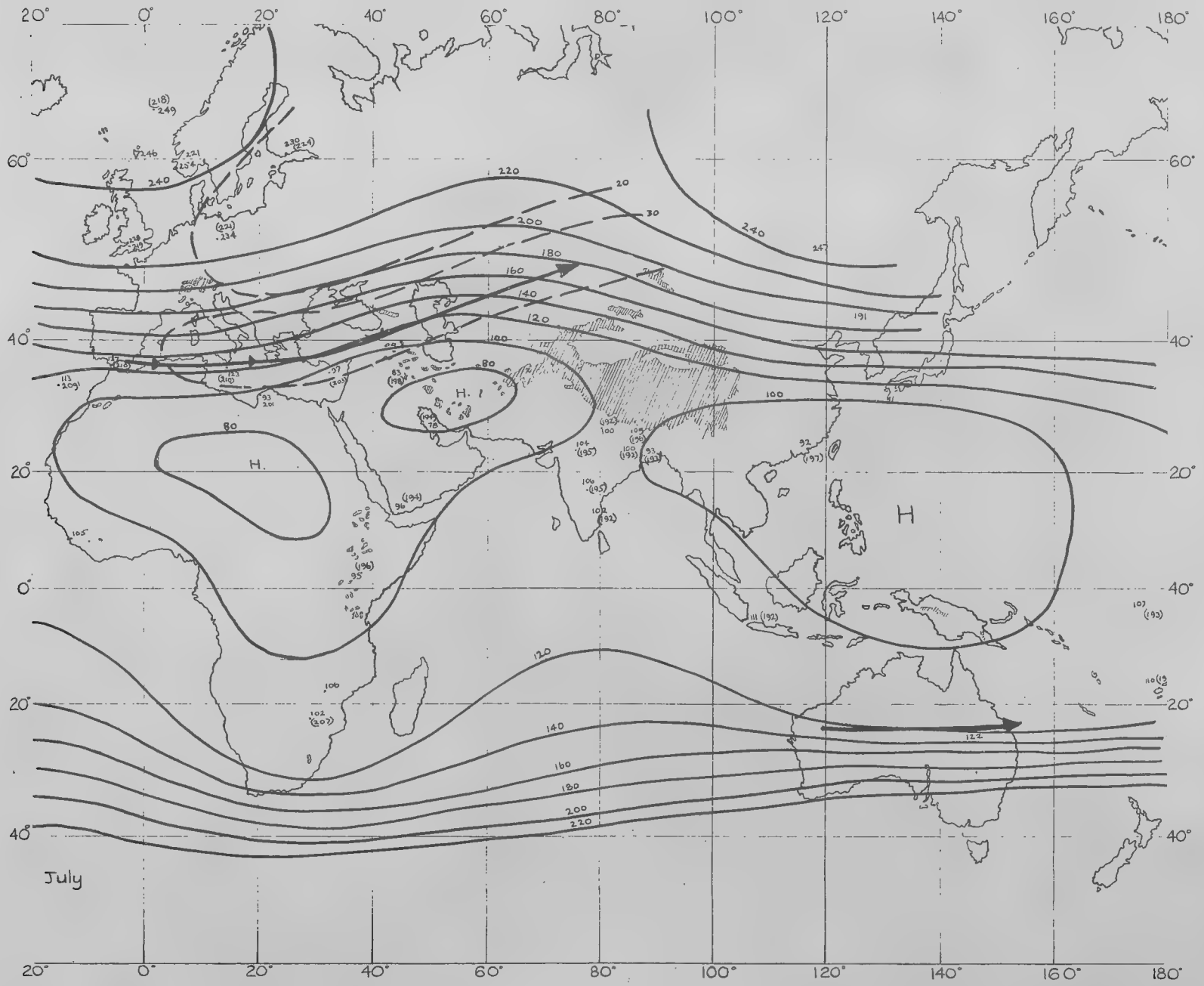

Fig.9. Mean pressure chart of the tropopause with isopleths of vector mean wind at 200mb. (Isobars are indicated by continuous lines, isopleths of vector mean wind by broken lines and the axis of the jet over India and Africa by a heavy line with arrows). Temperatures at the tropopause are in brackets.

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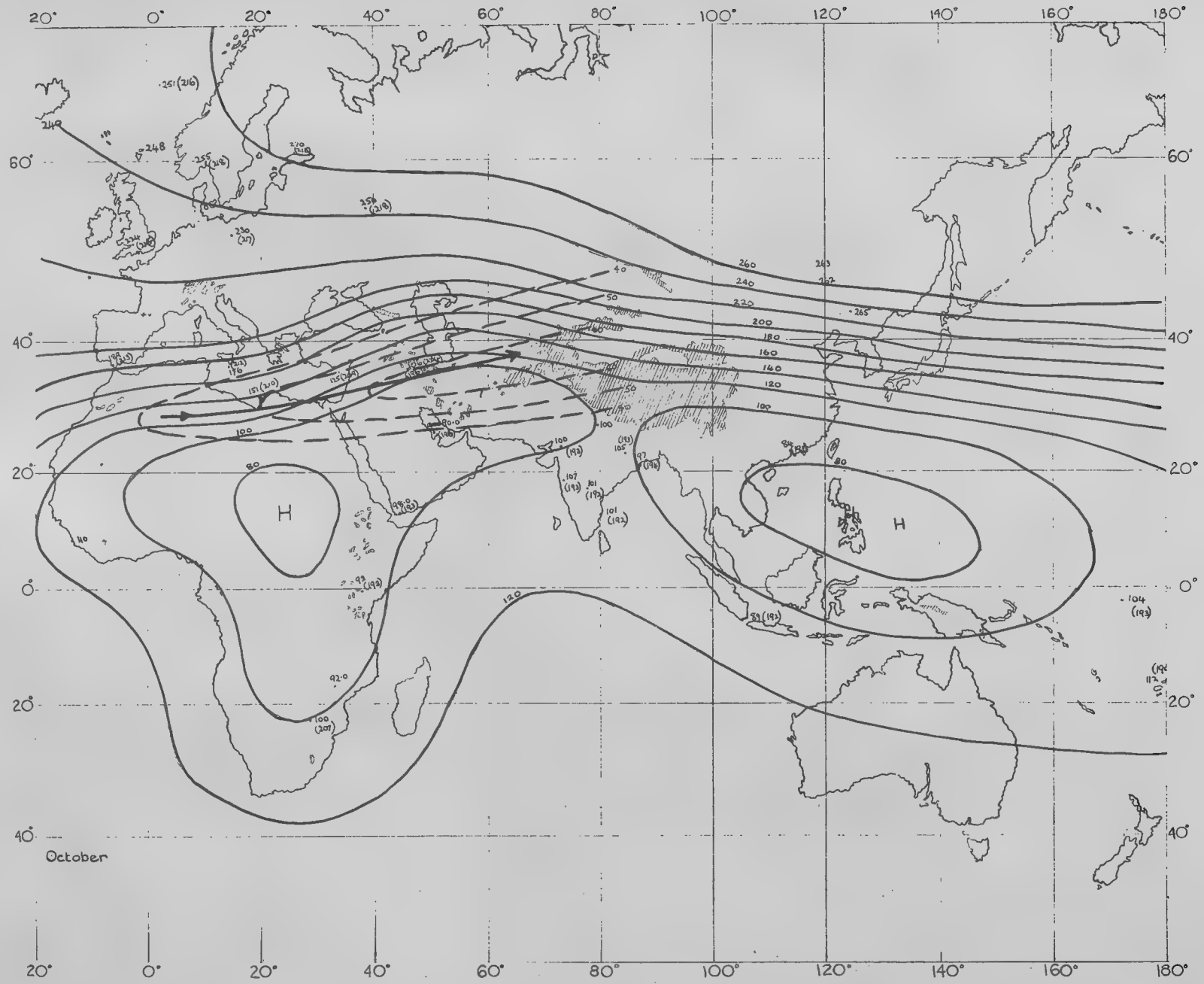


Fig-10 Mean pressure chart of the tropopause with isopleths of vector mean wind at 200mb. (Isobars are indicated by continuous lines, isopleths of vector mean wind by broken lines and the axis of the jet over India and Africa by a heavy line with arrows). Temperature at the tropopause are in brackets.

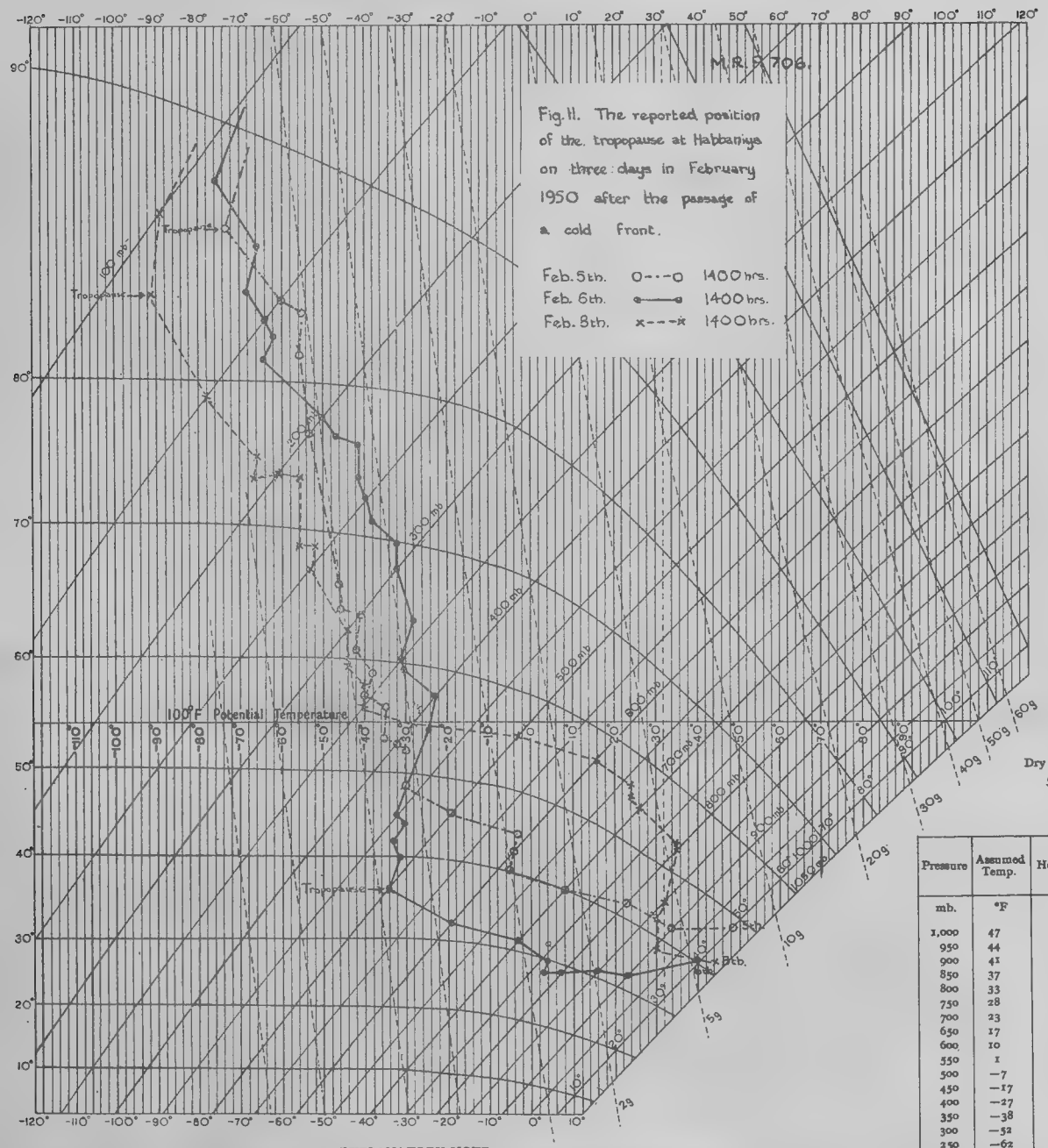
Form 2811 AIR MINISTRY, METEOROLOGICAL OFFICE

DATE.. 19

TEPHIGRAM OF RADIO-SONDE RECORDS

STATIONS	TIMES

REMARKS



Dry Adiabatic Lapse Rate is 5·3°F. per 1,000 feet.

Pressure	Assumed Temp.	H. Height above 1,000 mb.	Increase of height due to 10°F increase in mean temperature.
mb.	°F	Feet.	Feet.
1,000	47	0	—
950	44	1,380	30
900	41	2,830	60
850	37	4,350	90
800	33	5,950	120
750	28	7,640	150
700	23	9,420	190
650	17	11,320	230
600	10	13,340	270
550	1	15,490	320
500	−7	17,810	370
450	−17	20,330	430
400	−27	23,080	490
350	−38	26,120	560
300	−52	29,530	640
250	−62	33,440	740
200	−66	38,150	860
150	−66	44,200	1,000
100	−66	52,700	1,200

Adjust heights to other surface pressures by adding (subtracting) 28 feet for every millibar of pressure above (below) 1,000 mb.

EXPLANATORY NOTE

This chart is intended for use with radio-sonde ascents to show the general features of the complete results at a glance. It is not intended for use where the main purpose of plotting the result is to examine in details in the lower part of the troposphere. The scale of the form is approximately half that of the Tephigram Form 2810.

The temperature figures printed along the 1,000 mb. line apply both to the saturated adiabatics (the curved lines) and to the temperature scales (the vertical lines). The values printed at the left hand side of the diagram apply only to the saturated adiabatics.

A straight line on the diagram gives constant lapse rate. For an inversion the curve must cross the temperature scale from left to right. If the result of a radio sonde ascent is plotted it is usually easy to see where the stratosphere begins. It may begin as low as 400 mb.

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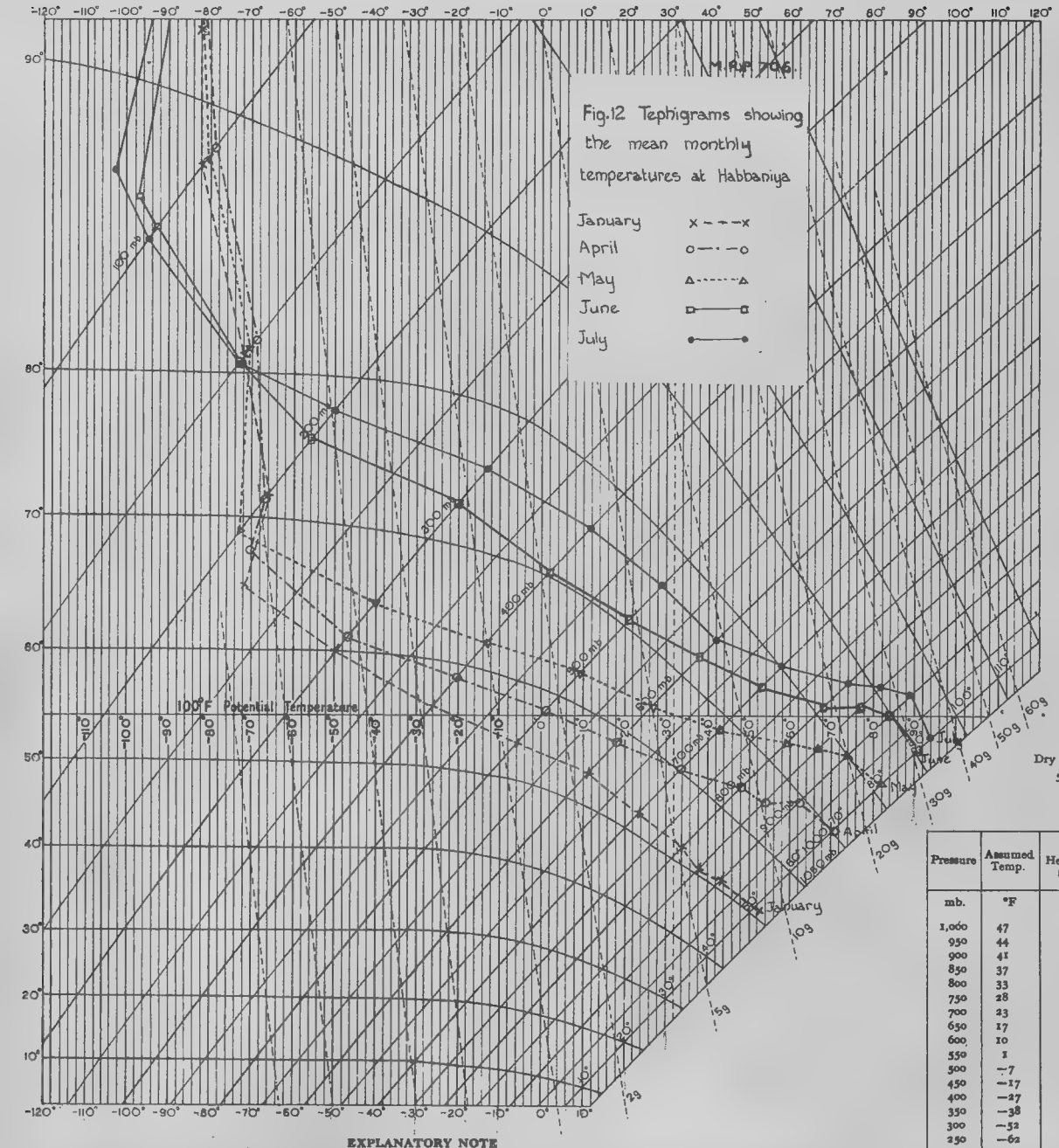
Form 2811 AIR MINISTRY, METEOROLOGICAL OFFICE DATE..... 19

TEPHIGRAM OF RADIO-SONDE RECORDS

STATIONS	TIMES

REMARKS



Dry Adiabatic Lapse Rate is 5·3°F. per 1,000 feet.

Pressure	Assumed Temp.	H. Height above 1,000 mb.	Increase of height due to 10°F increase in mean temperature.
mb.	°F	Feet.	Feet.
1,000	47	0	—
950	44	1,380	30
900	41	2,830	60
850	37	4,350	90
800	33	5,950	120
750	28	7,640	150
700	23	9,420	190
650	17	11,320	230
600	10	13,340	270
550	1	15,490	320
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Form 2811

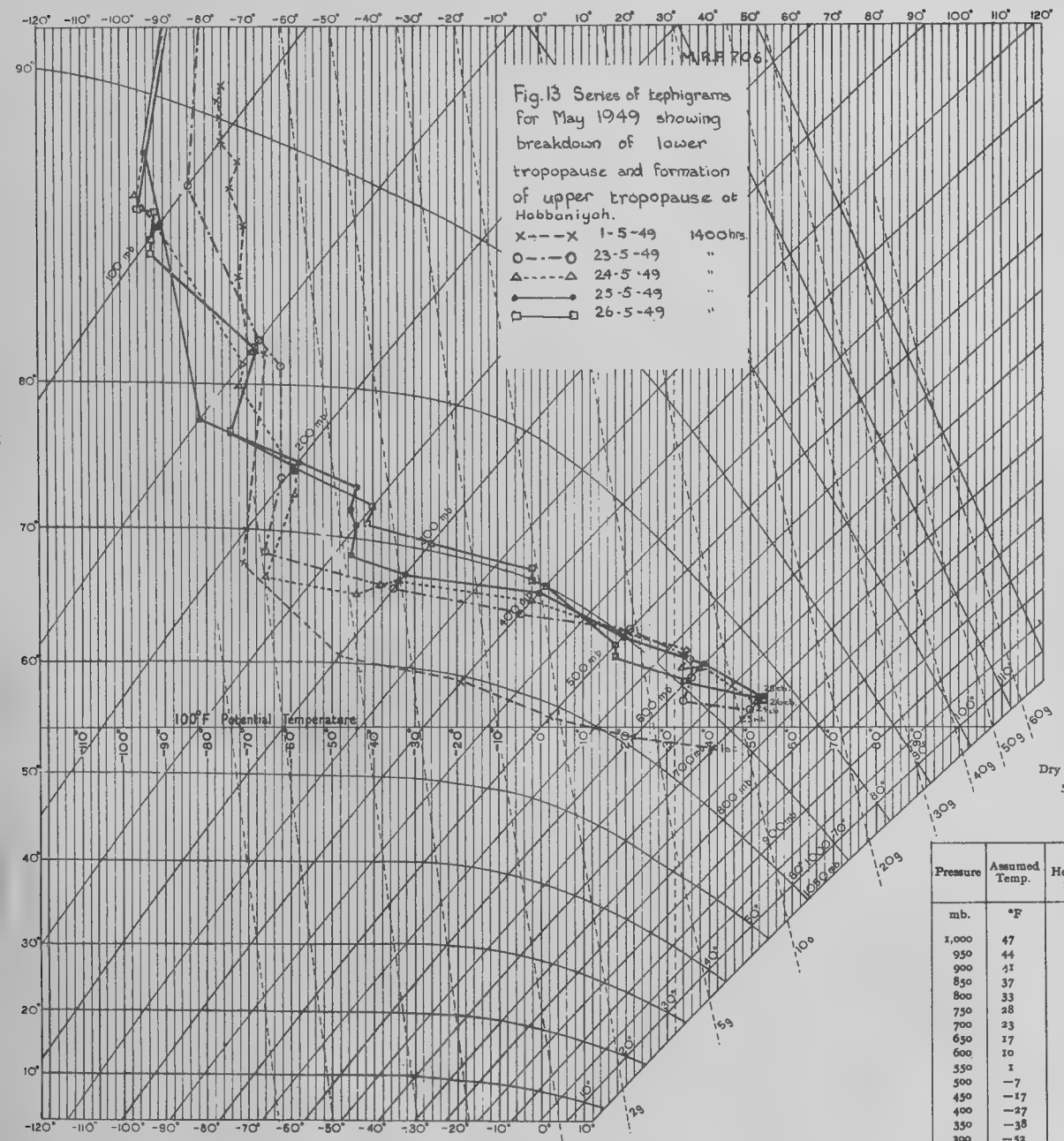
AIR MINISTRY, METEOROLOGICAL OFFICE

DATE..... 19

TEPHIGRAM OF RADIO-SONDE RECORDS

STATIONS	TIMES

REMARKS



Dry Adiabatic Lapse Rate is 5·3°F. per 1,000 feet.

Pressure	Assumed Temp.	H. Height above 1,000 mb.	Increase of height due to 10°F increase in mean temperature.
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600	10	13,340	270
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Probability Paper - an Application in Upper-air Climatology

by

N. Carruthers (Meteorological Office)

1. Introduction

The traditional picture of a tropopause surface sloping steeply in middle latitudes but extending continuously from pole to equator has tended in late years to be replaced, for each hemisphere, by that of two tropopause surfaces of which both the upper, characteristic of the tropics, and the lower, characteristic of poleward regions, extend into middle latitudes so that there is a zone of overlap between 30 and 40° north or south. Loewe and Radok1 show that this is consistent with observations of tropopause over Australia. Between 38°S and 32°S in winter, they find the frequency distributions of height of tropopause to be compounded each of two normal distributions with means in the neighbourhood of 10 and of 15 km, while to the south (at Macquarie, 54°S) the 10-km distribution prevails and to the north (at stations north of 27°S) the 15-km. Their analyses were performed by means of probability paper. This type of graph paper is being used now in the Upper-air Climatology Branch of the Meteorological Office in an attempt to sort out some of the irregularities found in the tropopause pressures reported at British stations overseas. The present note outlines the principles underlying the construction and use of probability paper (Section I) and shows its application in a discussion of the tropopause over Habbaniya (Section II).

Section I Probability Paper

2. Symbols used

The symbols used in this description of probability paper include the following:-

X - a variate, plotted as abscissa.

$\underline{x}$ - the departure of any specified value of X from the mean of the frequency distribution of X

σ - the standard deviation of X

N - the number of observations in the frequency distribution.

$\underline{t}$ - a standardised variate, i.e. $\underline{x}/\sigma$

P - the probability integral.

3. The probability scale

The use of probability paper for the examination of frequency distributions is analogous to the use of semi-logarithmic paper in examining a relationship of the form $Y = ab^X$ between the two variates X and Y. This relationship shows up as a straight line when Y on the logarithmic scale is plotted against X on the linear scale.

A normal frequency distribution shows up as a straight line on probability paper. The logarithmic scale of semi-logarithmic paper is replaced by a "probability" scale, the ordinate Y being the cumulative frequency of X.

/On

On semi-logarithmic paper, the scale of Y is linear with respect to log Y; on probability paper, the probability scale is linear with respect to the standardised normal deviate, i.e. with respect to $\underline{t}$ when $\underline{t}$ is distributed normally. It is easily seen that this arrangement of scale results in a straight line when a normal distribution is plotted; for,

$$\underline{t} = \frac{1}{\sigma} X + (\text{a constant}) \qquad (1)$$

The probability scale shows values of 100P where P is the "probability integral" defined by the expression

$$P = \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{\underline{t}} e^{-\underline{t}^2/2} \, d\underline{t} \qquad (2)$$

Corresponding values of 100P and $\underline{t}$ are given in Table I.

Table I. Corresponding Values of $\underline{x}/\sigma$ and 100P for a Normal distribution.

(a) For evenly-spaced values of $\underline{x}/\sigma$

$\underline{x}/\sigma$	-3.0	-2.5	-2.0	-1.5	-1.0	-0.5	0.0	+0.5	1.0	1.5	2.0	2.5	3.0
100P:	0.13	0.62	2.3	6.7	15.9	30.9	50.0	69.1	84.1	93.3	97.7	99.38	99.87

(b) For standard values of 100P.

100P:	0.1	0.2	1.0	2.0	5.0	10.0	20.0	30.0	40.0	50.0
$\underline{x}/\sigma$	-3.09	-2.88	-2.33	-2.05	-1.64	-1.28	-0.84	-0.52	-0.25	0.00

100P:	50	60	70	80	90	95	98	99	99.8	99.9
$\underline{x}/\sigma$	0.00	0.25	0.52	0.84	1.28	1.64	2.05	2.33	2.88	3.09

A more detailed table of 100P in terms of $\underline{x}/\sigma$ is given in Yule and Kendall[2]; and $\underline{x}/\sigma$ in terms of 100P in Table X of Fisher and Yates[3], under the heading "Probits".

4. Plotting a frequency distribution

In plotting a frequency distribution on probability paper, the ordinates are determined by numbering the observations in order of magnitude and multiplying the numbers so obtained by 100/N to convert them to percentages. In place of the usual ordinal numbers, 1, 2, 3, ... N, it is preferable to a adopt the series $\frac{1}{2}$, $1\frac{1}{2}$, $2\frac{1}{2}$ ....., $(N-\frac{1}{2})$. The ordinates are then symmetrical about the 50 per cent. line and the mean as well as the standard deviation can be estimated directly from the graph; also, this avoids the necessity of plotting a probability number of 100 per cent. which lies at an infinite distance from the 50 per cent. line.

The plotting of grouped observations is described in the Appendix.

5. Estimating the mean and standard deviation

In order to estimate graphically the mean and standard deviation of a normal, or almost normal, frequency distribution, a straight line is fitted by eye to the plotted points. The mean is then read off, being the abscissa of the intersection of this line with the 50 per cent. line. For example, for curve (b) of Fig. 5, the mean is 100 mb. and for curve (c) it is 224 mb.

The standard deviation is given by the slope of the fitted line. With the aid of Table I, σ can be determined from the difference between almost any two values of X; but the most accurate method is to read off values corresponding to the probability numbers, 100P = 6.7 and 100P = 93.3 and to divide the difference between them by 3. As a check, it may be verified that the result is the difference between the mean and the value of X corresponding to 100P = 84.

/6.

6. Effect of combining two normal frequency distributions

The straight lines (a) and (c) of Fig. 1 show on probability paper the plot of two normal frequency distributions with the same standard deviation (10 units) but different means (25 and 55 units). If in each there are 25 observations which fit exactly the straight line, they will have the following values:

Probability number:	2	6	10	14	..50..	..86	90	94	98
Curve (a)	4.5	9.5	12.2	14.2	..25..	..35.8	37.8	40.5	45.5
Curve (c)	34.5	39.5	42.2	44.2	..55..	..65.8	67.8	70.5	75.5

On plotting the whole fifty observations as one distribution the curve ABCD shown at (b) is obtained. The circles are observations comprising distribution (a) and the crosses those comprising distribution (c). The normal distribution with the same mean and standard deviation as the composite distribution is shown by a broken line; mean = 40; σ = 18. It will be seen that between A and B and between C and D the slope of the curve is steeper than that of the broken line whereas between B and C it is less steep. In other words, there are more observations between 0.8σ and 2.0σ on either side of the mean than would be expected in a normal distribution and fewer within 0.8σ of the mean. This type of distribution which has an excess of medium-sized deviations from the mean and a deficit of small deviations is termed platykurtic (flat-topped).

Fig. 2 shows, in similar fashion, the effect of combining two normal distributions with the same mean (40 units) and different standard deviations (15 and 2.5 units). In this, the composite curve shows an excess of small deviations (within 0.4σ of the mean) and a deficit of medium-sized deviations (between 0.4σ and about 2.0σ from the mean). This type is leptokurtic (humped).

7. Meteorological examples

Meteorological examples of the three types of distribution: nearly normal, platykurtic and leptokurtic, are shown in Fig. 3(a) and (b) and Fig. 4. In each, the equivalent normal distribution is shown by a broken line.

The nearly-normal distribution of upper-air temperatures at 300 mb. over Downham Market, shown in Fig. 3(a), has a slight tendency to positive skewness; for, the curve indicated by the crosses is slightly steeper than the broken line at the top of the diagram, showing that more observations fall within the same temperature range, and less steep at the foot, showing that the last few observations are more extreme, than would be expected in a normal distribution. Curve (b) for 200 mb. also shows lack of symmetry.

The curve in Fig. 4 is based on data given by Hesselberg[4]. These data are used also in the Handbook of Statistical Methods in Meteorology; leptokurtic distributions of meteorological elements appear to be somewhat rare.

Section II. Analysis of tropopause pressures

8. Tropopause pressures, 1948-50

During the period, January, 1948 to November, 1950, radio-sonde staff at British stations were required to report for each ascent only one tropopause; towards the end of 1950, those at British stations overseas were instructed to note also on their returns any second tropopause observed and to punch, on the appropriate Hollerith card, the lower of the two tropopauses.

When the observations for 1948 to 1950 were summarised, remarkable inconsistencies appeared in the monthly means of the tropopause pressures. At Bahrein for instance in December, 1950, the tropopause pressure which for

35 months had been in the neighbourhood of 75 to 100 mb. rose suddenly to about 180 mb. It was evident that before December, 1950 the upper tropopause had been reported almost invariably; but, in accordance with the new instruction, the lower tropopause now appeared in the Hollerith tabulations. This was confirmed by plotting the December data, as summarised, on probability paper. The resulting curve was conclusively platykurtic and the aggregate distribution split readily into two normal distributions with means at 99 mb. and 187 mb., all but two observations of the 187-mb. distribution having been recorded in December, 1950 and only three observations from this month being included in the 99-mb. distribution.

At Habbaniya, 33°22'N 43°34'E, similar discrepancies were found. For example, in the first half of 1949, considerably higher tropopauses were recorded than in the same months of 1948 and 1950. From even a cursory examination of the discontinuities of temperature in 1949 it was evident that lower tropopauses also were present. This and other conclusions as to the incompleteness of the recorded observations do not, however, negate the abrupt changes of regime discussed by Dewar[5]. These are undoubtedly real and are confirmed by Frost[6]. A further examination of data for April and May, 1949 alone is described in § 10.

The present brief investigation of the tropopause over Habbaniya commenced with an analysis of the tropopause pressures recorded in 1948-50 for the mid-season months, January, April, July and October. The probability graphs are shown as curves (a) of Figs. 5 and 6 and curves (a) and (b) of Fig. 7. They are plotted from observations grouped into 10-mb. classes.

Three of the four curves are definitely platykurtic and each approximates to two straight lines with a short transition region between. The numbers of observations in the different frequency classes, shown by small figures against the plotted points, are seen to fall off in the transition region. A break was made where the observations were fewest, as indicated in the diagrams, and each of the two portions so obtained was replotted, individual observations being used where they numbered less than 30. Replotting resulted in curves (b) and (c) of Figs. 5 and 6 and (c) and (d) Fig. 7. From these it is seen that the component distributions are roughly normal, departures from the normal distribution (full line) being sufficiently small to be attributable to the scantiness of the data which, for most of the component distributions, are drawn from one or two years only. A summary of the analysis is given in Table II(a); apart from those for the April lower tropopause, the means and standard deviations were estimated from the graphs by the method described in § 5.

The fourth of the original curves, that for July, Fig. 7(a), is the most striking. There is no doubt whatsoever as to the normality of this distribution. The lower tropopause seems to be absent in July; a fact which is confirmed by data for July, 1951.

Table II. Tropopauses recorded at Habbaniya

(a) 1948-50. Only one tropopause recorded each day.

Month	Upper Tropopause				Lower Tropopause			
	Mean (mb)	Number of obs.	Years	S.D. (mb)	Mean (mb)	Number of obs.	Years	S.D. (mb)
January	100	23	(1949) (1950)	10	224	82	(1948) (1950)	45
April	83	27	1949	13	212	117	(1948) (1950)	33
July	85	118	3 years	15	No lower tropopause			
October	106	106	3 years	19	181	26	(mainly 1950)	11

/(b)

(b) 1951. Two tropopauses recorded when found

Month	Upper Tropopause				Lower Tropopause				Ratio* (1951)
	Mean (mb)	Number of obs.	Years	S.D. (mb)	Mean (mb)	Number of obs.	Years	S.D. (mb)	
January	90	39	1951	16	236	55	1951	49	0.71
February	90	15	"	14	221	39	"	23	0.29
					13 incorrectly defined				
March	(128)	1	"	-	223	58	"	28	(0.02)
					2 incorrectly defined				
April	90	34	"	17	230	58	1951	33	0.59
May	85	42	"	15	187	58	"	28	0.72
June	99	58	"	11	(235)	4	"	-	(14.5)

* Ratio of number of upper to number of lower tropopauses, including those believed to be incorrectly defined. For July, this ratio is infinity.

It may be concluded that, for the greater part of the year, Habbaniya is situated in the region of overlap of the tropical and polar tropopauses, the latter being the more frequent in January and April and the former in October - provided, of course, that this concept of a discontinuous tropopause surface is true. In July, the tropical regime prevails. There is insufficient evidence to affirm any very definite seasonal change in the level of either the upper or the lower tropopause.

9. The first half of 1951.

Pressures at the tropopause were examined also for the first six months of 1951. During these months, both upper and lower tropopauses were recorded and, where there was but one, no difficulty was experienced in deciding to which regime it should be assigned. Graphing on probability paper was therefore very simple and, with the exception of February, no replotting was necessary. Values of means and standard deviations read from the graphs are given in Table II(b).

The frequency distributions for January and February were similar to the component distributions for January,1948-50 except for that of the lower tropopauses in February which, as the distribution stood, was certainly not normal. This is shown in Fig. 8 split into an approximately normal distribution of 39 observations and an irregular scatter of the remaining 13. It is possible that these represent temperature discontinuities in the troposphere which were noted incorrectly as tropopauses.

In April and May, both upper and lower tropopauses occurred but in March there appears to be no upper tropopause. Upper tropopauses were recorded in March,1949 but not in March of the other two years of 1948-50. The probability graph for March,1951 is given in Fig. 9. Three of the observations, labelled A, B and C, depart appreciably from the normal distribution indicated by the straight line. A may represent an isolated occurrence of the upper tropopause; the date was the 28th. B and C are probably not true tropopauses.

In June, the lower tropopause was observed only during the first three days of the month. It seems likely that the change to tropical regime found to be characteristic of July occurred, in 1951, about the 4th of June.

The ratio of the numbers of occurrences of upper and lower tropopauses is shown in the last column of Table II(b); from January to May, the lower is dominant but after the first few days of June, only the upper tropopause is observed.

/10.

10. Temperature discontinuities in April and May, 1949

A more detailed examination of the temperature discontinuities in one or two months at Habbaniya may throw light upon the controversy as to whether the tropopause is continuous or discontinuous in middle latitudes.

The concept of a continuous tropopause surface is not inconsistent with the occurrence of two tropopauses at a station, for these may arise equally well from the folding of such a surface as from the existence of two distinct surfaces; and the occurrence, at times of more than two tropopauses on the same ascent strengthens the argument in favour of folding. Frost[6] states that three tropopauses are frequently observed over Habbaniya in April and May and there may be as many as five.

Monthly returns of upper-air temperature give details of inversions and isothermals up to the level of the reported tropopause. In the first half of 1949, high tropopauses were reported at Habbaniya and the list of discontinuities is reasonably complete. The months of April and May, 1949 therefore were suggested by Mr. R. Frost for a detailed analysis by means of probability paper.

From the returns for 1400 G.M.T., pressures were extracted of all points at which a lapse rate of temperature extending over more than 5 mb. changed to an inversion or an isothermal. These data, which form the basis of the analysis, are all tropopauses of Type I or Type II by current definition. With two reported tropopauses not already included, they give a frequency distribution of 203 observations the probability graph of which is shown in Fig. 10. This approximates to a smooth curve concave to the right which suggests that the total frequency distribution is comprised of a number of component distributions with different means; attempts to split it into as few as two, or even three, normal distributions were unsuccessful.

A different form of graphical representation is shown in Fig. 11. Here, the ordinate gives the pressure on a logarithmic scale and the abscissa the day of the month. When the points had been plotted they were divided into groups, as nearly as possible by horizontal lines, such that no two observations from the same ascent occurred in each group. For clarity the members of each group are joined into a continuous graph in Fig. 11 although the continuity suggested in this way may not be realised in fact.

Probability graphs for the different groups taken separately could be fitted fairly closely by straight lines. It appears, therefore, that the data represent no less than ten normal distributions. (The last group shown in Fig. 11 comprises only 5 observations and was omitted from this analysis). Means and standard deviations, estimated from the graphs, are set out in Table III.

Table III. Habbaniya, April and May, 1949.

Parameters of component frequency distributions of pressure at significant points

Distribution number	1	2	3	4	5	6	7	8	9	10
Mean (mb.)	70	84	99	116	137	157	173	204	232	271
S.D. (mb.)	6.5	3.9	4.3	5.9	4.1	3.6	7.0	8.7	6.0	17
Number of observations	17	26	21	25	11	15	21	32	13	17

On two days in April, 1949, as many as six 'tropopauses' occurred. They are shown in lines (iii) and (iv) of Table IV, the group means of Table III being shown for comparison in lines (i) and (ii).

/Table IV.

Table IV. Tropopause pressures (mb.) at Habbaniya for two selected days in April, 1949

(i)	Group:	1	2	3	4	5	6	7	8	9	10	-
(ii)	Mean (mb.)	70	84	99	116	137	157	173	204	232	271	-
(iii)	April 6	70	87	-	123	-	-	-	211	-	252	343
(iv)	April 11	-	-	-	115	140	161	176	201	230	-	-

The pressure levels given by the means of Table III may each be considered a "most probable" position of the tropopause for the two months examined. They may be compared with the statement by Frost[6] that tropopauses in April and May tend to occur at levels of about 80, 160 and 220 mb.

The results of the foregoing analysis give some credence to the idea of a continuous tropopause surface. They also confirm that the reporting of a single tropopause, without careful verification from the remainder of the radio-sonde ascent, is insufficient in middle latitudes.

11. Acknowledgement. The author is much indebted to Miss E.E. Austin for her criticism of the first draft of this note and for her very helpful suggestions.

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/Appendix

Appendix

Method of plotting grouped observations on probability paper

In plotting grouped observations on probability paper, it is convenient to plot the probability number associated with the middle observation of each class against the middle point of the range covered by that class. Table III shows the tabulation for the January tropopause pressures plotted in Fig. 5, curve (a).

Table V. Method of plotting grouped observations on probability paper

Tropopause pressure over Habbaniya,
January, 1948-50

(1) Pressure (mb)	(2) Frequency	(3) Cumulative Frequency	(4) Mid-value of pressure range	(5) Probability Number
80- 89	3	$1\frac{1}{2}$	$84\frac{1}{2}$	1.43
90- 99	10	8	$94\frac{1}{2}$	7.62
100-109	4	15	$104\frac{1}{2}$	14.3
110-119	5	$19\frac{1}{2}$	$114\frac{1}{2}$	18.6
. .	.	.	.	.
. .	.	.	.	.
. .	.	.	.	.
. .	.	.	.	.
300-309	2	102	$304\frac{1}{2}$	97.2
310-319	1	$103\frac{1}{2}$	$314\frac{1}{2}$	98.6
320-329	1	$104\frac{1}{2}$	$324\frac{1}{2}$	99.5

Column (1) shows the range of pressure in each class and column (2) the frequency. The cumulative frequency in column (3) is the number of observations from the beginning of the table up to the middle observation of the class considered, the middle observation contributing $\frac{1}{2}$. The simplest method of computing the cumulative frequency corresponding to any specified class is to add the cumulative frequency assigned to the preceding class to the mean of the actual frequencies, column (2), of these two classes. For the class 80 to 89 mb., by taking the frequencies and comulative frequencies of preceding classes to be zero, the cumulative frequency is found to be

$$0 + (0+3)/2 = 1\tfrac{1}{2}$$

and, for the next class,

$$1\tfrac{1}{2} + (3+10)/2 = 8$$

and so on throughout the column.

Column (4) of Table III gives the central pressure value of each class; and column (5) gives the probability number, obtained by mutiplying the cumulative frequency, column (3), by 100/N. In this example, N = 105.

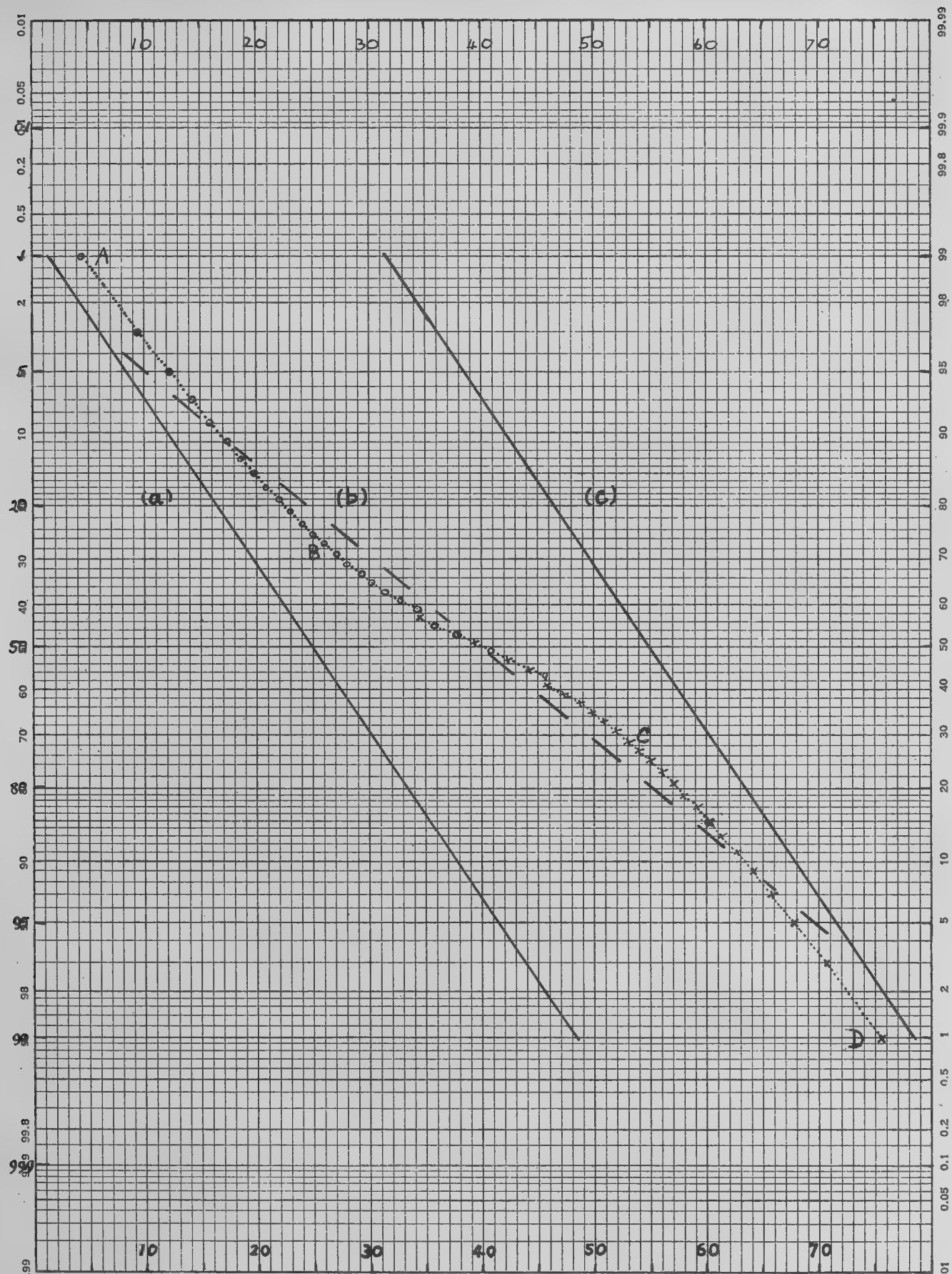

Fig. 1. Combination of normal distributions with different means.

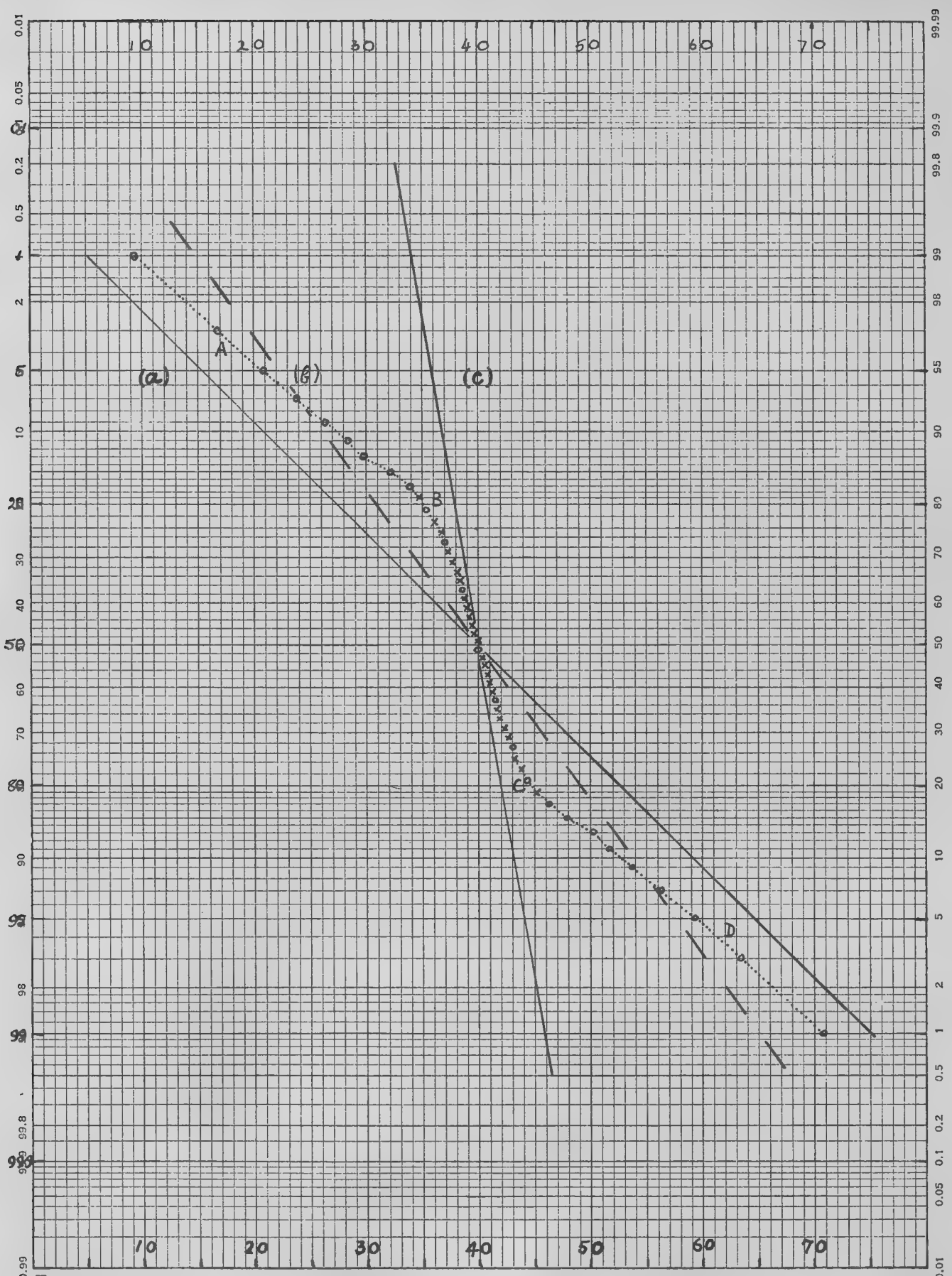

Fig. 2. Combination of normal distributions with different standard deviations.

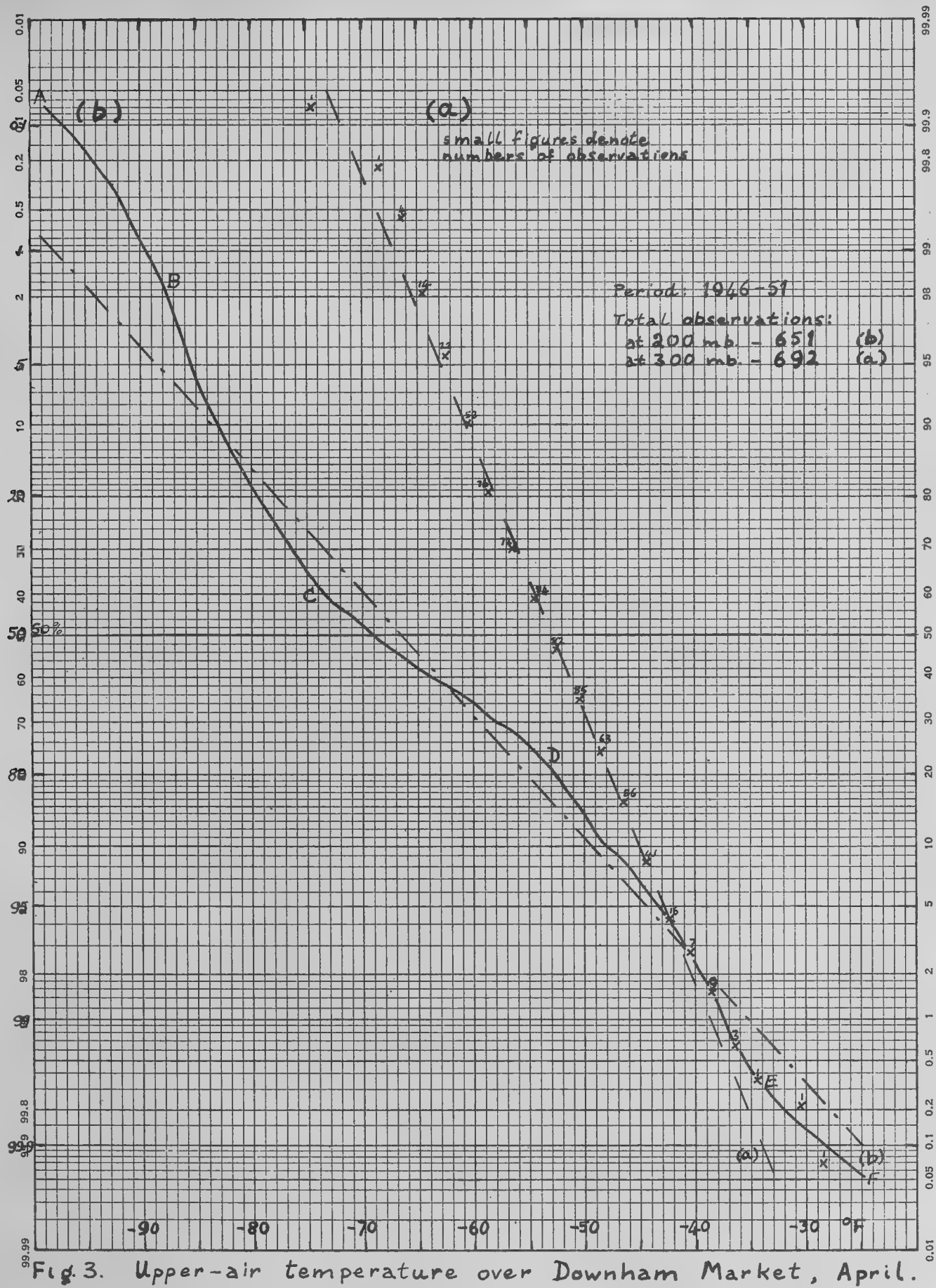

Fig. 3. Upper-air temperature over Downham Market, April.
(a) 300 mb. - distribution almost normal
(b) 200 mb. - distribution platykurtic (i.e. excess of medium-sized deviations)

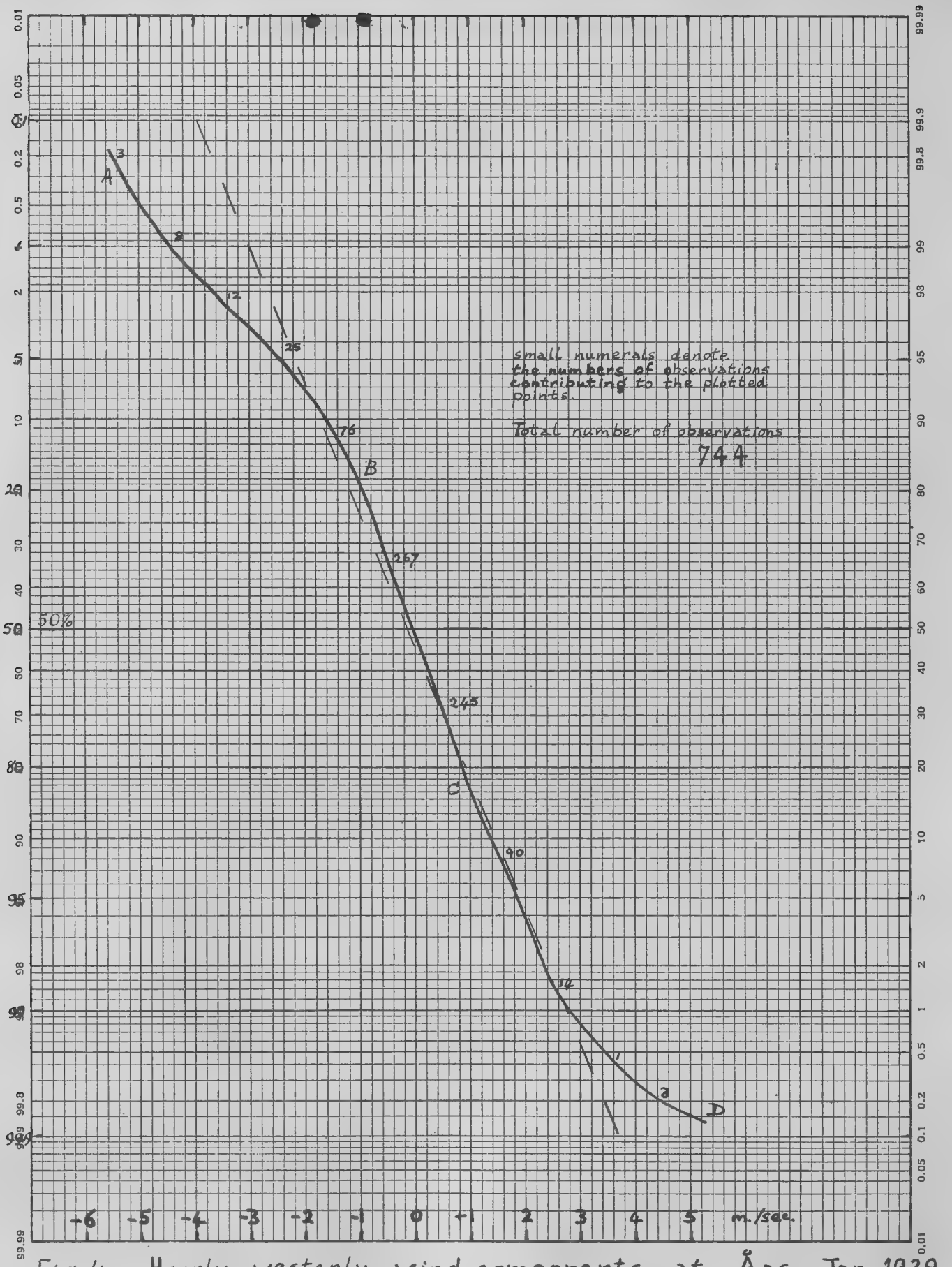

Fig. 4. Hourly westerly wind components at Ås, Jan. 1929. —distribution leptokurtic (i.e. excess of small and large deviations)

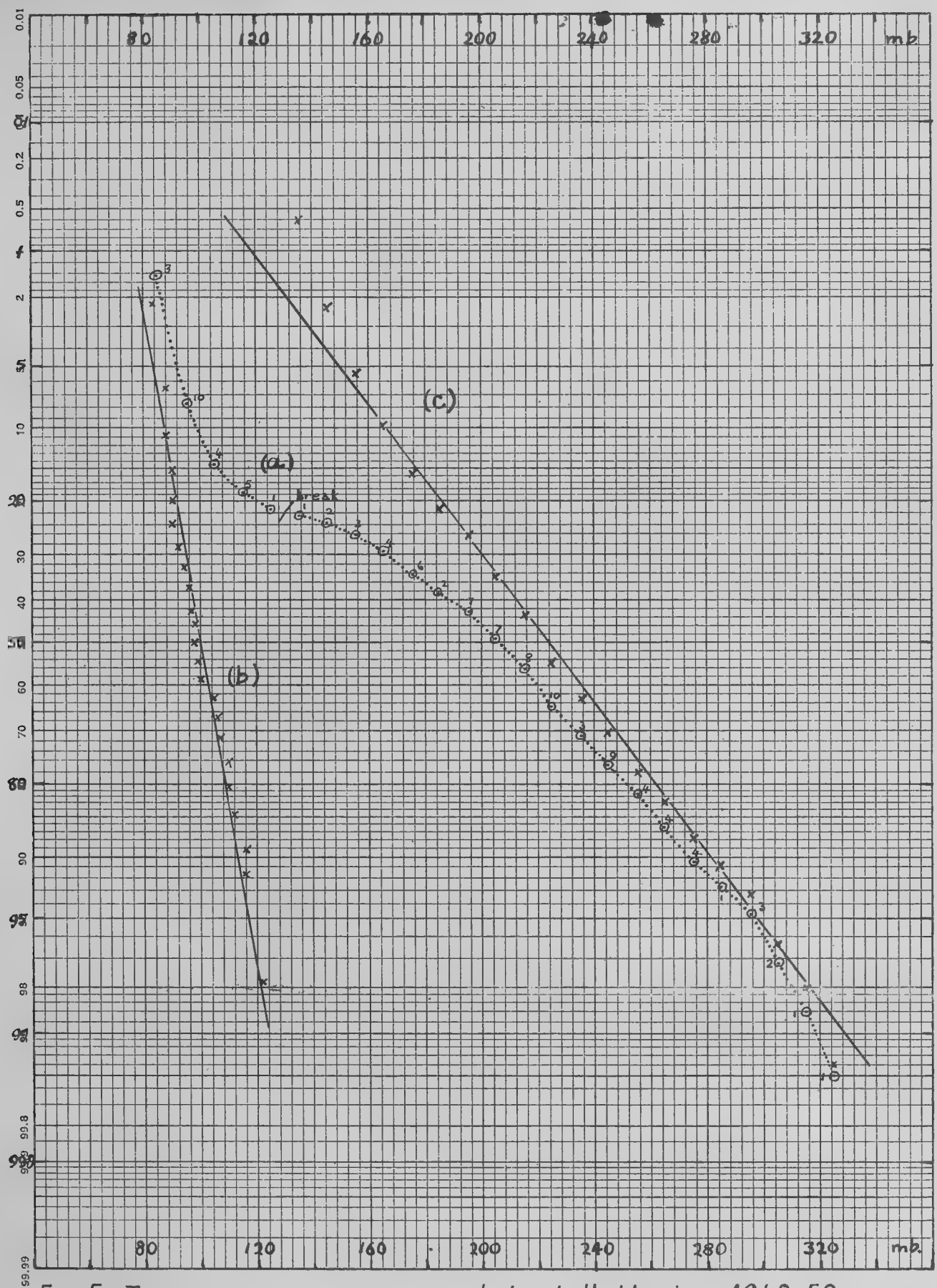

Fig. 5. Tropopause pressures recorded at Habbaniya, 1948-50.
(a) January ; (b) and (c) January replotted

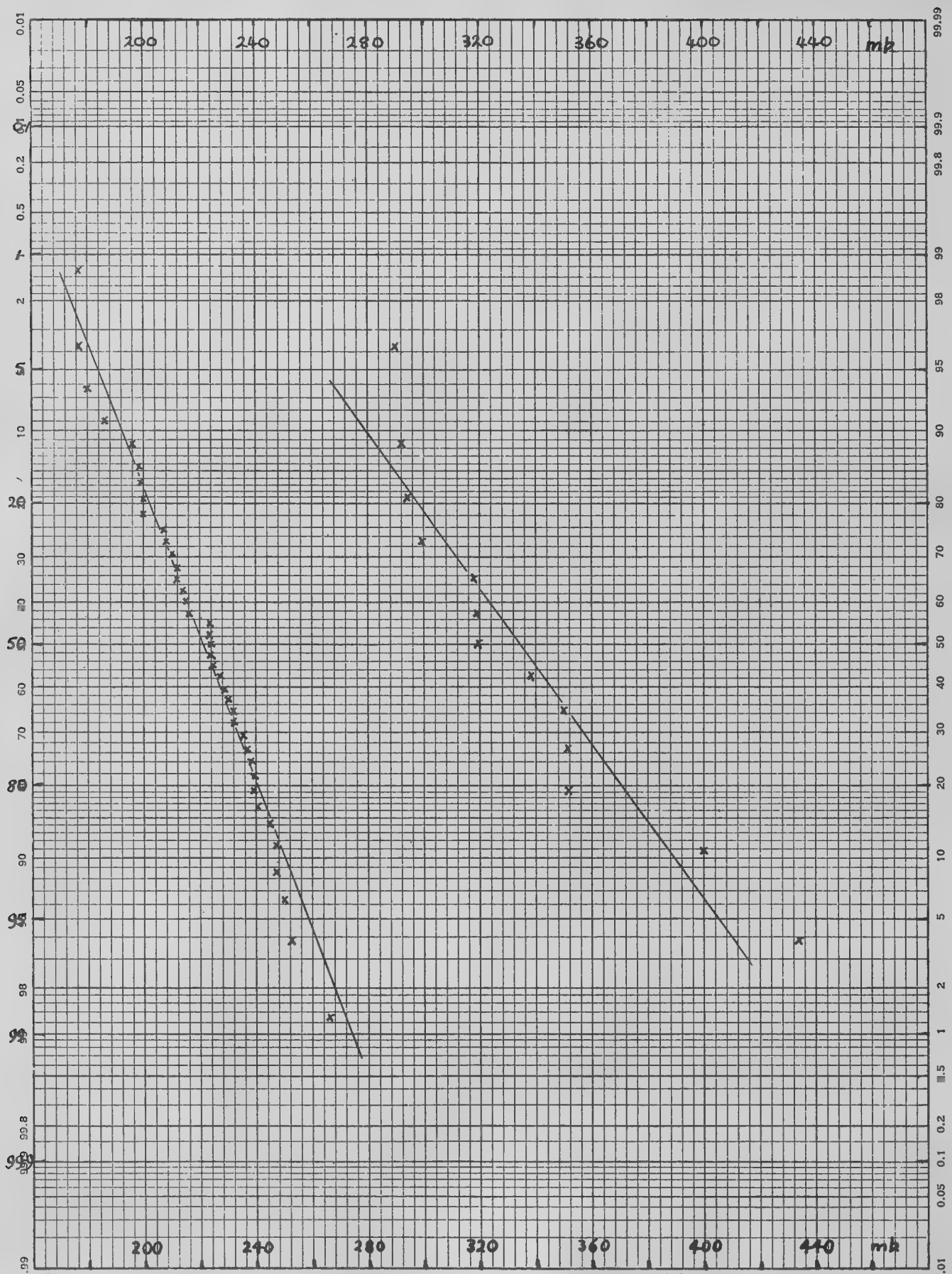

Fig. 8. Tropopause pressures recorded at Habbaniya, February 1951.
The upper tropopauses have been omitted from this diagram.

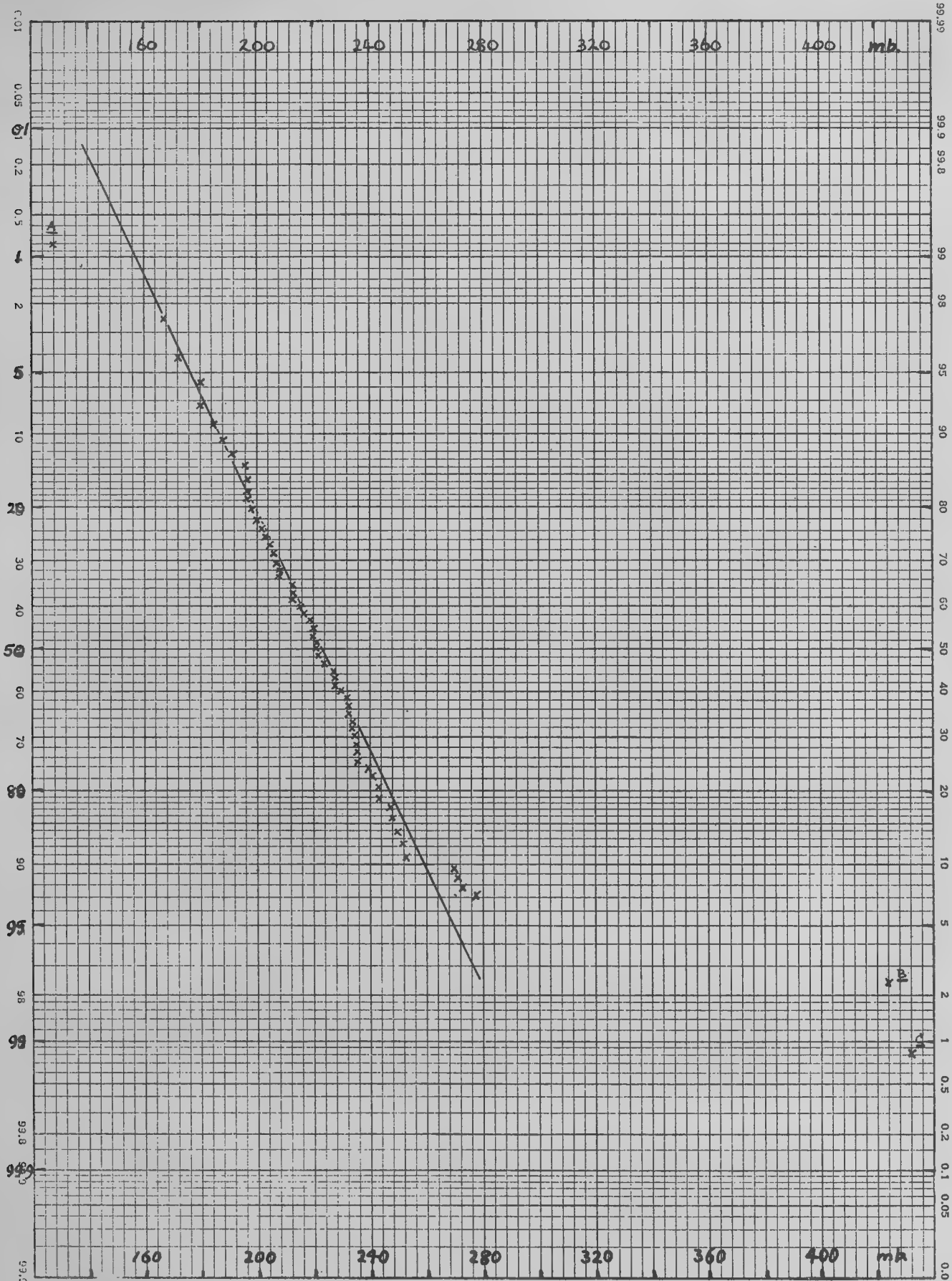

Fig. 9. Tropopause pressures recorded at Habbaniya, March 1951

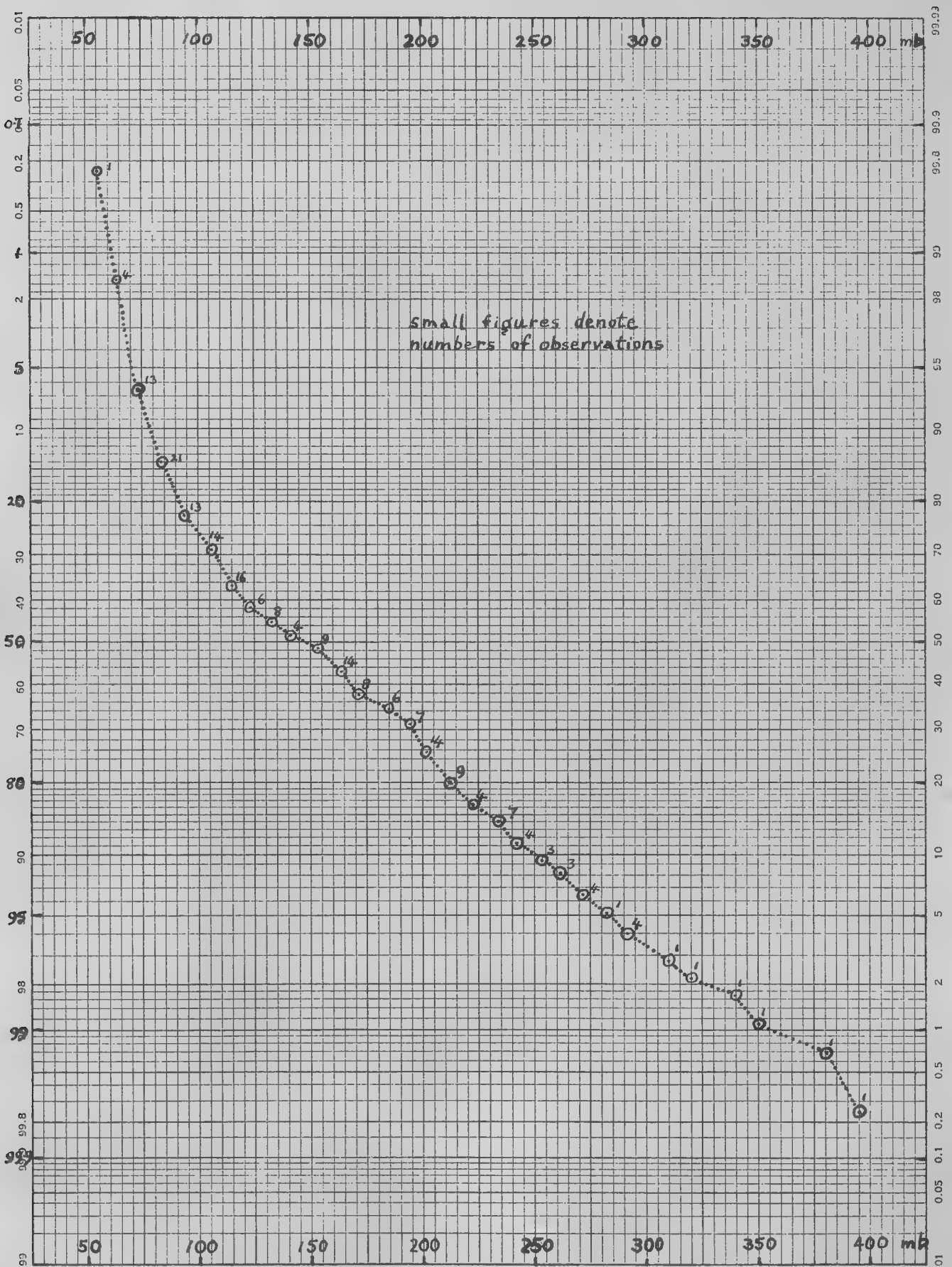

Fig. 10. Pressure levels of "tropopauses" at Habbaniya, April and May, 1949.

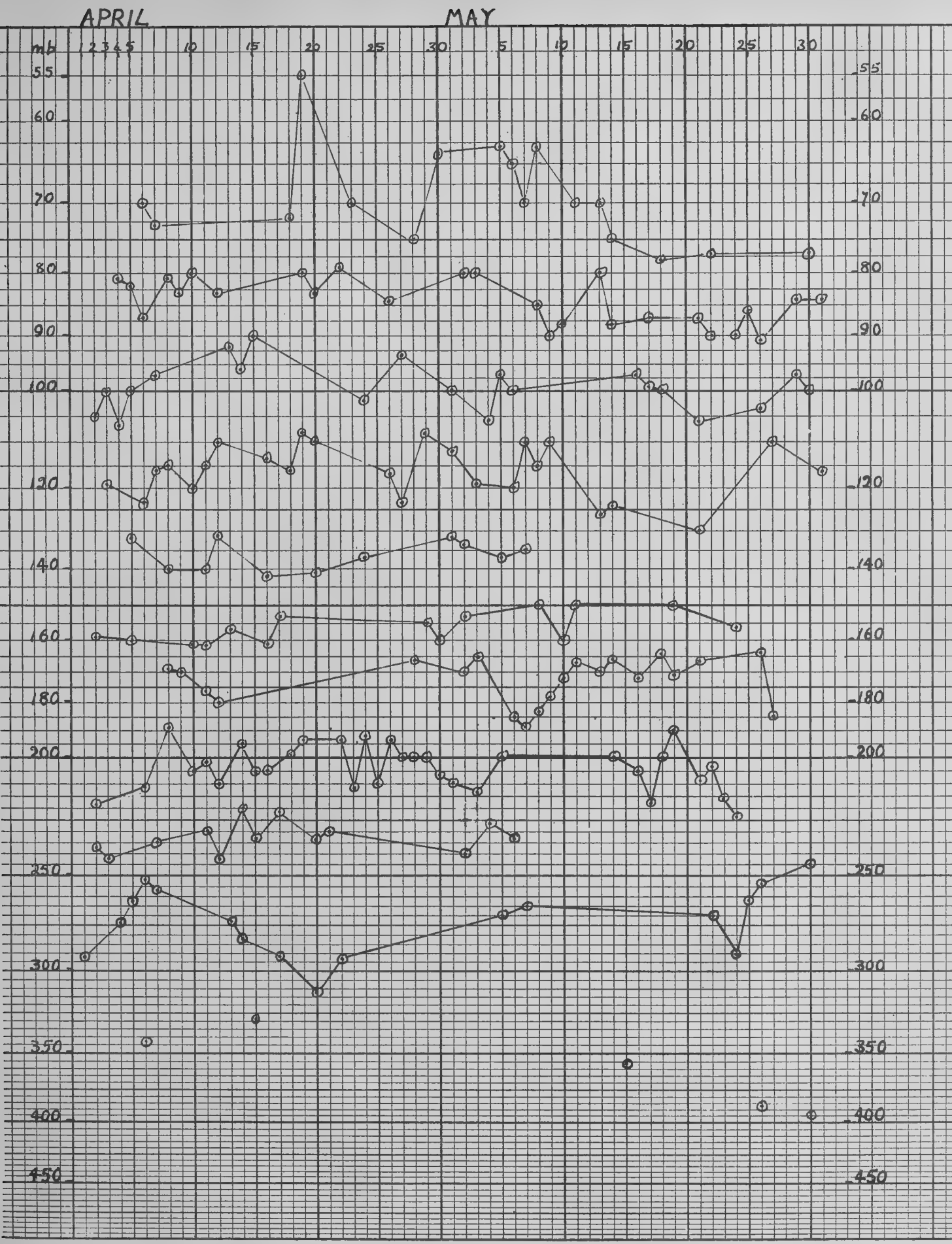

Fig. 11. Discontinuities of upper-air temperature over Habbaniya, April and May 1949; lower limits of inversions and isothermals occurring above the 450-mb level. Time: 1400 G.M.T.

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22nd February, 1952.

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Some Observations of Vertical Temperature and Humidity Gradients made from Ocean Weather Ships

by

C.S. Durst and A.H. Gordon (Meteorological Office)

1. Earlier Work

The vertical distribution of temperature and humidity over the sea has been studied by a number of meteorologists in recent years. Craig (1) (2) discusses observations of soundings of temperature and humidity in the lower 1000 feet of the atmosphere over Massachusetts Bay. The soundings were obtained in the summer and autumn of 1944 by means of an electrical-resistance psychrograph mounted on an aircraft. Emmons (3) describes a similar series of aircraft observations which were made up to 1500 feet in June, 1945 over the ocean between Nantucket and New Jersey. Montgomery (4), Sverdrup, Wüst and others have also published papers on this general subject, but generally speaking accurate observations of the variation of humidity with height are lamentably lacking in meteorological literature.

2. Observational Technique of Ocean Weather Ship Observations

It was decided in May, 1951 to arrange for a programme of regular observations of the vertical gradient of temperature and humidity to be carried out during the ensuing summer months from Ocean Weather Ships at stations JIG 52°30'N, 20°00'W and ITEM 59°00'N, 19°00'W. Previously observations of this kind had been made from dinghies at some distance from the ship with a psychrometer which was lifted by a balloon. However, occasions upon which these observations could be made were very few as they were limited to days of light winds and fairly calm seas. It was therefore planned to extend the observations by making them from the ship itself. Arrangements were accordingly made for observations to be made twice daily, once during daylight hours and once during night hours, at ten foot intervals above the sea surface up to 40 feet by hoisting the psychrometer by a pulley arrangement as illustrated in Figure 1. The rig was situated aft on whichever side was considered the windward side. Each ship had an aluminium pole "fished" as an extension to the main yard and a spar extending about 10-15 feet out from the stern of the ship, and angled somewhat on the same quarter on which the extension to the yard was situated. The travel of the psychrometer was then arranged from this spar up to a small block at the end of the aluminium pole at the end of the yard.

The observations covered the period May - September, 1951. They were discontinued after the end of September when rough weather conditions were expected to make the taking of further observations impracticable. No observations were received during part of the period, either because the Dutch Weather Ship was manning station JIG or because the instrument became unserviceable.

The instrumental observations are set out in the Appendix. Associated weather conditions for each observation have been plotted in station model form in Figure 2 along a vertical axis of sea temperature minus dewpoint.

3. Analysis of Observations

Such are the basic observations. We should welcome suggestions as to how best the data (and others which we hope will be similarly obtained) should be

/summarised

summarised and presented for we are aware that humidity observations are the most difficult of any to present in a satisfactorily useful manner.

It is clear that the discussion should take into account the humidity lapse rate in some manner since the prime object of the observations is to discover facts about the evaporation off the sea and the transport of water vapour upwards (and laterally). It is clear too that windspeed and consequent wave structure is of importance, since changes in the turbulent structure of the air are certainly important in the vertical transport of water vapour. However, the temperature lapse rate may have a fundamental effect through convective turbulence and on this account the time of day and the recent history of the air as shown by wind direction need to be taken into account. Further, we feel that as soon as breaking waves occur (with winds of force 5 or more) the evaporation off spray drops may assume a considerable importance in the transfer of water vapour upwards.

In a previous paper we have shown that, as would be expected, high humidity at bridge height is associated with winds between E.S.E. and S.W., low humidity with winds between W. and N., and that there is a tendency for humidity to increase at that height with stronger winds from the S.E., but this effect is not so apparent with N.W. winds.

Accordingly as a trial arrangement the observations were sorted into four groups (a) winds of force 3 or less from W'N to N.E., (b) winds of force 3 or less from E'S to S.W., (c) winds of force 4 or more from W'N to N.E. and (d) winds of force 4 or more from E'S to S.W. The mean temperatures and dew points at each height were calculated in each case with the results set out in Table I below.

Table I. Mean Temperature and Dew Point at various heights with winds of certain directions and speed.

Wind Force Wind Direction	0 to 3 W'N to NE Temper-ature	Dew Point	0 to 3 E'S to SW Temper-ature	Dew Point	4 or more W'N to NE Temper-ature	Dew Point	4 or more E'S to SW Temper-ature	Dew Point
Number of Obs.	13	13	8	8	28	28	22	22
Height (feet)								
50	52.2	44.1	52.7	45.6	50.7	40.9	58.0	53.8
40	52.2	44.3	52.7	45.6	50.9	41.4	58.0	53.9
30	52.3	44.5	52.6	45.9	51.0	41.8	58.0	54.0
20	52.5	45.0	52.7	46.3	51.1	42.2	58.0	53.9
10	52.7	46.0	52.9	47.5	51.6	43.5	58.1	54.0
0	54.2	54.2	54.8	54.8	54.8	54.8	58.3	58.3

Since O.W.S. Observer was unable to reach 50 feet, the temperatures and dew points at that height are calculated by adding the mean difference of temperature and dewpoints as observed by O.W.S. Recorder, Explorer and Watcher to the mean temperature and dewpoint recorded by all from O.W.S.

References

(1) Craig, R.A. Journal of Meteorology, Vol.6, No.2, pp. 123-133.
(2) Craig, R.A. Papers in Physical Oceanography and Meteorology, M.I.T. and Woods Hole Oceanographic Institute 10 No.1
(3) Emmons, G. " 10 No.3
(4) Montgomery, R.B. " 7 No.4

Appendix I. Observations of Temperature and Dew Point made at various heights by the Ocean Weather Ships

Ship	Postn.	Date	Time	Sea Temp.	10 Ft. Dry Bulb	10 Ft. Dew Point	20 Ft. Dry Bulb	20 Ft. Dew Point	30 Ft. Dry Bulb	30 Ft. Dew Point	40 Ft. Dry Bulb	40 Ft. Dew Point	50 Ft. Dry Bulb	50 Ft. Dew Point
Recorder		18.7.49	2005	58.2	56.2	50	56.0	51	55.6	52	55.4	50		
	Item	31.7.50	1630	59.1	57.7	48.7	56.3	46.6	56.2	46.3	55.8	45.8	55.4	46.3
	Jig	5.5.51	1500	48.1	46.9	45	47.3	44	47.0	45	47.1	45	46.8	45
		7.5.51	1500	48.6	48.4	44	47.9	44	46.3	44	46.2	43	46.2	43
		10.6.51	2030	56.1	54.9	48	54.9	49	55.0	49	54.8	49	54.9	48
		11.6.51	1000	55.8	54.3	51	54.3	51	54.4	51	54.4	51	54.5	50
			2130	56.2	55.4	51	55.4	51	55.3	51	55.4	51	55.5	50
		12.6.51	1000	56.1	54.6	50	54.5	50	54.7	49	54.8	49	54.9	49
			2140	56.8	53.6	52	53.8	51	53.8	51	53.8	51	53.8	52
		13.6.51	1000	56.8	55.0	53	55.0	53	54.9	53	54.9	53	54.9	53
			2135	56.1	51.1	43	51.0	42	51.1	42	51.1	43	51.4	43
		14.6.51	0930	55.2	-	-	49.1	36	49.2	35	49.2	36	49.1	36
			2130	55.8	50.9	39	50.7	38	50.3	39	50.3	39	50.2	37
		15.6.51	1000	54.7	51.6	-	51.2	-	51.0	-	50.8	-	50.6	-
			2205	55.6	48.3	38	47.9	39	48.0	36	47.8	36	47.1	35
		16.6.51	1005	55.5	49.9	37	49.4	34	49.7	34	49.3	33	48.2	32
			2100	55.6	51.8	38	51.6	38	51.2	37	51.2	36	50.8	28
		17.6.51	1030	55.0	48.0	40	48.2	39	48.0	38	48.0	38	48.0	39
			2120	55.4	48.6	42	48.1	39	47.7	40	47.7	40	47.3	39
		18.6.51	1015	55.6	47.3	46	47.2	45	47.3	44	47.7	44	48.1	43
			2115	55.5	50.0	45	49.7	45	49.3	44	49.1	44	47.8	45
		19.6.51	1200	55.1	47.8	47	47.6	45	47.5	44	47.4	44	47.3	44
			2130	54.2	48.6	39	48.2	38	48.2	35	48.1	35	47.9	35
		20.6.51	1000	55.3	50.4	37	50.4	34	50.5	35	50.5	34	50.3	33
			2030	55.7	52.0	45	52.0	43	51.7	43	51.6	41	51.2	42
		21.6.51	1015	55.3	52.9	37	53.0	37	53.0	35	53.1	34	53.2	32
			2130	56.1	52.2	41	52.1	41	52.1	41	52.1	41	52.3	39
		22.6.51	0945	56.1	51.3	45	51.4	43	51.3	43	51.2	42	51.1	42
			2100	56.3	52.1	47	51.7	48	51.2	47	50.9	47	50.8	46
		23.6.51	1030	55.9	48.2	45	48.2	45	48.3	44	49.3	41	49.2	40
			2045	55.8	51.1	42	51.1	41	51.0	40	50.8	40	50.6	41
		24.6.51	1030	55.8	50.1	47	49.3	46	49.2	46	49.2	46	49.2	46
			2115	55.8	51.2	48	51.2	46	51.2	46	51.2	46	51.6	48
		25.6.51	0930	56.6	52.8	46	52.9	46	53.0	46	52.8	45	53.0	45
			2105	56.1	52.9	42	52.6	42	52.6	42	52.1	43	52.0	43
		26.6.51	1000	56.1	51.0	47	51.3	46	51.1	46	51.1	46	51.0	46
			2100	57.0	51.6	48	50.8	49	50.8	49	50.4	49	50.6	49
		27.6.51	1045	56.1	51.2	49	50.8	49	50.7	49	50.6	49	50.6	49
			2020	56.8	53.5	41	53.5	37	53.5	37	53.9	38	53.9	38
Observer	Item	2.6.51	1600	51.3	50.1	50	50.0	49	50.1	49	50.1	49	50.1	49
Explorer		9.6.51	1830	52.0	49.7	39	49.5	38	48.8	39	49.3	38	48.9	36
		10.6.51	1000	52.0	48.8	38	48.7	36	48.3	36	48.7	36	48.6	40
		11.6.51	1000	50.9	48.7	38	48.7	38	48.9	35	48.9	35	48.9	35
		12.6.51	1030	52.2	51.6	43	51.3	45	51.3	42	51.3	41	51.1	42
			2130	52.0	49.8	43	49.8	43	49.5	42	49.5	41	49.3	40
		13.6.51	1100	52.1	49.9	41	49.8	39	49.6	40	49.1	39	49.2	39
			2200	52.3	50.1	41	50.1	39	49.7	40	49.2	40	49.3	39
		15.6.51	2200	53.3	51.5	34	51.4	34	51.0	34	50.9	36	51.1	36
		16.6.51	1030	53.0	48.5	41	48.5	41	48.2	40	48.1	40	47.8	40
			2200	52.2	49.7	39	49.1	39	48.9	39	48.9	40	49.0	41
		17.6.51	1100	52.7	50.6	40	50.5	40	50.5	40	50.6	40	50.1	39
			2200	52.8	51.2	45	50.9	43	50.8	43	50.8	43	50.7	43
		18.6.51	1000	53.0	50.8	42	50.6	41	50.5	41	50.7	41	50.8	41
			2230	53.1	50.5	48	50.3	47	50.0	47	49.9	47	50.0	47
		19.6.51	1000	53.0	51.7	47	51.6	47	51.5	47	51.3	48	51.7	49
		20.6.51	1000	53.0	50.9	42	50.6	42	50.6	42	50.6	41	50.3	41
		21.6.51	1030	53.1	51.9	43	51.8	42	51.8	41	51.8	41	51.5	41
		22.6.51	1230	53.3	51.9	47	51.7	47	51.5	46	51.3	47	50.8	47
Observer		2.7.51	1000	53.8	51.2	42	51.1	42	51.2	42	51.2	42	-	-
			2359	53.8	51.1	45	51.0	43	50.9	46	-	-	-	-
		3.7.51	1000	53.5	52.4	46	52.5	45	52.5	45	52.5	43	-	-

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Ship	Postn.	Date	Time	Sea Temp.	10 Ft. Dry Bulb	10 Ft. Dew Point	20 Ft. Dry Bulb	20 Ft. Dew Point	30 Ft. Dry Bulb	30 Ft. Dew Point	40 Ft. Dry Bulb	40 Ft. Dew Point	50 Ft. Dry Bulb	50 Ft. Dew Point
Observer		3.7.51	2359	53.6	52.5	44	52.2	43	52.1	42	52.0	41	-	-
		4.7.51	1000	53.4	52.4	42	52.3	41	52.4	41	52.3	40	-	-
		5.7.51	1000	54.0	54.2	53	54.3	53	54.5	53	54.5	53	-	-
			2359	54.3	53.3	47	53.1	47	53.4	46	53.4	45	-	-
Observer	Item	6.7.51	1000	54.3	54.9	54	54.8	53	54.7	53	55.0	51	-	-
			2359	54.2	53.2	50	53.0	50	53.0	48	53.0	48	-	-
		7.7.51	1000	55.0	54.9	50	54.3	50	54.4	49	54.1	49	-	-
			2359	55.2	55.4	52	55.2	50	55.1	50	55.0	49	-	-
		8.7.51	1000	56.1	55.8	50	55.5	48	55.5	47	55.4	48	-	-
			2359	56.0	56.3	53	56.1	54	56.1	54	56.0	53	-	-
		9.7.51	1000	55.7	55.9	50	55.7	50	55.2	50	55.1	50	-	-
			2359	55.3	55.1	47	55.0	47	55.1	46	54.9	46	-	-
		10.7.51	1000	55.1	53.2	45	53.1	42	53.1	42	53.3	41	-	-
		11.7.51	1000	55.6	52.3	42	52.4	38	52.4	37	52.3	37	-	-
			2359	55.2	53.4	45	53.3	43	53.2	43	53.2	42	-	-
		12.7.51	1000	56.1	57.2	42	55.6	42	53.6	42	53.2	41	-	-
			2359	56.1	55.9	51	55.9	51	55.7	51	55.8	50	-	-
		13.7.51	1000	55.3	55.6	53	56.0	54	56.0	54	56.3	54	-	-
			2359	55.4	56.4	54	56.5	52	56.7	56	56.7	56	-	-
		14.7.51	1000	55.5	54.9	50	54.8	50	54.9	49	55.0	48	-	-
			2359	55.8	55.6	51	55.4	50	55.3	50	55.3	50	-	-
		15.7.51	1000	55.9	55.7	50	55.2	49	55.2	49	55.2	49	-	-
			2359	55.5	55.5	53	55.5	52	55.7	53	55.6	53	-	-
		16.7.51	1000	55.5	58.0	50	57.4	50	57.1	50	56.7	50	-	-
			2359	55.3	55.1	54	55.2	54	55.1	55	55.3	54	-	-
		17.7.51	1000	55.1	53.1	41	52.8	41	52.8	41	52.8	41	-	-
			2359	55.3	56.0	50	56.3	49	56.0	49	56.1	49	-	-
		18.7.51	1000	55.6	55.3	55	55.3	55	55.3	55	55.3	55	-	-
			2359	55.7	55.8	56	55.1	55	54.6	55	54.5	55	-	-
		19.7.51	1000	55.2	53.9	48	53.7	48	53.6	48	53.1	46	-	-
Watcher	Jig	22.7.51	1030	59.9	56.5	47	56.3	47	56.1	47	55.9	46	55.9	45
			2130	59.8	56.3	43	55.8	45	55.5	42	55.2	42	55.1	42
		23.7.51	1010	59.8	57.7	49	57.5	47	57.4	46	57.6	46	57.6	48
			2200	60.2	58.8	56	58.8	56	58.8	56	58.8	56	59.6	56
		24.7.51	1030	60.6	60.6	57	59.9	58	59.9	58	60.0	58	60.2	58
			2200	61.0	60.3	58	60.1	58	60.1	58	60.0	58	59.9	59
		25.7.51	1015	60.2	60.6	59	60.6	60	60.7	60	60.7	60	60.8	60
			2150	60.9	60.5	60	60.6	61	60.7	61	60.7	60	60.6	61
		26.7.51	1010	60.6	58.8	57	58.7	57	58.3	56	58.3	56	58.1	57
			2215	59.6	59.7	56	59.6	56	59.4	56	59.5	56	59.5	57
		27.7.51	1000	60.2	58.2	47	57.7	49	57.6	49	58.0	50	58.0	50
			2000	60.2	58.7	50	58.4	50	58.6	51	58.6	51	58.2	49
		28.7.51	1030	60.8	57.7	50	57.9	50	58.0	50	58.7	50	58.6	51
			2130	61.8	58.7	56	58.6	55	58.6	55	58.5	55	58.3	56
		29.7.51	1000	61.0	59.8	51	59.8	51	59.7	51	59.7	51	59.7	51
			2205	60.0	59.8	54	59.9	54	59.7	53	59.6	53	59.2	54
		30.7.51	0955	60.5	61.2	53	60.9	52	60.6	53	60.8	52	60.4	50
			2200	60.0	58.8	58	59.1	51	59.0	57	59.0	57	59.0	57
		31.7.51	1030	60.0	56.0	55	56.4	51	56.2	51	56.1	51	56.0	51
Observer		23.9.51	0001	52.8	53.4	53	53.4	52	53.3	52	53.2	52	-	-
			1000	52.8	52.1	48	52.0	48	52.1	48	52.1	48	-	-
		24.9.51	0001	52.8	54.2	50	54.1	50	54.1	49	53.9	50	-	-
			1000	52.9	53.2	51	53.1	51	52.9	50	53.0	50	-	-
		25.9.51	0001	53.0	53.2	51	53.1	51	53.1	51	52.9	50	-	-
			1000	53.1	56.1	47	54.9	48	54.8	48	54.6	49	-	-
		26.9.51	0001	53.2	52.7	50	52.5	48	52.3	49	52.2	49	-	-
			1000	52.8	52.6	50	52.5	49	52.2	50	53.1	48	-	-
		27.9.51	0001	52.7	52.4	50	52.8	49	52.8	49	52.8	49	-	-
			1000	52.9	53.3	49	53.5	47	53.7	49	53.8	49	-	-
		28.9.51	0001	52.7	52.8	50	52.1	49	52.3	50	52.1	49	-	-
			1000	53.0	52.4	51	52.4	51	52.4	51	52.4	51	-	-
		29.9.51	0001	52.7	52.8	50	52.1	49	52.3	49	52.1	49	-	-
			1000	53.0	52.4	49	52.4	49	52.4	49	52.4	49	-	-
		30.9.51	0001	52.9	55.7	53	55.8	53	55.8	52	55.4	52	-	-
			1000	52.9	54.6	53	54.6	53	54.8	52	54.9	53	-	-

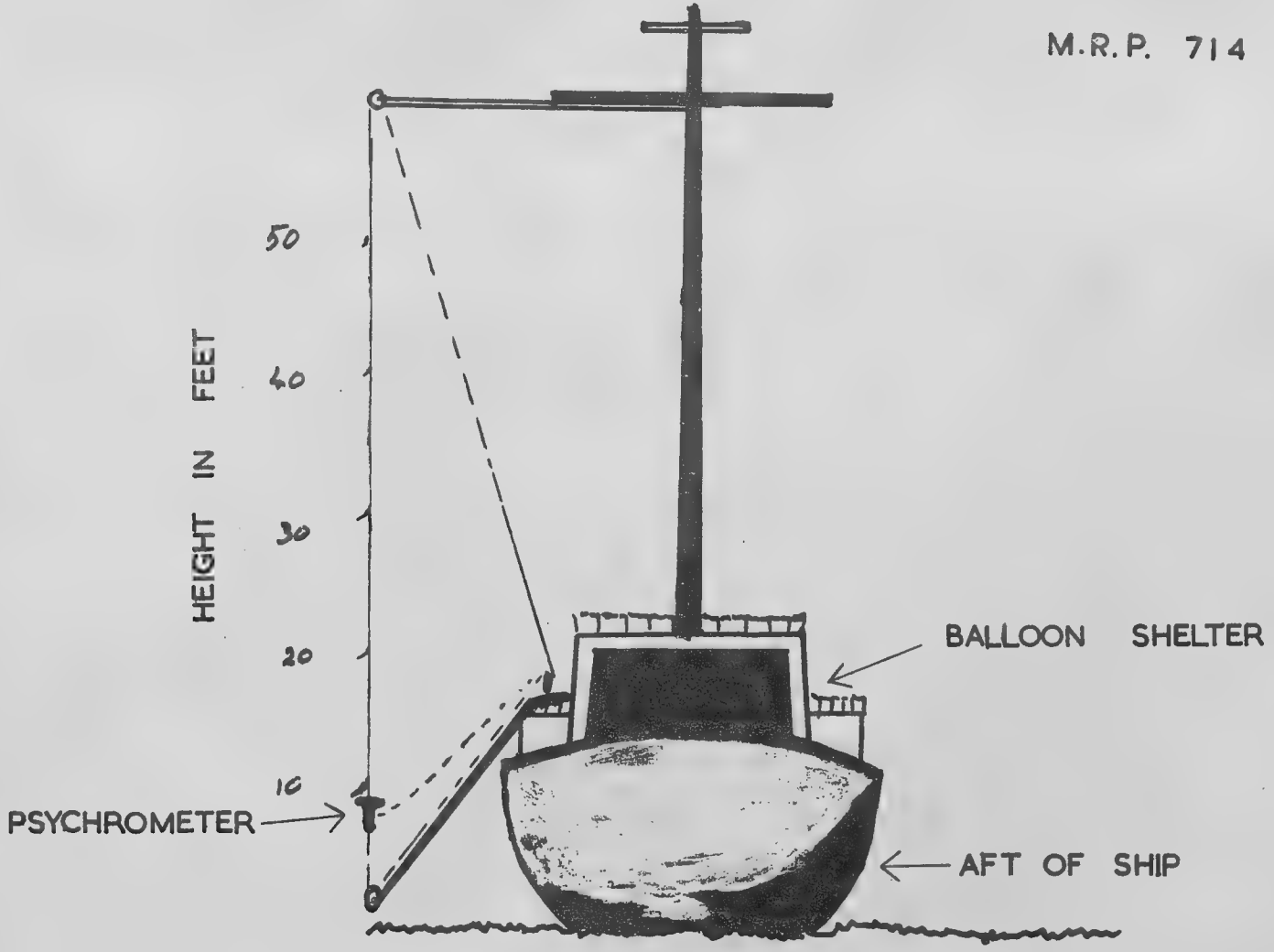

FIG.1. SKETCH OF PSYCHROMETER RIGGING.

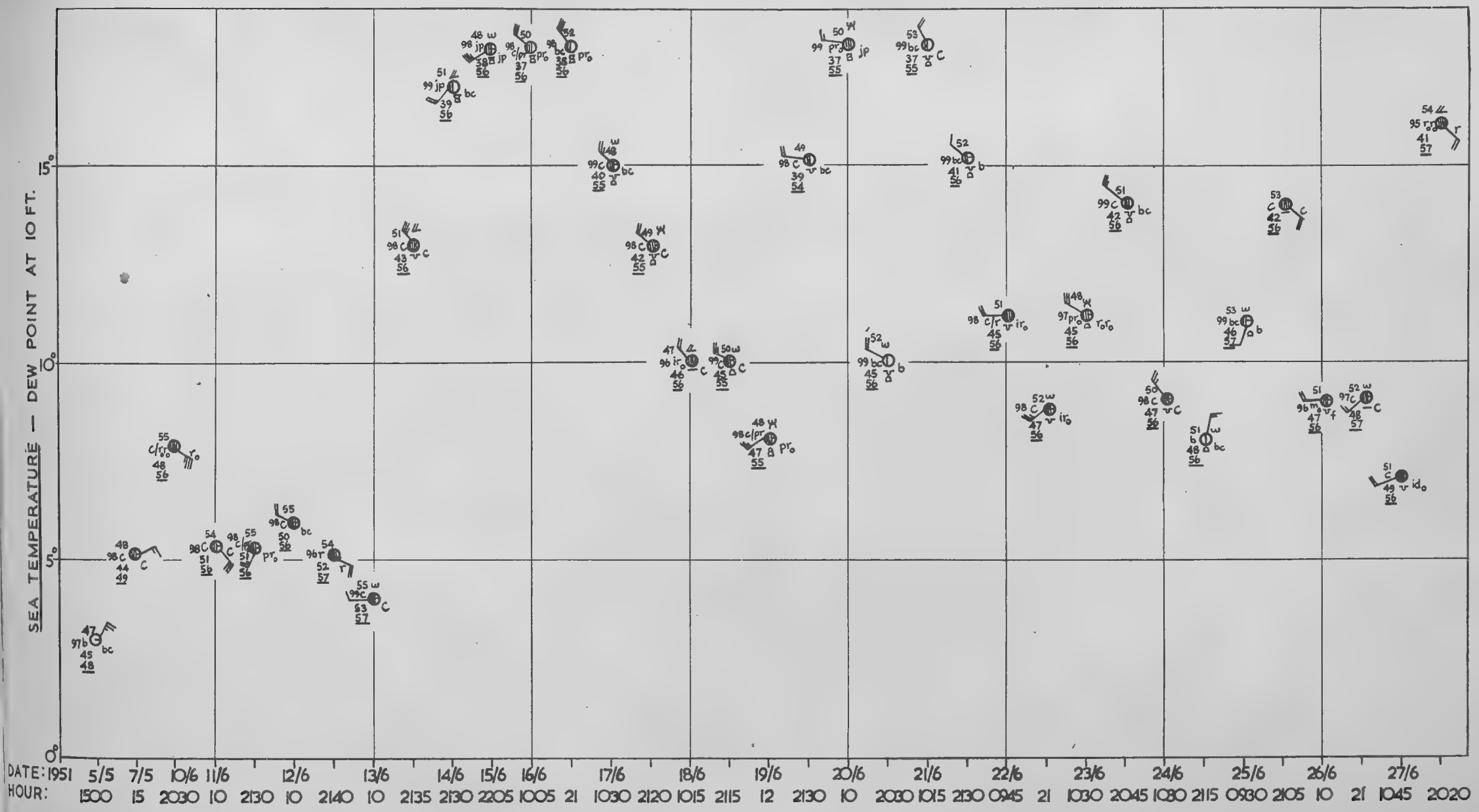

FIG. 2(a). STATION MODELS OF WEATHER OBSERVATIONS IN RELATION TO VERTICAL GRADIENT OF DEW POINT.

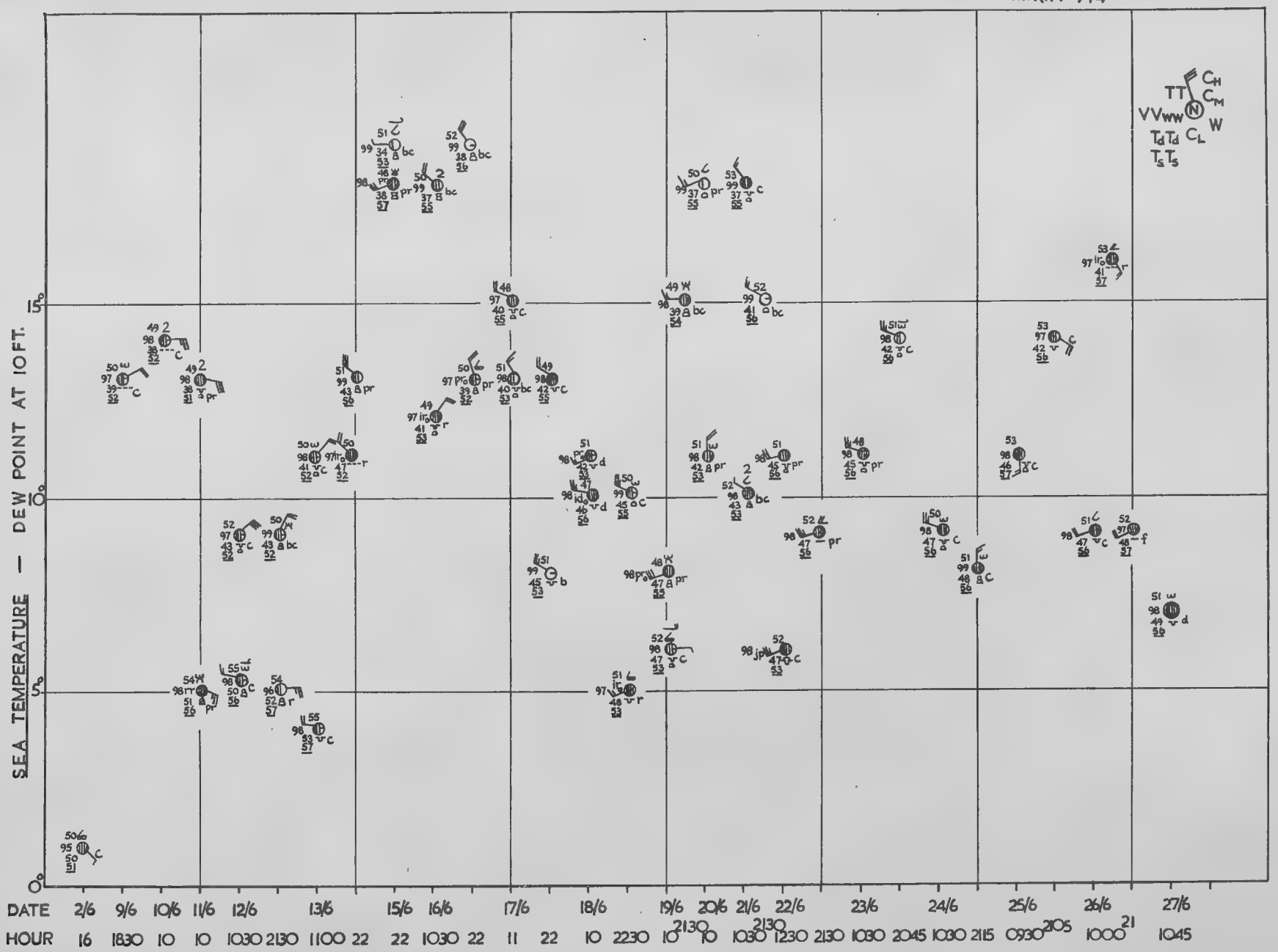

FIG. 2(b). STATION MODELS OF WEATHER OBSERVATIONS IN RELATION TO VERTICAL GRADIENT OF DEW POINT.

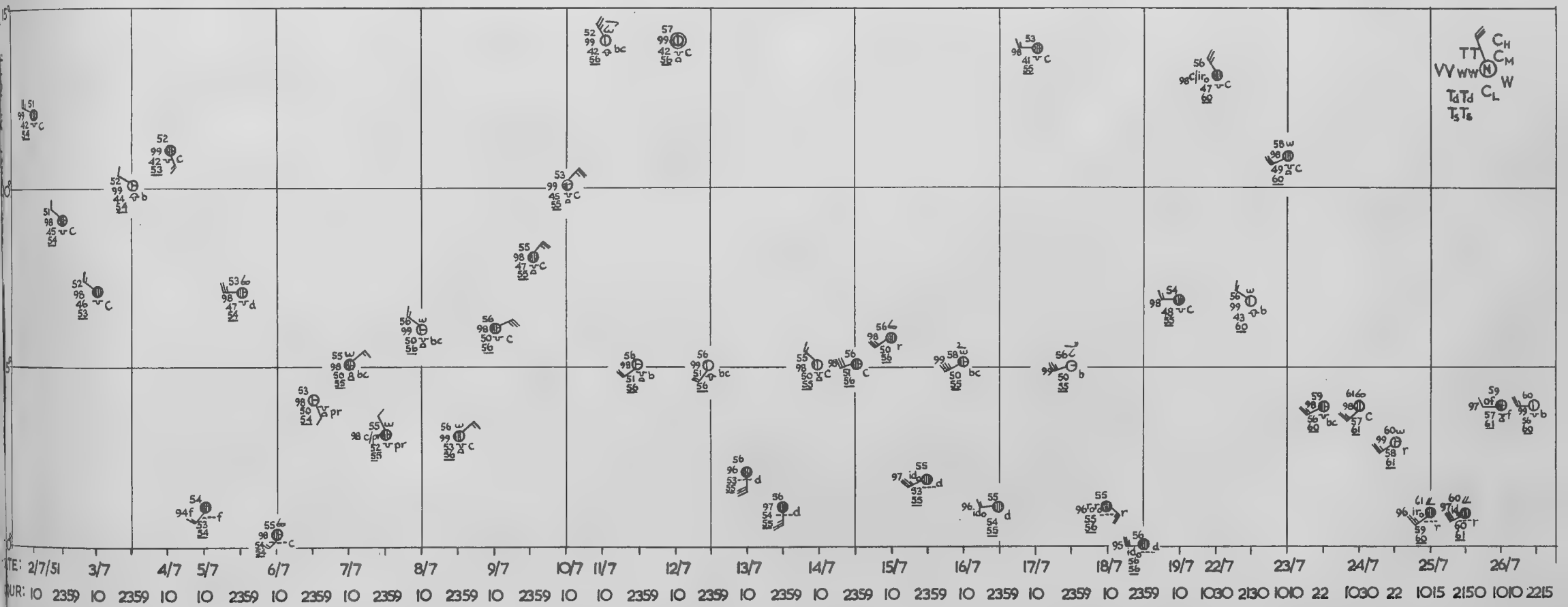

FIG. 2(c). STATION MODELS OF WEATHER OBSERVATIONS IN RELATION TO VERTICAL GRADIENT OF DEW POINT.

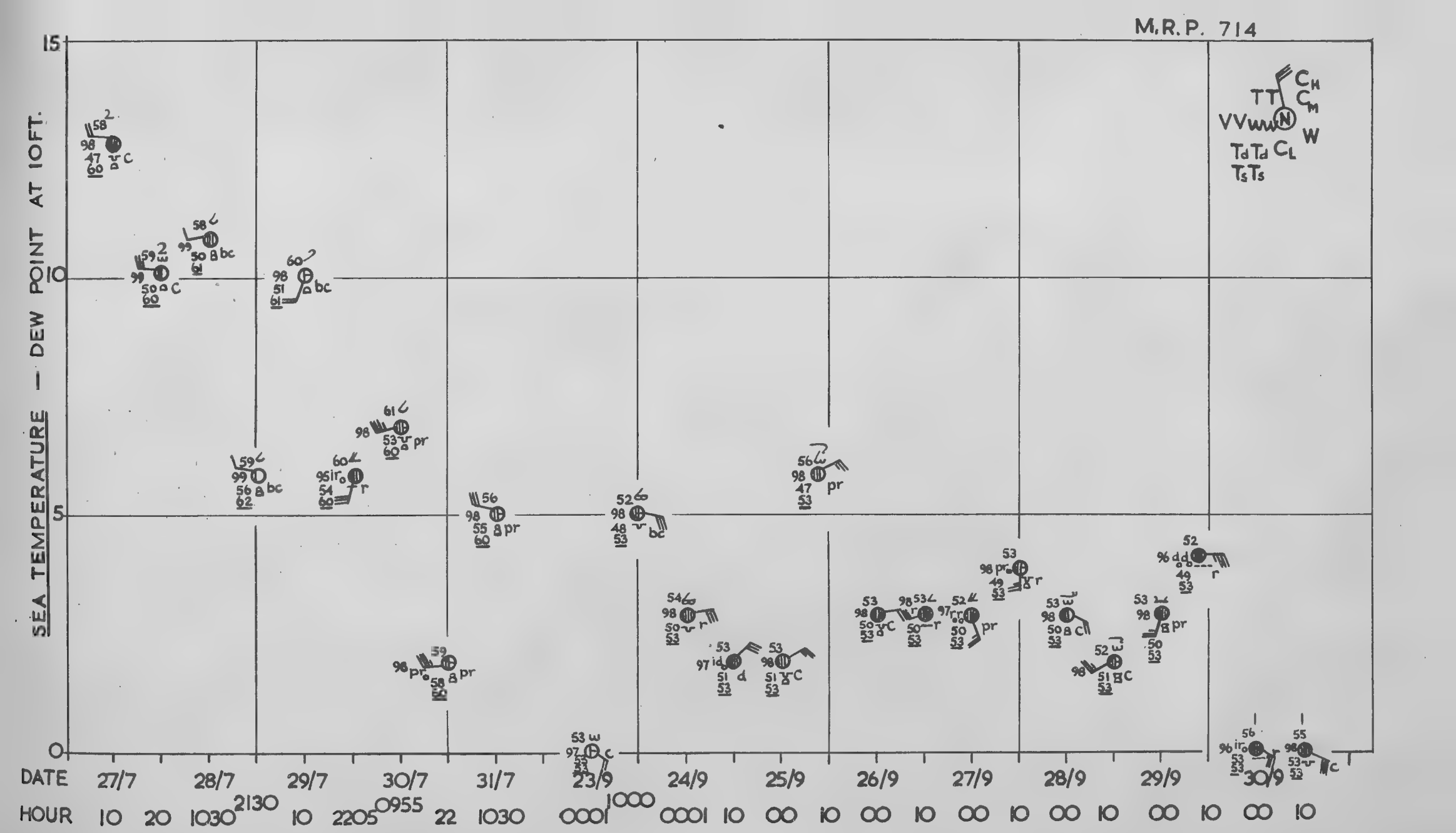

FIG. 2(d). STATION MODELS OF WEATHER OBSERVATIONS IN RELATION TO VERTICAL GRADIENT OF DEW POINT.

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A baroclinic model atmosphere suitable for numerical integration

by
J.S. Sawyer and F.H. Bushby (Meteorological Office)

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SUMMARY

Subject to various approximations a system of 3 simultaneous partial differential equations is derived to predict the height of a contour surface, the thickness of the layer from 1000 mb. to that surface, and the vertical velocity averaged throughout a vertical column. The equations are in a form suitable for numerical integration and a brief outline of a method of solution is given.

Introduction.

Several attempts have recently been made by Charney and others (1,2,3) to predict by numerical methods the large scale atmosphere motions. In contrast to the early attempt by L.F. Richardson (4) who endeavoured to treat the most general type of atmospheric motion, Charney and his collaborators have assumed that the wind is unidirectional with respect to height, and that the increase of wind with decreasing pressure is constant along a vertical. In this way they have been able to treat the mid-tropospheric flow as the motion of an equivalent barotropic fluid on a rotating globe. This simplification has considerable mathematical advantages, but it has proved inadequate as a means of predicting future atmosphere movements. This is not surprising when it is recalled that many of the most vigorous atmospheric developments take place in strongly baroclinic regions where isobaric contours and thickness lines intersect.

We are therefore led to study whether a more general baroclinic model of the atmosphere can serve as a basis for numerical integration without the need to proceed to the complete generality of L.F. Richardson's approach. A significant generalisation of the barotropic model of the atmosphere is obtained if it is assumed that the direction of the wind shear (but not necessarily the direction of the wind itself) is the same throughout a vertical column. In the present note we shall discuss the suitability of such a model for study by numerical methods and we shall derive a set of equations suitable for numerical integration. The model has certain theoretical disadvantages, in as much as the equations derived are not exact even for the ideal model. Nevertheless, since the simpler models omit effects of vertical motion which have been demonstrated to be important, it is considered that the numerical integration should be attempted on the basis of the present model. The work of integration will, no doubt, prove very long and laborious but, with the possibility of using electronic computing methods, this aspect of the problem can be considered later.

The basic assumptions.

The model of the atmosphere which is adopted for study consists of a baroclinic fluid in which the thermal wind is constant in direction in any vertical column but not necessarily parallel to the wind direction at any level. We also assume that the thermal wind speed is proportional to the pressure difference through the layer concerned. We consider the atmosphere as bounded by two pressure surfaces

$p = p_1$ corresponding to the upper limit of the troposphere and $p = p_0$ corresponding

to a level near the surface and we assume that the movement of air across the isobaric surfaces at these levels is negligible.

/Following

Following Charney (5) and Sutcliffe (6) we also regard the vorticity of the air movement as given with adequate approximation when geostrophic winds are substituted for the actual winds. Such substitution is not legitimate for the determination of horizontal divergence, because the divergence of the geostrophic wind field, ignoring variations of the Coriolis parameter, is zero. The divergence is therefore eliminated from the equations before the geostrophic substitution is made. Justification of this procedure will be found in the original papers of Charney and Sutcliffe and is also discussed by Eady and Sawyer (7).

Since the geostrophic wind closely approximates to the actual wind, the state of motion in our model atmosphere is adequately specified by the height of some selected isobaric surface throughout the area considered, together with the thermal wind through any selected layer (or, what is equivalent, the distribution of thickness of the layer). It is convenient to use for this purpose the two variables h_m, the height of the central pressure level of the layer at $p = p_m$, and h', the thickness of the layer from $p = p_0$ to $p = p_m$ (i.e. the thickness of the lower half of the layer).

The atmosphere does not, of course, precisely satisfy the assumptions we have made and we shall use the parameters h_m and h' to identify a particular state of the atmosphere with a corresponding state of our model. We shall then use the theoretical model to derive expressions for $\frac{\partial h_m}{\partial t}$ and $\frac{\partial h'}{\partial t}$ thus defining the rate of change of our basic variables and providing the basis for integration with respect to time, since the solutions for $\frac{\partial h_m}{\partial t}$ and $\frac{\partial h'}{\partial t}$ are determined by the values of h_m and h' themselves.

A disadvantage of the model which we have adopted is that although we can specify that initially the thermal wind does not vary in direction with height this condition will not necessarily be fulfilled subsequently. This difficulty could be partially overcome if it were assumed that the temperature in the free atmosphere were solely the result of horizontal advection as in a model suggested by Charney (2). However it has been demonstrated by Bushby (8) that the vertical motion makes a substantial contribution to temperature changes and can be assessed with reasonable approximation on a dynamical basis. It was therefore considered essential to retain the effect of vertical motion on the thickness, h' in our model and to rely on the observational evidence that the real atmosphere retains a structure broadly in accord with the assumptions of our model in order to justify the integration with respect to time of $\frac{\partial h_m}{\partial t}$ and $\frac{\partial h'}{\partial t}$.

In fact what we have done is to replace the atmosphere by a simpler model for the purpose of computing the rates of change of the basic variables h_m and h'. These rates of change we hope may be found applicable to the real atmosphere also, and when integrated through time, should give an approximate description of the behaviour of the real atmosphere.

/Notation

Notation.

We use isobaric co-ordinates, the vertical co-ordinate being the pressure p ; the horizontal co-ordinates are r and s which are curvilinear distance co-ordinates on the earth's surface (treated as a sphere).

We denote by u and v the components of the horizontal velocity, $\underline{V}$, in the directions r and s respectively, and for a measure of the vertical component we define $\tilde{w} = \frac{dp}{dt}$. The suffix g will be used to indicate a geostrophic wind or a value based on geostrophic winds. The suffixes 0, 1, and m denote values of the variables at the pressure levels p_0 , p_1 and $\frac{p_0 + p_1}{2}$ respectively.

It is convenient to use as a vertical co-ordinate a variable α defined by the equation

$$\alpha = \frac{p_0 + p_1 - 2p}{p_0 - p_1} \qquad (1)$$

α then varies from - 1 at the base of the layer to + 1 at the top of the layer.

The assumption of a uniform thermal wind per unit pressure difference through a vertical column implies,

$$\underline{V}_g = \underline{V}_{gm} + \alpha \underline{V}'_g \quad \& \quad \xi_g = \xi_{gm} + \alpha \xi'_g \qquad (2)$$

where $\underline{V}'_g$ represents the thermal wind through the layer from pressure level p_0 to pressure level p_m , and where ξ_g , ξ_{gm} and ξ'_g are the vertical components of the vorticity of the wind fields $\underline{V}_g$, $\underline{V}_{gm}$ and $\underline{V}'_g$ respectively. The relations (2) are also approximately true in respect of the actual winds.

In this notation the vorticity equation reduces after some approximation to

$$\frac{d}{dt}(\xi + l) = -(\xi + l)\, \mathrm{div}_s \underline{V} \qquad (3)$$

where l is the Coriolis parameter and

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} + \underline{V} \cdot \mathrm{grad}_s + \tilde{w} \frac{\partial}{\partial p}$$

The suffix s after a differential operator refers to differentiation with respect to r and s. All such differentiation is taken in an isobaric surface.

Equation (3) is widely used in dynamical meteorology to describe the large-scale air movements and the approximations need not be further discussed here.

In the present co-ordinate system the equation of continuity is

$$\frac{\partial \tilde{w}}{\partial p} = -\mathrm{div}_s \underline{V} \qquad (4)$$

and we also note that

$$d\alpha = -\frac{2\,dp}{p_0 - p_1} \qquad (5)$$

The equation for the mean motion of the atmospheric layer.

In order to form an equation for the mean horizontal motion of the atmospheric layer we substitute from (4) in (3), The resulting equation does not contain the divergence explicitly and we may approximate by substituting the geostrophic winds for the actual winds. Thus

$$\frac{\partial}{\partial t}\left(\zeta_g + f\right) + \underline{V}_g \cdot \mathrm{grad}_s\left(\zeta_g + f\right) + \tilde{\omega}\frac{\partial}{\partial p}\left(\zeta_g + f\right) = \left(\zeta_g + f\right)\frac{\partial \tilde{\omega}}{\partial p} \tag{6}$$

Substitution from (2) in (6) and integration with respect to α from - 1 to + 1 then gives

$$\int_{-1}^{+1}\left\{\frac{\partial}{\partial t}\left(\zeta_{gm} + \alpha\zeta_g'\right) + \left(\underline{V}_{gm} + \alpha\underline{V}_g'\right)\cdot\mathrm{grad}_s\left(\zeta_{gm} + \alpha\zeta_g' + f\right) + \tilde{\omega}\frac{\partial}{\partial p}\left(\zeta_{gm} + \alpha\zeta_g' + f\right)\right\} d\alpha$$
$$- \int_{-1}^{+1}\left(f + \zeta_{gm} + \alpha\zeta_g'\right)\frac{\partial \tilde{\omega}}{\partial p}\, d\alpha = 0 \tag{7}$$

Recalling that ζ_{gm}, $\underline{V}_{gm}$, ζ_g' and $\underline{V}_g'$ are independent of α, and substituting $\frac{\partial}{\partial p} = -\frac{2}{p_0 - p_1}\frac{\partial}{\partial \alpha}$, the following equation is obtained, viz:-

$$\frac{\partial \zeta_{gm}}{\partial t} + \underline{V}_{gm}\cdot\mathrm{grad}_s\left(\zeta_{gm} + f\right) + \tfrac{1}{3}\underline{V}_g'\cdot\mathrm{grad}_s\zeta_g' - \frac{2\zeta_g'}{p_0 - p_1}\int_{-1}^{+1}\tilde{\omega}\, d\alpha = 0 \tag{8}$$

Here we have made use of the fact that $\tilde{\omega} = 0$ at $\alpha = \pm 1$. Charney's equation for the motion at the "equivalent barotropic level" was obtained by a similar technique.

The equation for the thickness change.

An equation for the local rate of thickness change of an isobaric layer has been given by Sutcliffe and Forsdyke (9). They separate the thickness change into three parts; advective, dynamical (due to vertical motion) and non-adiabatic.

In the present notation the advective component of thickness change in the layer from p_0 to p_m is given by

$$C = -\overline{\underline{V}}\cdot\mathrm{grad}_s h', \tag{9}$$

where $\overline{\underline{V}}$ is the mean wind in the layer considered. If we make use of the geostrophic approximation and use equation (2), we obtain

$$\overline{\underline{V}} = \underline{V}_{gm} + \underline{V}_g'\int_{-1}^{0}\alpha\, d\alpha$$

Thus
$$C = -\underline{V}_{gm}\cdot\mathrm{grad}_s h' \tag{10}$$

In the present notation the dynamical component of the thickness tendency is

$$D = -\frac{R}{g}\int_{p_m}^{p_0}\Gamma_p\,\tilde{\omega}\, d(\log p) = -(p_0 - p_1)\frac{R}{2g}\int_{-1}^{0}\Gamma_p\frac{\tilde{\omega}}{p}\, d\alpha \tag{11}$$

where R is the gas constant;

/ Γ_p is

T_p is a measure of the departure of the lapse rate from the adiabatic given by $T_p = \left(\frac{\partial T}{\partial p} - \frac{\gamma}{g\rho}\right)$; γ is the appropriate wet or dry adiabatic lapse rate, and ρ is the density.

For the present we assume that the atmospheric processes are adiabatic and we consequently ignore the non-adiabatic component of the thickness change. However, if a reasonable estimate can be made of the local rate of heating, there is no reason why the term should not be retained. It would not introduce any significant complication in the final numerical integration. Combining (10) and (11) the equation for the thickness tendency therefore becomes

$$\frac{\partial h'}{\partial t} = -\underline{V}_{gm} \cdot \mathrm{grad}_s h' - \frac{p_o - p_1}{2g} R \int_{-1}^{0} T_p \frac{\tilde{w}}{p} \, d\alpha \qquad (12)$$

The development equation.

Substitution from (2) in (6) and subsequent differentiation with respect to α gives

$$\frac{\partial \xi_g'}{\partial t} + \underline{V}_g' \cdot \mathrm{grad}_s(\xi_{gm} + \ell) + \underline{V}_{gm} \cdot \mathrm{grad}_s \xi_g' + 2\alpha \underline{V}_g' \cdot \mathrm{grad}_s \xi_g' = \left(\xi_{gm} + \alpha \xi_g' + \ell\right) \frac{\partial^2 \tilde{w}}{\partial \alpha^2} \frac{d\alpha}{dp} \qquad (13)$$

Since $$\frac{\partial \xi_g'}{\partial t} = \frac{\partial}{\partial t}\left(\frac{g}{\ell} \nabla_s^2 h'\right) = \frac{g}{\ell} \nabla_s^2 \left(\frac{\partial h'}{\partial t}\right)$$

where ∇_s^2 is the surface spherical Laplacian operator,

we can substitute from (12) in (13) and we obtain

$$-\frac{2}{p_o - p_1}\left(\xi_{gm} + \alpha \xi_g' + \ell\right) \frac{\partial^2 \tilde{w}}{\partial \alpha^2} = \underline{V}_g' \cdot \mathrm{grad}_s(2\xi_{gm} + \ell) + 2\alpha \underline{V}_g' \cdot \mathrm{grad}_s \xi_g' + \frac{p_o - p_1}{2} \frac{R}{\ell} \int_{-1}^{0} \nabla_s^2 \left(\frac{T_p \tilde{w}}{p}\right) d\alpha \qquad (14)$$

This equation corresponds to that given by Sumner (10) for the relative divergence. It can be regarded as a differential equation for $\tilde{w}$ as a function of α and a similar equation has been solved as such in some simple cases. For our model to correspond with the behaviour of baroclinic systems in the atmosphere, it should provide for convergence at low levels to be balanced by divergence at high levels and vice versa. Thus the vertical velocity (and therefore $\tilde{w}$) should have a maximum somewhere near the middle of the layer falling off to zero at the upper and lower levels, p_1 and p_o. Solutions of equations corresponding to (14) in some simple cases suggest that $\tilde{w}(\alpha)$ does not depart very far from parabolic form. Thus in order to render numerical calculations tractable, it seems reasonable to endeavour to obtain an approximate solution to our problem by evaluating integrals which involve $\tilde{w}$ as a function of α on the approximate assumption that

$$\tilde{w} = \tilde{w}_m(1 - \alpha^2) \qquad (15)$$

/Returning

Returning to the vorticity equation (3) and applying it to the levels $p = p_0$ and $p = p_1$,

$$\frac{\partial \xi_1}{\partial t} + \underline{V}_1 \cdot \mathrm{grad}_s (\xi_1 + l) + \tilde{w}_1 \frac{\partial}{\partial p}(\xi_1 + l) = (\xi_1 + l)\left(\frac{\partial \tilde{w}}{\partial p}\right)_1 \quad (16)$$

$$\frac{\partial \xi_0}{\partial t} + \underline{V}_0 \cdot \mathrm{grad}_s (\xi_0 + l) + \tilde{w}_0 \frac{\partial}{\partial p}(\xi_0 + l) = (\xi_0 + l)\left(\frac{\partial \tilde{w}}{\partial p}\right)_0 \quad (17)$$

and subtracting (17) from (16) and making use of the geostrophic-assumption and the fact that $\tilde{w}_1 = \tilde{w}_0 = 0$, we obtain

$$\frac{\partial \xi_g'}{\partial t} + \underline{V}_{g_m} \cdot \mathrm{grad}_s \xi_g' + \underline{V}_g' \cdot \mathrm{grad}_s (\xi_{g_m} + l) = -\tfrac{1}{2}\left[(\xi_{g_m} + l)\left\{\left(\frac{\partial \tilde{w}}{\partial p}\right)_0 - \left(\frac{\partial \tilde{w}}{\partial p}\right)_1\right\} - \xi_g'\left\{\left(\frac{\partial \tilde{w}}{\partial p}\right)_0 + \left(\frac{\partial \tilde{w}}{\partial p}\right)_1\right\}\right]$$

$$= -\frac{\xi_{g_m} + l}{p_0 - p_1}\int_{-1}^{1} \frac{\partial^2 \tilde{w}}{\partial \alpha^2}\, d\alpha + \frac{\xi_g'}{p_0 - p_1}\int_{-1}^{1} \frac{\partial}{\partial \alpha}\left(\alpha \frac{\partial \tilde{w}}{\partial \alpha}\right) d\alpha \quad (18)$$

The variable $\tilde{w}$ enters equations (8), (12) and (18) only through various vertical mean values of functions of $\tilde{w}$ or its derivatives. The exact solution of the problem would involve a knowledge of the functional form of $\tilde{w}$ as a function of α (or height). This would involve the difficult problem of solving equation (14), the solutions of which vary with the horizontal co-ordinates. Therefore, as suggested above, it is proposed to evaluate all the integrals in equations (8), (12) and (18) on the assumption that $\tilde{w}$ has the form

$$\tilde{w} = \tilde{w}_m (1 - \alpha^2)$$

a form which is suggested by observation and is a rough approximation to the solution of (14) in simple cases.

From equation (15) it follows that

$$\left.\begin{aligned} &\int_{-1}^{1} \tilde{w}\, d\alpha = \frac{4}{3}\tilde{w}_m, \quad \int_{-1}^{1} \frac{\partial^2 \tilde{w}}{\partial \alpha^2} d\alpha = -4\tilde{w}_m, \quad \int_{-1}^{1} \frac{\partial}{\partial \alpha}\left(\alpha \frac{\partial \tilde{w}}{\partial \alpha}\right) d\alpha = 0 \\ &\text{and} \quad \int_{-1}^{0} \frac{\tilde{w}}{p}\, d\alpha = \frac{2\tilde{w}_m}{p_0 - p_1}\left\{\tfrac{1}{2}\frac{p_0 + 3p_1}{p_0 - p_1} - \frac{4p_0 p_1}{(p_0 - p_1)^2}\log_e \frac{2p_0}{p_0 + p_1}\right\} = \frac{2A\tilde{w}_m}{p_0 - p_1} \end{aligned}\right\} \quad (19)$$

where $$A \equiv \tfrac{1}{2}\frac{p_0 + 3p_1}{p_0 - p_1} - \frac{4p_0 p_1}{(p_0 - p_1)^2}\log_e \frac{2p_0}{p_0 + p_1}.$$

It is also assumed that T_p is independent of p and substitution from (19) in (8), (12) and (18) then gives

$$\frac{\partial \xi_g'}{\partial t} + \underline{V}_{gm} \cdot \text{grad}_s \xi_g' + \underline{V}_g' \cdot \text{grad}\left(\xi_{gm} + \ell\right) = \frac{4\tilde{w}_m}{p_0 - p_1}\left(\xi_{gm} + \ell\right) \tag{20}$$

$$\frac{\partial \xi_{gm}}{\partial t} + \underline{V}_{gm} \cdot \text{grad}_s\left(\xi_{gm} + \ell\right) + \tfrac{1}{3}\underline{V}_g' \cdot \text{grad}_s \xi_g' - \frac{8\tilde{w}_m \xi_g'}{3(p_0 - p_1)} = 0 \tag{21}$$

$$\frac{\partial h'}{\partial t} = -\underline{V}_{gm} \cdot \text{grad}_s h' - \frac{RAT_p \tilde{w}_m}{g} \tag{22}$$

Substitution of the geostrophic wind equation then gives

$$u_g = -\frac{g}{\ell}\frac{\partial h}{\partial s}, \quad v_g = \frac{g}{\ell}\frac{\partial h}{\partial r}, \quad \xi_g = \frac{g}{\ell}\nabla_s^2 h \quad \text{etc.}$$

and equations (20), (21) and (22) then become

$$\nabla_s^2 \frac{\partial h'}{\partial t} + J_s\left(h_m, \frac{g}{\ell}\nabla_s^2 h'\right) + J_s\left(h', \frac{g}{\ell}\nabla_s^2 h_m + \ell\right) = -\frac{4\tilde{w}_m\left(\nabla_s^2 h_m + \frac{\ell^2}{g}\right)}{p_0 - p_1} \tag{23}$$

$$\nabla_s^2 \frac{\partial h_m}{\partial t} + J_s\left(h_m, \frac{g}{\ell}\nabla_s^2 h_m + \ell\right) + \tfrac{1}{3}J_s\left(h', \frac{g}{\ell}\nabla_s^2 h'\right) = \frac{8\tilde{w}_m \nabla_s^2 h'}{3(p_0 - p_1)} \tag{24}$$

$$\frac{\partial h'}{\partial t} - \frac{g}{\ell}J_s\left(h', h_m\right) = -\frac{RAT_p \tilde{w}_m}{g} \tag{25}$$

where J_s is the surface spherical Jacobian operator,

Equations (23), (24) and (25) form a set of 3 simultaneous differential equations in h', h_m and $\tilde{w}_m$ and given sufficient boundary conditions they may be solved for h', h_m and $\tilde{w}_m$.

Method of solution.

It is convenient to represent the surface of the northern hemisphere on a plane by a conformal transformation. A suitable transformation is a stereographic projection from the south pole on to a plane perpendicular to the earth's axis and passing through the north pole. Then

$$\nabla_s^2 = \beta^2 \nabla^2, \quad \text{and} \quad J_s = \beta^2 J$$

where ∇^2 is the plane Laplacian operator and J the plane Jacobian

/operator

operator referred to rectangular Cartesian co-ordinates x and y. β is the magnification factor $\sec^2\left(\frac{\pi}{4}-\frac{\theta}{2}\right)$ where θ is the geographical latitude.

Referring to the Cartesian co-ordinate system, equations (23), (24) and (25) become

$$\nabla^2\frac{\partial h_2'}{\partial t} + J\left(h_m, \beta^2 g l^{-1}\nabla^2 h'\right) + J\left(h', \beta^2 g l^{-1}\nabla^2 h_m + l\right) = -\frac{4\tilde{w}_m\left(\nabla^2 h_m + \frac{l^2}{\beta^2 g}\right)}{p_0 - p_1} \quad (26)$$

$$\nabla^2\frac{\partial h_m}{\partial t} + J\left(h_m, \beta^2 g l^{-1}\nabla^2 h_m + l\right) + \frac{1}{3}J\left(h', \beta^2 g l^{-1}\nabla^2 h'\right) - \frac{8\tilde{w}_m\nabla^2 h'}{3(p_0 - p_1)} = 0 \quad (27)$$

$$\frac{\partial h'}{\partial t} = \beta^2 g l^{-1} J\left(h', h_m\right) - \frac{RAT_p\tilde{w}_m}{g} \quad (28)$$

For the upper and lower levels at which $\tilde{w}$ is to be zero we take a level near the surface and another near the tropopause, for example p_0 = 1000 mb. and p_1 = 200 mb. Then h_m refers to the height of the 600 mb. contour surface and h' to the thickness of the 1000-600 mb. layer. The set of equations can then be solved for h', h_m and $\tilde{w}_m$ at a finite set of points on a square grid on our map projection.

Eliminating $\tilde{w}_m$ between equations (26) and (28) we get an equation in $\frac{\partial h_2'}{\partial t}$ of the form

$$\nabla^2\frac{\partial h_2'}{\partial t} + F_1(x, y, t)\frac{\partial h_2'}{\partial t} + F_2(x, y, t) = 0 \quad (29)$$

Initially F_1 and F_2 are known at all grid points. Specifying $\frac{\partial h'}{\partial t}$ along our boundary we can then solve equation (29) for $\frac{\partial h_2'}{\partial t}$ by the relaxation technique (Southwell (11)). New values of h' at time $t+\Delta t$ can then be computed from the relationship

$$h'(t+\Delta t) = h'(t-\Delta t) + 2\Delta t\left(\frac{\partial h'}{\partial t}\right)_t \quad (30)$$

By substituting the values of $\frac{\partial h'}{\partial t}$ obtained as a solution to equation (29) in equation (28), we can obtain values for $\tilde{w}_m$. Substituting these values

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in equation (27) we get an equation of the form

$$\nabla^2 \frac{\partial h_m}{\partial t} + G_1(x,y,t)\frac{\partial h_m}{\partial t} + G_2(x,y,t) = 0 \qquad (31)$$

From this we can solve for $\frac{\partial h_m}{\partial t}$ and hence obtain h_m at time $t+\Delta t$.

This process can be repeated to obtain values of h_m and h' at time $t+n\Delta t$, but it is then necessary to specify $\frac{\partial h'}{\partial t}$, $\frac{\partial h_m}{\partial t}$, F_1, F_2, G_1, and G_2 along the boundary for the forecast period. The specification of F_1, F_2, G_1, and G_2 entails the specification of ζ_{gm} and ζ'_g along the boundary.

It has been estimated by Charney (2) when solving a similar set of equations that Δt should not be greater than 2 hours, and thus the whole process would have to be repeated 12 times before a 24 - hour forecast could be produced.

Conclusion.

Although the equations (26), (27) and (28) can be solved by hand, it would be an extremely lengthy and laborious process. Probably the most that can be hoped from hand computations is an attempt to evaluate the instantaneous thickness and height tendencies. Nevertheless it does not seem likely that a simpler set of equations can be derived which will satisfactorily predict the future behaviour of the atmosphere, and it may therefore be necessary to invoke the aid of modern electronic computing machinery to solve equations (26), (27) and (28) for a number of cases. In view of the many assumptions made in deriving these equations the results of the computations may well show that the present model is not sufficiently close to actuality to enable a better forecast to be produced than that obtained by conventional methods. It is, however, only by studying the results obtained by solving these equations that it will be possible to decide which are the most important restrictions inherent in our model and by removing them thus proceed to a more realistic model of the atmosphere.

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FEB 24 1953

Review of Long Range Forecasting Methods

by

J.M. Stagg (Meteorological Office).

Introduction.

This review is concerned with methods that have been proposed or explored for the preparation as a regular procedure of forecasts for periods longer than can be dealt with by extension of normal day-to-day synoptic practice. To emphasise the difference between the precise and definite forecasts aimed at in short period methods and the kind of statement in general terms which is the most that can be expected from any long range system at present in sight, new words should be introduced. "Forecast" and "forecasting" will however be used in this review with their usual meteorological sense appropriately broadened.

The principal processes and phenomena that have been tried or suggested, singly or in combination, in the preparation of long range forecasts are:

1. Regression equations and multiple correlations.
2. Pressure waves.
3. Symmetry patterns.
4. Climatic singularities.
5. Pressure trends and other trend phenomena.
6. Long waves in the zonal circulation.
7. Kinematics of air flow and pressure cells.
8. Analogues and the repetition of synoptic patterns.
9. Solar phenomena.
10. Atmospheric ozone.

1. Regression equations. Relationships expressed in the form of multiple correlations or regression equations between future weather events in one region and present or past events in the same or other regions have been applied to forecasting in various parts of the world, particularly India, U.S.A. and Germany. The method depends on finding close connections between the element to be forecast and other elements or processes (such as the Southern Oscillation), and on the persistence of each of the constituent correlations. Because few of the relationships among meteorological events are simple or direct, especially when they are

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separated in time as they have to be for application to forecasting, the correlations are liable to be unstable, and therefore the regression equations require to be modified from time to time so that new or improved correlations may replace those whose validity has deteriorated.

In regions where large scale processes like a monsoon dominate weather for several months at a time, regression procedures have proved of value in assessing whether such elements as temperature or rainfall are likely to be substantially above or below normal. But even in these regions, the correlations are seldom stable and though the physical processes behind the regression equation may be broadly understood the relationships are purely statistical so that the forecaster who uses them can exercise only such discretion and judgement as is allowed him in the formulation of the forecast.

The search for correlations among meteorological parameters in different parts of the world can improve understanding of the functioning of the atmosphere, and the use of regression equations can be helpful in conjunction with other methods, for example in relating the amount of ice in the north west Atlantic in the spring and early summer months to the mean latitude of depression tracks in the north east Atlantic later in the year. But forecasting by this means alone cannot lead far, especially in a meteorologically complex area like the British Isles, even if the correlations were elaborated and extended to take account of the more recently available upper air parameters.

2. Pressure Waves. Analysis of barograms of surface pressure from stations in middle and high latitudes can provide evidence for the existence of apparently regular wavelike oscillations whose principal periods are approximately 72 and 48 days and their submultiples. If the existence over a sufficiently wide area of one or more of these wave trains can be diagnosed early enough, it may be used to estimate future pressure at a network of stations and so lead to a forecast surface pressure chart. A similar procedure has been applied to pressure height contours at 500 mb. and higher levels. Pressure waves have been extensively investigated and applied to long range forecasting in Germany where indeed other important properties have been attributed to particular waves. For example the 72 and 36 day waves are believed to extend into the stratosphere and exercise control over the direction of travel of the 6 day wave and the movements of the centres of the 24 hour isallobaric pattern: the amplitude of the 24 day wave on the other hand decreases rapidly with height and is conspicuous only in the high latitudes. Pressure waves have also been investigated in this country.

Postulating a single or double layered atmosphere having prescribed kinds of temperature structure it may be shown that the atmosphere can oscillate in a number of periods ranging from 6 to 72 days, and also that the oscillations are propagated from west to east in middle and high latitudes. There is no unanimity about the origin of these oscillations: they have been variously attributed to unstable gravitational waves between troposphere and stratosphere induced by extension upwards of instability waves on the polar front, to roughly periodic outbreaks of cold air from the polar calotte or the breaking away of cold pools, to waves dependent upon the temperature differences between the main land and sea masses around zones of the earth, and even to precession movements in the wind circulation zones around the earth set up by unsymmetrical pressure fields, tilting the wind belts out of their appropriate latitude. No one of these explanations is physically satisfactory or complete.

But even without a satisfactory physical basis the existence of regular trains of periodic oscillations might be of value in forecasting if they were sufficiently regular, of big enough amplitude and sustained over long enough intervals. Unfortunately these requirements are seldom, if ever, all met.

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The wavelengths of the dominant waves vary over a considerable range and a sequence of oscillations seldom continues more than three or four cycles: the use of the word wave (in the sense of a sustained and regular recurrence pattern) in connection with the tendency of the atmosphere to oscillate is misleading.

It has to be concluded that wave-like oscillations of the atmosphere probably do exist and that further study may well be valuable as likely to give further information about the structure and behaviour of the atmosphere; but pressure waves are too transient, irregular, and perhaps localised, and variable from season to season and year to year to be of assistance in forecasting.

3. Symmetry Patterns. From time to time the pattern formed by a sequence of barograms at a station over an interval of a month or more from a particular date is roughly a direct, or less frequently an inverted, mirror image of the pattern before that date. When this occurs the patterns at other stations in the same region show similar characteristics, and the pattern as a whole appears to be translated into other areas as if it were embedded in the circulation. If a pattern of this kind can be recognised before, or by the time of, the central date as likely to be a symmetry pattern, harmonic or other analysis can be employed to extend the pattern into the future at a network of stations and so allow it to be applied to forecasting the surface pressure distribution.

The chief components in a symmetry pattern are found to be the same as in pressure waves, viz. 72 days and its submultiples, and the pattern is therefore explained by the simultaneous existence of pressure waves, phased so that the extreme minimum or maximum values all coincide at the point of symmetry.

If this is the only physical explanation of symmetry patterns - and it fits the facts - then the value of their application to forecasting is as precarious as is the use of pressure waves. There are however some further aspects. The 24 day wave is one of the most dominant constituents of every symmetry pattern and this, as explained in the preceding section, has been attributed to major outbursts of polar air from the Arctic basin: if there is a tendency for these outbursts to occur at about 24 day intervals, the occurrence of a roughly symmetrical pattern a few times each year may express little more than the tendency for one prolonged weather type (e.g. south westerly, with its series of depressions each having its particular family characteristics) to be interrupted by a polar air outburst. This leaves open the questions whether the polar air surges do have a 24 day rhythm, even if only for two or three cycles, and whether, if there is an approximate rhythm, it is inherent in the build up and break down of pressure over the polar cap, or whether it is started by a smaller and less spectacular tendency for the atmosphere to oscillate naturally at about 24 day intervals. There is so much doubt about the answers to these questions, and indeed about any system which depends on the analysis of an element like pressure having intrinsic coherence in its day to day variation that some authorities have concluded that symmetry patterns are the result of fortuitous circumstances in the pressure field. All but a very few patterns are recognisable only after detailed analysis; they are found only a few times a year, mainly in winter.

To be of value in forecasting the existence of a pattern and the symmetry date must be known as soon as possible after the central point of symmetry has been reached and its main features must be reproducible by a few of the harmonic components. But there is no known method of forecasting the symmetry date, and even if there were the lack of persistence in the pattern and uncertainty about its form and amplitude after the central date make the information of very doubtful value. Methods have been devised and applied both in Germany and in this country by which the

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pressure field over a whole region can be analysed in means of one to three or more days but such statistical devices do not compensate for the inherent weaknesses of the method both in physical interpretation and practical application.

4. Climatic singularities. In probably every region of the world there is a tendency for particular types of weather or abrupt changes in its annual course to recur round about the same calendar dates each year. These singularities (as the German meteorologists who have studied them most have called them) include large scale phenomena like the beginning of the Indian monsoon or, at the other extreme, discontinuities in the otherwise smooth trend of mean temperature through the year such as were first discussed by Mossman and Buchan for south east Scotland. By classifying the type of surface pressure distribution over the British Isles on each day of the year for a long series of years Lamb has recently shown that many of the singularities listed by German meteorologists show up also in the weather of this country.

When the singularities are restricted to a comparatively few well-marked events each with a genuine physical basis there can be little doubt of their value in forecasting: the reality and value of many singularities become more questionable when their number is increased so that 6 or 8 are expected each month. It is admittedly dangerous to discount a phenomenon because it has no obvious explanation, and until more is known of the mechanism which leads to the major changes in the zonal circulation in middle latitudes of the troposphere, or what is probably the same thing, to the irregular interchanges of heat and momentum between the zonal circulations, the explanation of the high incidence of particular pressure distributions over an area on days linked with the calendar must be lacking. But the reality of individual singularities can be accepted and used as an auxiliary factor in forecasting only when their frequency of occurrence over a long series of years is substantially higher than random, and when some explanation, at least proximate, can be offered when they fail to occur or are much displaced from their place in the calendar.

Up to now the only explanation of singularities is in terms of pressure waves, particularly the 72-day wave and its submultiples. According to this, the wave processes determine the pattern of surface pressure distribution and through it the major changes from zonal to meridional circulation: on this theory the absence or displacement of singularities can be taken as a warning that another system of waves has been established or the phase altered and this in itself is a tool in the experienced forecaster's hand.

In current synoptic language many of the more important singularities in this part of the hemisphere are probably associated with "blocking" of the zonal westerlies: an intensive study of this phenomenon might therefore throw light on the cause of authentic singularities. Alternatively the study of real singularities, critically segregated from the merely fortuitous, might well be extended to the simultaneous events in the middle and higher troposphere and lower stratosphere: even though it might not lead to conclusions directly applicable to long range forecasting, a study of this kind would be one method of obtaining further experience of the three-dimensional behaviour of the atmosphere which could hardly fail to have applications to forecasting practice.

5. Pressure trends and other trend phenomena. As the amount of detailed information in a series of synoptic charts at 6, 12 or even 24 hour intervals is apt to conceal the slow though widespread changes that may be in progress it has been considered that the use of charts drawn to represent the deviation of the mean surface pressure distribution for a number of days from the long term normal distribution would bring these slower trends into prominence. Charts of pressure anomaly for various combinations of days have been used in

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this way to identify centres of abnormally high or low pressure, and to study their movements with the object of extrapolating their trajectories. But the results of at least one experiment on these lines conducted by C.E.P. Brooks in this country using 6, 12 and 24 day mean charts were not encouraging.

A similar kind of procedure, though not using deviations from normal, forms part of other systems of forecasting. For example trends of index figures representing the strength of the zonal circulation have been applied to help in assessing the future circulation pattern, and surface pressure profiles at a network of stations are similarly used as a clue to the general trend of pressure over an area.

In these and other applications trend procedures are quite empirical, and for want of knowledge of the causes of the trend the bias in extrapolation must always be towards normal. It is therefore not surprising that the contribution of trend methods to long range forecasting procedures has not been found helpful; at best they can be only auxiliary aids.

6. Long waves in the zonal circulation. The working hypotheses which allow long wave ideas to be applied to forecasting are, firstly that the waves are more slowly variable and their immediate behaviour is a little more amenable to forecasting than the surface pressure field, and secondly that the main features of, and changes in, the surface field are related to the pattern of the upper tropospheric circulation. Unfortunately these hypotheses are not conspicuously reliable beyond the second or third day. The only approximately realistic theory of long waves so far developed rests on assumptions both as regards the physical characteristics of the medium and the nature of the undulations which the atmosphere seldom recognises, so that substantial empirical and subjective factors have to be superposed when the theory is applied in practice. Even then inferences can be made about speed and wavelength only of waves that already exist: little help can be given about the time and place of formation of new waves or about the intensification or decay and disappearance of old ones. For these aspects of the long wave pattern the forecaster must use the second hypothesis in reverse, and assess the changes in the pattern of tropospheric circulation that are likely to be brought about by the development and movement of surface features. And reliance on this interaction is impaired in that its extent depends among other things on the scale and intensity of the surface developments. Small and shallow depressions and high pressure cells are steered by the circulation pattern without altering it substantially, but the larger and more vigorous features of the pressure field impress their effects on the circulation.

Notwithstanding these weaknesses, techniques based on circulation theory and experience have contributed to the preparation of 5 and, more recently, 30 day forecasts in U.S.A. and 10 day forecasts in Germany. In Germany the technique adopted is a grafting of a step-by-step extension of daily synoptic practice for the first five days on to a forecast derived from a combination of other methods for the second half of each 10 day period. In U.S.A. 5 and 30 day mean configurations of the surface and upper air circulation patterns are dealt with as units, using step-by-step daily practice for the first 2 or 3 days as a check in the preparation of the 5 day forecast.

Apart from the uncertainties inherent in the use of circulation ideas, adaptation to 5 or more day mean patterns introduces further doubts. It can be contended that mean circulation charts for even as many as 5 days can have little physical significance, or more precisely, that the equations of circulation theory cannot be satisfied by means of meteorological elements which do not correspond with any real state. On the other hand

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those who use them claim that 5 and even 30 day mean circulation charts and their surface field counterparts have real physical individuality and that only mean charts can disclose the "centres of action" which regulate the longer period trends in weather: they claim that the results of long wave theory can be applied to mean charts with only a little more empirical modification than is already necessary for daily charts, and, further, that the broad features of weather, e.g. departure of temperature and rainfall from normal, can be inferred from the mean surface charts deduced from the mean charts of circulation. And in this connection it is appropriate to recall that monthly mean surface pressure charts for the British Isles have been officially published for many years.

Without experience of the use of mean charts it is difficult to be categorical about them: if experience were to confirm that they have physical significance, it might be expected that methods of dealing with them, either empirically or even theoretically, would be devised, keeping in mind that the inferences to be made would relate only to substantial departures from normal.

7. Kinematics of air flow and pressure cells. With their weather dominated for much of the year by anticyclones and their vast plains providing a natural laboratory for the study of air mass and pressure cell movements, it is not surprising that Russian meteorologists should have been led to base their system of forecasting on trajectories of high pressure cells. Though analogues and rhythms and to a minor extent dynamical reasoning also play an important part in the system begun by Multanovski (M) and elaborated by Pagava (P), the movements of anticyclonic cells and the pressure fields associated with them form the "natural" basis of their "natural" processes within "natural" regions.

According to the M-P school the anticyclonic nuclei that invade U.S.S.R. territory, come predominantly either from the Azores, from Greenland or from the Arctic basin via Taimyr. They have preferred axes along which they move and the frequency of invasion from each of the main centres and the orientation of their trajectories depend on the time of year and on the kind of season. In some years, for example those characterised by unusually mild autumns, the frequency and the orientation of the tracks followed are consistently abnormal.

Once a high pressure cell has started to migrate its future track is approximately known and by reference to charts which have been constructed to show the positions of the other high and low pressure cells relative to similar tracks followed about the same time in other years, the general form of the surface pressure distribution linked with a particular anticyclonic axis can be forecast. As the likely orientation of a high pressure cell trajectory can be determined within 2 or 3 days after the nucleus of the cell has appeared and as the average life of a cell is 10-12 days, it is possible by the M-P system to forecast 7 or 8 days ahead.

This period of 7-8 days can be extended by other considerations. At a time varying from 30 to 35 days before the start of a high pressure cell process, there are recognisable premonitory symptoms in the surface field, and in the interval between the appearance of these warnings and the start of the process there are several recognisable phases of 6 to 15 days each characteristic of a stage in the progress towards the main (high pressure cell) process. In addition the Russian long range forecasting school has discovered that there is a tendency to rhythms with still longer periods in the behaviour of the atmosphere. The principal rhythms are of 3 and 5 months' duration and are related to major outbreaks of air south or south westwards from the Taimyr area.

It is difficult to form a just appreciation of the M-P system of forecasting, and even if it were proved to be successful in Russia it does not

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follow that it could be applied to any other areas because Russia's extent (east to west and into the Arctic basin) and surface topography relative to the general circulation, are unique. Nevertheless there are lessons to be learnt:

(1) Russian meteorologists have clearly made an intensive study of dynamical climatology on the lines sketched by Bergeron.

(2) As a sequel to this they have recognised and applied the idea of natural periods and natural regions; rejecting the week, month and season as their units of time they accept as their intervals for study the particular kind of weather process likely to be dominant. They make their composite charts, and base their procedure on these more (meteorologically) natural intervals and issue their forecasts accordingly.

(3) Stress is laid on the genesis, development and track of the anticyclonic cells. This may be as natural and as necessary in Russia as it is natural and necessary in an area situated like the British Isles to stress the role of depressions. But even in less anticyclonically controlled areas than Russia it might be rewarding to give more attention to the growth, migration and decay of high pressure cells.

8. Analogues and the repetition of synoptic patterns. In a sense the use of analogues is an important feature of all forecasting, short and long range alike: no forecaster can afford to ignore experience of apparently similar situations in the past, and as a means of conserving that experience and making it readily available when required it is no more degrading to use cards with holes or photographs on them than to use regression equations, or other arithmetical means of putting unexplained relationships into number form. The regression equation may indeed be regarded as the less helpful, whether it be used for assessing the speed and direction of upper wind at a place four hours after an observation or for the foreshadowing of rainfall in the Indian monsoon: the result of the arithmetic excludes judgement, but a well devised and efficient system of analogues leaves the forecaster scope for discretion.

In short range forecasting frequent observations, efficient communications and simple extrapolation go a long way to fill blanks in the forecaster's knowledge of the physical processes at work so that the use of mechanical means of aiding his memory or making up for lack of experience is not necessary, and is even to be deprecated. Physical reasoning should replace memory. But in the present state of knowledge of the major, longer term processes which control the weather over an interval of 10 days or a month, and until at least as much is known about them as about the 6, 12 and 24 hour processes, some system of summarising and recording experience is necessary if forecasts are to be made in turn by members of a team. The officer who devises the system may become sufficiently saturated with weather history and its modes of evolution to be able to rely on his experience as a guide for the future: but he has to make that experience available to others, and in default of an adequate theory a scientifically devised system of analogues may be as useful as any other.

Hitherto analogue systems have been mainly concerned with aspects of the surface pressure pattern and have used as their basis the relative dispositions of the main cells, or the origin and orientation of anti-cyclonic cells, or direction of the dominant air flow, or combinations of these. And in all intelligent use of analogues care has been taken (1) to ensure that the similarity has extended beyond instantaneous patterns to the immediately preceding development, and also (2) to take account of the part likely to be played in future development by the

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difference in the earlier histories of the air masses in the pattern. Further, analogues have seldom been used as the sole guide. But from what is now known about the importance for surface weather of the upper troposphere and lower stratosphere, it is likely that these systems have been of indifferent value because they could not be extended to take into consideration the thermal or contour patterns relating to those higher layers.

It is not to be inferred from this that the present reviewer regards analogue systems as a necessary and much less a permanent technique in forecasting: on the contrary he has seen how valueless and misleading they can be in practice. But if it be accepted that a preliminary to starting any system of forecasting for periods more than 2 or 3 days ahead must be an intensive study (from a wider perspective than short period forecasting requires) of modes of evolution of particular types of thermal and flow pattern in the middle and higher troposphere and lower stratosphere the information derived from the study must be systematised. The best way to systematise it is to devise a comprehensive theory which takes account rigorously of the dynamics and thermodynamics of the whole field of interaction. But it is unlikely that that will be available for many years. So unless each member of the forecasting team is to make his own study, and build up his own background, an adequate system for comparing and grouping situations of like evolution is necessary - always keeping in mind that the system is only an interim measure and that the forecasting team must be weaned away from it at least as fast as physical knowledge of the underlying processes replaces empiricism.

9. Solar Phenomena. As regular seasonal changes in solar radiation so obviously affect the general temperature in each hemisphere and the tempo and intensity of weather changes, it is contended that the irregular changes in the sun's output of radiation which are assumed to vary in parallel with particular indices of its surface activity should affect weather to an appropriate degree. Much work has been done in many countries to discover relationships; mean annual and monthly and daily values of sunspot numbers, and of measures of the solar constant, numbers of faculae and other indices have been correlated with a great range of climatic and weather parameters. The results, though of interest for the light they shed here and there (or fail to shed) on indirect relationships between solar activity and terrestrial weather, are of no value for forecasting: the only reliable and persistent correlation so far found is between sunspots and temperature in the more cloudy regions of the tropics and this (negative) correlation shows up only in the mean annual values. More must be known about the relationships between measures of solar activity and the output of radiation from the sun in various spectral regions, particularly in the ultra violet, and about the possible effects of corpuscular radiation, before solar phenomena can be of use in forecasting: and even then it is not clear that it will be any easier to forecast behaviour on the sun than the behaviour of our own atmosphere, except perhaps in a general way for the 11 (or 22) year cycle.

10. Ozone. Although much is now known about the life history of atmospheric ozone, its distribution over the globe, the seasonal changes in that distribution and the intimate association between the distribution and the patterns of the surface pressure field especially in moderate and high latitudes, measurements of ozone are so far of value to the forecaster only as pointers to the nature of the upper air processes that accompany the birth, growth and decay of low and high pressure cells in the lower troposphere. Even if the redistribution of ozone by some extra-terrestrial influence preceded rather than accompanied the changes in the higher circulation pattern and surface field it would still be necessary to forecast the behaviour of the mechanism which effects the irregular changes in the amount and distribution of the ozone.

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11. Conclusions. The only valid inference from the foregoing brief reviews is that no ready made procedure or combination of procedures exists which recommends itself as suitable for adoption in detail for long range forecasting in the British Isles. The objections are not only that most of the methods in use or that have been explored are primarily applicable, if at all, to large continental areas and also that they rely too much on empirical and impermanent relations with the subjective factor too dominant for them to be acceptable as a basis for a regular system in our area: the objection rather is that too few of them are founded on lines which will allow organic development into sound physical methods as knowledge of the atmosphere grows. Even with present knowledge of the complexity of the atmosphere and the interactions of its parts, any worthwhile system of forecasting beyond 2 or 3 days, like any modern system of day-to-day synoptic forecasting, requires the building up of an elaborate procedure and the accumulation of much experience in its use. Unless much effort is to be wasted it is therefore necessary to ensure that any new method to be started has an adequate physical background and is conceived with sufficient elasticity to permit modification for absorbing new ideas without wholesale abandonment of technique and experience.

Guidance on the choice of a system fulfilling this requirement would be at hand if a confident answer could be given to the question, where is the principal seat of those physical processes that result in, or have as their necessary concomitants, the changes in surface pressure distribution and associated fronts which have our weather as end products. Unfortunately that question remains open. But increasingly in recent years the zonal circulation in the middle and higher troposphere, its day-to-day, seasonal and annual changes in intensity and latitude, its maintenance and role in equalising the gains and losses of heat and momentum in adjacent zones and the relationship of its varied configurations with the surface pressure field - these have rightly assumed great importance. Progress in any of these aspects of the subject has potential significance not only in helping to provide an answer to the basic question: it cannot avoid having practical application to long range (as indeed also to short range) forecasting. It may well be that the zonal circulation in the upper troposphere is as dependent for its complete specification on what takes place in the lower stratosphere as it probably is on events nearer the earth's surface: but sufficiently detailed and extensive knowledge of the modes of interaction between the troposphere and stratosphere must await improvement of observational practice at high levels and can be grafted on to an adequately designed forecasting system as that knowledge becomes available.

If the foregoing line of thought is broadly right one inference appropriate to the present stage of our knowledge about what goes on in the higher (meteorological) atmosphere might be to defer setting up a long range forecasting system until we have a more generally acceptable conception of how the circulation works. A more justifiable inference is that the sooner a start is made in planning the strategy and the tactics of a system the sooner will the main problems fall into their proper priority for solution.

The design of a forecasting system is in part also determined by the period to be covered by the forecast and the intervals of issue. As explained earlier the Multanovski-Pagava school adheres to "natural" periods whose duration is defined by the time taken by the main weather processes to complete themselves: forecasts are issued at variable times adapted to these natural periods and any change from that procedure has resulted in reduced accuracy. A distant relative of this kind of procedure has been long in use in this country in the form of further outlooks, added to forecasts when circumstances allow. It is probably not regarded by the users as satisfactory and hardly constitutes a forecasting system. The aim

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should be the issue at regular intervals of forecasts for fixed periods even though the period of issue has at first to be restricted to a particular season of the year.

In spite of the advantages of the variable "natural" period, it will be difficult in this country to ignore the month (or 30 day period) and its subdivisions of 10 and 5 days: they have been found to be the most satisfactory elsewhere from the user's point of view and also for the practical aspects of the routine in preparing the forecasts and of the adaptation of existing statistics. But as the same basic system is to be used for all periods, it is clear on this reasoning that it should be designed with the aim of producing forecasts for 5 days in the first instance, developing to 10 and 30 day periods as opportunity and increasing knowledge allow.

Seasonal forecasts of useful value for this country are unlikely to be attainable for many years. And yet, by application of by-products (e.g. long term fluctuations in the mean latitude and intensity of flow of the westerly zonal circulation) of the studies that will be necessary for the main forecasting system it might be found that some very general information could be offered about departures from normal of such elements as temperature and precipitation.

It might of course turn out that the basic supposition underlying these conclusions is wrong and that the zonal circulation is unamenable to any physical or statistical treatment which will allow its intensity, position and configuration to be determined even 5 days in advance. It is already believed in some quarters that forecasting 2 or 3 days ahead may be impracticable because the zonal circulation, like the frontal waves in the lower troposphere is inherently unstable. If that were confirmed it might still be possible to give seasonal guidance and to supplement that at irregular intervals by information derived from a statistical examination of the various modes by which the zonal flow can change from one broad pattern to another.

12. Summary of Recommendations.

1. Any system of long range forecasting to be developed in this country should be based on the modes of behaviour of the zonal westerly circulation in the middle and higher troposphere, using contours at the 500, 300 and, if practicable, the 100 mb. levels, or the corresponding thermal patterns as the main working tools. The aim should be to issue forecasts in general terms regularly for 5, 10 and ultimately 30 day intervals, though not necessarily throughout the year.

2. As one method of determining the future configuration of the circulation pattern may be by mean charts, a study should be made of charts of contours at the 500 mb. and higher levels meaned for 5 or more days, with a view to (a) assessing their significance and potentialities for forecasting, and (b) devising ways of deriving the general form of each mean chart from the preceding sequence of mean charts.

3. Every use should be made of advance in knowledge about the processes (of heat and momentum interchange with the circulation of neighbouring zones) by which the middle latitude circulation is maintained and caused to vary. If practicable, contributions should be made to knowledge of these processes. The object would be to achieve a procedure by which abnormal influxes of heat or momentum into the zone could be detected as a routine procedure while in progress, and their effects on the intensity, latitude, distribution, etc. of the flow assessed.

4. As a means of gaining further insight into the modes of evolution of patterns of zonal flow in association with selected classes of surface pressure distribution, sequences of 500 mb. and higher level contour charts

/should be

should be analysed on days preceding the same authentic singularities in each year, and also on days preceding major changes of type.

5. Study should be made (perhaps by the Climatological Division working in close co-operation with the Forecasting Research Division) of the broad features and trend of weather, in terms of temperature, precipitation, etc. anomalies from normal, associated with mean surface pressure distribution for 5, 10 and 30 days. The purpose of this would be to allow statements of the expected weather to be quickly formulated after the mean surface pressure distribution has been deduced from corresponding mean contour charts. It might be

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Footnote:- Gee-fixes are positions obtained by Gee, i.e. radio transmissions from a master and two slave stations on the ground, the probable error of such positions being about $\frac{1}{4}$ mile in the best circumstances; at 300 miles distant it may possibly be 5 miles.

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Sheet #2

found that the use of the intermediary surface pressure charts could in time be dispensed with.

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6. In addition to continuing to accumulate experience of the ways in which the zonal circulation in the middle troposphere is affected by the development and movement of high and low pressure circulations in the lower troposphere and vice versa, study should be made wherever the network of observations allows of the corresponding interactions between the lower stratosphere and the main zonal circulation.

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Committee (London),
M.R.P. No..... by Mr.....
A copy is available in the Library

M.R.P. 722
S.C.II/107
8th April, 1952.

AIR MINISTRY

METEOROLOGICAL RESEARCH COMMITTEE

A.D. 1695

The Profile of a Jet Stream observed 18th January, 1952.

AIR WEATHER SERVICE
TECHNICAL REFERENCE FILES

by

G. W. Hurst (Meteorological Office).

1. Introduction.

During the course of a discussion of M.R.P. 615 Mr. Sawyer suggested that a profile of a jet stream over the Persian Gulf might be obtained by flying an aircraft straight and level which was photographing the ground at regular intervals. It was not found possible at the time to perform this experiment in that locality but a suggestion was made to the R.A.F. Benson that a specimen run might be made in this country. The result is given in the following note.

2. The Brief.

On the morning of 18-1-52, a very well-marked jet stream was affecting the western half of England and Wales. Speeds of up to 178 knots had been reported (Aldergrove 300 mb at 0300 GMT), with speeds in the east of England of the order of 50+ knots. It was a day of solid northerlies at all heights, but with the prospect of only well broken cloud over the mainland of England and Wales. Accordingly, arrangements were made with the C.O. 540 Squadron for a Mosquito to fly at 30,000' (indicated) from the Yarmouth area to Pembrokeshire to take photographs, to fly across to Lands End, and to take more photagraphs from there on a track towards Benson; these two tracks were chosen to be more or less at right angles to the 300mb winds. The crew, Fg. Off. Slaymaker and Flt.Lt. Vigers, was impressed with the need to maintain constant height, speed and course throughout both these two legs. The photographic equipment consisted of two cameras: a vertical (F52, 10 inch focal length) and a port facing oblique (F24, 8 inch focal length). The latter was carried for interest only, and as it was unfortunately mounted with a $17\frac{1}{2}^{\circ}$ depression instead of the normal 10°, with a consequent loss of horizon, its photographs were of little interest, and are not discussed below. The time intervals used between exposures was 12 seconds (vertical) and 36 seconds (port oblique). The navigator was asked to keep an accurate position log, and to obtain Gee-fixes* and winds as often as possible.

3. The Execution.

The flight took place very much as planned. Photography started near Lowestoft and continued without break on a course of 292° (magnetic) to just beyond 5°W, south of Pembrokeshire. No ground detail was visible in the east, where there was extensive and well-developed cumulus cloud, but from about the Greenwich Meridian to the point of leaving the Welsh coast at the Gower Peninsula, the ground was clearly discernible and identifiable in all save a handful of photographs. From that point to the end of the first leg the cloud over the sea was well broken. The time taken for these 282 photographs was 1436-1537 GMT. Photography recommenced near Helston at 1550 with the aircraft on a course of 041° (magnetic) and a further 158 photographs were taken over southwest England, the run ceasing near to Taunton at 1621 GMT; ground detail is unfortunately far poorer on this leg, and in many prints, no detail at all could be picked out.

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Footnote:- Gee-fixes are positions obtained by Gee, i.e. radio transmissions from a master and two slave stations on the ground, the probable error of such positions being about $\frac{1}{4}$ mile in the best circumstances; at 300 miles distant it may possibly be 5 miles.

It was not possible to use the photographs to derive winds for the prints east of Greenwich, but the navigator obtained accurate Gee-fixes to this point, and there is no reason to doubt the winds which he obtained; accordingly, his winds have been accepted to this point. From 0-4°W (prints 5070-5236) it was possible in nearly all cases to identify the centre point of the print (i.e. the point at right angles to the plane of the aircraft) and winds were in the first instance calculated between each successive pair of prints (just over a mile). The resulting winds proved very erratic, as might be expected, in view of the effect of the slightest change in attitude of the aircraft magnified over 30,000'. Accordingly, five print means were taken. These gave much more consistent results, but finally means were taken for five print intervals for which the end points had been adjusted slightly if necessary if they appeared out of harmony with the neighbouring points; very reasonable winds indeed emerged. There is a short break between 5140-150, due to obvious divergence of the aircraft momentarily from its track, and the run between prints 5144-147 has been ignored. The navigator's Gee-winds (meaned over much broader intervals than derived from the photographs) agreed very well as far as $2\frac{1}{2}$°W, but at this point the navigator changed Gee-stations and obviously the winds to the west are far too low; it was decided to explore the possibility of assessing the rate of drift across the cloud (mainly thin and well-broken stratocumulus) in the west, and deriving winds. Accordingly winds from rate of drift were obtained near and just to the east of the Gower Peninsula, and compared with the winds derived from ground detail; it was found that if the cloud were given a speed of 56 knots, agreement was very close. Accordingly the photographs from 5236-282 were analysed in this manner, mean rate of drift of cloud over five prints being obtained. A very reasonable answer was obtained apart from 5281 and 5282 which appeared to be on a turn.

The winds on the return run were obtained from ground fixes, again utilising approximately every fifth print. There were some long gaps with no recognisable ground detail (notably 5357-5390) and no attempt was made to smooth out the points. The interest of this run was only secondary, as the wind speed increased only slightly as the axis of the jet stream was approached.

4. Results.

Enclosed with this note are an outline chart (Fig. 1) shewing the surface synoptic chart for 1500 GMT for that day, with the 1500 winds from the radio-sonde stations for the 300mb level plotted; a plotting chart shewing the track of the aircraft, times of flight and vectorial representations of the winds (Fig.2) and a histogram shewing the wind strength distribution on the outward run (Fig. 3).

On the synoptic chart has been inserted the line of maximum winds at the 300mb level derived from consideration of winds reported at that level on the 18th and 19th from the various radio-sonde stations. This line is necessarily somewhat uncertain as it is difficult to say with confidence when the maximum winds would have been operative at the stations.

The plotting chart (Fig. 2) is largely selfexplanatory. Entries in full line are those corresponding to definite fixes on the ground, and entries in broken line those from assumptions of correctness of Gee or of cloud fixes. Winds are shewn vectorially, with directions as derived except to the west of Wales, where a direction of 350° was assumed. It will be noticed that the wind directions for the adjusted print centres on the northern track gives a much more consistent wind direction than the unadjusted winds on the southern track.

The histogram (Fig. 3) is for the outward run only and shews the profile of the wind strength across the jet stream. Again full lines correspond to measured winds, whereas broken lines indicate winds from Gee-fixes and the broken lines from 240 to 280 show the winds derived from rate of drift across cloud. It will be seen that to almost exactly the point where ground detail became plain, the wind is of the order of 80 knots, and has not been subject to appreciable change. From this point (60 n.m. after the start of the run) there is a fairly steady increase of wind which at the 120 n.m. point has reached 130 knots (a gradient of 50 knots/60 n.m.). Between this point and

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the 140 n.m. point there is a further increase to 170 knots (a gradient of 40 knots in 20 n.m.). A maximum speed of just over 170 knots is maintained for about 20 n.m., and this is followed by a slow fall to the 250 n.m. mark where it is 150 knots, or perhaps less, as this figure depends on there being no reduction in the speed of the wind at cloud height (5-6000 feet) to the west of the Gower coast. These winds relate well to those actually reported on that day; the maximum wind reported was 178 knots (Aldergrove 300 mbs at 0300 GMT) compared with a derived 180 knots. The Camborne winds were 156 and 139 knots at 0900 and 1500 GMT respectively, corresponding to positions near, and about 100 n.m. to the west of the maximum wind line. The jet stream as a whole was moving east at about 15 knots, so that on the derived figures, the increase from 80 to 130 knots would take approximately 4 hours at a particular place, and the increase to 170 knots a further 1-2 hours. This high speed would only be maintained for an hour or so, after which a slow decrease could be expected. It will be noticed that the Gee-winds shew very good agreement for the first five fixes, but agreement is poor in the west. It is felt that the westerly Gee-winds are in error, and this is supported by the difficulties the navigator experienced with the chain in the west, and by the fact that there was faulty identification of ground detail in the vicinity of Fairwood Common near Swansea. In view of the poorer detail in the return run, its much shorter nature and the absence of any striking change, a similar plot has not been made for this flight. It is noteworthy that there is a slight increase of wind during the flight (of the order of 12-15 knots) which corresponds very closely with the change of wind derived from rate of drift across cloud in the outward leg.

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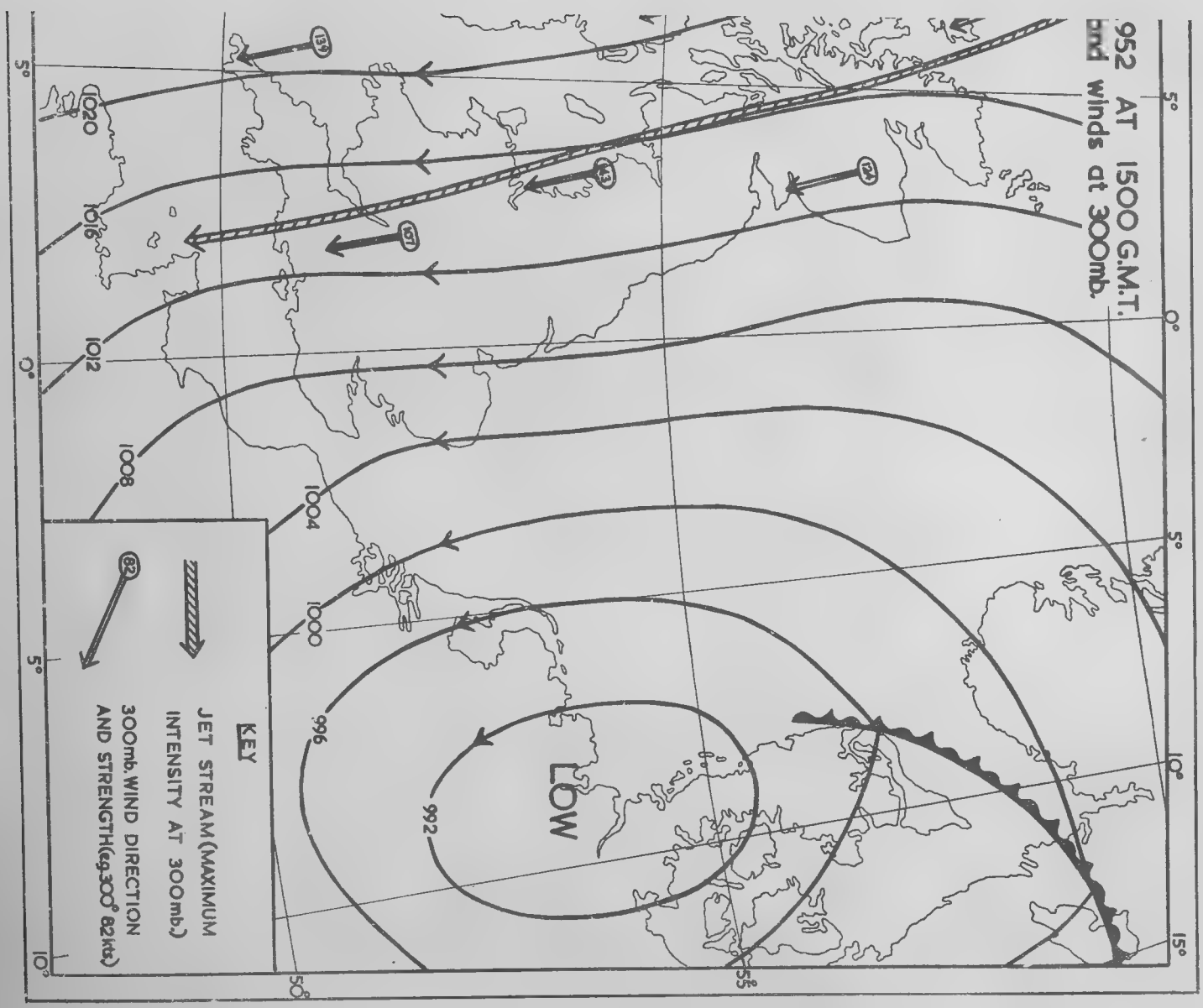


M.R.P. 722.

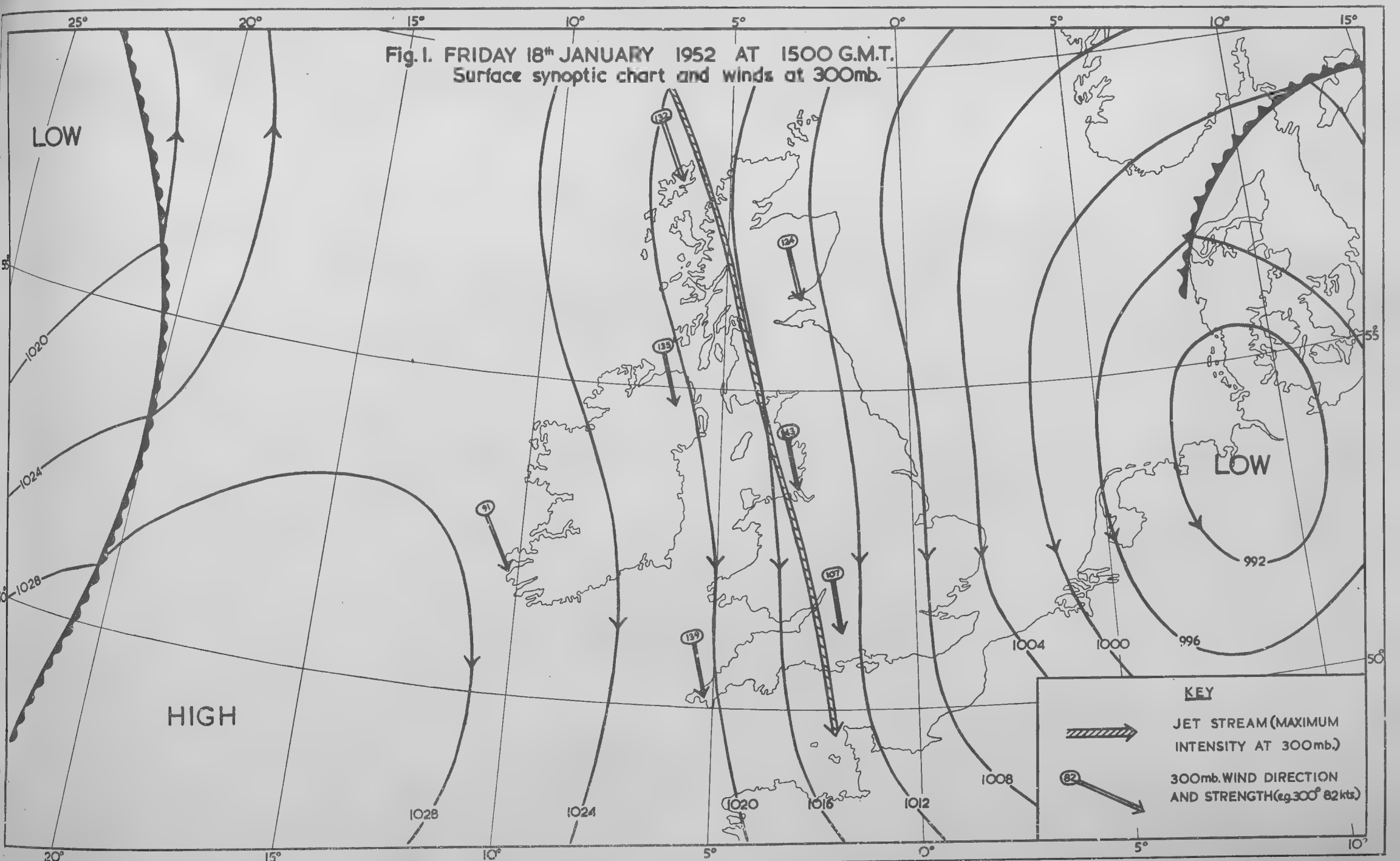


Fig. 1. FRIDAY 18th JANUARY 1952 AT 1500 G.M.T.
Surface synoptic chart and winds at 300mb.
LOW
HIGH
LOW
1020
1024
1028
1028
1024
1020
1016
1012
1008
1004
1000
996
992
132
124
135
143
91
107
139
25°
20°
15°
10°
5°
0°
5°
10°
15°
55°
50°
KEY
JET STREAM (MAXIMUM INTENSITY AT 300mb.)
82
300mb. WIND DIRECTION AND STRENGTH (e.g. 300° 82 kts.)

Fig. 2. Plot of Mosquito PF 678 on flying across jet stream on 18/1/52

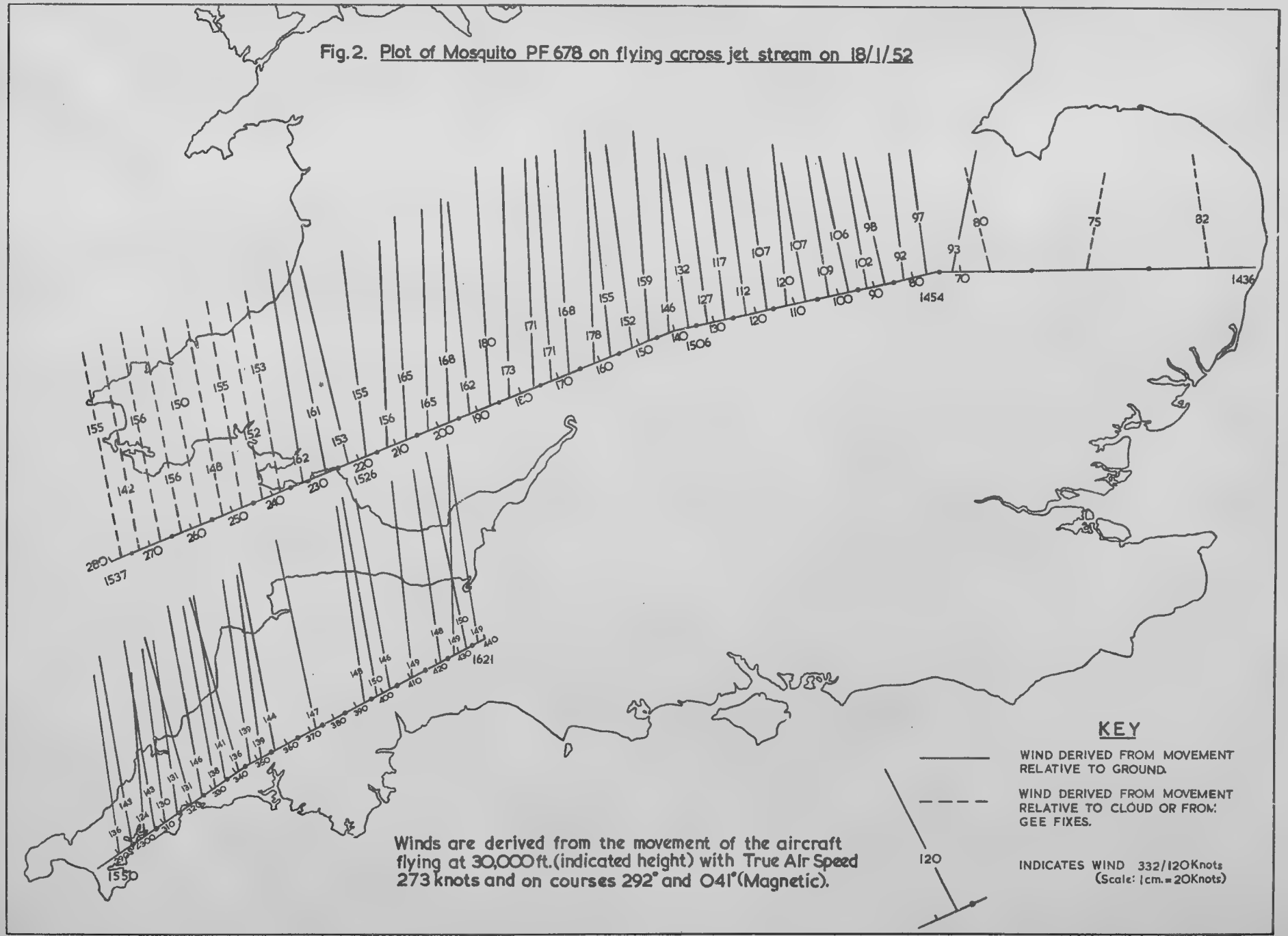
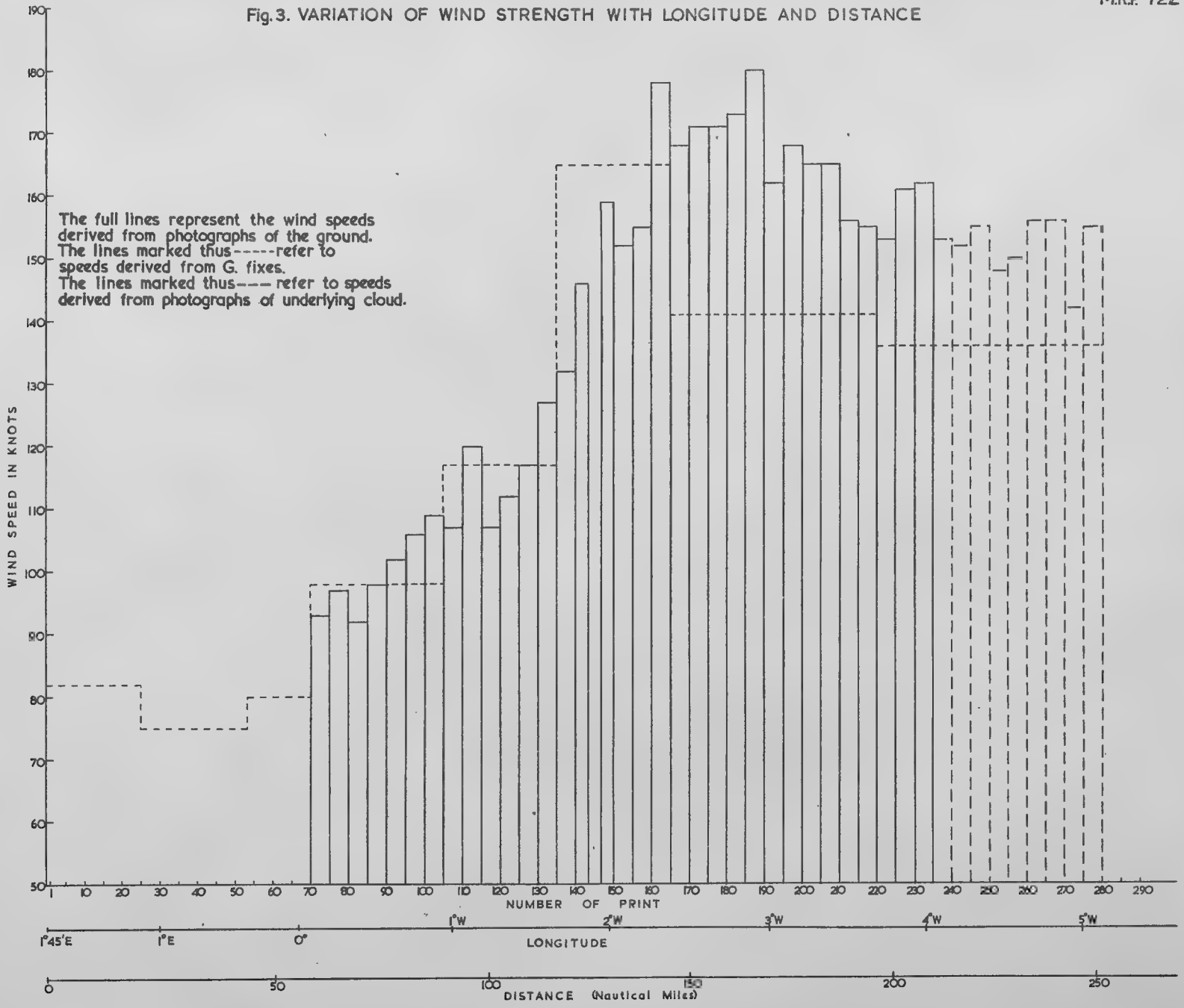


Fig. 3. VARIATION OF WIND STRENGTH WITH LONGITUDE AND DISTANCE



Committee (London).
M.R.P. No ..... by Mr. .....
A copy is available in the Library of the Meteorological Office."

M.R.P.724
S.C.III/127
30th April, 1952.

AIR MINISTRY

AD 223037

METEOROLOGICAL RESEARCH COMMITTEE

The Evaporation from an Oat Field during the Late Spring and Summer of 1951

by
N.E. Rider, (Meteorological Office)

DEC 5 1952

Summary

Evaporation from a natural land surface may be evaluated by a method based upon aerodynamic considerations. These considerations lead to the formula

$$E = \frac{\rho k^2 (u_2 - u_1)(q_1 - q_2)}{(\log_e z_2/z_1)^2}$$

where E, ρ , and k are the rate of evaporation, the air density, and von Karman's constant and u1, u2, q1 and q2 are the wind speeds and specific humidities at the two heights z1 and z2. This formula has been used to determine the hourly values of E over a field of oats during a number of periods between May and September. The apparatus is described and the need to introduce the constant d, the zero displacement, in the above expression is indicated. The values of d are shown to increase with increasing wind speed and crop height, the latter increasing eightfold over the total period of observation. Daily water losses are calculated and vary from as little as 0.05 mm. to 5.65 mm.. The hourly values of the rates of evaporation obtained from the above expression are shown to depend on the heights of exposure of the apparatus z1 and z2, the magnitude of the rates of evaporation decreasing in unstable conditions as z1 and z2 increase. A modified form of the equation given above based on the Deacon generalised wind profile law is used to compute the hourly rates of evaporation in unstable conditions from the observations obtained with two sets of apparatus during the last period of exposure. The use of this modified form improves the agreement between the rates of evaporation calculated from the two sets. This modified form of the expression could not be generally applied since the zero displacement could not be specified with sufficient precision and the accuracy of the determination of the mean wind speeds was insufficient. In general the daily water losses given are considered to be underestimates to an extent not exceeding 15 per cent. Finally it is stated that attempts to specify the water losses from a study of normal weather data failed, and a suggested programme for future work on this problem is given.

Introduction

The measurement of evaporation from natural land surfaces has always been regarded as a difficult undertaking. At certain periods of the year when a clay soil, which has no water table, is drying out it is possible to extract cores of suitable diameter and depth, expose them in their natural positions in evaporimeters and obtain the rate of evaporation over periods ranging from an hour to the order of a few days directly by weighing. The soil has to be bare or covered with a short crop with shallow roots such as grass so that in extracting the cores the roots are not broken and the general appearance of the surface is not changed. The site must also be reasonably uniform in character so that the mean result from a number of evaporimeters holding isolated specimens of the surface may be considered to be representative of the whole area. When the surface is covered with a tall crop, or where a water table exists a procedure on these lines is not practicable and other methods have to be employed. At present there are two lines of approach to this problem. The first is the aerodynamic method mainly developed in this country by Pasquill (1949a) and the second is the heat balance method favoured by Penman (1948). The first method appeals to the imperfectly understood diffusive properties of the atmosphere, and the second depends esentially on the performance of the conventional open water-tank evaporimeter. Pasquill (1950c) has recently given a critical survey of the two methods and he regards

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the aerodynamic method as the more satisfactory, at any rate for the dependable estimation of evaporation from specific areas for short periods of time. He also suggests that an accumulation of such data might eventually lead to the development of a satisfactory method, less exacting in its demand for observations of a delicate nature, which would provide practically useful determinations of this quantity.

The observational programme to be described in this paper is a first attempt in the U.K. to apply the aerodynamic method to the determination of natural evaporation from an area of tall crops, namely an oat field, during the period over which the crop grew from a height of 10 cm. to 80 cm. and was finally reaped. The observations were not made continuously throughout the period but the hourly rates of evaporation in a number of periods ranging from 1 to 13 days was determined.

The Aerodynamic Method

The method is dependent on the validity of three propositions. These are that in the normally turbulent atmosphere near the ground;

(a) the wind speed, u_z, at a height z may be expressed as

$$u_z = \frac{1}{k}\left(\frac{\tau_o}{\rho}\right)^{1/2} \log_e \left(\frac{z}{z_o}\right) \quad \ldots\ldots\ldots\ldots (1)$$

where τ_o = the shearing stress per unit area of the surface
ρ = the density of the air
z_o = a constant for each type of surface, known as the roughness parameter.
k = a dimensionless constant, known as von Karman's constant

(b) the shearing stress is constant with height

$$\tau = \tau_o$$

(c) the eddy diffusivity for momentum is the same as that for matter, in this case water vapour

$$Km = Kv$$

(a) is an experimental result derived initially from the study of the turbulent flow of fluids over artifically roughened surfaces in wind tunnels, see e.g. Nikuradse (1933) and Schlichting (1936). Deacon (1948) has demonstrated that the functional form of equation 1 holds over natural surfaces in the open provided that the atmosphere near the ground is in the adiabatic or neutral equilibrium state, i.e. there is no thermal stratification or instability. More recently Pasquill (1950) and Rider (1951) have established the explicit truth of this equation over grassland with differing roughness characteristics and the value of k found by both was approximately the same as that determined for artificial surfaces in wind tunnels. Sheppard (1947) obtained a similar result for a concrete surface in the open. All these determinations were made in effectively neutral conditions, and it is now safe to consider equation 1 to apply independently of the nature of the surface in these conditions. The consistency of the shearing stress with height was first proved by Ertel (1933) and has been confirmed by Calder (1939) but some doubt must still attach to this as experimental results obtained by Scrase (1930) are not in agreement. Pasquill (1950b) has advanced a possible explanation. The identity of the eddy diffusivity for momentum and matter has been the subject of experimental investigations by Pasquill (1949a) and Rider and Robinson (1951) and while the evidence for this may not be conclusive in some respects, there is, as far as the present writer is aware, no evidence to the contrary. In any event many expressions for diffusion in the atmosphere have been based on the validity of (b) and (c) and have been found to be in good agreement with observation. In this connection we may particularly note that Pasquill (1950a) has applied the aerodynamic method to the determination of evaporation from a clay pasture in effectively neutral conditions and his results were in good agreement with the evaporation determined directly from the use of evaporimeters of the type mentioned in the introduction.

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The eddy diffusivity for momentum, Km, is defined by

$$K_m = \frac{\tau_o}{\rho \left(\frac{\partial u}{\partial z}\right)} \qquad 2$$

which with equation 1 and the assumption that the shearing stress is constant with height gives

$$K_v = K_m = \frac{u_1 k^2 z}{\log_e z_1/z_o} \qquad 3$$

where u1 is the wind speed at the height z1. In steady horizontal conditions the rate of evaporation E is given by

$$E = - \rho K_v \frac{\partial q}{\partial z} \qquad 4$$

where q is the specific humidity. If now the expression for Kv at 3 above is substituted in 4 and the expression is integrated over the heights z1 and z2 and rearranged we obtain the formula

$$E = \frac{\rho k^2 (q_1 - q_2)(u_2 - u_1)}{(\log_e z_2/z_1)^2} \qquad 5.$$

This is the familiar formula for the determination of the rate of evaporation by the aerodynamic method. Since equation 1 is only applicable in neutral conditions the same restriction applies to this expression. Here it is convenient to note that when the height of the roughness elements above the solid air earth interface is comparable with the heights z1 and z2 it is necessary to introduce a further constant d, the zero displacement, in equation 1 to provide the observations of the wind speed with the correct functional form when z1 and z2 are measured from the ground. The qualitative explanation of this is that the turbulent process does not extend right down to the interface but to some level within the roughness elements. The precise level for a particular surface covering depends on the height, density and general character of the roughness elements. Under these conditions equation 1 becomes

$$u_z = \frac{1}{k} \left(\frac{\tau_o}{\rho}\right)^{1/2} \log_e \left(\frac{z-d}{z_o}\right) \qquad 6$$

and equation 5

$$E = \frac{\rho k^2 (q_1 - q_2)(u_2 - u_1)}{\left(\log_e \frac{z_2 - d}{z_1 - d}\right)^2} \qquad 7.$$

As has already been noted equations 1 and 6 and therefore equations 5 and 7 are strictly applicable only when the lower atmosphere is in the neutral or adiabatic state, and in the ordinary day this state occurs for a few short periods only. Thus the general use of equation 7 without qualification would lead to errors in the computed values of E. The degree of departure of the atmosphere from a neutral state has been shown to depend on the value of the Richardson Number and not on the gradient of temperature alone.

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The Richardson Number is defined by

$$Ri = \frac{\frac{g}{T}\left(\frac{\partial T}{\partial z} + \Gamma\right)}{\left(\frac{\partial u}{\partial z}\right)^2}$$

where g = the acceleration due to gravity
T = the absolute temperature of the air
Γ = the dry adiabatic lapse rate

and in the lowest few metres of the air the numerical size of the number decreases as the surface is approached (see for example Pasquill (1949a) and Deacon (1948)). If the humidity and wind speed measurements required to compute E with the aid of equation 7 are made close to the surface the errors are kept to a minimum. Pasquill (1949c) estimated that by making observations in the first 50 cm. above a grass pasture during fine spring weather the systematic error in the form of an underestimation of E in unstable conditions and an overestimation of E in stable conditions was of the order of ± 10 per cent. Pasquill (1949a) has also shown that the Deacon generalised wind profile law

$$\frac{\partial u}{\partial z} = a\, z^{-\beta}$$

where a and β are constants, β taking values >1, $=1$, and <1 in unstable, neutral and stable conditions respectively provides a basis for the modification of equation 7 which proved to be satisfactory in unstable conditions. This equation is

$$E = \frac{\rho k^2 (q_1 - q_2)(u_2 - u_1)(1 - \beta)\, z_o^{2(1-\beta)}}{\left[(z_2 - d)^{1-\beta} - (z_1 - d)^{1-\beta}\right]^2} \qquad 8.$$

In stable conditions the position remains obscure and at present we are forced to rely on the observed decrease in the magnitude of the influence of the stability effect as the ground is approached. The present writer (1951) undertook an experimental study into the aerodynamic drag wind profile relationship which was conducted with a view to throwing some light on this problem, but the results obtained did not enable any definite conclusions to be reached. These considerations will have further attention later.

The Site

An oat field of some 10 acres was available for the observations. It was on a river gravel soil and formed part of the University Farm at Cambridge. Its shape was roughly rectangular and its limits were marked by a track to the south, by an open wire fence to the west and by a low hedge to the north and east. Apparatus was sited in this field some 80 metres from the eastern boundary and was always approached along the same path to keep damage to the crop and disturbance to the general character of the surface to a minimum. The path was about 50 cm. wide and ran to the apparatus from the nearest point on the eastern boundary of the field. All cables to the apparatus were laid along it. From the position of the apparatus there was a uniform surface of the crop for at least 150 metres in an arc from SE through S, W, and N to NNE. When the observations were started the crop had grown to a height of 10 cm. and for the rest of the season a record of the height of the crop was kept. This is shown in fig.1. An attempt was made to specify the general height of the crop rather than that at the foot of the apparatus. The crop had been sown in drills about 12 cm. apart parallel to the eastern hedge and the growth was patchy to some extent. The hut used to house the apparatus such as recorders etc. was outside the field and below its level in a shallow disused gravel pit and could not be seen from the position of the apparatus.

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The Apparatus

The minimum observational requirement for the applicant of equations 7 and 8 is the measurement of the wind speed at three heights and of humidity at two heights. (The determination of the wind speed at the third height is required to specify d and/or z_o.) Records of the total run of wind were obtained by the use of Sheppard (1940) type anemometers suitably modified to operate electromagnetic counters at a position remote from the position of exposure, the more familiar counter mechanism having been discarded. The electromagnetic counters associated with each of three anemometers were mounted side by side with a fourth counter in a Deacon type recorder. A description of these photographic recorders is given by Hartley (1951). The construction of the contact mechanism in the anemometers was such that two revolutions of the cups produced one contact and moved the remote counters on one. The fourth counter in the recorder was provided to act as a register from which it could be seen whether any record had been missed. Photographs of the counters were taken every half hour although in the final analysis only the hourly runs of wind were used. It must be mentioned that for 40 sec. of each hour (the period occupied by two openings of the camera shutter) the anemometers were out of circuit with the counters, so that an underestimation of the run of wind amounting to roughly 1 per cent occurred. As in this application we are mainly interested in the differences in wind speed between the various levels no correction was applied. In any case the accuracy of the final result does not warrant its application. Experience with the anemometers has shown that they become unreliable at wind speeds much below 40 cm./sec. Masts were constructed to carry the three anemometers and were provided with suitable wiring and junction boxes.

Measurement of the humidity at two heights was made with aspirated thermocouple thermometers of the same type as those used by Pasquill (1949b) and Rider and Robinson (1951). Since only two psychrometers were required in one set of apparatus it was possible to mount them vertically at the extremities of a hollow mast and to provide each with a set of plates to make sure that the air was being drawn from the required level. In this way the apparatus was given a multidirectional characteristic and the need to rotate the mast to prevent the direct ingress of sunshine to the thermometer bulbs eradicated. This is a desirable feature as it is convenient to set up the mast and leave it for a day without attention. The general construction of one of these masts is shown in fig.2. The tubes A and A' are made to be sliding fits in tube B so that the height interval between the levels of measurement can be adjusted as required. By fitting various lengths of main supporting mast to C the height of the centre of the apparatus can also be adjusted. The material used was tufnol and the whole mast was sprayed with white cellulose. A vacuum flask was attached to the main support together with a junction box from which a 7-cored 22 s.w.g. single copper conductor cable led away to the hut. In use the vacuum flask was filled with melting ice and a standard junction was fitted through the cork. A wiring diagram of the apparatus is given in fig.3. It will be noted that the dry and wet thermometers forming the psychrometer at the higher level were each provided with two thermojunctions. One from each thermometer was connected in a standard circuit, and the other junction in each was connected in a differential circuit with the corresponding junction in the lower psychrometer. A study of the usual psychrometric formulae and of equations 7 and 8 shows that, in this application, it is not necessary to know the absolute values of the wet and dry bulb temperatures at either height to any great accuracy, but as accurate a knowledge as possible of the dry and wet bulb temperature differences between the two heights is required. The absolute temperature of the dry and wet bulbs at the greater height (t and t') was therefore recorded on a comparatively insensitive thread recorder with reference to the standard junction in the ice flask, and a photographic recorder working in conjunction with two Tinsley reflecting galvanometers provided a record of the differences in temperature of the dry and wet bulbs (dt and dt') at the two levels. The overall temperature sensitivity of the thread recorder when connected to the mast with 200 yd. of cable was such that t and t' could be read with difficulty to 0.1°F, and are considered to be reliable to 0.2°F only. A thread recorder with a current sensitivity about 50 per cent greater would have been better but such was not available.

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The temperature differences dt and dt' could be obtained to 0.01°F with the photographic recorders. These limits of temperature measurement enabled the vapour pressure differences between the two levels to be calculated to 0.005 mm. Hg. The thread recorder has six ways and switched at intervals of 2 min.. Two ways were connected in the dry bulb circuit and two in the wet bulb circuit, the remaining two ways being used to provide a closed circuit zero. The two galvanometers in the photographic recorders gave continuous records of dt and dt' with the qualification that every 30 min. the movements were disconnected from the external circuits and shorted through their critical damping resistances. This was executed by a time switch and relay system, and provided a record of the zeros of the instruments as well as good time marks. In all the wiring care was taken to avoid the introduction of dissimilar metals and therefore of spurious e.m.f's., and the merit of the apparatus in this connection was tested by placing all the thermoelectric thermometers in a stirred water bath and taking records of t, t', dt and dt' with the recorders in their normal working positions with relay circuits etc. operating. No deflections were registered by the thread recorder movement but small deflections, corresponding to temperature differences of up to 1/20°F, were noted on the developed photographic recorder record. As there was no systematic variation of these deflections with time they were considered to arise from small temperature differences within the stirred bath.

The whole apparatus was subjected to lag and ventilation tests as described by Pasquill (1949b) and the lag coefficient of the dry bulbs was found to be about 80 secs. and that of the wet bulbs slightly less at the ventilation rates of 3m./sec.. Curves were constructed relating the aspiration fan motor current to the ventilation over the bulbs and it was found that ample ventilation, about 3.5m./sec., was obtained with a motor current of 2.5amp.. There was a slight but insignificant variation from motor to motor and mast to mast.

The photographic recorders were provided with 24-hour drums for normal use but it was found that a satisfactory calibration of the differential circuits could not be obtained with this drum as the temperature of the water baths did not remain constant for a sufficient time to produce an identifiable trade on the developed record. A quickrun drum driven by a synchronous motor which revolved once in 50 min. was obtained and used for the calibrations. It was found advantageous to place the thermometers to be calibrated together with standard mercury-in-glass thermometers in vacuum flasks about three quarters full with water and to close the flasks with corks and shake them vigorously during calibration. This method seemed superior to the use of stirred open baths in which suprisingly large temperature differences can exist. Consideration was given to the application of a correction to eliminate the overestimation of large deflections by the cylindrical drum in the photographic recorders, but the path length from the galvanometers to the drum was sufficiently large to make the error insignificant for deflections of less than 3 cm., which corresponded to a temperature difference of 1.5°F. Temperature differences as large as this were rarely met in the series of field observations.

A number of sets of apparatus was constructed and calibrated and on some occasions two sets were exposed simultaneously. Each set consisted of a psychrometer mast, an anemometer mast with three anemometers, a thread recorder, a photographic galvanometer recorder and a run of wind recorder together with the appropriate time switches, relays, wiring etc..

As all the recorders were fitted with 24-hour drums it was only necessary to disturb the apparatus once a day. A period of about 10 min. was required in which to change the photographic papers and replenish the wet bulb water holders and this amount of temperature record was lost. As far as possible the operation was carried out in the middle of a half hour period so that no run of wind record would be lost.

Occasions of Observation

As has already been indicated, continuous recording over the whole period was not undertaken but the apparatus was set out on occasions when fine weather was expected. The periods of recording are set out in table 1 together

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with the heights of exposure of the instruments, periods of rain and the heights of the crop. It will be seen that for all periods after the first, two anemometer masts were exposed. The reason for this is given below. The wind directions during observational periods 2 to 8 inclusive were within the arc which provided a fetch of at least 150 metres over the oat field, with the exception of an odd hour here and there. In group 1 the wind directions were mainly easterly and the fetch was as little as 80 metres at times. The greatest height of humidity measurement was 60 cm. and previous experience (Rider 1952) has indicated that horizontal homogeneous flow conditions would have been established up to this height. The greatest height of wind speed measurement in this period was 100 cm. and it is doubtful whether steady horizontal conditions were established at this height. The wind speed determination at 100 cm. was required to determine d and did not otherwise enter into the evaluation of E.

Reduction of the Records

The developed records from the anemometer recorders consisted of columns of figures which represented the number of revolutions of the cups during each half hour. It was therefore a simple matter to obtain the hourly runs of wind and hence the mean wind speeds for each hour by referring to the calibration curves for the anemometers. The thread recorder records consisted of the familiar series of dots, and the mean height of the curves formed by the dots was a measure of $(t - 32)$ and $(t' - 32)\,^{\circ}F$. A transparent scale provided with the instruments was used to measure the mean heights of the curves for the hourly periods. The mean height was judged by eye and occasionally the eye estimates were checked by measuring the heights of the individual dots. The eye estimates were very satisfactory except during periods when the dots showed considerable scatter. During such periods it was thought advisable to employ the longer method of measuring the height of each dot. A further check on the validity of the eye estimates was obtained by having each record measured by two observers and repeating any estimates which showed a difference. Once the mean hourly heights had been measured the values of $(t - 32)$ and $(t' - 32)$ were obtained from the calibration factors.

The determination of the temperature differences between the two dry and wet bulbs (dt and dt') was carried out with a planimeter. The hourly areas were measured by running round each ten times with the planimeter, the planimeter being read after 5 circuits as well as after the 10. The areas so found were then divided by the drum rate which was indicated by the automatic time marking device, and the mean hourly ordinates obtained. Recourse to the calibration factors provided the required values of dt and dt'. In every period of 24 hours the record for one hour was split between two traces. There was also a time of about 10 min. during which no record was taken. For these hours the hourly areas were obtained by simple proportion.

The Determination of d

Before any computations of the rates of evaporation could be made it was necessary to obtain values of d, the zero displacement. Two methods are available, the first consisting of the graphical solution of the following expression which is derived from equation 6

$$\frac{u_3 - u_2}{u_2 - u_1} = \frac{\log (z_3 - d) - \log (z_2 - d)}{\log (z_2 - d) - \log (z_1 - d)}$$

Wind speeds at three heights are required for this. Alternatively we may plot values of uz against $\log(z - d)$ using likely values of d. When the best approach to a straight line is obtained we have the required value of d. When the experiments were started it was intended to rely on the first method but it soon became apparent that considerable scatter was arising in the results, presumably due to instrumental errors in the wind speed measurements. A method in which graphical smoothing could be applied was thought to be necessary and so the longer method was adopted. Wind speed measurements at three heights are not sufficient for the critical application of the second method and consequently, after the first group of observations, two anemometer masts

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were always exposed so that wind speed determinations at six heights were available. The zero displacement can only be determined from observations in neutral conditions to which equation 6 applies, and it was necessary to select the hourly periods in which this state was approached. The condition finally decided upon was that the dry bulb temperature gradient dt/dz should not exceed + or - .001°F/cm. at the mean logarithmic height of z1 and z2, assuming a logarithmic variation of temperature with height. In practice this meant that suitable hours could be identified by an examination of the magnitude of the values of dt. Strictly the criterion for the suitability of an hourly observation for the determination of d is the magnitude of the Richardson Number, but to restrict the analytical work to reasonable proportions the condition above was adopted as an acceptable approximation. On some days two or more values of d were obtained, but there were a few days on which no periods of an hour were suitable. Previous work on the form of the wind profile over long grass by Deacon (unpublished) which has been discussed by Calder (1949) indicates that the value of d varies with the crop height and wind speed, and this feature soon became apparent in the present observations. The determinations of d were therefore grouped according to the wind velocity at 2 m., the latter being estimated from plots of uz against log (z - d). The groups adopted were for wind speeds at 2m. of < 1, 1 to 2, 2 to 3, 3 to 4, and > 4 m./sec.. This feature provided an additional complication in the evaluation of E, it being necessary to use a value of d depending on the wind speed and crop height. A plot of the values of d divided according to the wind speed groups given above is shown in fig.1. in which a smooth curve has been constructed for each group. The values of d used in the computations of E from equation 9 (see below) were taken from these smooth curves. It will be seen that d increases with wind speed and crop height. It was also noticed that there was a tendency for d to vary with the wind direction. When the wind was blowing along the lines of the crop, d tended to be less than when the direction was at right angles to them. The observations were not sufficiently numerous to permit any indication of the magnitude of this effect and it has been neglected, the results plotted on fig.1 being for all wind directions experienced.

The Evaluation of E

Equation 7 may be rewritten as

$$E = \frac{k^2 M (e_1 - e_2)(u_1 - u_2)}{RT \left\{\log_e \frac{z_2 - d}{z_1 - d}\right\}^2}$$

where e is the vapour pressure, M is the molecular weight of water, R is the universal gas constant, and T the absolute temperature. Here we are neglecting the variation of T over the height interval z1 to z2. Substituting the usual values of k, M, and R we have

$$E = \frac{4.62 \times 10^{-5} (e_1 - e_2)(u_2 - u_1)}{T \left\{\log_e \frac{z_2 - d}{z_1 - d}\right\}^2} \qquad 9$$

If u is in cm./sec., e is in mm.Hg, and z and d are in cm., then E is in gm./cm^2/sec. Values of E were calculated from this equation for all the periods of observation, and when two sets of apparatus were exposed the E's were calculated from the results provided by each set. T was always taken as the absolute temperature at the greater height and e1 - e2 was calculated on the basis of the usual psychrometric formulae (M.O.265).

The equation corresponding to equation 9 constructed on the basis of equation 8 is

$$E = \frac{4.62 \times 10^{-5} (e_1 - e_2)(u_2 - u_1)(1 - \beta) z_o^{2(1-\beta)}}{T \left[(z_2 - d)^{1-\beta} - (z_1 - d)^{1-\beta}\right]^2} \qquad 10$$

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but before it can be applied it is necessary to know z_o and β. z_o, like d, depends on the crop height and wind speed, and it would be possible to construct a figure similar to fig. 1 for z_o, the required values being obtained from the plots of uz against log (z - d) in neutral conditions. A short discussion of the methods available for the determination of β has been given by the present writer (1951 para. 15) and we may note here that either of the following expressions may be used.

$$\frac{u_3 - u_2}{u_2 - u_1} = \frac{(z_3 - d)^{(1-\beta)} - (z_2 - d)^{(1-\beta)}}{(z_2 - d)^{(1-\beta)} - (z_1 - d)^{(1-\beta)}} \qquad 11$$

$$\frac{u_2}{u_1} = \frac{(z_2 - d)^{(1-\beta)} - z_o^{(1-\beta)}}{(z_1 - d)^{(1-\beta)} - z_o^{(1-\beta)}} \qquad 12$$

The zero displacement d appears in both these equations and the roughness parameter z_o in equation 12. Confining our attention to equation 11 we see that the R.H.S. may be evaluated for a range of likely values of β. The β corresponding to the observed value of the L.H.S. of the equation can then be read from a plot of the calculated values of the R.H.S. against β. It would be necessary to draw a series of curves for a range of values of d corresponding to the range shown in fig. 1. To facilitate this it is possible to use fig. 1 as the basis of another figure relating d to the crop height for each wind speed from which, by interpolation, the d corresponding to any combination of wind speed and crop height could be read. It would, however, be necessary to assume that the smooth curves drawn for the wind speed groups in fig.1 give the correct values of d for the mean wind speed of each group. Once the curves of fig. 1 are accepted as a true representation of the values of d there is no difficulty. But we must consider the likely error in d and its influence on the determined values of β. It appears that it is necessary to know d with certainty to at least the nearest cm. before β can be determined with confidence. An example will make this clear. On 13th July, 1400 to 1500 hours, the mean wind speeds at 145, 110, and 70 cm. were 317, 291, and 241 cm./sec. respectively, and the plot of these observations together with the wind speeds at 245, 195, and 95 cm. indicates that the wind speed at 200 cm. was 350 cm./sec.. The value of d taken from the smooth curve D of fig. 1 is 28 cm. which with equation 11 leads to a value of β of 1.04. The scatter of the individual determinations of d suggests that its value might have been as low as 26 cm. or as high as 30 cm., and these values of d lead to values of β of 1.07 and 1.01 respectively.+ An uncertainty in β of this order results in turn in a large range in the values of E determined from equation 10 even when the variation in z_o is neglected. Thus without a detailed knowledge of the variation of d with wind speed and crop height, a knowledge which the present observations do not in general provide since the crop height was changing from day to day and insufficient determinations were obtained on any one day, we are unable to determine β with any confidence.

The reliability of the observations of the mean wind speeds also deserve attention in this respect. Consider the example given above. If d is fixed at 28 cm. β has the value 1.04 as already stated. The anemometers used probably do not provide a measure of the mean wind speed to an accuracy greater than 1 per cent, and if we allow the wind speeds given to vary by this amount the resulting variation in β is alarming. The effect is greatest for a variation in the wind speed at the intermediate height and if we allow the indicated wind speed at this level to increase and decrease by $\frac{1}{2}$ per cent (the wind speeds at the other two heights remaining as stated) the value of β changes from 1.04 to approximately 1.15 and 0.9 respectively. A number of the hourly observations were examined and some were found to lead to impossible values of β which is not surprising in view of the above. Thus, while it is theoretically possible to specify the rate of evaporation in unstable conditions by the use of equation 10, the uncertainties in the observations do not permit this to be done.

+ This range of possible values for d also produces uncertainties in the values of E calculated from equation 9. For the example given this possible error is 5 per cent.

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There was one period of observation, the last, after the crop had been reaped when the height of the surface covering was constant. Moreover, such values of d as were obtained in this period showed no systematic variation with wind speed. This may have been partly due to the wide limits within which d could be determined, but at least any variation in d with wind speed was small compared with the heights of measurement. The application of equation 10 has therefore been restricted to the observations made in this period. In order to obtain values of β which did not depend on the accuracy of a single observation of the wind speeds, equation 12 was used in the following way. Two heights of wind speed measurement were 160 and 60 cm., and the ratio u160/u60 was plotted against the Richardson Number at 40 cm. for each hour. The height of 40 cm. was chosen as it is the approximate logarithmic mean of 60 and 25 cm., two heights for which the difference in temperature was measured. The Richardson Number was calculated from

$$Ri_{40} = \frac{122 \times (t_{60} - t_{25})}{(u_{60} - u_{25})^2}$$

which is arrived at by neglecting Γ , the dry adiabatic lapse rate (-1×10^{-4} °A/cm.), assuming an absolute temperature of 288°A, and a logarithmic variation of temperature and wind speed with height. The latter assumption cannot lead to any appreciable error in the gradients of these elements at the mean logarithmic height and it avoids the need to plot the elements against height so that the gradients may be found by tangent drawing. From the plot of u160/u60 against Ri40 the value of the ratio corresponding to Ri40 = 0 was read. This was 1.34 and by substituting this together with d = 6 cm. (see fig.1) in equation 6 a value of z_o = 2.47 cm. was obtained. Using this value of z_o, the R.H.S. of equation 12 was then calculated for a range of values of β and by plotting the results and combining this curve with that already obtained for the variation of u160/u60 with Ri40 a third curve of Ri40 against β was drawn. This curve was then used to read off the value of β appropriate to each hour of observation in period 8.

Equation 10 was then used to calculate values of E for the hours during which the atmosphere was unstable in the layer concerned, i.e. hours for which Ri40 was negative and $\beta > 1$. The equation was not applied directly to the observations of e1 - e2, u2 - u1, etc. in order to get E but the following process was employed. Writing the E of equation 9 as En and the E of equation 10 as Eu we have

$$\frac{E_n}{E_u} = \frac{\dfrac{1}{\left(\log_e \dfrac{z_2 - d}{z_1 - d}\right)^2}}{\dfrac{(1-\beta)\, z_o^{2(1-\beta)}}{\left[(z_2 - d)^{(1-\beta)} - (z_1 - d)^{(1-\beta)}\right]^2}}$$

Values of this ratio for z1 and z2 appropriate to both sets of apparatus exposed during period 8 were calculated for a range of values of β . A plot is shown in fig. 6 which, for general interest, has been drawn for $\beta > 1$ and $\beta < 1$ although equation 10 is not applicable in the latter instance. The hourly values of En had already been calculated and in order to obtain Eu these values were divided by the ratio En/Eu corresponding to the β for the particular hour.

Discussion of the Results

The hourly rates of evaporation calculated by means of equation 9 for the first 9 days of observational period 1 are shown in fig.4. Curves of e25 - e60, u60 - u25, t60 and td60 are also shown. The normal diurnal range of temperature is well illustrated together with the much smaller range of the dew point temperature. The values of u60 - u25 are generally greatest during daylight hours, and this feature together with the very marked variation of e25 - e60 from night to day leads to the high values of E during the day and the low values at night. When e25 - e60 is negative it follows from equation 9 that E must be negative, indicating a downward transport of water vapour to the crop and ground.

/During

During observational period 7 two sets of apparatus were exposed and the hourly values of E calculated using equation 9 from the results provided by both sets are shown in fig.5. The same diurnal range of E is apparent but the main point to note from these curves is that during the day when the rate of evaporation is high the values of E calculated from the results provided by apparatus set B (heights of measurement 210 and 110 cm.) are lower than those calculated from the results provided by set A (heights of measurement 125 and 85 cm.). This is in accordance with Pasquill's result, namely that the use of equation 9 in unstable conditions must lead to an underestimation of E, the magnitude of the underestimation depending on the degree of instability as indicated by the Richardson Number. Now the numerical value of the Richardson Number increases with height in the layer of air near the ground and the underestimation in E should therefore increase on any unstable occasion as the height of measurement of the wind speeds and humidities increase. This feature is thought to be well illustrated by the curves of fig.5. In all the observations there is casual error in addition to the systematic error arising from the neglect of the stability effect, and in the stable case when the rates of evaporation were low the casual error has hidden the effect of the systematic error.

Fig. 7 shows a plot of E from the results provided by apparatus sets A and B on 23rd. and 24th August, two days in period 8. In the hours in which the layer of air concerned was unstable the values of E, calculated by means of equation 10, have been added. Here again we see that lower values of E are obtained from the results provided by apparatus set B when equation 9 is used in unstable conditions. For both days the agreement between the values of E by set A and set B is improved by the use of equation 10. The underestimation to be expected from the use of equation 9 for any one value of β decreases as the heights of measurement decrease. That this should be so is shown by fig. 6. For example, if $\beta = 1.03$ and z2 and z1 are 125 and 85 cm, respectively $En/Eu = 0.805$ whereas when z2 and z1 are 60 and 25 cm. $En/Eu = 0.855$. The mean height of measurement of the humidities and wind speeds and not the separation of the levels is the main factor controlling the magnitude of this ratio.

Since the majority of the hourly rates of evaporation have been calculated by the use of equation 9 it is necessary to enquire into the errors likely to be present. The total error for any hour will be partly composed of a casual error arising from instrumental imperfection and analytical inaccuracies and partly due to the systematic error introduced by the neglect of the stability influence. The casual error, taking into account the possible errors in the assumed values of d, is thought to be of the order of 10 per cent except during hours when the rate of evaporation is small. An estimate of the systematic error in unstable conditions based on the results for group 8 suggests that on a few occasions this will be as large as 20 per cent. No estimate of the magnitude of this systematic error in stable conditions can be made from the present results but we might expect the maximum error to be of the same order.

The daily totals of evaporation are given in table II, and it is desirable to know the likely error in these results. To some extent the underestimation during the day will be balanced by the overestimation at night, but since the rates of evaporation at night are very small compared with those in the day, the daily totals will all be underestimates. Since the nocturnal evaporation makes such a small contribution to the daily totals we may neglect its overestimation as a first approximation and assume that the daily totals for the three complete days of group 8 for which equation 10 was used in the unstable hours are satisfactory. The mean differences between the daily totals for the three complete days from the results provided by the two sets of apparatus when equation 10 is used in the unstable conditions is about 1 per cent. For set A the mean underestimation arising from the use of equation 9 for all hours is 9 per cent and the corresponding figure for set B is 16 per cent. An examination of the approximate values of the Richardson Number throughout the whole series of observations suggests that the underestimation in period 8 is typical. Although, when the crop reached its maximum height (group 7) it was necessary to expose the set A apparatus at heights equal to those used for set B in group 8, the error is not so large as the effective height of exposure

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was less, the much larger values of d during these times having to be taken into account. It is considered that the daily totals of evaporation calculated from the results of apparatus set A by the use of equation 9 are underestimates to an extent not exceeding 15 per cent. On many days the error is less.

During periods of rain and showers the accuracy of measurement of the wind speeds and dry and wet bulb temperatures is doubtful and the daily totals for days with rainfall should be treated with caution. The daily totals of evaporation were examined in conjunction with the weather information given in daily weather reports and a connection between such data as mean wind speed, hours of sunshine, vapour pressure, etc. and evaporation was looked for with disappointing results. General trends could be traced but all attempts to set up an empirical equation to specify the daily evaporation failed.

Conclusion and Suggested Future Investigations

The present work suggests that the daily evaporation from cropped surfaces may be obtained to within 15 per cent when the simple aerodynamic formula is used to determine the hourly rates of evaporation from the results provided by apparatus suitably exposed. An improvement in the accuracy of the result can be obtained by the use of the modified form of the simple formula for such hours of the day as have unstable conditions of flow. Unfortunately the difficulties which arise when an attempt is made to apply this modified formula make it of limited practical value, and it is doubtful if it will ever be of use in those cases where the surface roughness characteristics are changing from day to day and with wind speed. Apart from the difficulties of specifying the roughness characteristics of any surface the accuracy with which it is possible to determine the mean wind speeds with cup anemometers is generally insufficient to yield reliable estimates of the value of the index in the Deacon wind profile law. Experience in the field suggests that, even if a better instrument were available, its use would not be justified as the horizontal variation of mean wind speed appears to be of the same order as the accuracy of the present anemometers. It might be possible to determine the corrections to be applied to the rates of evaporation, determined by the use of the simple formula, by exposing three or more sets of apparatus simultaneously at different levels. Since the errors due to the stability effect decrease as the surface is approached it might be found that the computed values of E approach a limit as the heights of exposure of the apparatus decrease. The Richardson Number could then be computed and the correction to be applied to the value of E calculated from the results provided by apparatus exposed at any height could be stated in terms of the Richardson Number. Unfortunately it seems likely that, even if this proved possible, it would be necessary to repeat the procedure for each different surface as it is by no means certain that there is one relationship between β and the Richardson Number which is applicable to all surfaces irrespective of their roughness characteristics.

The overriding requirement is to find some simple method of obtaining reliable figures of daily evaporation from natural land surfaces. The present outlook is not encouraging.

The aerodynamic formula suggests that, other factors being equal, the rate of evaporation will increase with increasing crop height and density since

$$E \propto (q_1 - q_2)\, u_2\, (1 - u_1/u_2)$$

and u_1/u_2 is known to decrease with increasing roughness. It is hoped to use the apparatus described in this paper for the simultaneous determination of the rates of evaporation from various land surfaces. It may be found that although u_1/u_2 decreases with increasing roughness there is a compensating decrease in the value of $(q_1 - q_2)$.

On the fundamental side the most urgent requirement is to find a relationship which will provide a satisfactory modification of the simple aerodynamic formula for use in stable conditions. The most promising line of approach appears to be an investigation of the relation between the

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aerodynamic drag and the form of the wind profile, a relation which has to date only been established explicitly for adiabatic flow conditions. Even when a satisfactory relationship has been established it might prove of limited practical use in the determination of the rates of evaporation as has been shown to be the case when the aerodynamic formula is suitably modified for use in unstable conditions.

G.332233/JWP/4/52/40.

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TABLE I

The Instruments etc.

	Periods during which Rain or Showers fell
bers	
1645 26/5/51 to 0400 27/5/51 0200 4/6/51 to 0500 4/6/51	
2000 to 2200 11/6/51 0100 to 0200, 0500 to 0800, 1600 to 1900 12/6/51 1630 to 1900 14/6/51	
0000 to 0130, 0700 to 1000 26/6/51	
0500 to 0730 4/7/51 2200 to 2400 5/7/51 1400 to 1700 8/7/51	
0830 to 1000 11/7/51 1300 to 1630, 1700 to 2400 12/7/51	
0800 to 0900, 1530 to 1700, 23/7/51 0000 to 0300, 1300 to 1700, 2100 to 2400 23/7/51	
1800 to 1900 9/8/51 0900 to 2400 11/3/51 0000 to 0200, 0600 to 0900, 1200 to 1300, 1600 to 1800. 12/8/51	
2000 to 2200 21/8/51 0600 to 1600 22/8/51 0900 to 1000, 1600 to 1700 24/8/51	

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	1947 Proc. R. Soc., A.188, 208

TABLE I

Periods during which the Instruments etc.

Numbers	Periods during which Rain or Showers fell	
1645	26/5/51 to 0400	27/5/51
0200	4/6/51 to 0500	4/6/51
2000 to 2200	11/6/51	
0100 to 0200, 0500 to 0800,		
1600 to 1900	12/6/51	
1630 to 1900	14/6/51	
0000 to 0130, 0700 to 1000	26/6/51	
0500 to 0730	4/7/51	
2200 to 2400	5/7/51	
1400 to 1700	8/7/51	
0830 to 1000	11/7/51	
1300 to 1630, 1700 to 2400	12/7/51	
0800 to 0900, 1530 to 1700,	23/7/51	
0000 to 0300, 1300 to 1700,		
2100 to 2400	23/7/51	
1800 to 1900	9/8/51	
0900 to 2400	11/3/51	
0000 to 0200, 0600 to 0900,		
1200 to 1300, 1600 to 1800.	12/8/51	
2000 to 2200	21/8/51	
0600 to 1600	22/8/51	
0900 to 1000, 1600 to 1700	24/8/51	

TABLE I

Periods of Observation and Details of the Heights of Exposure of the Instruments etc.

Period No.	Hours and Dates	Crop Height cm.	Apparatus set A Heights of Exposure of Anemometers cm.	Apparatus set A Heights of Exposure of Psychrometers cm.	Apparatus set B Heights of Exposure of Anemometers cm.	Apparatus set B Heights of Exposure of Psychrometers cm.	Periods during which Rain or Showers fell
1.	G.M.T. 1300 25/5/51 to 1000 6/6/51	10 to 19	25 60 100	25 60	Not exposed	Not exposed	1645 26/5/51 to 0400 27/5/51 0200 4/6/51 to 0500 4/6/51
2.	1000 11/6/51 to 1000 16/6/51	23 to 28	40 75 115	40 75	40 65 165	Not exposed	2000 to 2200 11/6/51 0100 to 0200, 0500 to 0800, 1600 to 1900 12/6/51 1630 to 1900 14/6/51
3.	1100 25/6/51 to 1000 26/6/51	35	55 90 130	55 90	55 80 180	Not exposed	0000 to 0130, 0700 to 1000 26/6/51
4.	1500 2/7/51 to 1200 9/7/51	45 to 61	70 105 145	70 105	95 195 245	Not exposed	0500 to 0730 4/7/51 2200 to 2400 5/7/51 1400 to 1700 8/7/51
5.	1200 9/7/51 to 1000 14/7/51	61 to 71	70 110 145	70 110	95 195 245	Not exposed	0830 to 1000 11/7/51 1300 to 1630, 1700 to 2400 12/7/51
6.	1000 19/7/51 to 1100 26/7/51	75 to 79	85 125 160	85 125	110 210 245	Not exposed	0800 to 0900, 1530 to 1700, 23/7/51 0000 to 0300, 1300 to 1700, 2100 to 2400 23/7/51
7.	0900 9/8/51 to 0900 17/8/51	80	85 125 160	85 125	110 210 245	110 210	1800 to 1900 9/8/51 0900 to 2400 11/3/51 0000 to 0200, 0600 to 0900, 1200 to 1300, 1600 to 1800. 12/8/51
8.	1400 20/8/51 to 1000 25/8/51	Stubble 10	25 60 100	25 60	85 125 160	85 125	2000 to 2200 21/8/51 0600 to 1600 22/8/51 0900 to 1000, 1600 to 1700 24/8/51

TABLE II

Computed Daily Evaporation (mm. water)

a.	Date	Amount(b)
Group 1.		
1300 - 2400	25/5/51	1.48
	26	0.70
	27	0.74
	28	0.52
	29	1.28
	30	1.38
	31	1.52
	1/6/51	1.92
	2	1.40
	3	1.18
	4	1.01
	5	1.55
0000 - 1000	6	0.50
Group 2.		
1000 - 2400	11/6/51	2.38
	12	3.98
	13	3.62
	14	2.49
	15	2.03
0000 - 1000	16	0.36
Group 3		
1100 - 2400	25/6/51	2.09
0000 - 1000	26	1.14
Group 4		
1500 - 2400	2/7/51	2.28
	3	5.83
2400 - 1000	4	2.73
1600 - 2400	5	1.09
	6	5.06
	7	3.50
	8	3.40
0000 - 1200	9	3.12
Group 5		
1200 - 2400	9/7/51	2.53
	10	4.87
	11	2.92
	12	4.28
	13	3.22
0000 - 1000	14	0.11
Group 6		
1000 - 2400	19/7/51	4.46
	20	3.79
	21	1.95
	22	1.22
	23	0.05
	24	1.31
	25	2.53
0000 - 1100	26	0.43

a.	Date	Amount(b)	Amount(c)
Group 7.			
0900 - 2400	9/8/51	2.17	2.03
	10	3.06	2.43
	11	1.91	1.69
	12	0.04	0.17
0000 - 0900	13	0.25	0.22
1600 - 2400	14	0.15	0.17
	15	2.60	2.31
	16	1.41	1.23
0000 - 0900	17	0.09	0.10
Group 8			
1400 - 2400	20/8/51	1.82	1.81
	21	5.48	5.25
0000 - 0900	22	0.69	0.69
1600 - 2400			
	23	1.30	1.26
	24	2.90	2.70
0000 - 1000	25	0.22	0.20

		Amount(d)	Amount(e)
Group 8			
1400 - 2400	20/8/51	2.09	2.15
	21	5.97	5.97
0000 - 0900	22	0.70	0.70
1600 - 2400	23	1.49	1.51
	24	3.18	3.06
0000 - 1000	25	0.20	0.23

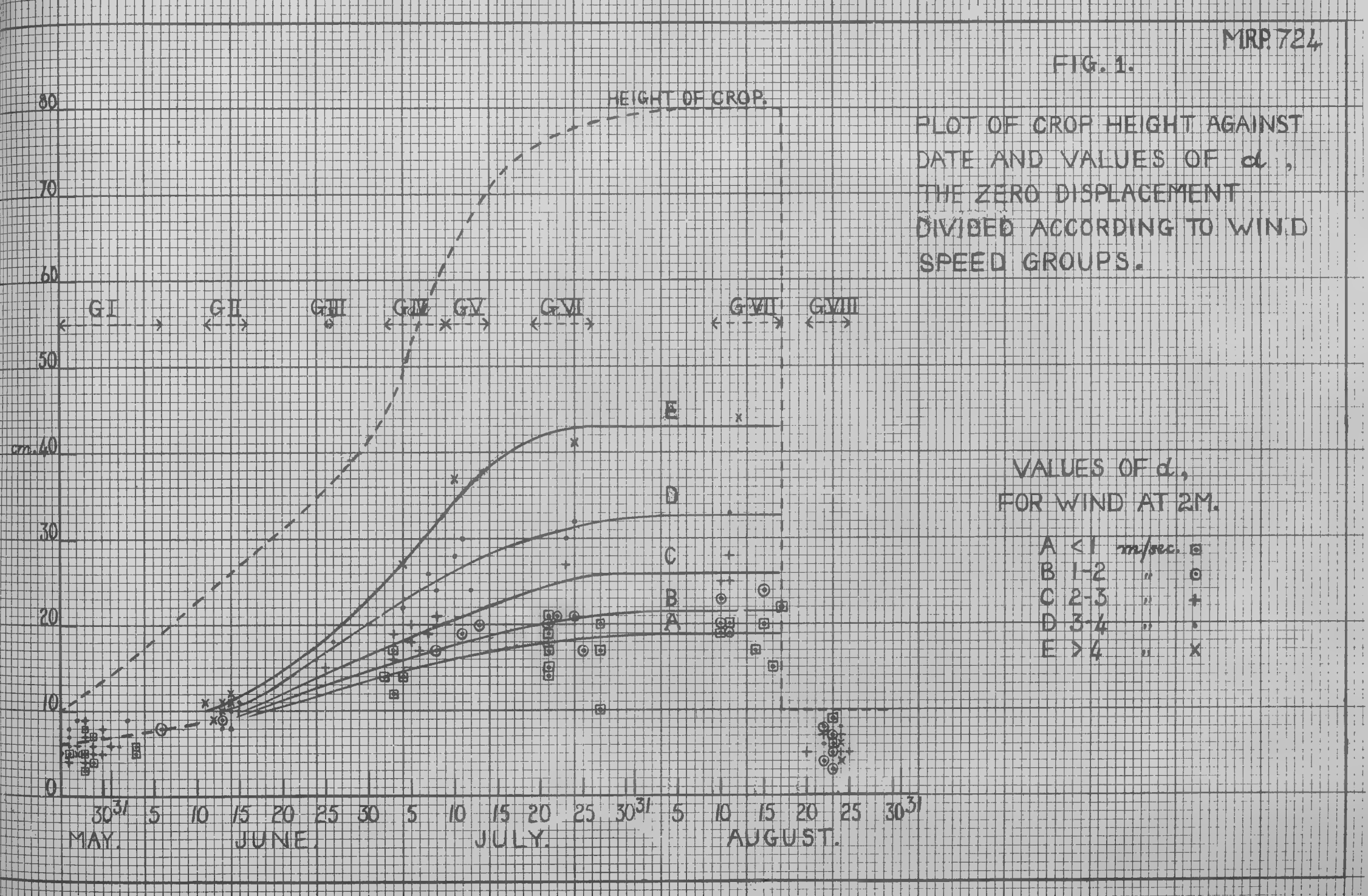
Notes.

On a few occasions records were lost due to clock stoppages, relay failures, etc..
Therefore when the water loss is for a period other than the 24 hours the period is at a.
b is the amount calculated from the results of apparatus set A by the use of equation 9.
c is the amount calculated from the results of apparatus set B by the use of equation 9.
d is the amount calculated from the results of apparatus set A by the use of equation 10 for the unstable hours and of equation 9 for the neutral and stable hours.
e is the amount corresponding to d but calculated from the results of apparatus set B

MRP.724

FIG. 1.

PLOT OF CROP HEIGHT AGAINST DATE AND VALUES OF d, THE ZERO DISPLACEMENT DIVIDED ACCORDING TO WIND SPEED GROUPS.



VALUES OF d, FOR WIND AT 2M.

A	<1	m/sec.	⊡
B	1-2	"	⊙
C	2-3	"	+
D	3-4	"	•
E	>4	"	x

M.R.P 724

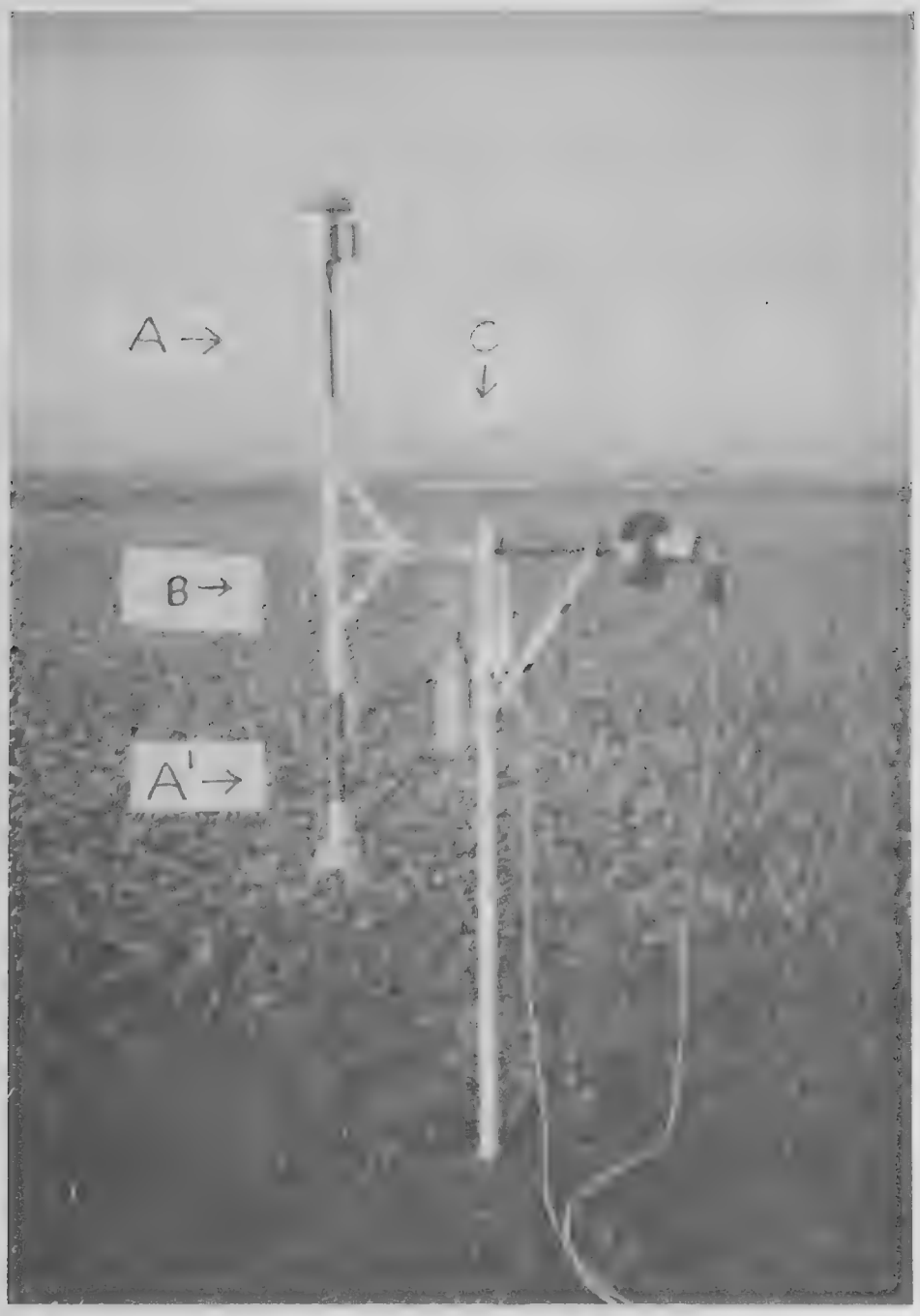
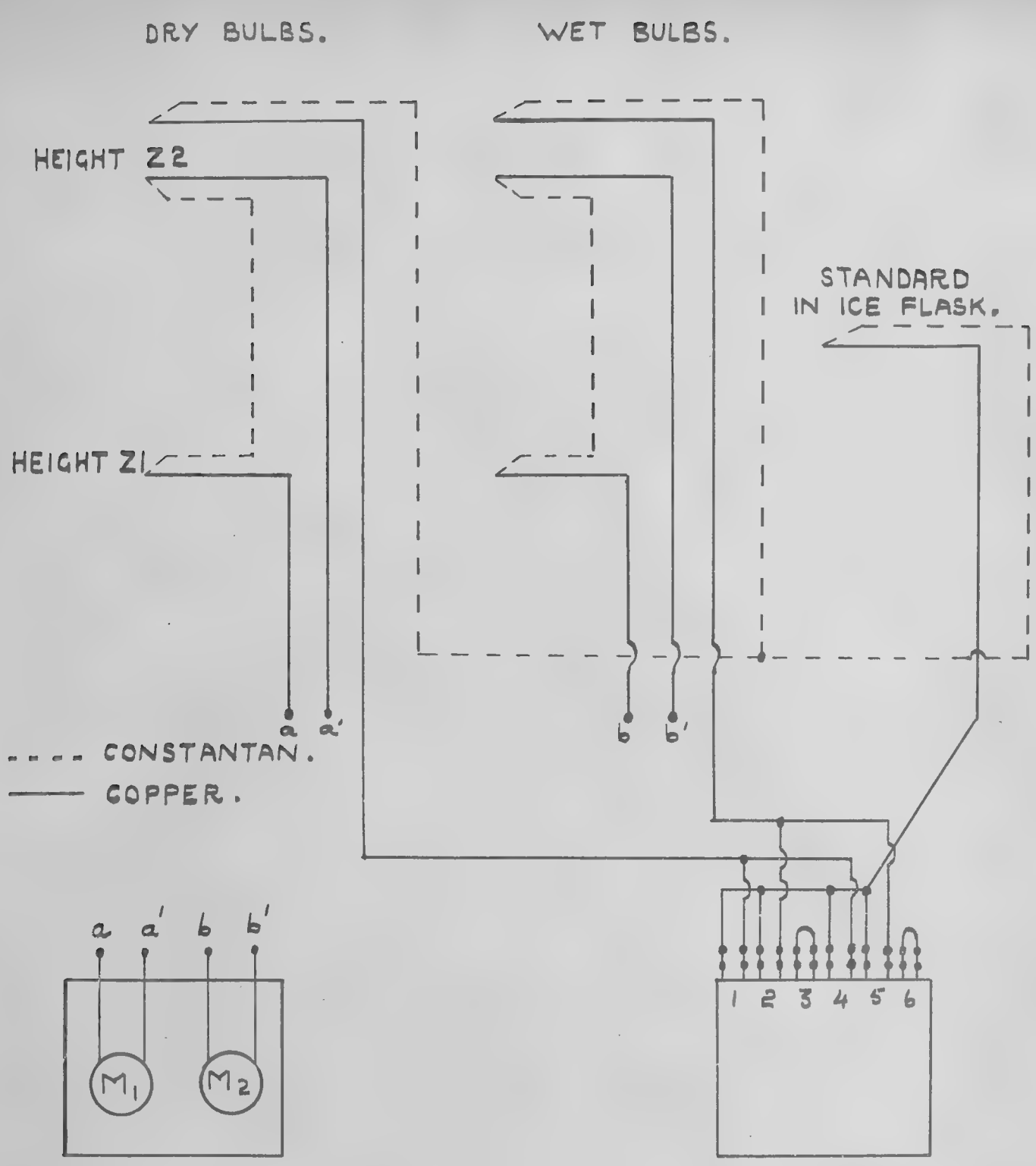


FIG. 2



PHOTOGRAPHIC GALVANOMETER RECORDER.

$M_1 \quad T_{2_2} - T_{2_1} = dt$

$M_2 \quad T'_{2_2} - T'_{2_1} = dt$

THREAD RECORDER.

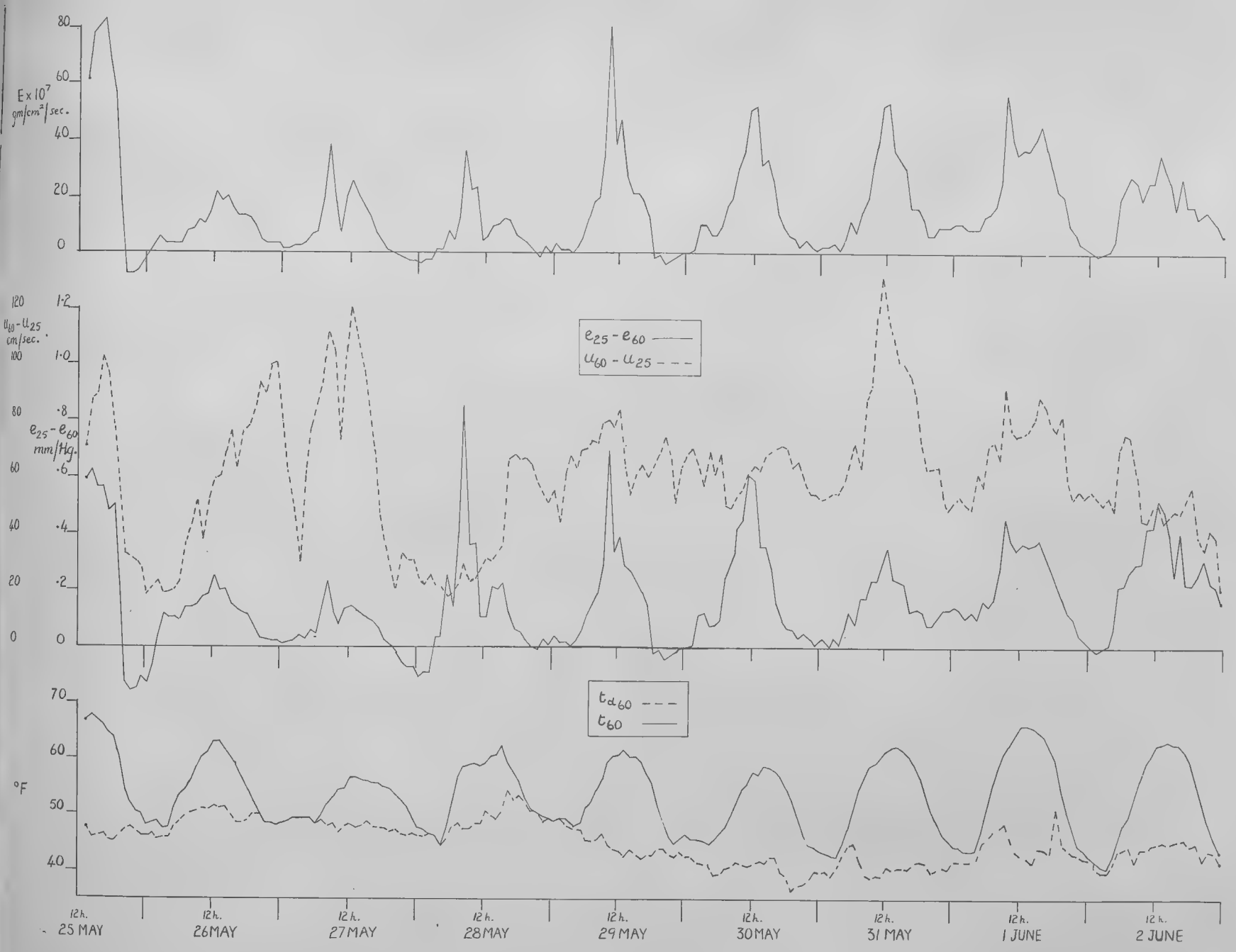
1 & 4 $T_{2_1} - T_S$

2 & 5 $T'_{2_1} - T_S$

3 & 6 CLOSED CIRCUIT ZEROS.

Fig. 3. Wiring Diagram of Humidity Masts.

FIG. 4.



A HOURLY RATES OF EVAPORATION FOR THE PERIOD 1300 hrs. 25·5·51 TO 2400 hrs. 2·6·51

B HOURLY VAPOUR PRESSURE DIFFERENCES AND WIND VELOCITY DIFFERENCES OVER THE HEIGHT RANGE 25 cms. TO 60 cms. FOR THE SAME PERIOD.

C HOURLY DRY-BULB AND DEW-POINT TEMPERATURES AT 60 cms. OVER THE SAME PERIOD.

FIG. 5 HOURLY RATES OF EVAPORATION DURING OBSERVATION PERIOD NO. 7, COMPUTED FROM THE RESULTS PROVIDED BY APPARATUS SET A AND SET B, USING THE NORMAL AERODYNAMIC FORMULA.

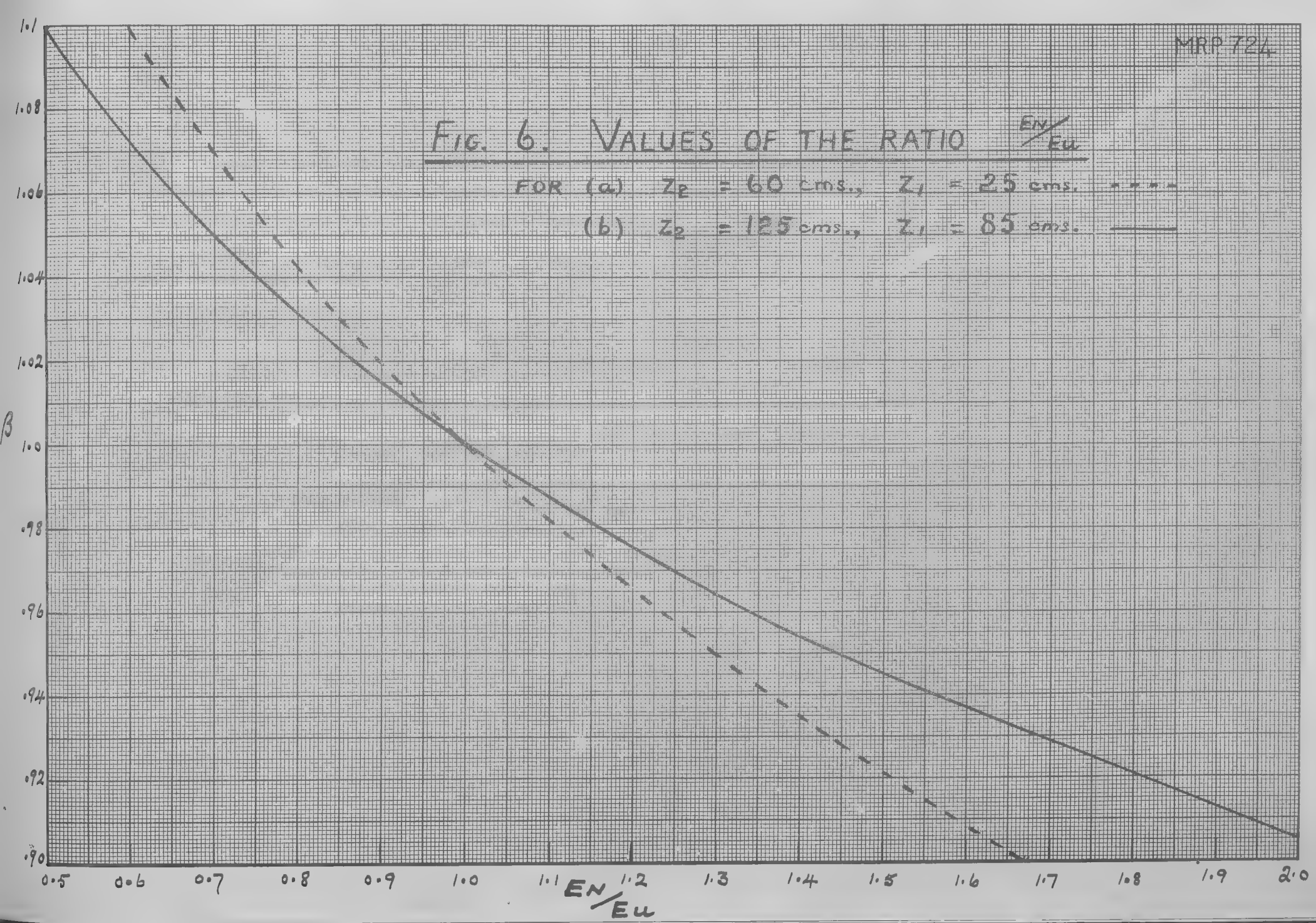


FIG. 6. VALUES OF THE RATIO EN/Eu
FOR (a) Z2 = 60 cms., Z1 = 25 cms.
(b) Z2 = 125 cms., Z1 = 85 cms.
β
EN/Eu
1.1
1.08
1.06
1.04
1.02
1.0
.98
.96
.94
.92
.90
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2.0

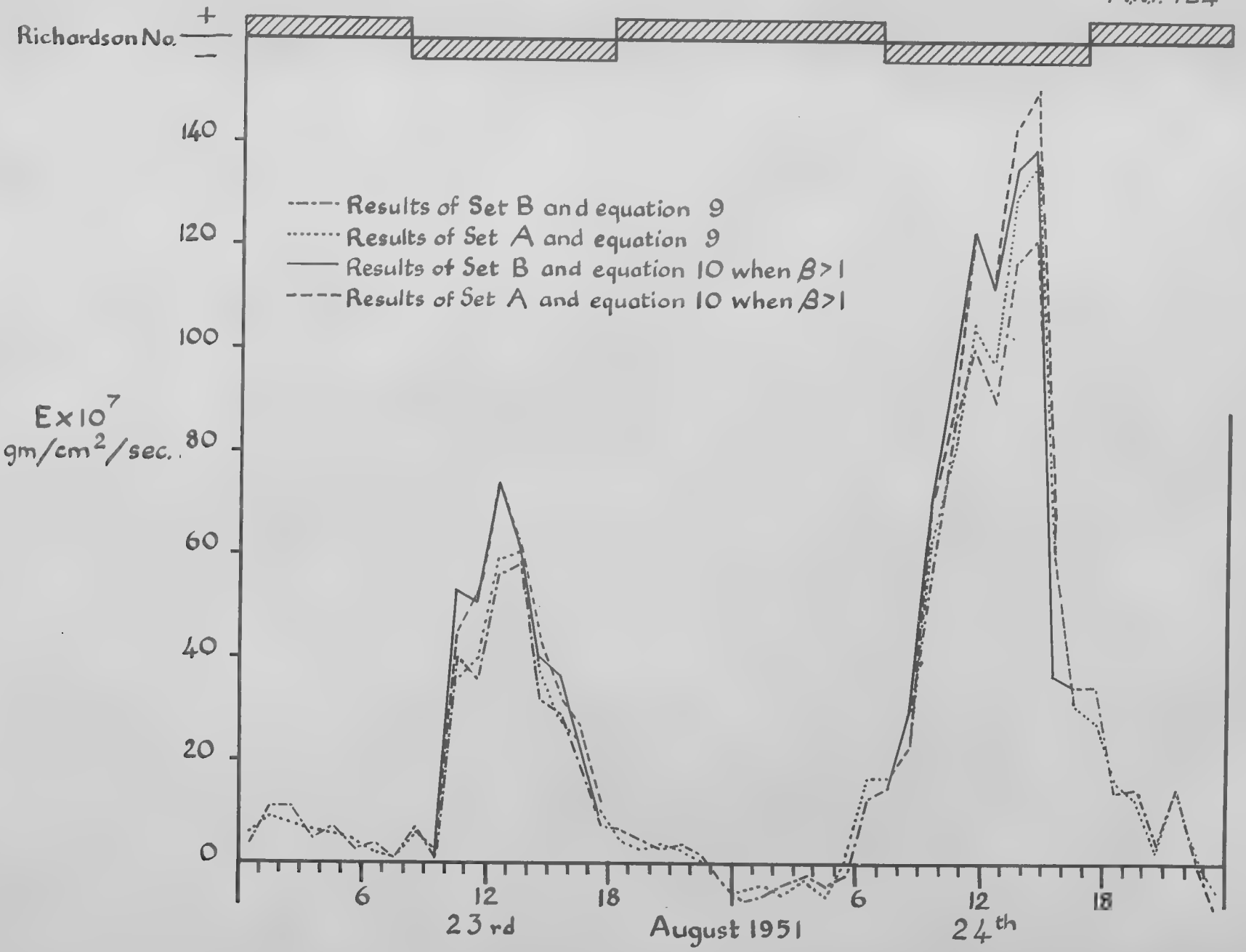


Fig 7. Hourly Rates of Evaporation Computed from Observations with Apparatus Sets A and B on 23rd and 24th August.

M.R.P.725
S.C.III/128
12th May, 1952.

AD 223038

AIR MINISTRY

METEOROLOGICAL RESEARCH COMMITTEE

THE VARIATION OF HUMIDITY WITH HEIGHT OVER THE OCEAN

by
C.R. Darlington

Observations.

1. During a number of cruises in H.M. Ships during 1948, 1949, 1950 and 1951, readings of dry and wet bulb temperatures were taken at different levels on a number of occasions in exposed positions to windward, using an Assman Psychrometer. Each reading recorded in the 1948-50 series was the average of three readings taken during a two minute period. In the 1951 series, when temperatures were often below freezing point, great care was taken to use a clean wick on the wet bulb and the readings at each level were made only after waiting for several minutes. A further difficulty in this series was the height measurement in a small ship, particularly in rough weather. Some of the irregularity of the vapour pressure results is probably due to this.

In the calculations it has been assumed that the dew point temperature at the surface is equal to the sea surface temperature: vapour pressure at the surface has been taken to be 98% of the vapour pressure over fresh water to allow for salinity.

The readings have been grouped according to area, stability of the air and state of the surface of the sea (rough or smooth).

2. Theory.

(a) State of the surface. Rossby[1] has shown that for wind speeds less than 12 kt. as measured at heights between about 20 and 40 feet above the surface, the surface can be regarded as hydrodynamically smooth. Anemometer heights in the ships concerned in the 1948-1950 readings were about 100 ft. but this makes very little difference to the criterion. If for a smooth surface with a thin laminar layer the flow in the turbulent layer above is assumed to be given by von Karman's equation

$$\frac{1}{\gamma_z} + \frac{1}{k_o} \log_e \frac{1}{\gamma_z} = 5.5 + \frac{1}{k_o} \log_e \frac{W_z\, z}{\nu} \quad \ldots\ldots(1)$$

it is easily shown that for heights between 100 and 130 feet, 13 kt. may be taken as the limiting speed for hydrodynamically smooth flow.

In the above equation ν and k_o are the kinematic viscosity of air and von Karman's constant (0.4) respectively, γ_z and W_z the resistance coefficient and wind speed referred to height z .

(b) The evaporation coefficient. To describe the vapour pressure gradient Montgomery[2] introduced a non-dimensional quantity which he called the 'evaporation coefficient' Γ defined as $\Gamma_a = \frac{1}{e_s - e_a} \frac{\partial e}{\partial (\log h)}$ where e_s and e_a are the vapour pressures at the surface and height a respectively.

Montgomery also showed that if the distribution of water vapour varies as the logarithm of height above the surface and if the flux of water vapour is independent of height in this layer, then the rate of evaporation from the surface is given by

$$E = \rho\, k_o\, \gamma_a\, \Gamma_b\, (q_s - q_b)\, W_a \quad \ldots\ldots\ldots\ldots\ldots(2)$$

ρ is air density, the q's are specific humidities at the surface and the height b , W_a is the wind speed at height a and the other quantities have been defined. This formula holds for a rough or smooth sea surface. The units of E are mass per unit area per unit time.

/(c)

(c) Smooth surface. Montgomery obtained a theoretical expression for Γ, for a hydrodynamically smooth surface, given by

$$\Gamma_a = \left[\frac{\lambda \nu k_o}{\kappa} + \log_e \frac{k_o \gamma_a W_a a}{\kappa} \right]^{-1}$$

where κ is the kinematic coefficient of diffusion of water vapour through air, λ is a constant and the other symbols have already been defined. According to von Karman $\lambda = 11.5$, but Montgomery uses $\lambda = 7.8$. With $\lambda = 11.5$ and $a = 6$ metres, and taking the values of ν and κ to be .15 $\frac{cm^2}{sec}$ and .25 $\frac{cm^2}{sec}$, which are applicable to a temperature of 68°F, an average value for Γ_{6m} is .08 approximately. This result applies to neutral conditions only. If λ is taken to be 7.8 the value for Γ_{6m} is about ten per cent higher. Variation of temperature has a negligible effect on the result.

(d) Rough surface. For a hydrodynamically rough surface, Montgomery uses a 'roughness length' (z_o) for the sea of .6 cm, independent of the wind velocity. He deduces theoretically that the evaporation coefficients for a rough surface are about half those for a smooth surface.

Sverdrup[3] has adopted a simpler theoretical approach by assuming that the turbulent layer extends right down to the sea surface, for a rough surface. He then obtains $\Gamma_a = \left[\log_e \frac{a+z_o}{z_o} \right]^{-1}$ independent of wind speed. For $a = 6$ metres and $z_o = .6$ cm. this gives $\Gamma_{6m} = .145$, and Sverdrup's experimental results, for all unstable cases with wind speed greater than about 16 kt., appear to be in reasonable agreement with this value. Γ_{6m} does not vary in any obvious way with wind speed and the average value is .156, about four times Montgomery's theoretical value. Norris[4], using a variation of Montgomery's approach and adopting the same 'roughness length' (.6 cm), deduces a theoretical value $\Gamma_{6m} = .166$ in near agreement with Sverdrup.

(e) Stable conditions. None of the above applies to stable conditions, for which no satisfactory theory is available, but the value of Γ_{6m} was found by Sverdrup to be generally higher than for unstable air.

3. The present readings.

The individual readings are given in Tables I(a) to I(j) of the Appendix, and the means of each group in Tables II(a) to II(j). The grouping is explained in Section I. The graphs are plotted from Tables II(a) to II(j).

(a) Arctic. These readings were taken between 5 and 30 feet. As already mentioned, difficulty in measuring height accurately in a small ship in rough weather makes it dangerous to regard the results as showing anything but broad features. The vapour pressure gradients with strong winds are irregular; for calmer conditions the gradient is approximately logarithmic between 5 and 15 feet in the unstable groups. The contrast between the mean vapour pressure and temperature gradients above and below five feet is very marked in these groups; for vapour pressure the ratio is about one to twenty, and for temperature higher still. The few readings for stable conditions indicate little change of temperature or vapour pressure with height, except near the surface.

(b) Northern Waters. Readings were taken between 22 and 70 feet. For 'smooth' conditions the vapour pressure is practically constant with height, but for 'rough' seas the logarithmic relation is approximately true.

(c) Sub-tropics (S.W.North Atlantic). Readings were taken between 21 and 73 feet. In all cases the sea was 'smooth'. The logarithmic relation for vapour pressure is almost exact for instability and holds up to about 60 feet for stability.

4. Results.

For those groups in which the lapse of vapour pressure is approximately logarithmic the value of Γ has been worked out for a height of 20 feet, in order to compare results with those previously published for 6 metres. The corresponding

/value

value of E , the rate of evaporation from the surface, and the equivalent in calories per square cm. per day, is also given, derived from equation (2). The value of $\gamma_{a_{k_o}}$ in equation (2) is given by

$\gamma_a = \frac{}{\log_e \frac{a+z_o}{z_o}}$ for a rough surface and is deduced from equation (1) for a smooth surface.

The results are summarised below:

Arctic

Type	Mean Wind (kt.)	No. of readings	$\Gamma_{20'}$	$\frac{gm.}{E_{cm.^2 sec}} \times 10^{-6}$	$\frac{cal.}{E_{cm.^2 day}}$
Unstable Smooth	8.3	7	.165	2.96	150
Unstable Rough (a)	13	13	.102	5.18	262
" " (b)	18	12	.052	3.80	192
(c)	-	-	Too erratic	-	-
(d)	-	-	"	-	-
" " (overall mean) (e)	18.7	36	.063	4.67	236

Northern Waters

Unstable Rough (a)	13	3	.249	6.75	341
" " (b)	18	6	.103	5.88	297
" " (c)	23	5	No gradient	-	-
" " (overall mean) (d)	18.7	14	.095	5.30	268

S.W.North Atlantic

Unstable Smooth	7.3	16	.131	2.60	132

Although the theory does not apply to stable conditions there were two groups for which the mean vapour pressure gradient was approximately logarithmic between about 20 feet and 60 feet. Values for $\Gamma_{20'}$ were:

Northern Waters

Type	Mean Wind (kt.)	No. of readings	$\Gamma_{20'}$
Stable Rough	18.6	8	.332

S.W.North Atlantic

Stable Smooth	8.6	7	.098

5. Comments

It will be noted that the values of Γ are very erratic. Whether this is due to the difficulty of obtaining representative readings on board ship or to true variability, or a combination of both, is impossible to say from the information available.

One interesting comparison can be made however: the results for Arctic waters suggest that Γ for unstable smooth surfaces is rather more than double the value for unstable rough surfaces, which supports Montgomery's theoretical result. Norris and Sverdrup deduce theoretically (and Sverdrup produced results in support) that the relationship was the other way round. Unfortunately in the present series the other regions do not afford a similar comparison.

/E

E is not so erratic as Γ, but it is noticeable that its value decreases with increasing wind speed, for rough surfaces both in Arctic and Northern waters.

As the expression for E is in essentials equivalent to $E \propto W_a(e_s - e_b)$ this must mean that the intense mixing caused by strong winds results in a decrease in the value of $(e_s - e_b)$ which more than compensates for the increase in W_a.

The numerical values of E are comparable with those given by Sverdrup in "Oceanography for Meteorologists", page 233.

References:

1. Rossby, C.G., 1936: On the momentum transfer at the sea surface. Papers in Physical Oceanography and Meteorology, 4, No. 3.

2. Montgomery, R.B., 1940: Observations of vertical humidity distribution above the ocean surface and their relation to evaporation. Papers in Physical Oceanography and Meteorology, 7, No. 4.

3. Sverdrup, H.U., 1946: The humidity gradient over the sea surface. Journal of Meteorology, 3, No. 1.

4. Norris, R., 1948: Evaporation from extensive surfaces of water roughened by waves. Q.J.R. Met. Soc., 74, No. 319.

TABLE I(a)

ARCTIC WATERS - UNSTABLE ROUGH

Date	Time	Position	Cloud	Wind	P_σ	Sea Surface T	Sea Surface e	5 ft. T	5 ft. e	10 ft. T	10 ft. e	15 ft. T	15 ft. e	20 ft. T	20 ft. e	25 ft. T	25 ft. e	30 ft. T	30 ft. e	Remarks
18.10.51	1000Z	67°N 10°E	2/8 CL1	290/15	1011	48.0	11.15	45.4	8.06	45.3	7.96	44.1	7.55	43.8	7.65	43.0	7.82	44.6	7.01	
"	1500Z	67°22'N 11°5'E	4/8 CL2	260/16	1010	48.4	11.31	45.2	7.86	45.3	8.03	44.6	7.72	44.6	7.38	44.0	7.18	43.2	6.94	
23.10.51	2100Z	70°30'N 21°E	CL NIL	175/12	1007	42.4	9.03	36.9	4.26	35.9	3.93	35.4	3.70	35.4	3.70	35.4	3.59	36.2	3.80	Wind gusty
24.10.51	1500Z	71°11'N 28°17'E	CL NIL Prefrontal CM	165/14	1001	43.1	9.25	37.0	5.49	36.6	5.52	36.1	5.22	35.8	5.32	36.0	5.59	35.5	5.42	
"	2100Z	70°43'N 29°34'E	8/8 Low CU.	110/20	997	42.8	9.15	35.8	6.74	35.1	6.70	35.0	6.30	35.1	6.60	35.1	6.27	35.1	6.70	Showery
25.10.51	1500Z	70°N 34°E	5/8 ST.CU.	015/17	1002	43.2	9.28	38.0	5.42	36.3	4.57	35.9	4.06	36.1	4.40	36.0	5.11	36.0	5.11	
"	2200Z	70°51'N 33°51'E	1/8 CU.	360/12	1007	43.1	9.25	36.3	5.25	37.0	4.54	36.2	4.71	37.0	4.37	35.2	4.40	36.3	4.09	Wind gusty, strong sea
26.10.51	2200Z	73°10'N 34°30'E	5/8 ST.CU.	360/13	1017	40.5	8.40	31.3	5.79	31.0	4.71	30.1	4.20	30.0	4.23	30.0	3.83	29.5	3.67	Slight snow falling
27.10.51	1000Z	71°15'N 38°20'E	6/8 ST.CU. 4000 ft.	360/12	1018	38.8	7.83	31.9	4.81	31.2	4.64	31.0	4.50	30.5	4.64	29.9	4.09	31.9	4.23	Gusty
"	1500Z	70°30'N 38°32'E	5/8 ST.CU. 3500 ft.	360/13	1017	39.0	7.90	30.5	4.98	30.0	4.81	29.3	4.50	29.3	4.54	29.4	4.43	28.8	4.37	Gusty
2.11.51	2200Z	70°29'N 22°44'E	8/8 ST. 1700 ft.	040/31	998	45.1	9.99	35.1	6.36	34.0	6.17	32.6	5.79	31.9	6.07	32.7	5.76	32.2	5.90	Gusty
3.11.51	1600Z	71°N 26°20'E	4/8 CU.NB. 2000 ft.	030/32	1010	45.1	9.62	30.0	4.94	29.4	4.91	30.2	5.18	29.5	5.28	30.4	5.18	30.6	5.18	Intermittent snow showers
"	2100Z	70°51'N 26°51'E	3/8 CU.NB 2000 ft.	340/14	1011	44.2	9.65	29.0	5.05	29.2	4.81	29.2	4.64	28.6	4.26	28.5	4.43	29.2	4.40	
5.11.51	0900Z	70°52'N 29°32'E	7/8 CU.NB. 600 ft.	270/12	1027	42.1	8.93	33.3	4.54	32.1	4.13	32.8	4.64	32.3	5.73	31.7	5.76	30.2	5.62	Continuous light snow. Gusty. Sea rough
"	1500Z	71°32'N 27°38'E	8/8 CU.& ST.CU 1000 ft.	295/16	1027	43.8	9.53	36.2	4.91	34.2	4.71	32.8	5.56	31.2	5.59	32.0	5.69	32.3	5.66	Soft hail showers.
"	2100Z	72°10'N 25°50'E	8/8 CU.& ST.CU. 1000 ft.	330/18	1026	43.8	9.53	35.2	4.40	34.2	4.13	33.8	3.73	33.8	4.98	34.0	4.84	34.0	4.71	Slight hail showers
6.11.51	1000Z	73°30'N 21°40'E	2/8 FR. ST. 1000 ft.	250/16	1020	40.4	8.37	34.9	6.17	33.8	5.83	33.6	5.76	33.8	5.56	34.1	5.08	34.0	5.28	Snow showers
"	1500Z	73°48'N 20°56'E	4/8 ST. CU. 7000 ft.	270/13	1015	38.0	7.60	33.8	5.83	33.4	5.90	33.4	5.90	33.6	5.83	34.0	5.86	34.2	6.00	
7.11.51	1200Z	75°50'N 17°E	8/8 FR. ST. 1500 ft.	060/26	1017	30.9	5.67	23.1	3.83	21.0	3.39	21.0	3.19	19.3	3.29	19.4	3.08	20.5	3.02	Intermittent snow
"	1600Z	75°15'N 17°15'E	8/8 FR. ST. 1400 ft.	060/25	1018	34.0	6.47	21.3	3.29	23.3	3.86	17.8	2.85	16.6	2.81	16.6	2.78	17.0	2.81	Snow showers
"	2200Z	74°30'N 18°30'E	4/8 ST. CU. 3000 ft.	100/19	1014	33.4	6.32	21.7	3.08	20.7	2.88	20.4	2.85	20.2	2.64	20.4	2.78	21.7	2.20	
9.11.51	1000Z	71°05'N 27°E	2/8 CU. NB.	200/20	1028	45.2	10.03	31.6	4.33	30.0	4.23	29.9	3.96	30.8	4.06	29.8	3.76	30.1	3.83	
16.11.51	1600Z	71°N 30°20'E	5/8 CU.& large CU.	090/19	1021	40.7	8.43	31.5	5.09	30.3	5.15	30.1	5.22	30.1	4.54	29.5	3.86	30.0	3.05	Showers of snow.
"	2200Z	70°35'N 31°50'E	3/8 CU. 1600 ft.	100/13	1023	41.8	8.82	30.1	4.20	29.6	3.59	29.6	3.02	29.5	2.78	28.9	2.91	28.9	2.98	Gusty
17.11.51	1100Z	70°35'N 31°50'E	6/8 ST. CU. 2400 ft.	180/21	1023	41.2	8.62	28.3	4.47	27.5	4.57	27.3	4.23	26.9	3.83	26.7	3.70	26.8	3.12	Occasional hail
"	1600Z	70°08'N 34°30'E	7/8 ST. 2500 ft.	180/15	1021	41.0	8.55	24.6	3.89	22.8	3.80	22.4	3.36	22.2	3.39	21.3	3.12	21.9	2.95	Gusty
"	2200Z	69°31'N 33°39'E	7/8 ST. CU. 3300 ft.	180/19	1018	41.1	8.60	26.3	4.06	25.6	3.93	25.3	4.03	24.5	3.39	24.1	3.36	25.1	3.05	Gusty
20.11.51	1100Z	70°39'N 31°32'E	4/8 CU.& CU. NB. 600 ft.	110/21	998	42.3	8.99	33.0	5.25	33.0	5.25	31.4	5.83	31.6	5.86	31.6	5.59	30.7	5.76	Slight intermittent snow.
28.11.51	1500Z	68°53'N 16°24'E	8/8 ST. CU. 800 ft.	310/23	970	42.6	9.09	36.5	4.23	36.1	4.43	35.8	4.54	36.5	4.43	35.4	4.67	35.9	4.50	
"	2100Z	69°28'N 16°12'E	5/8 CU. & CU.NB.2100ft.	010/36	977	43.0	9.22	31.5	4.26	31.3	4.20	29.7	4.57	30.9	3.36	31.5	3.66	31.3	4.03	
29.11.51	0900Z	67°51'N 11°52'E	8/8 CU.& CU. NB. 1500 ft.	350/22	992	43.3	9.22	32.0	5.59	31.6	5.28	28.8	5.40	30.6	5.66	29.9	5.32	30.2	5.62	Showers of snow, very gusty
29.11.51	1500Z	67°11'N 10°29'E	8/8 CU.& CU. NB. 1500 ft.	330/27	994	43.7	9.87	30.9	3.42	30.9	3.12	30.2	3.99	29.8	3.83	30.9	3.23	30.9	3.12	Showers of snow
"	2100Z	66° 10'N 08°30'E	8/8 CU.& CU. NB.	350/19	993	44.4	9.72	34.2	3.05	33.6	3.29	33.4	3.32	32.7	3.46	30.9	3.80	32.7	3.39	Showers of snow.
1.12.51	1000Z	61°N 0°15'W	4/8 ST. CU. 1800 ft.	340/25	996	47.8	11.08	41.0	5.32	39.8	4.33	39.0	3.73	37.0	4.57	36.9	4.13	39.8	3.76	Heavy swell

ARCTIC WATERS - UNSTABLE ROUGH (contd.)

Date	Time	Position	Cloud	Wind	P_o	Sea T	Surface e	5 ft. T	5 ft. e	10 ft. T	10 ft. e	15 ft. T	15 ft. e	20 ft. T	20 ft. e	25 ft. T	25 ft. e	30 ft. T	30 ft. e	Remarks
1.12.51	1500Z	60°32'N 0°31'W	5/8 CU. 1500 ft.	340/16	1003	49.1	11.61	42.1	5.42	41.0	5.05	40.8	4.84	40.1	5.25	40.1	4.98	39.7	5.18	Slight showers of snow
"	2100Z	59°43'N 1°15'W	5/8 CU. 1500 ft.	350/12	1007	49.1	11.61	44.1	6.03	43.7	5.69	43.3	5.66	43.0	5.08	43.4	4.84	43.0	5.08	

TABLE I(b)

NORTHERN WATERS - UNSTABLE ROUGH 36° - 60°N.

Date	Time	Position	Cloud	Wind	P_o	Sea T	Surface e	22 ft. T	22 ft. e	49 ft. T	49 ft. e	79 ft. T	79 ft. e
26.10.49	1230Z	Off N.Scotland	4/8 CU.NB.	360/25	1005	49.0	11.35	46.0	8.30	46.5	8.12	45.5	7.74
16.11.49	0900Z	Off Portland	ST.CU. 4000 ft.	160/16	1020	55.5	14.44	52.0	11.11	52.0	11.11	50.5	10.45
28. 1.50	1100Z	W. Channel	6/8 ST.CU. 3000 ft.	080/23	1015	45.5	9.96	42.8	7.48	41.8	7.38	41.0	7.32
1. 2.50	0900Z	36°30'N 8°10'W	-	350/25	1020	61.0	17.61	54.5	7.96	51.7	7.67	50.5	7.74
1. 3.50	1030Z	37°25'N 02°30'E	5/8 CU. 2500-1000 ft.	015/20	1010	58.4	16.02	53.1	7.96	52.8	7.86	52.1	7.58
2. 3.50	0900Z	38° 06'N 7°56'E	4/8 CL3 3000 ft.	325/16	1010	57.5	15.53	52.1	7.38	52.5	7.38	52.2	7.48
14. 3.50	0900Z	41°49'N 7°07E	3/8 ST.CU. 3000 ft.	290/18	1015	55.5	14.44	55.5	12.34	55.5	10.64	53.6	10.35
22. 5.50	0900Z	48°10'N 5°05'W	2/8 CU.NB. 2500 ft.	170/24	1010	55.5	14.44	55.8	12.22	55.4	12.21	55.5	12.34
26. 5.50	0800Z	46°22'N 4°36'W	6/8 CU. 2000 ft.	275/15	1010	57.4	15.48	56.0	13.04	56.0	12.60	56.0	12.60
2. 6.50	0900Z	S. of Isle of Wight	6/8 CM 7	090/16	1020	56.8	15.15	57.7	14.22	56.5	14.12	56.2	14.14
15. 6.50	0800Z	56°14'N 12°44'W	Cloudy	330/15	1015	52.0	12.68	50.4	8.77	50.7	8.05	50.6	8.08
16. 6.50	0900Z	58°14'N 8°44'W	"	330/18	1000	51.6	12.51	50.8	9.79	50.5	9.67	50.7	9.37
25. 7.50	0830Z	57°20'N 7°20'E	5/8 CU. 2000 ft.	305/25	1000	61.4	17.86	59.1	13.29	59.0	13.08	58.7	13.09
26. 7.50	0730Z	55°N 5°45'E	4/8 Ci.	300/14	1010	59.5	16.69	59.2	14.57	59.0	14.19	58.5	13.62

TABLE I(c)

ARCTIC WATERS - STABLE ROUGH

Date	Time	Position	Cloud	Wind	P_o	Sea T	Surface e	5 ft. T	5 ft. e	10 ft. T	10 ft. e	15 ft. T	15 ft. e	20 ft. T	20 ft. e	25 ft. T	25 ft. e	30 ft. T	30 ft. e
29.10.51	1000Z	71°03'N 26°40'E	6/8 FR.ST. 1000 ft.	250/25	1010	43.0	9.22	44.5	9.99	44.9	10.06	43.5	9.38	44.2	9.65	45.2	9.89	44.6	9.72
"	1600Z	70°57'N 24°18'E	8/8 ST. 1000 ft.	250/25	1009	44.8	9.89	45.9	10.06	45.5	10.06	45.8	9.85	45.9	9.89	46.4	10.02	46.5	10.13
"	2200Z	70°26'N 22°10'E	8/8 ST. 2600 ft.	250/20	1010	44.3	9.69	47.8	9.65	47.5	9.62	47.8	9.82	47.5	10.09	48.1	9.79	48.8	10.16

/TABLE I(d)

Date	Time	Position	Cloud
10.11.49	0900Z	Off Scilly Isles	-
30. 1.50	0930Z	44°N 10°11'W	8/8 ST. 800 ft.
5. 3.50	0900Z	38°34'N 3°20'E	-
15. 3.50	1500Z	27°55'N 2°57'E	-
1. 6.50	1200Z	S. of I. of W.	-
14. 5.50	1030Z	50°15'N 5°55'W	4/8 Ci.
9. 7.50	0900Z	64°50'N 11°E	7/8 Cu. 2000 ft.
15. 7.50	0800Z	60°40'N 4°E	3/8 ST.CU. 4500 ft.

Date	Time	Position	Cloud	Remarks
27.10.51	2130Z	69°45'N 39°E	3/8 ST.CU. 3500 ft.	light snow fall.
28.10.51	1500Z	70°13'N 32°40'E	7/8 ST.CU. 1000 ft.	
2.11.51	1500Z	70°26'N 21°50'E	3/8 ST.CU. 3000 ft.	
8.11.51	1000Z	73°N 22°52'E	8/8 ST.CU.	
"	1600Z	72°23'N 24°48'E	8/8 CU.& large Cu.	howers of snow
"	2100Z	71°40'N 26°30'E	8/8 Cu.& large Cu.	now showers
15.11.51	2100Z	70°46'N 23°50'E	5/8 ST.CU. 5000 ft.	light occasional snow.

Date	Time	Position	Cloud
8.11.49	0900Z	Off W.Ireland	Ci. ST.
15.11.49	0900Z	Off Portland	4/8 ST.CU. base 1500
22. 3.50	0930Z	36°1'N 04°49'W	NIL
24. 5.50	0800Z	48°20'N 05°W	2/8 Med.
5. 7.50	0830Z	Moray Firth	7/8 ST.CU. 1500 ft.

/TABLE I(g)

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ABLE SMOOTH. 13°N - 31°N.

Sea	Surface	21 ft.		41 ft.		57 ft.		73 ft.	
T	e	T	e	T	e	T	e	T	e
77.0	30.45	77.0	22.90	76.6	22.66	75.9	23.20	75.1	22.52
78.0	31.18	77.5	25.37	76.5	24.98	75.8	24.21	76.5	24.45
78.0	31.18	75.6	24.16	75.0	24.14	75.2	24.57	74.8	23.24
78.0	31.18	79.0	26.13	78.0	25.98	77.0	25.43	76.7	25.68
78.0	31.18	78.3	26.13	77.2	25.88	76.7	25.30	76.5	25.09
78.0	31.18	77.8	27.23	77.2	25.88	76.7	25.68	76.6	25.84
79.0	32.53	79.0	27.74	78.5	26.83	78.0	26.25	78.4	26.48
79.0	32.53	78.2	27.64	77.8	27.43	77.4	27.38	77.4	27.24
79.0	32.53	79.4	26.25	78.8	25.28	78.2	24.70	78.4	24.89
80.0	33.63	77.8	28.32	77.8	27.50	78.0	26.49	77.6	26.78
82.0	35.87	82.9	26.38	81.1	26.11	80.2	26.67	80.5	26.33
82.0	35.87	79.4	27.30	79.2	26.96	79.0	27.33	78.9	26.75
81.0	34.72	81.9	27.85	80.9	27.53	80.7	26.80	80.7	26.92
81.0	34.72	80.0	28.71	79.9	27.51	79.9	26.85	79.6	26.16
83.0	37.03	81.4	30.75	80.6	30.62	80.7	30.28	80.3	29.88
83.0	37.03	81.2	33.02	81.0	32.51	81.0	32.35	80.8	32.00

)

LE SMOOTH 36° - 60°N

Sea	Surface	22 ft.		49 ft.		79 ft.	
T	e	T	e	T	e	T	e
54.0	13.65	56.5	14.20	55.5	13.65	55.8	13.55
43.0	9.04	45.3	5.93	44.5	6.34	44.0	6.20
43.8	9.34	44.0	6.87	42.5	6.08	42.5	6.08
52.8	13.07	54.6	11.38	54.2	10.94	53.0	10.82
55.0	14.18	55.8	10.60	55.6	10.52	55.0	10.57
59.2	16.54	60.0	14.92	60.1	14.69	59.5	14.73
52.4	12.88	55.8	13.46	55.5	13.39	55.0	13.41
54.4	13.87	55.3	13.21	55.2	13.07	55.1	12.94
55.6	14.49	56.3	14.63	56.5	14.66	56.0	14.39
54.9	14.13	56.0	13.04	56.1	13.26	55.6	13.11

/TABLE I(j)

TABLE I(j)

S. WESTERN NORTH ATLANTIC - STABLE SMOOTH (13°N - 31°N)

Date	Time	Position	Cloud	Wind	P_o	Sea Surface T	Sea Surface e	21 ft. T	21 ft. e	41 ft. T	41 ft. e	57 ft. T	57 ft. e	73 ft. T	73 ft. e
1.10.48	1500 0	30°N 33°W	3/8 CL2 2500 ft.	175/10	1020	79.0	32.53	79.9	26.41	79.1	26.75	78.9	26.56	79.2	25.79
2.10.48	1200 0	26°30'N 35°30'W	3/8 CL2 2/8 CM3	170/8	1021	79.0	32.53	81.0	27.23	80.3	26.53	79.8	27.80	79.0	24.14
3.10.48	1200 0	25°30'N 38°W	1/8 CL7 3-5000 ft. 3/8 CH9	140/3	1020	79.0	32.53	82.1	25.88	80.5	26.87	80.1	26.63	79.2	25.37
4.10.48	1200 0	22°30'N 42°30'W	2/8 CL3 2000 ft. 4/8 CM7	160/11	1019	80.0	33.59	80.9	28.93	81.0	28.19	80.0	27.74	80.0	28.45
5.10.48	1800 0	20°N 50°W	2/8 CL3 3000 ft. 8/8 CM1	120/10	1016	81.0	34.72	82.8	29.19	82.2	29.02	81.5	27.17	81.0	27.09
6.10.48	1200 P	17°30'N 52°W	Trace CL1 3000 ft.	110/12	1016	81.0	34.72	85.2	29.85	85.9	29.31	81.4	29.31	83.1	28.53
8.10.48	1200 P	13°30'N 58°30'W	4/8 CL2 2000 ft. 2/8 CM5.	130/6	1015	84.0	38.24	87.8	34.96	86.0	32.68	88.0	33.46	85.9	32.90

SUMMARY OF ARCTIC RESULTS

TABLE II(a) UR GROUP

Wind ≥ 12 kt.

Height-ft.	MEAN e (mb.) 12-15 kt. a	16-20 kt. b	21-25 kt. c	26 kt. & over d	Mean of whole	MEAN T(°F) a	b	c	d	Mean of whole
0	9.07	9.33	8.88	8.81	9.10	42.5	43.0	41.9	41.4	42.4
5	5.24	5.04	4.69	4.56	4.99	34.2	34.4	32.0	30.1	33.3
10	4.93	4.88	4.62	4.36	4.78	33.7	33.3	31.9	29.3	32.7
15	4.74	4.78	4.43	4.54	4.67	33.3	33.0	30.0	28.7	32.0
20	4.73	4.82	4.53	4.37	4.68	33.2	32.7	29.9	28.3	31.8
25	4.67	4.73	4.36	4.18	4.57	32.8	32.5	29.5	29.0	31.6
30	4.59	4.59	4.26	4.25	4.49	33.1	32.8	30.1	29.1	31.9
No. of readings	13	12	6	5	36					
Mean wind speed kt.	13	18	23	30	18.7					

TABLE II(b)

SR GROUP

	Mean e (mb.)	Mean T(°F)	
0	9.60	44.0	Wind ≥ 12 kt.
5	9.90	46.1	
10	9.91	46.0	Mean Wind 23 kt.
15	9.68	45.7	
20	9.88	45.9	3 readings
25	9.90	46.5	
30	10.00	46.6	

TABLE II(c)

US GROUP

Mean e(mb)	Mean T(°F)	
8.58	40.9	Wind < 12 kt.
5.11	30.9	
4.54	30.1	Mean Wind 8.3 kt.
4.33	29.9	
4.33	29.9	7 readings
4.38	29.6	
4.47	30.0	

SUMMARY OF RESULTS FROM NORTHERN WATERS (36° – 60°N)

TABLE II(d) UR GROUP WIND ≥ 13 kt.

HEIGHT– FT.	MEAN e(mb) 12–15 kt. a	16–20 kt. b	21–25 kt. c	Mean of whole	MEAN T(°F) a	b	c	Mean of whole
0	14.96	14.70	14.24	14.61	56.3	55.9	54.5	55.5
22	12.13	10.47	9.85	10.60	55.2	53.5	51.6	53.2
49	11.61	10.13	9.69	10.29	55.2	53.3	50.9	52.9
79	11.43	9.90	9.65	10.13	55.0	52.6	50.2	52.3
No. of readings	3	6	5	14	3	6	5	14
Mean wind speed kt.	13	18	23	18.7	13	18	23	18.7

TABLE II(e) SR GROUP

	MEAN e(mb.)	MEAN T(°F)	
0	14.43	55.4	Wind ≥ 13 kt.
22	13.15	57.9	Mean Wind 18.6 kt.
49	12.87	57.7	8 readings
79	12.63	57.5	

TABLE II(f) SS GROUP

	MEAN e(mb.)	MEAN T(°F)	
0	13.13	52.5	Wind < 13 kts.
22	11.82	54.0	Mean Wind 8.8 kt.
49	11.66	53.6	10 readings
79	11.58	53.2	

TABLE II(g) US. GROUP

	MEAN e(mb.)	MEAN T(°F)	
0	14.86	56.2	Wind < 13 kt.
22	11.67	54.9	Mean wind 7.2 kt.
49	11.71	54.6	5 readings
79	11.67	54.2	

SUMMARY OF RESULTS FROM SOUTH WESTERN N. ATLANTIC

TABLE II(h) US GROUP

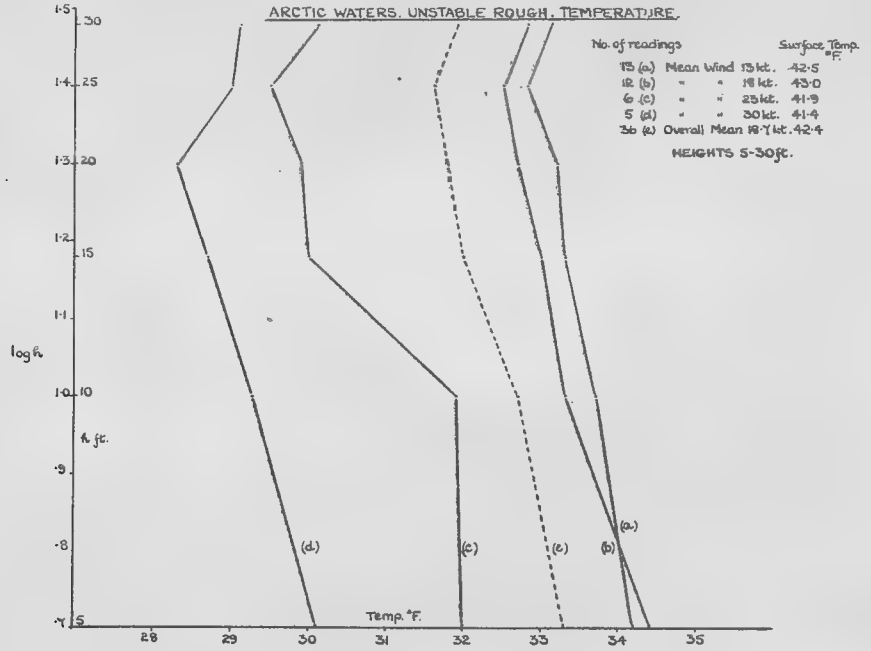
HEIGHT– FT.	MEAN e(mb.)	MEAN T(°F)	
0	33.30	79.8	Wind < 13 kt.
21	27.24	79.2	
41	26.74	78.5	Mean Wind 7.3 kt.
57	26.53	78.2	
73	26.27	78.1	16 readings

TABLE II(j) SS GROUP

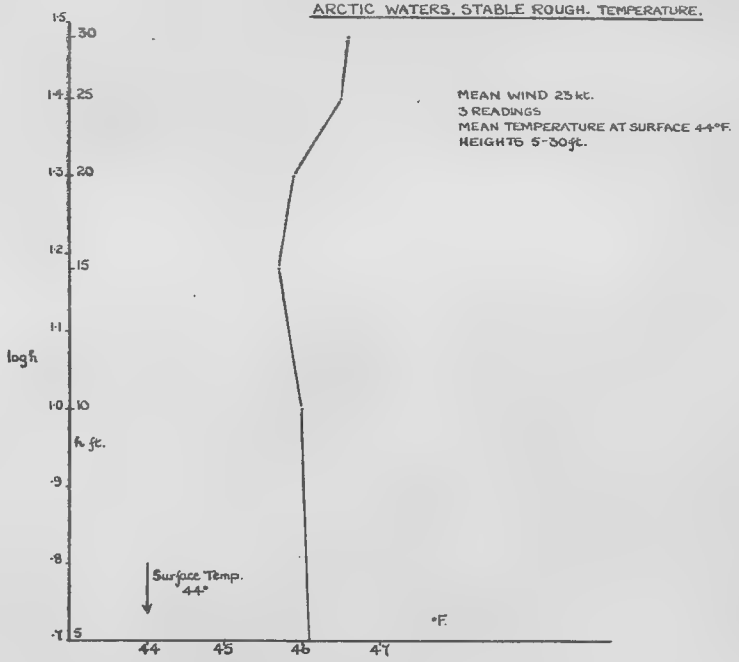
	MEAN e(mb.)	MEAN T(°F)	
0	34.12	80.4	Wind < 13 kt.
21	28.92	82.8	
41	28.48	82.1	Mean Wind 8.6 kt.
57	28.38	81.4	
73	27.47	81.1	7 readings

UR = Unstable Rough
US = Unstable Smooth
SR = Stable Rough
SS = Stable Smooth

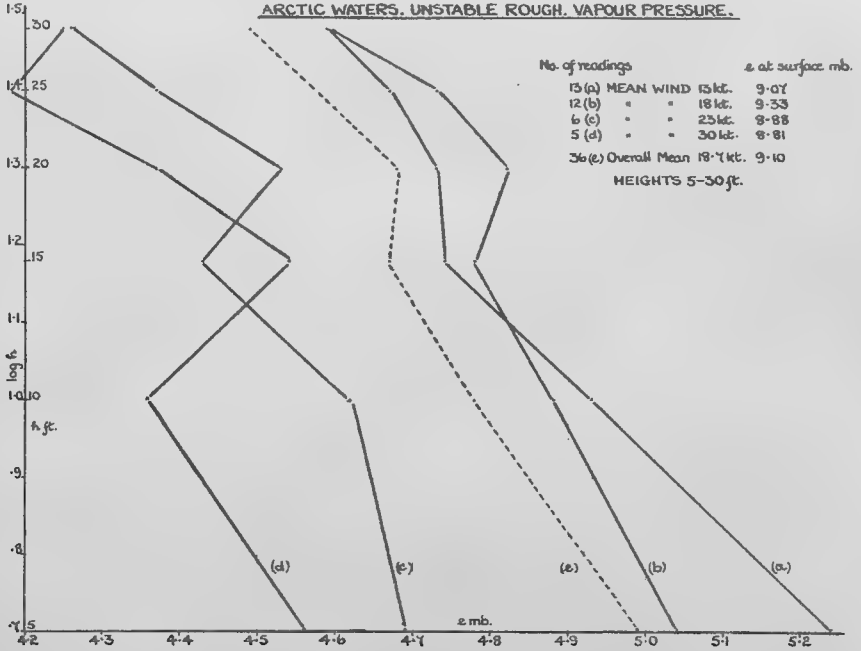
ARCTIC WATERS. UNSTABLE ROUGH. TEMPERATURE.



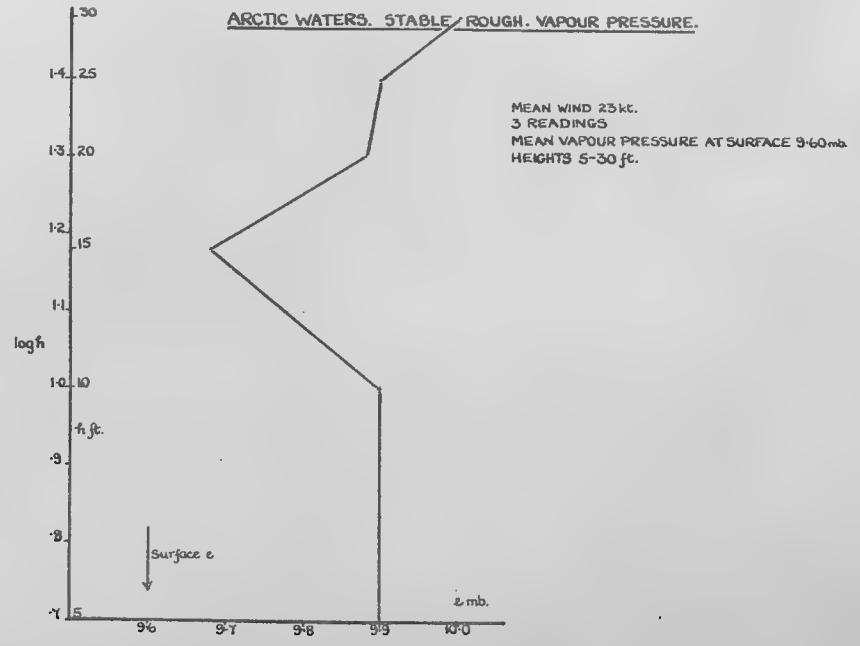
ARCTIC WATERS. STABLE ROUGH. TEMPERATURE.

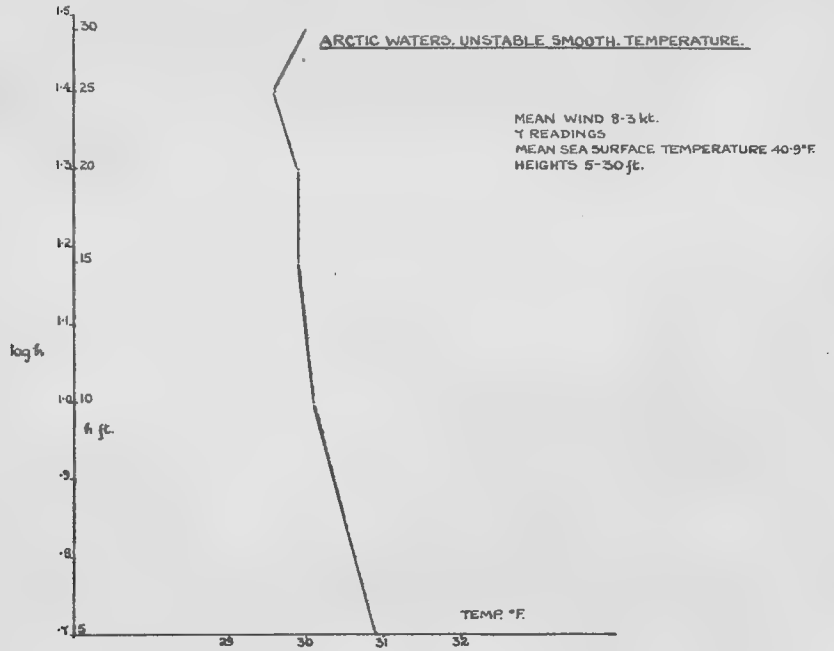


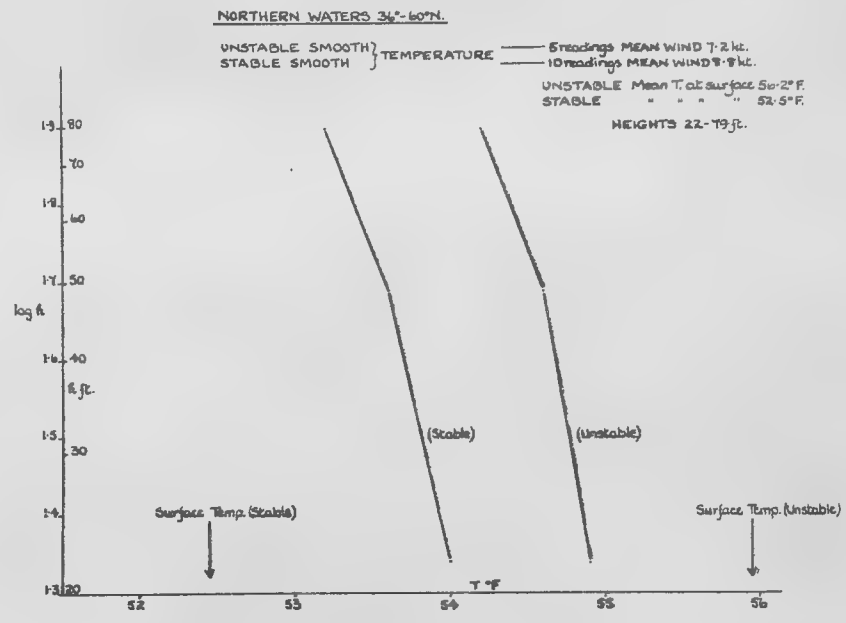
ARCTIC WATERS. UNSTABLE ROUGH. VAPOUR PRESSURE.

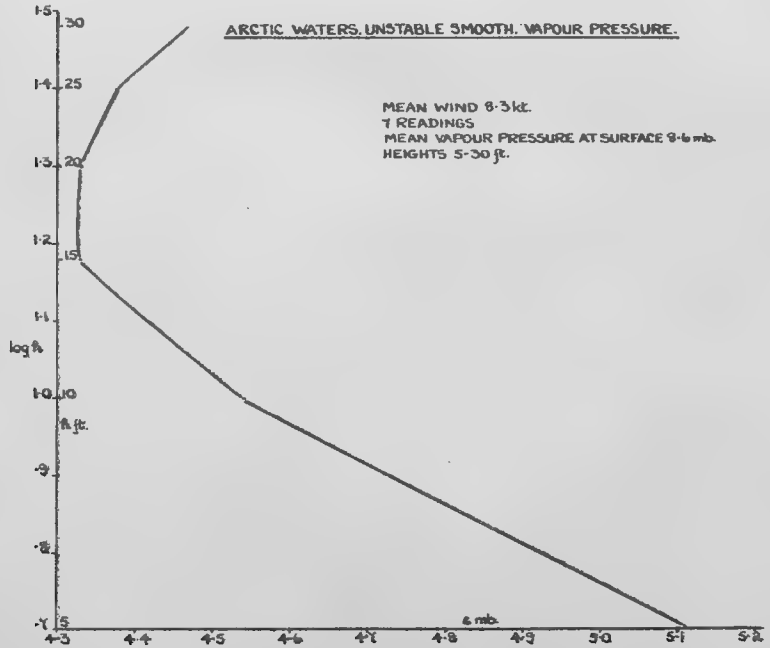


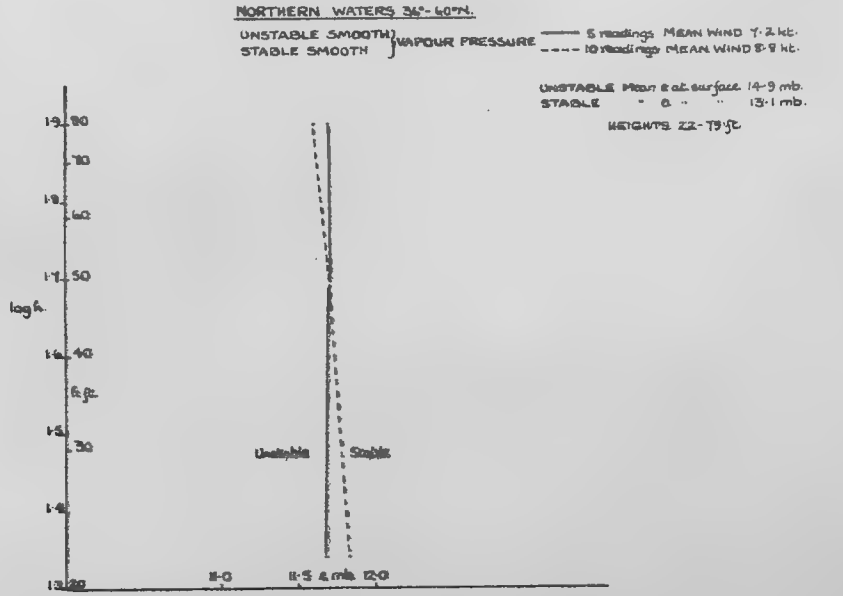
ARCTIC WATERS. STABLE ROUGH. VAPOUR PRESSURE.



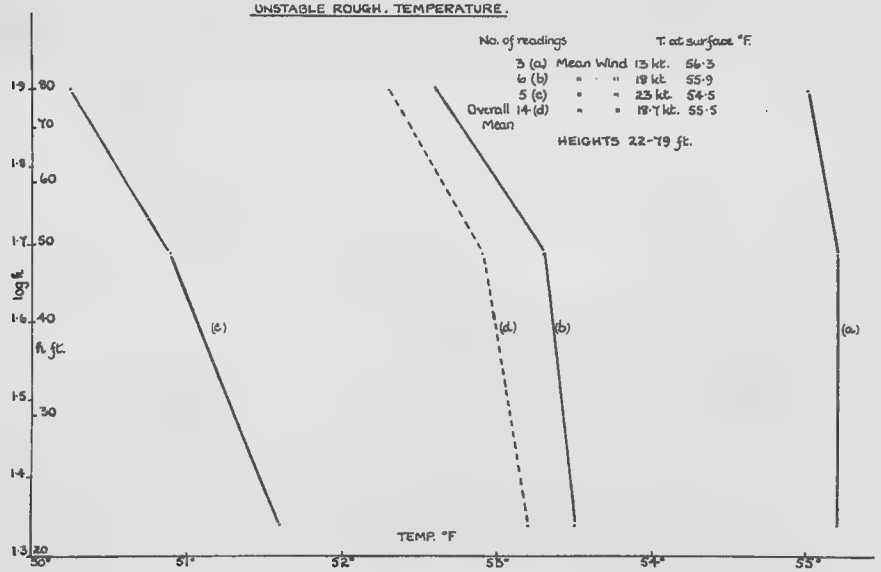

ARCTIC WATERS. UNSTABLE SMOOTH. TEMPERATURE.
MEAN WIND 8·3 kt.
7 READINGS
MEAN SEA SURFACE TEMPERATURE 40·9°F.
HEIGHTS 5-30 ft.
log ft.
ft ft.
TEMP °F.
28
30
31
32


NORTHERN WATERS 36°-60°N.
UNSTABLE SMOOTH
STABLE SMOOTH
TEMPERATURE
5 readings MEAN WIND 7·2 kt.
10 readings MEAN WIND 9·8 kt.
UNSTABLE Mean T. at surface 54·2°F.
STABLE " " " " 52·5°F.
HEIGHTS 22-79 ft.
log ft.
ft ft.
(Stable)
(Unstable)
Surface Temp. (Stable)
Surface Temp. (Unstable)
T °F
52
53
54
55
56

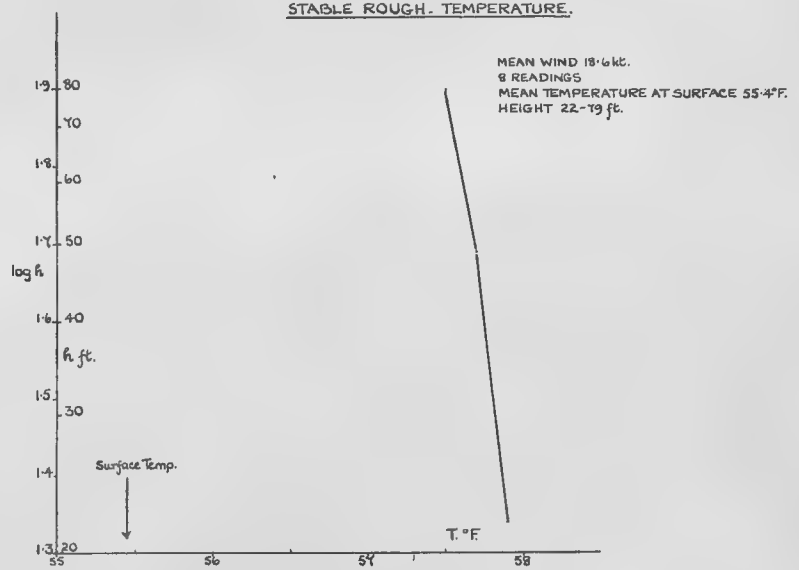

ARCTIC WATERS. UNSTABLE SMOOTH. VAPOUR PRESSURE.
MEAN WIND 8·3 kt.
7 READINGS
MEAN VAPOUR PRESSURE AT SURFACE 9·6 mb.
HEIGHTS 5-30 ft.
log ft.
ft ft.
e mb.
4·3
4·4
4·5
4·6
4·7
4·8
4·9
5·0
5·1
5·2


NORTHERN WATERS 36°-60°N.
UNSTABLE SMOOTH
STABLE SMOOTH
VAPOUR PRESSURE
5 readings MEAN WIND 7·2 kt.
10 readings MEAN WIND 8·9 kt.
UNSTABLE Mean e at surface 14·9 mb.
STABLE " e " " 13·1 mb.
HEIGHTS 22-79 ft.
log ft.
ft ft.
Unstable
Stable
11·0
11·5 e mb.
12·0

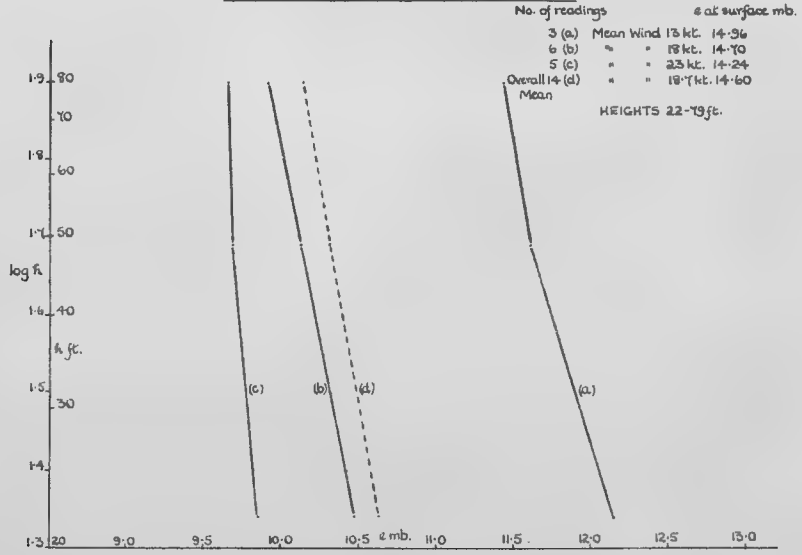
NORTHERN WATERS. 36°-60°N.
UNSTABLE ROUGH. TEMPERATURE.



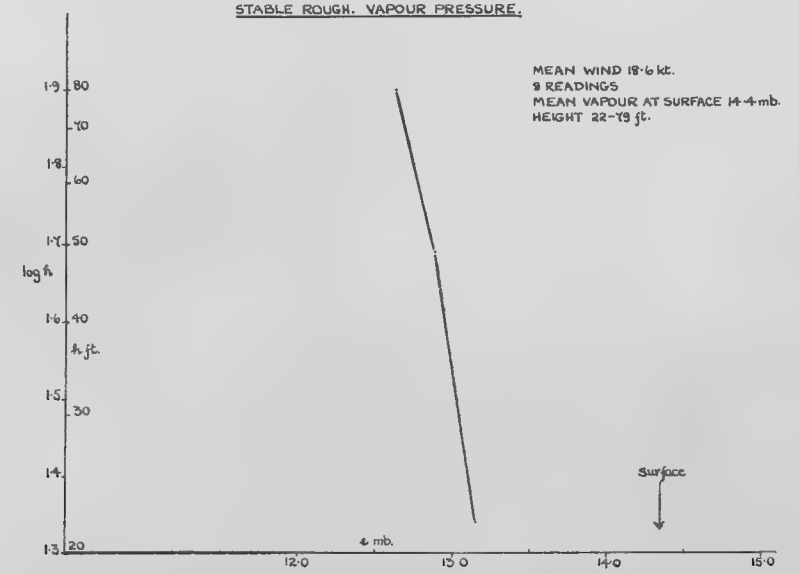
NORTHERN WATERS 36°-60°N.
STABLE ROUGH. TEMPERATURE.



NORTHERN WATERS 36°-60°N.
UNSTABLE ROUGH. VAPOUR PRESSURE.



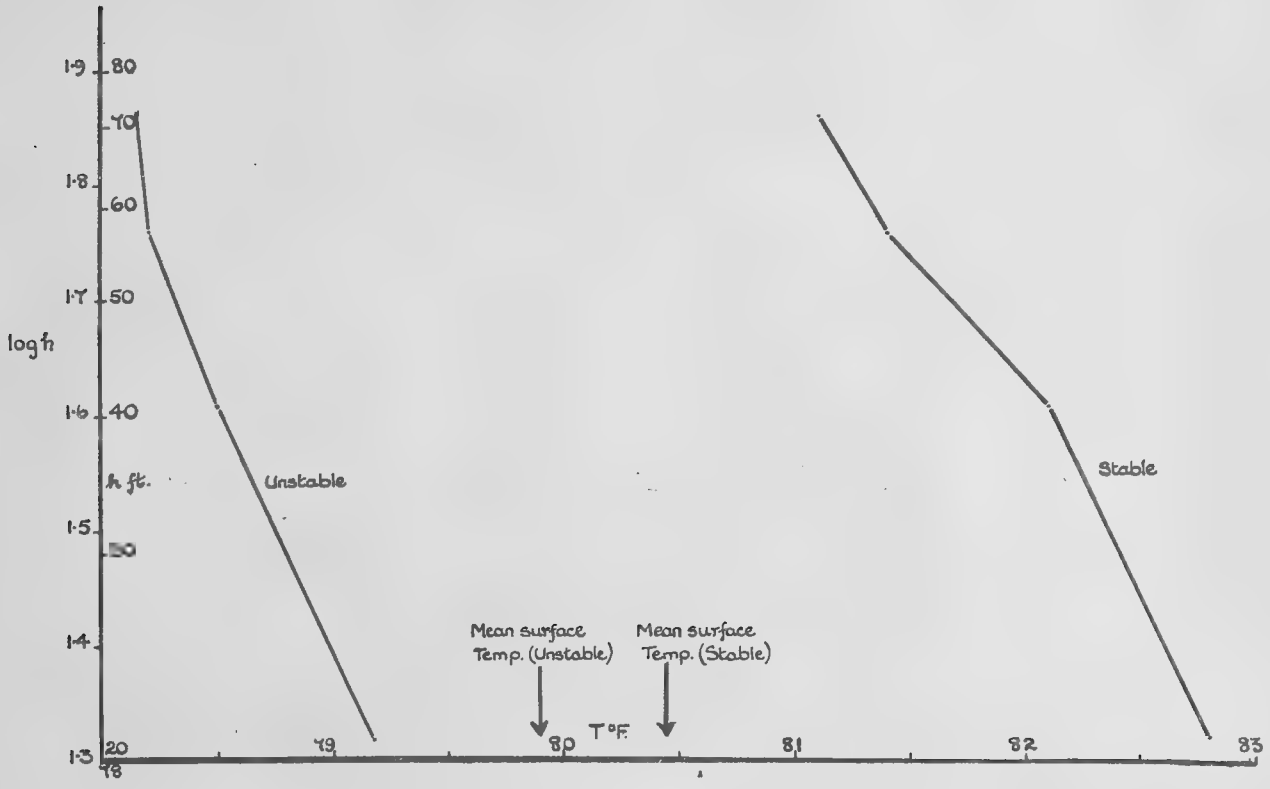
NORTHERN WATERS 36°-60°N.
STABLE ROUGH. VAPOUR PRESSURE.



S.W. NORTH ATLANTIC 13°N.–31°N.

UNSTABLE SMOOTH } TEMPERATURE 16 Readings Mean Sea Temperature 79·8°F.
STABLE SMOOTH } " " " " " 80·4°F.

HEIGHTS 21–73 ft.

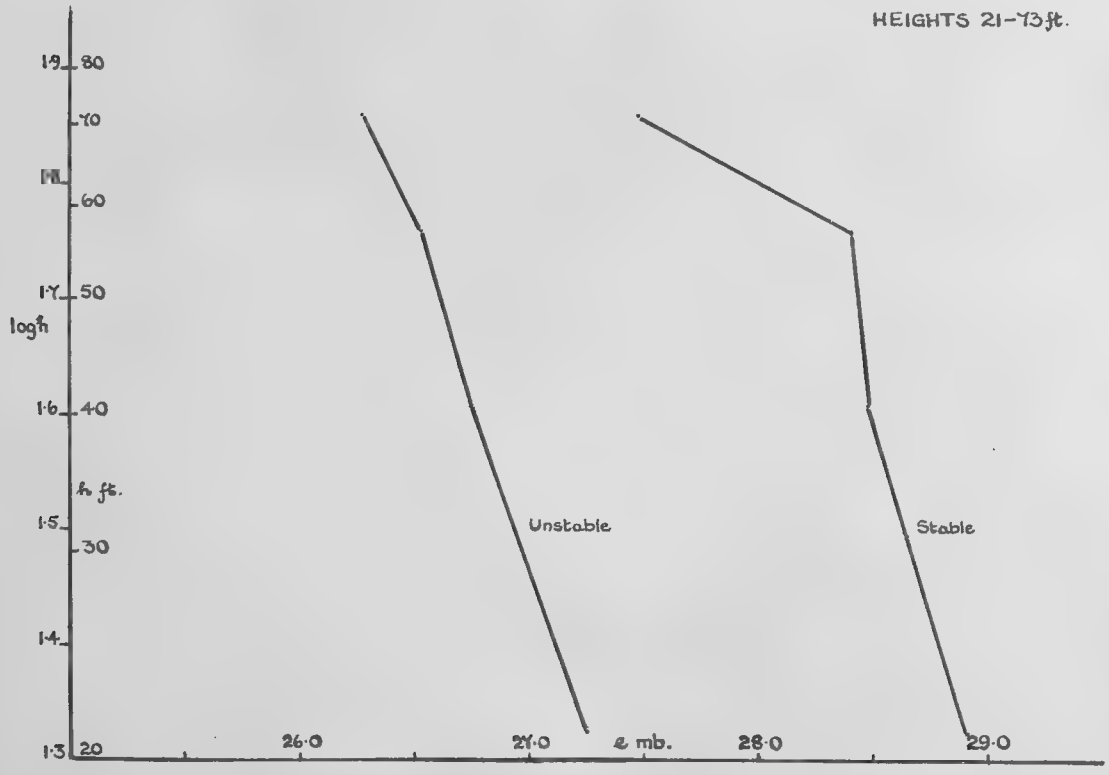


S.W. NORTH ATLANTIC 13°N.–31°N.

UNSTABLE SMOOTH } VAPOUR PRESSURE 16 Readings Mean vapour pressure at surface 33·4 mb.
STABLE SMOOTH } " " " " " 34·1 mb.

UNSTABLE MEAN WIND 7·3 kt.
STABLE " " 8·6 kt.

HEIGHTS 21–73 ft.



quote this paper should refer to it as "A paper of the Meteorological Research Committee (London). M.R.P. No..... by Mr..... A copy is available in the Library of the Meteorological Office.

M.R.P. 727
S.C.III/129
12th May, 1952.

AIR MINISTRY

METEOROLOGICAL RESEARCH COMMITTEE

High Level Cloud in the Tropics

by

C.S. Durst (Meteorological Office)

1. During a recent flight on 18th-19th March, 1952 in a Comet Aircraft from Wadi Seidna Airfield (Khartoum) to Entebbe and thence to Livingstone, it was observed that there appeared to be a continuous sheet of cirrus or Cirrostratus at about 45000 to 50000 feet extending from Latitude 10° 40' N Longitude 32° 40' E to Latitude 16° 10' S Longitude 27° 30' E a distance of over 1600 n miles. Seen from between 30000 and 35000 feet, it looked very thin when observed at a small angle with the vertical; indeed, it often could not be recognised above the aircraft but when observed at a small angle of elevation it appeared quite definitely as a comparatively thick sheet all round the horizon with a layer of clearer air below.

On the return trip the first appearance of high cirrus, though not dense, was at 1220 G.M.T. on 21st March in about Latitude 15°S. The veil did not seem to be so continuous as on the outward trip but it was present at about 45000 feet. At about Latitude 7°S there was the appearance of a definite thin veil of Cirrostratus far above the cirrus thrown off by large cumulonimbus with all the appearance of a clear layer in between. This persisted to about 3°S. On 22nd March Cirrus was seen above the aircraft soon after leaving Entebbe and appeared to be rather patchy until Latitude 6°N. However, at 7°N there was no cirrus veil. The veil was renewed again between Latitudes 9° and 10°N but was again very patchy.

Some photographs are shown of the cloud and annotations are given. I am informed by officers of B.O.A.C. and others who have travelled by Comet in the tropics that this cirrus layer is the rule rather than the exception.

2. It has been the custom to think that cirrus cloud seen in the tropics was either directly associated with the cumulonimbus of the tropical thunderstorms or was the residual cloud from these storms which persisted after the violent convection had ceased. But the great extent of the cirrus or cirrostratus which in the case reported above extended for 1600 miles (and even beyond the limits of normal rainfall in March) make it most improbable that it is to be attributed to the residual cloud of convectional storms.

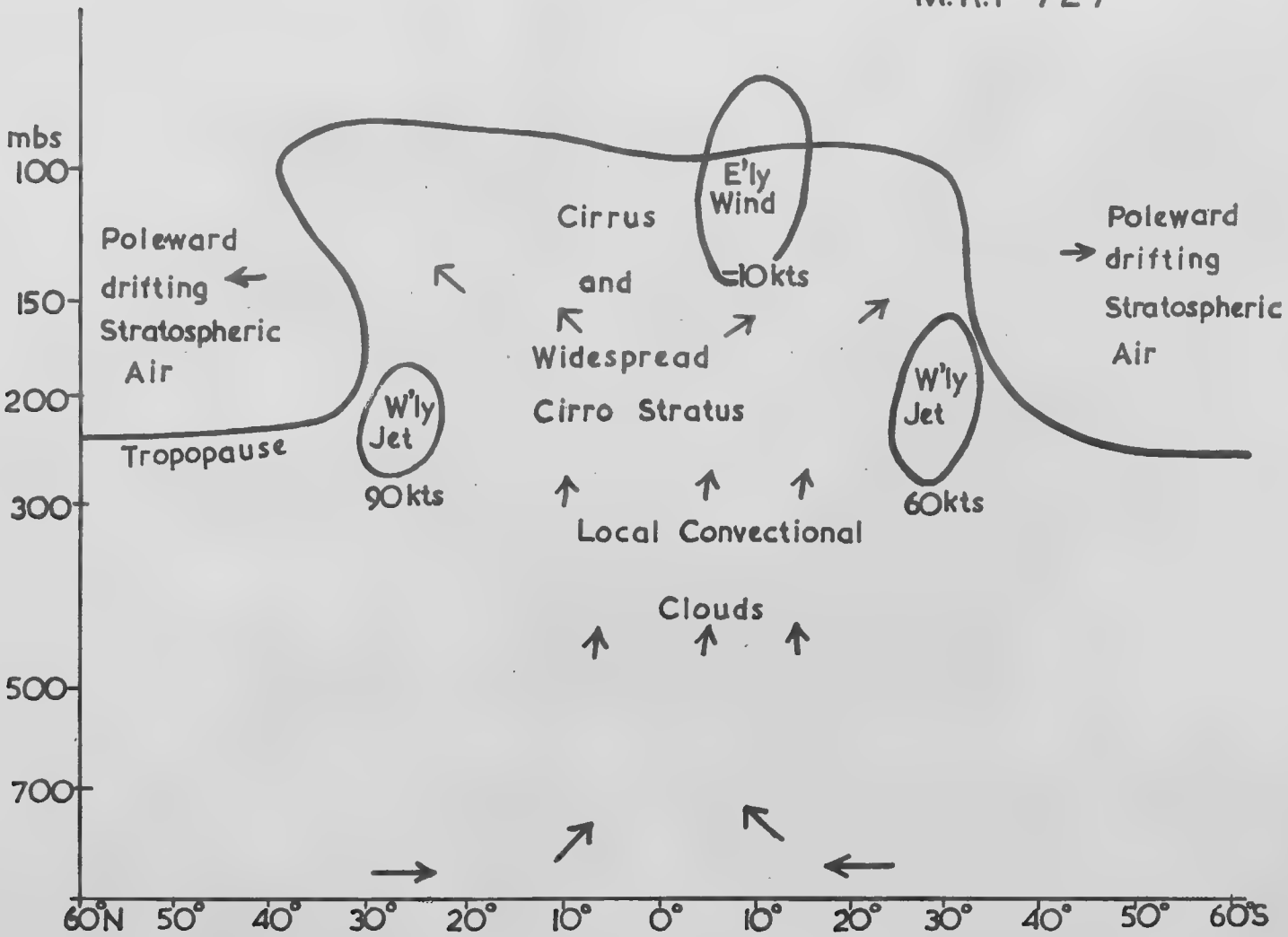
3. The section of the atmosphere postulated by Frost in MRP.706 shows an abrupt rise in the normal level of the tropopause in about latitude 30 N. and S. at the equinoxes, the equatorial tropopause being above the 100 mb level (say 55000 ft) the extratropical tropopause being below the 250 mb level (say 35000 feet). At the positions where the tropopause starts to rise abruptly there are the two semi-permanent jet streams with maximum speeds at about 40000 feet (200 mb). Moreover, it has been supposed that the general circulation of the atmosphere is such that air flows towards the equator near the surface, rises between the tropics, and flows outwards high up in the atmosphere.

4. It is therefore suggested that the cirrus seen at high levels is the result of condensation as the intertropical air rises. When this suggestion was put forward, objection was raised that the condensation would happen not only in the layers between 45000 and 50000 feet but throughout the whole of the atmosphere up to the tropopause. It has, however, been observed by Frith on a number of occasions that when cirrus was visible from the ground it was invisible when the aircraft was amongst it, the reason being that the ice needles of which it was composed were too sparse to be noticeable at a near view. Therefore it is possible that the Comet was, in fact, among ice needles during the flight from 10°N to 16°S. This supposition is supported to some extent by the observation that the cloud was not easily visible except when one was looking through a great quantity of it, i.e., when the line of vision was nearly horizontal.

5. The diagram is intended to show how the general vertical circulation would take place if the speculation I have put forward above is true.

6. It is to be noted that in many of the photographs taken by the Comet when flying from Calcutta to Karachi in September high cirrus cloud far above the aircraft (flying at 40000 feet or so) was visible to the southward. It is probable that this cloud too is the expression of the general rising of the air in the inter-tropical region.

M.R.P 727


mbs
100
150
200
300
500
700
Poleward drifting Stratospheric Air
Tropopause
W'ly Jet
90kts
Cirrus and Widespread Cirro Stratus
E'ly Wind
=10kts
W'ly Jet
60kts
Poleward drifting Stratospheric Air
Local Convectional Clouds
60°N 50° 40° 30° 20° 10° 0° 10° 20° 30° 40° 50° 60°S

Some photographs of Cirrus and Cirrostratus
Cloud in the Tropics.

18 March, 1952. 0635 G.M.T. Lat. 10°40'N Long. 32°40'E. Height 32,500 feet.

This photograph taken over the Sudan shows a dust haze stretching up to about 18,000 feet with the top of a rising column revealed by a cloud at the top of the dust. Some 20,000 feet above the top of the dust is the sheet of cirrostratus.

18 March, 1952. 0657 G.M.T. Lat. 08°18'N Long. 32°45'E. Height 33,000 feet.

This photograph taken over the Sudan shows a sheet of altostratus or altocumulus cloud, one of a number of patches, an individual patch extending over some 1,000 square miles. The Ac layer has been measured at a height of about 16,500 feet above MSL. Above can be seen the cirrus or cirrostratus layer, the height of which was estimated by eye to be 45,000 feet or more.

18 March, 1952. 0745 G.M.T. Lat. 03°00'N Long. 33°00'E. Height 34,600 feet.

This photograph taken over Uganda shows typical cumulus cloud with base 6,000 feet above MSL (3,000 feet above ground level) and tops to 11,000 feet but mainly 7,000 or 8,000 feet. It was notable how the Cu could be seen to hang over the hill tops of the broken country beneath. Above is the cirrus or cirro-stratus veil the height of which was estimated at 45,000 feet or more. The measurements from the photographs gave an approximate value of 43,000 feet for the height of the Cs cloud.

19 March, 1952. 0736 G.M.T. Lat.04°20'S Long. 30°53'E. Height 30,000 feet.

This photograph shows a mass of cumulus and cumulonimbus cloud beneath. The aircraft had climbed through and past cumulus and cumulonimbus cloud over Lake Victoria and during the ten minutes preceding this photograph had passed by a big cumulonimbus head on the starboard side towering above the aircraft. Ci and Cs had been passed through at 30,000 feet. Above these convectional clouds can be seen in the photograph the Ci or Cs far above the aircraft.

M.R.P. 728
S.C.III/130
16th April, 1952

AD 223034

AIR MINISTRY

METEOROLOGICAL RESEARCH COMMITTEE.

Random motion and atmospheric turbulence

by

E. Knighting.

(Meteorological Office)

AIR WEATHER SERVICE
TECHNICAL REFERENCE FILES

Summary

An attempt is made to consider atmospheric turbulence as an aspect of the random walk problem. Some formulae are deduced and compared with existing formulae in concentration analysis. It is found that the length of the step of the random walk must vary in order to give formulae which are in agreement with those now used and the implications are discussed.

1. Introduction. A feeling of satisfaction with a theory is largely a personal affair and is rarely widely shared in a developing subject. The theories which satisfy many people are usually those in which there is close agreement upon what is postulated and upon the syllogistic processes to be employed; portions of mathematics or astronomy are good examples. In considering atmospheric turbulence there is, in my view, difficulty in defining some of the postulated entities, for example an eddy.

There are at least two views upon the essence of physics. The first is that its function is to formulate laws which enable one to predict. That is, when an observation is made upon a given system, certain laws will predict what the result of an observation will be at some future time - and that is all. What happens is of no import and indeed not within the province of physics. The essence is the relata of numbers given by certain rules and the aim of physics is to find these rules. This viewpoint gives rise to abstract mathematical theories and in a way is allied to the search for invariants under transformation of certain equations; this is as it should be, for the physical facts are independent of the frame of reference.

The second view is that physics is concerned with what goes on; it builds a picture, very simplified of course, in terms of some concepts such as perfectly elastic billiard balls. The rules about the concepts are formulated and then, frequently, a mass-rule is deduced and compared with experimental results. If the agreement is good, then the concepts are satisfactory; the danger lies in taking the short step to considering that the concepts are factual. The last step is rarely admitted to have been taken for a mechanistic view is no longer fashionable.

In practice it is the first view which predominates but the second which is used; what happens is that the new rules are found by imagining some elementary concepts but when the rules are tested and found to be useful, the concepts are rubbed out and the first view is held.

Statistical turbulence theories are mainly concerned with number-relations and the first view widely held. I want here to incline to the second view and in a sense 'make a picture', in order to prevent our attention from wandering owing to the difficulty of keeping in mind long chains of equations. This picture will eventually be rubbed out and I will believe that I have nothing to say about the process of turbulence, but merely have some predictive formulae.

It is clear when watching marked atmospheric particles, say tobacco smoke, that the motion is more or less a chaotic one superimposed on a more definite motion. Now a more or less chaotic motion that is fairly well-known is that of the random walk; the elementary form is that a walker moves in constant steps such that the direction in which he moves is arbitrary. If many walkers start from the same point there will be a concentration of walkers in any given area, providing that we select a proper-sized area, neither too large nor too small; in the former case we may get an unchanging concentration, and in the latter a discontinuous concentration. I have set out in this note to investigate the concentrations of walkers in a less elementary random-walk process and to compare the results with known expressions used in turbulence theory.

It is also clear when watching marked particles that the motion varies with distance from the source. In the initial cluster only small eddies have a chance of affecting the particles but as the cluster spreads the larger eddies come into play and have more effect. There is a difficulty here if we attempt to interpret results in terms of a diffusion coefficient, especially if there are no boundaries (in the mathematical sense). We do not want to have a diffusion coefficient which is a function of position for this implies an absolute origin of co-ordinates distinguishable from other points, whereas we want results which are independent of the co-ordinate system used. In the presence of boundaries the difficulty is not so acute because the boundaries do often indicate a particular point, e.g. near the surface of the earth it is easy to believe that the motion depends upon distance from the surface. We will bear in mind this difficulty of a particular origin and will use neither a diffusion coefficient nor a differential equation.

2. One-dimensional motion. Although one-dimensional motion is not of great interest it is the simplest to treat and illustrates the basic ideas. Consider a single particle starting at the origin and moving either forward or backwards along a line. If a_n is the length of the nth step and the probability of the particle being in the neighbourhood of the point x after n steps is $F_n(x)$ then

$$F_n(x) = \tfrac{1}{2}\left[F_{n-1}(x-a_n) + F_{n-1}(x+a_n)\right] \quad \ldots\ldots\ldots(1)$$

This equation simply states that the particle must have come from one or another of two regions, distant a_n from its position, and that the probability of it coming from either is the same.

To solve this equation we multiply by $\exp(-px)$ and integrate over the range $(-\infty, \infty)$, writing

$$\bar{F}_n(p) = p\int_{-\infty}^{\infty} \exp(-px)\, F_n(x)\, dx \quad \text{or} \quad F_n(x) \supset \bar{F}_n(p) \quad \ldots\ldots(2)$$

noting that

$$F_n(x+a) \supset e^{ap}\,\bar{F}_n(p) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Thus

$$\begin{aligned}\bar{F}_n(p) &= \tfrac{1}{2}\,\bar{F}_{n-1}(p)\left[\exp(-a_n p) + \exp(a_n p)\right] \quad \ldots\ldots\ldots(4)\\ &= \bar{F}_{n-1}(p)\cosh(a_n p)\\ &= \bar{F}_0(p)\prod_1^n \cosh(a_n p)\\ &= p\prod_1^n \cosh(a_n p) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)\end{aligned}$$

since the particle is originally at the origin.

We will approximate to $\bar{F}_n(p)$ by writing

$$\bar{F}_n(p) = p\prod_1^n \cosh(a_n p) = p\prod_1^n \left[1 + \frac{a_n^2 p^2}{2} + \ldots\right]$$

$$\sim p \exp\left[\frac{p^2}{2}\sum a_n^2\right] \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

In some cases there may be difficulty over the convergence of the product if we propose to make $n \to \infty$, the difficulty is resolved by noting that

$$F_n(\alpha x) \supset \bar{F}_n(p/\alpha) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

and we can usually choose α such that convergence is assured.

One very important point should be noticed immediately. In passing from equation 1 to equation 4 we have assumed that a_n is independent of x or that the length of step does not depend upon the co-ordinate of the point, which is in accordance with the point made in the previous paragraph. So far, this is the only restriction on a_n.

We may write

$$\sum a_n^2 = n\sigma^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(8)$$

and obtain

$$\bar{F}_n(p) \sim p \exp\left[\frac{n\sigma^2 p^2}{2}\right] \qquad \ldots\ldots\ldots\ldots\ldots\ldots(9)$$

which yields, either from the inversion integral or from tables of transforms,

$$F_n(x) \sim \frac{1}{\sigma\sqrt{2\pi n}} \exp\left[\frac{-x^2}{2n\sigma^2}\right] \qquad \ldots\ldots\ldots\ldots(10)$$

The effect of approximating is seen in the form of equation 10, which gives a probability density for large values of x, although the particle could not have reached such a position, since

$$F_n(x) = 0 \qquad x > \sum a_n$$

Approximating gives a 'thin' probability density at large distances and reduces the actual probability density nearer the origin.

To give useful formulae we require that results should be expressed in terms of time rather than the number of steps. I am now going to assume that all the steps are made in the same unit of time, τ, and define t by

$$t = n\tau \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(11)$$

so that t is a measure of time since n is a pure number. The velocity with which the nth step is made is a_n/τ so that the assumption is that different steps are made with different velocities; σ^2/τ^2 is now simply the variance of the velocities, and equation 10 becomes

$$F(x,t) = \frac{1}{\sigma(2\pi t/\tau)^{1/2}} \exp\left[\frac{-x^2}{2\sigma^2 t/\tau}\right] \qquad \ldots\ldots\ldots\ldots(12)$$

3. The many-particle problem. So far we have considered the motion of only one particle, but clearly if there are many particles released at the same time $F(x,t)$ is still proportional to the probability density at (x,t) providing that the particles do not interact, for from equation 12

$$\int_{-\infty}^{\infty} F(x,t)\,dx = 1 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(13)$$

If we put $\sigma^2/2\tau = k$, equation 12 becomes

$$F(x,t) = \frac{1}{2\sqrt{\pi kt}} \exp\left[\frac{-x^2}{4kt}\right]$$

which is precisely the expression for the conduction of heat from an instantaneous point source, providing that k is a constant. Here we have made contact with orthodox physics and shown the relation between the conduction of heat and the random walk problem when all the steps are of the same length. a_n is as yet undefined save that it is not a function of position; it may well be a function of n and we leave this for discussion later.

4. Biased motion. In considering the one-dimensional random walk the probability of the particle moving either one way or the other was taken as equal. Suppose now that the motion is biased so that the probability of moving in one direction is P. Then equation 1 becomes

$$F_n(x) = P\,F_{n-1}(x - a_n) + (1 - P)\,F_{n-1}(x + a_n) \quad \ldots\ldots(14)$$

leading to

$$\overline{F}_n(p) = p \exp\left[\sum a_n(1 - 2P)p + \frac{n\sigma^2 p^2}{2}\right] \quad \ldots\ldots\ldots\ldots\ldots(15)$$

using the same approximation as before. From equation 3 we see that we have the same solution as before except that the origin of the cluster has moved a distance $\sum a_n(1 - 2P)$; this is equivalent to giving the whole system a velocity $\sum a_n(1 - 2P)/n\tau$ along the x axis.

5. Two-dimensional motion. In two-dimensional motion the equation corresponding to equation 1 states that the particle must have come from the circumference of a circle of radius a_n and is

$$F_n(x,y) = \frac{1}{2\pi}\int_0^{2\pi} F_{n-1}(x - a_n\cos\theta,\ y - a_n\sin\theta)\,d\theta \quad \ldots\ldots\ldots(16)$$

We write

$$\overline{F}_n(p_1,p_2) = p_1p_2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F_n(x,y)\exp(-p_1x - p_2y)\,dx\,dy \quad \ldots\ldots(17)$$

and when we deal with three-dimensional motion we will introduce p_3 corresponding to z.
Thus

$$\overline{F}_n(p_1,p_2) = \frac{\overline{F}_{n-1}}{2\pi}\int_0^{2\pi} \exp(-a_np_1\cos\theta - a_np_2\sin\theta)\,d\theta$$

$$= \overline{F}_{n-1}\,I_0(a_n\sqrt{p_1^2 + p_2^2})$$

$$= p_1p_2\prod I_0(a_n\sqrt{p_1^2 + p_2^2}) \quad \ldots\ldots\ldots\ldots\ldots\ldots(18)$$

As before, we use an approximation to the series for I_0 and get

$$\overline{F}_n(p_1,p_2) \sim p_1p_2 \exp\left[\frac{n\sigma^2}{4}(p_1^2 + p_2^2)\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots(19)$$

$$F_n(x,y) \sim \frac{1}{\pi n\sigma^2}\exp\left[-\frac{x^2 + y^2}{n\sigma^2}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(20)$$

which reduces to the expression for the conduction of heat from an instantaneous point source if all the path lengths are the same. It is a trivial matter to introduce a bias such that the particle must have come from an ellipse.

6. Three-dimensional motion. The analogue of equation 1 must state that the particle came from the surface of a sphere of radius a_n, and is

$$F_n(x,y,z) = \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} F_{n-1}(x-a_n\sin\theta\cos\phi, y-a_n\sin\theta\sin\phi, z-a_n\cos\theta)\sin\theta\, d\theta\, d\phi \quad \ldots\ldots\ldots\ldots\ldots\ldots(21)$$

A bias in the direction of motion can be introduced by the insertion of a factor $f(\theta,\phi)$ in the integral; it is easy to adjust the expression to express that the particle must have come from an ellipsoid.

Equation 21 gives

$$\bar{F}_n(p_1,p_2,p_3) = \frac{\bar{F}_{n-1}}{4\pi}\int_0^{2\pi}\int_0^{\pi}\exp\left[-a_n(p_1\sin\theta\cos\phi + p_2\sin\theta\sin\phi + p_3\cos\theta)\right]\sin\theta\, d\theta\, d\phi \quad \ldots\ldots\ldots\ldots\ldots\ldots(22)$$

The integration can be performed (first with respect to ϕ and then by expansion of the integrand in series; it is rather tedious and best omitted) to give

$$\bar{F}_n(p_1,p_2,p_3) = \frac{\sinh a_n\sqrt{p_1^2+p_2^2+p_3^2}}{a_n\sqrt{p_1^2+p_2^2+p_3^2}}\,\bar{F}_{n-1}$$

$$= p_1p_2p_3\prod\left[\frac{\sinh a_n\sqrt{p_1^2+p_2^2+p_3^2}}{a_n\sqrt{p_1^2+p_2^2+p_3^2}}\right]$$

$$\sim p_1p_2p_3\exp\left[\frac{n\sigma^2(p_1^2+p_2^2+p_3^2)}{6}\right] \quad \ldots\ldots\ldots(23)$$

$$F_n(x,y,z) \sim \left[\frac{3}{2\pi n\sigma^2}\right]^{3/2}\exp\left[-\frac{3(x^2+y^2+z^2)}{2n\sigma^2}\right] \quad \ldots\ldots\ldots(24)$$

Now consider the values of a_n. If all a_n are the same then equation 24 reduces to the expression for the conduction of heat from an instantaneous point source when we put $t = n\tau$; we know, however, that such an expression does not give the correct concentration when atmospheric turbulence is acting. The only restriction on a_n, which we have already stressed, is that it should not be a function of position, a very proper restriction from a physical point of view. There are certain other restrictions placed upon it by the physical problem that we are investigating. Firstly, a_n is essentially positive, as it plays the role of a radius vector. Secondly, we know that turbulence spreads an initial concentration over a large volume, so that we require $\sum a_n$ to be divergent; if $\sum a_n$ converges to a constant K then no particle can lie outside a sphere of radius K. The expression for the probability density would give a value outside this sphere, but it would be small.

We want to express that as time goes on the larger eddies somehow come more into play; the simplest way of doing so is to put

$$a_n = f(n), \qquad \sum f(n) \rightarrow \infty \text{ with } n.$$

For example, let us write

$$a_n = kn^{\alpha}, \qquad n\sigma^2 \sim \frac{k^2 n^{2\alpha+1}}{2\alpha+1} \qquad \ldots\ldots\ldots(25)$$

Writing $n\tau = t$ and inserting in equation 24

$$F(x,y,z,t) = \left[\frac{3(2\alpha+1)\,\tau^{2\alpha+1}}{2\pi k^2\, t^{2\alpha+1}}\right]^{3/2} \exp\left[-\frac{3(2\alpha+1)\,\tau^{2\alpha+1}(x^2+y^2+z^2)}{2k^2\, t^{2\alpha+1}}\right]$$

$$\ldots\ldots\ldots\ldots(26)$$

This equation is formally the same as that derived by O. G. Sutton (1) except that Sutton writes the index $2\alpha+1$ as $2 - N$ so that we have

$$2\alpha + N = 1$$

The values of N are limited to $0 \leq N \leq 1$ so that in order to make a second contact with current physical theory we must have $0 \leq \alpha \leq 1/2$. The result is interesting in that Sutton chose a particular form of equation involving the squares of the co-ordinates in the exponential term; he might, for example, have chosen the fourth powers and still satisfied his equations (although the special case of the conduction of heat would lead one naturally to think of squares). The random- walk method naturally produces the squares of the co-ordinates.

Having reproduced a currently used formula we must pay more attention to the implications of taking $a_n = k\, n^{\alpha}$. It would appear that we have made each step larger than the preceding one and asserted that each of the particles takes a step of the same length in the interval $(n-1)\tau$, $n\tau$, and thus has the same velocity. However, I do not think that this is quite the case. We have really assigned a value to σ^2 implying that each particle on an average takes the same sized steps as any other particle (so that there is no favoured particle) but not necessarily in the same order. In addition we have implied, since σ^2 increases with n or time, that on an average the particles tend to take larger steps the longer the time from commencement; this does not prohibit some of the particles from taking small steps at a late time, but these small steps are counterbalanced as a whole by other particles taking longer steps. This agrees with observation, usually expressed in the form that the larger eddies have more effect as the cluster expands.

7. A particular problem. Consider now particles which can only move in the vertical such that $z_1 \geq z_2$ if $t_1 \geq t_2$ where z_r is the vertical height at time t_r. Such a motion may occur in a convective situation such as an explosion at the surface producing a temperature rise sufficient to make all particles rise. Corresponding to equation 1 we have

$$F_n(x,y,z) =$$

$$\frac{1}{2\pi}\int_0^{2\pi}\int_0^{\pi/2} F_{n-1}(x-a_n\sin\theta\cos\phi,\ y-a_n\sin\theta\sin\phi,\ z-a_n\cos\theta)\sin\theta\, d\theta\, d\phi$$

$$\ldots\ldots\ldots\ldots\ldots\ldots(27)$$

which gives

$$\bar{F}_n = \frac{\bar{F}_{n-1}}{2\pi} \int_0^{2\pi} \int_0^{\pi/2} \exp\left[-a_n(p_1 \sin\theta\cos\phi + p_2 \sin\theta \sin\phi + p_3\cos\theta)\right] \sin\theta \, d\theta \, d\phi \quad \ldots\ldots(28)$$

$$= \bar{F}_{n-1} \int_0^{\pi/2} \exp(-a_n p_3 \cos\theta)\, I_0(a_n \sin\theta \sqrt{p_1^2 + p_2^2})\, \sin\theta \, d\theta \quad \ldots\ldots(29)$$

I have not succeeded in evaluating the integral in finite terms, but the first three terms are

$$\bar{F}_n = \bar{F}_{n-1}\left[1 - \frac{a_n p_3}{2} + \frac{a_n^2(p_1^2 + p_2^2 + p_3^2)}{6}\right] \quad \ldots\ldots(30)$$

This gives

$$\bar{F}_n \sim p_1 p_2 p_3 \prod \left[1 - \frac{a_n p_3}{2} + \frac{a_n^2(p_1^2 + p_2^2 + p_3^2)}{6}\right]$$

$$\sim p_1 p_2 p_3 \exp\left[-\frac{n\bar{a}\, p_3}{2} + \frac{n\sigma^2(p_1^2 + p_2^2 + p_3^2)}{6}\right] \quad \ldots\ldots(31)$$

where $n\bar{a} = \sum a_n$. Now this differs from equation 23 only in the presence of $-n\bar{a}p_3/2$ in the exponential term, and using equation 3 we can write down immediately

$$F_n(x,y,z) \sim \left[\frac{3}{2\pi n\sigma^2}\right]^{3/2} \exp\left[-\frac{3(x^2 + y^2 + (z - n\bar{a}/2)^2)}{2n\sigma^2}\right] \quad \ldots(32)$$

This means that the distribution is the same as when the particles can travel in any direction but that the centre of the cluster moves along the z axis through a distance $n\bar{a}/2$ with an average velocity of $n\bar{a}/2t$.

Equation 32 gives a probability density for negative values of z where, according to the conditions of the problem, the probability must be zero. All our previous expressions for F have been such that the integral taken through all the relevant space is unity. Here we must consider

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_0^{\infty} F_n(x,y,z)\, dxdydz = \left[\frac{3}{2\pi n\sigma^2}\right]^{1/2} \int_0^{\infty} \exp\left[-\frac{3(z-n\bar{a}/2)^2}{2n\sigma^2}\right] dz$$

$$= \frac{1}{2}\left[1 + \operatorname{erf}\left\{\frac{\bar{a}}{2\sigma}\sqrt{\frac{3n}{2}}\right\}\right] \quad \ldots\ldots(33)$$

This expression is very nearly unity when $\bar{a}(3n/2)^{1/2}/2\sigma = 3$. If all steps are of the same length $\bar{a} = \sigma$ and $n = 24$. If we take $a_n = kn^{\alpha}$ the total probability becomes almost unity when

$$\frac{\sqrt{2\alpha+1}}{\alpha+1}\sqrt{\frac{3n}{2}} = 3$$

and the maximum value of n for $0 \leq \alpha \leq 1/2$ is $n = 27$. Thus after about 30 steps equation 32 gives a reasonable representation of the distribution.

Conclusion. We set out to see if the spread of a cluster in the atmosphere could be described as the random walk motion of the cluster. In deriving expressions for probability density we did not assume that all steps of the random walk are of the same length. We did assume

(a) The length of step is not a function of position
(b) All steps are taken in a certain unit of time
(c) A certain variation in the length of step which allowed us to insert, in a vague way, the condition that larger eddies have more effect when the cluster is larger. This does not altogether forbid small steps at a late time, rather indicating that on the whole the step size increases.

A particularisation of (c) led to a formula already obtained by O. G. Sutton.

The proposal is that it is the law of step size that characterises the turbulence. Once this law is given it is possible in theory to obtain prediction formulae. The law itself is intimately connected with the departures of the velocity from a mean velocity, since there is no difficulty in superimposing a mean velocity on all the particles.

The central problem remaining is motion in the presence of a fixed barrier, say motion above the plane $z = 0$. We have seen in two cases that the presence of a constraint effectively gives a velocity to the centre of the cluster, but it is too much to expect that the cluster always behaves in the same way with its centre displaced. If we make the assumption that the particle reaching the barrier is reflected, continuing to its full path length, then the problem is simple enough, but it is doubtful if the assumption is appropriate. It may be, for example in evaporation, that a particle reaching the surface is lost. I have not yet succeeded in properly formulating this problem.

Reference

1. Sutton, O. G., A theory of eddy diffusion in the atmosphere, Proc. roy. Soc., London, A 135, 1932, p. 143.

AIR MINISTRY

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UPPER WINDS OF ADEN

by
Elaine Austin (Meteorological Office)

1. The Observations

Summaries of the monthly frequencies of wind at Aden (Lat. 12° 50'N Long. 45°01'E) in the period from August 1948 - December 1950 have recently been compiled in the Upper Air Climatological Branch with a view to publication in the series of volumes of Upper Air Data now in course of preparation for British stations. The results show many points of interest, and in view of the scarcity of information about wind in the tropics a summary of the main features in advance of the publication of the complete tables may be of value.

Radar-wind observations began at Aden in August 1948. For the first nine months (August 1948 to April 1949) observations were made only once a day at 1400Z (1700C), but from May 1949 onwards a second observation was made at 0200Z (0500C).

2. Mean Winds and Frequencies

The number of observations available at the higher levels is shown in the table below:

mb.	J	F	M	A	M	J	J	A	S	O	N	D	Year
100	63	57	66	78	87	70	72	84	114	130	122	122	1065
200	84	74	85	86	103	95	96	124	134	145	140	147	1313
850	87	80	88	89	106	102	103	128	143	150	144	151	1371

From these observations frequency tables were compiled for fourteen levels viz. Surface, 900, 850, 800, 700, 600, 500, 400, 300, 200, 150, 100, 60, 40 mb. Only 208 of the 1371 ascents reached 60 mb. and only 5 reached 40 mb.

In working up the frequency tables all winds of less than 10 kt. were counted as calms. This was done mainly for ease in mechanical computations, but partly also because the direction of the very light winds is often doubtful.

From the frequencies (treating winds of less than 10 kt. as calms) vector mean winds were computed, the results are shown in Table I(b) and Fig.1. There are many interesting features, but outstanding is the strong easterly wind in the levels above 300 mb. in the summer months which, as first noted by Frost (M.R.P.706)1, reaches its maximum between 150 and 100 mb. in July when it averages over 70 kt.

The "components" from which the vector resultants were computed are shown in Fig.2. These roses differ from the ordinary wind roses in that they represent wind speed multiplied by frequency, i.e. the total flow from each direction. The constancy of the upper easterlies is obvious.

3. Transition from westerlies to easterlies

The onset of the upper easterlies takes place at Aden in May, and is fairly abrupt; if, for example, the 200 mb. values are divided into two groups, May 1-18 and May 19-31, with almost no exception all the westerlies are concentrated in the first group and the easterlies in the second.

A rough diagram of the daily observations for Aden in May 1949 and 1950 for levels 300, 200, 150 and 100 mb. is given in Fig. 3 in which wind direction (as ordinate) is plotted against time (as abscissa). The figure plotted for each observation is the speed in knots. The separation of westerlies in the early part of the month from easterlies in the latter part is clearly shown, at the 200 mb. level.

/At

At Bahrein, the onset of the easterlies is fairly abrupt and occurs in mid June. The easterlies reach their maximum of between 35 and 40 kt. above 100 mb. in July and cease fairly abruptly in mid August.

Bahrein

Dates of change (as found by Mr. N.E. Davis)	1949	1950
300 mb.	June 16th	June 24th
200 mb.	June 12th	June 20th.

4. Persistent easterlies in July

The horizontal extent of the easterlies in July can be seen from Fig.5 of Frost's paper (M.R.P. 706).

The strength of the upper easterlies at their period of greatest development is illustrated in Fig. 4 which gives daily observations of wind speed at 200, 150, 100 mb. in July 1950. A separate diagram shows also the speed at the level of maximum wind - of the 52 occasions on which a maximum wind was recorded only 3 gave speeds of less than 80 kt. and on all these the speed exceeded 70 kt.

The level at which the maximum wind was reached in July 1950 is shown in the diagram immediately below the speed. Frequencies with which the maximum wind was recorded at different pressure levels are shown also in Fig. 5 - but the interpretation is complicated by the fact that in many of the ascents the wind was still increasing when the ascent terminated. For the months June - September the heights at which the ascents terminated on the occasion when the maximum wind was not reached are shown below the diagram, the level of maximum wind occurred at an unknown height above these levels. The position of the tropopause is also indicated.

5. Transition from easterlies to westerlies

By October, there is a weak belt of easterlies at about 100 mb. about 10 kt. extending from about 20° to the equator and by January this has retreated to the equator.

At Aden at 200 mb. the easterlies persist throughout the greater part of October though they become lighter and more variable in direction. The approximate dates of cessation of the persistent easterlies were:

1949 October 21st
1950 October 23rd
1951 October 12th

After their cessation there was in 1949 and 1951 a period of light variable winds lasting for two or three weeks before the westerlies became established.

At Bahrein the change at 200 mb. in 1950 took place in the last week in August.

Reference

1. Frost, R. 1952 M.R.P. 706.

/Table I

Table I

(a) Values of average scalar wind speed (knots) at Aden

mb.	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Year
100	16	14	15	14	18	55	66	54	40	21	13	18	28
150	23	21	24	23	22	50	71	61	47	21	20	29	35
200	26	23	31	26	23	37	50	46	37	19	22	36	31
300	22	18	24	22	19	22	25	24	22	14	18	25	21
400	16	13	12½	17	16	16	14	16	15	14	14	17½	15
500	12	10	8	12	14	19	16	16	13	15	12	13	14
600	11	9	9½	11	18	22	16½	18	20	16	11	10½	15
700	10	10	9	13	14	11	9	9	11	17	14	11	12
800	9	9	9	12	9	7	9	8½	8	12	15	12½	10
850	10	10	9	13	7	8	10	10	7	10	15	14	10
900	11½	10½	11	14	7	9	11	10½	7½	10	15	14	11
Surface	8	9	9	9	6	5	9	7	6	6	8½	9½	8

(b) Vector resultant

mb.		Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
100	°	270	227	276	247	116	94	94	93	93	101		264
	kt.	5	2	7	6	12	55	66	54	39	18	calm	7
150	°	293	257	291	272	147	99	93	93	100	124	275	273
	kt.	15	10	21	18	3	49	71	60	46	15	14	23
200	°	279	241	291	273	290	96	90	91	98	125	281	273
	kt.	17	11	25	19	2	36	48	45	36	13	14	26
300	°	289	270	296	268	279	88	80	87	94	114	269	262
	kt.	15	7	18	16	6	21	23	24	20	8	8	17
500	°	59	(77)	(166)	(62)	66	48	45	51	53	85	99	192
	kt.	3	1	1	1	9	16	13	13	10	12	3	3
700	°	34	56	101	56	64	53	29	49	52	35	43	39
	kt.	3	4	3	8	9	7	3	4	7	15	9	5
850	°	63	64	86	87	(99)	236	278	267	(258)	80	68	63
	kt.	7	6	5	10	1	2	5	5	1	7	13	11

Addendum by C.S. Durst

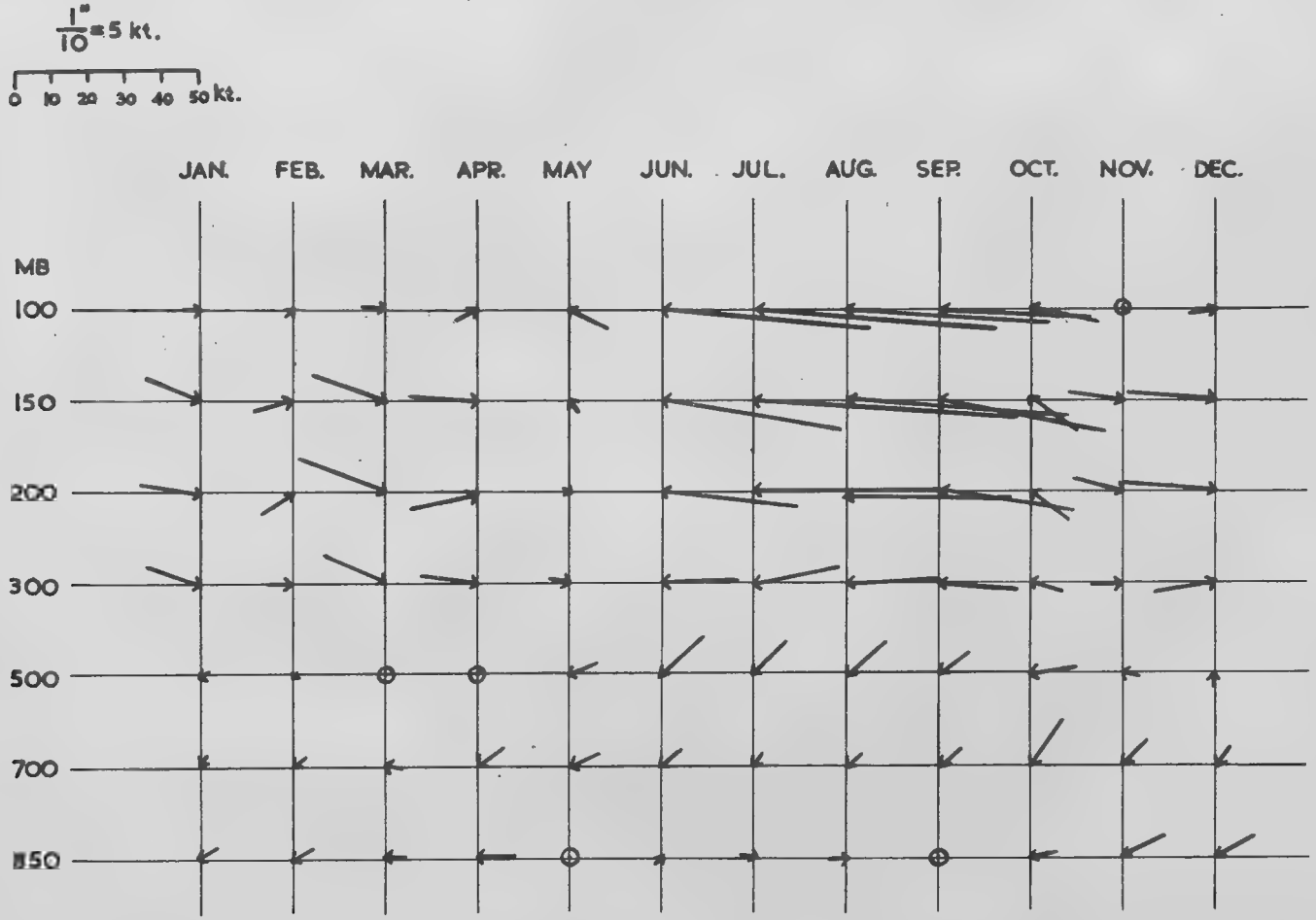
The following winds were reported by a Comet Aircraft flying between Cairo (Lat.30°.05'N) and Entebbe (Lat. 00° 02'N) in July 1951.

Date	Times G.M.T.	Positions Lat.	Long.	Lat.	Long.	Average height	Distance run (n. miles)	Wind
3.7.51	0808 to 0845	13°35'N	32° 25'E	09° 54'N	32° 20'E	34000ft	221	005° 20 kt.
8.7.51	1241 to 1317	09°45'N	31° 55'E	13° 08'N	32° 18'E	35000ft	143	325° 36 kt.
	1317 to 1341	13°08'N	32° 18'E	15° 38'N	32° 28'E	35000ft	150	075° 37 kt.
	1341 to 1414	15°38'N	32° 28'E	19° 11'N	32° 30'E	37000ft	213	105° 31 kt.
	1414 to 1447	19°11'N	32° 30'E	22° 37'N	32° 17'E	38000ft	206	064° 20 kt.
	1447 to 1518	22°37'N	32° 17'E	26° 08'N	32° 10'E	39000ft	211	150° 16 kt.
17.7.51	0029 to 0123	27°50'N	31° 18'E	21° 55'N	31° 25'E	29000ft	355	350° 22 kt.
	0123 to 0232	21°55'N	31° 25'E	14° 46'N	31° 44'E	33000ft	429	123° 10 kt.
	0232 to 0333	14°46'N	31° 44'E	09° 04'N	32° 05'E	37000ft	342	040° 12 kt.
	0333 to 0403	09°04'N	32° 05'E	05° 37'N	32° 02'E	39000ft	207	058° 35 kt.
28.7.51	0751 to 0816	03°34'N	32° 23'E	06° 15'N	32° 16'E	34000ft	161	065° 20 kt.
	0816 to 0844	06°15'N	32° 16'E	09° 22'N	31° 50'E	35000ft	187	090° 45 kt.

Clearly between 35,000 and 40,000 feet easterly winds of the same magnitude as at Aden extend over Abyssinia and the Sudan.

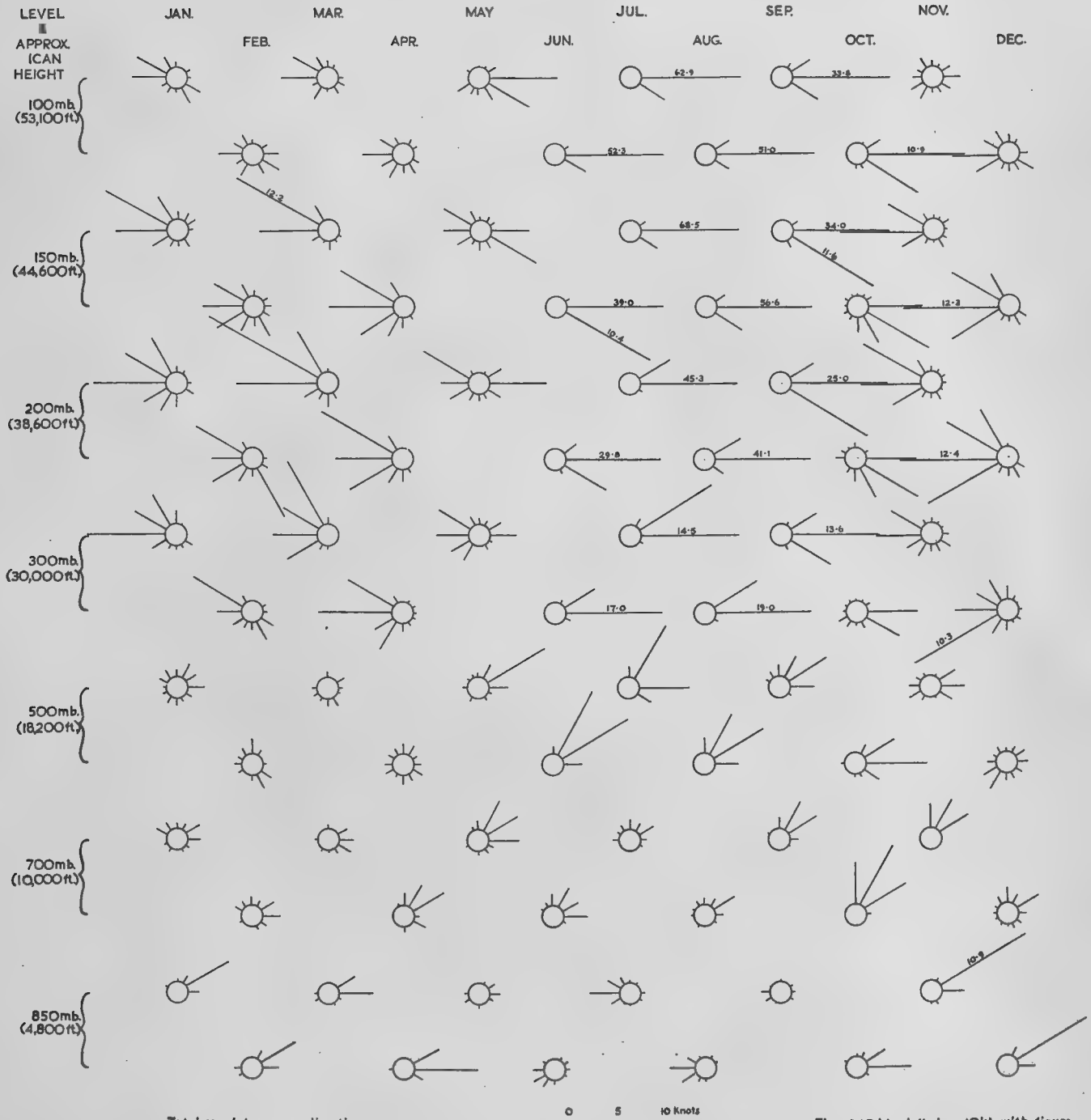
In a map of the normal upper winds in summer at 200 mb. and 300 mb. issued in May 1951 easterly winds were shown over Aden and Khartoun but they were not represented as being as strong as is now evidenced by the Aden observations.

FIGURE 1. Vector resultant winds at Aden.



Computed from all observations but treating winds of <10knots as calm.

FIGURE 2. Wind roses at Aden. August 1948 – December 1950.



Total flow = $\frac{\text{Total speed from any direction}}{\text{Total observations from all directions}}$

Flow >10 kt. plotted as 10kt. with figure written on the line.

Note: Flow in these roses is computed regarding winds of <10 kt. as calm.

FIGURE 3. Transition from upper westerlies to upper easterlies at Aden.

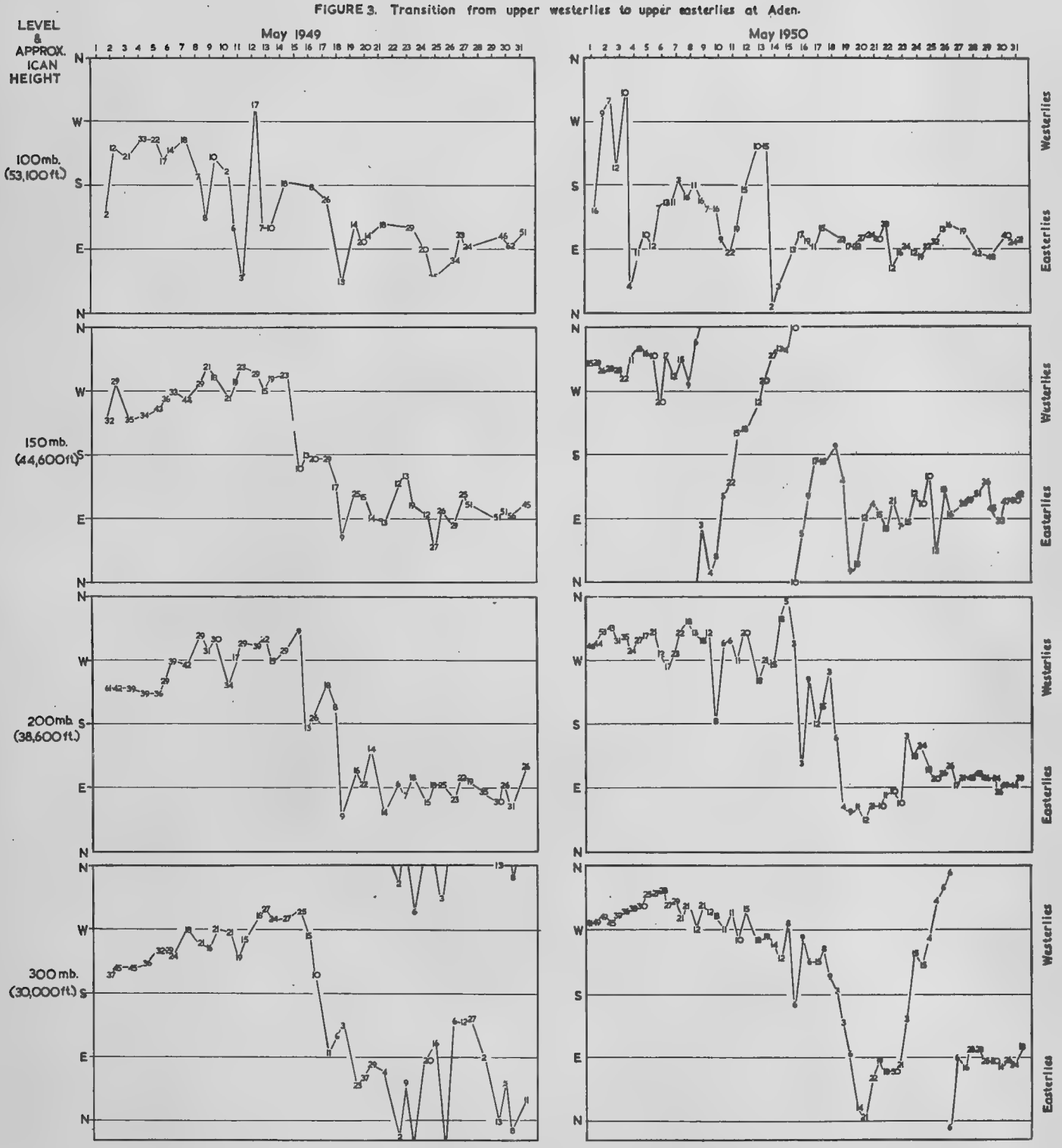


FIGURE 4. Daily observations of the speed of the upper easterlies.
Wind speed at Aden.
July 1950

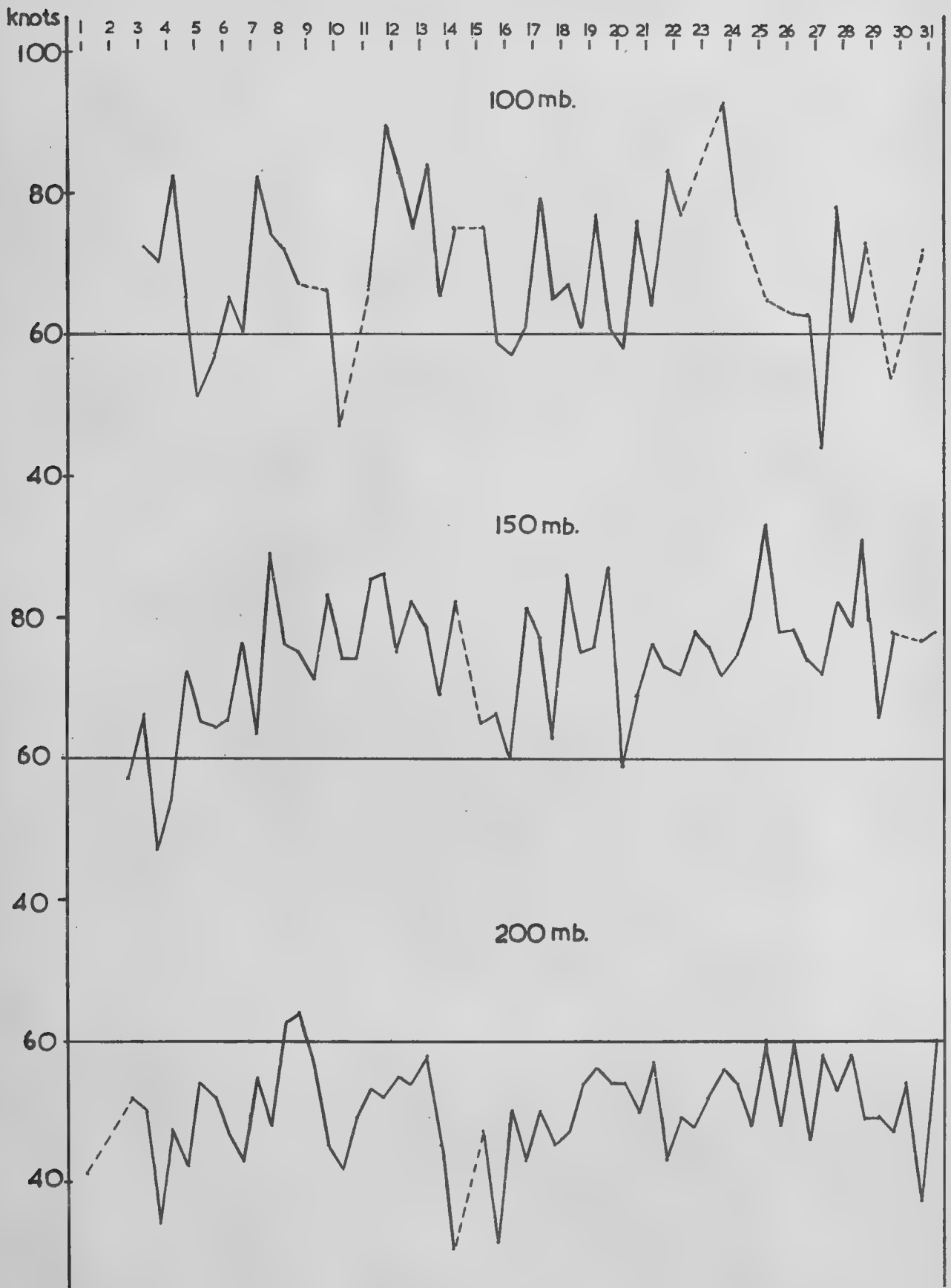
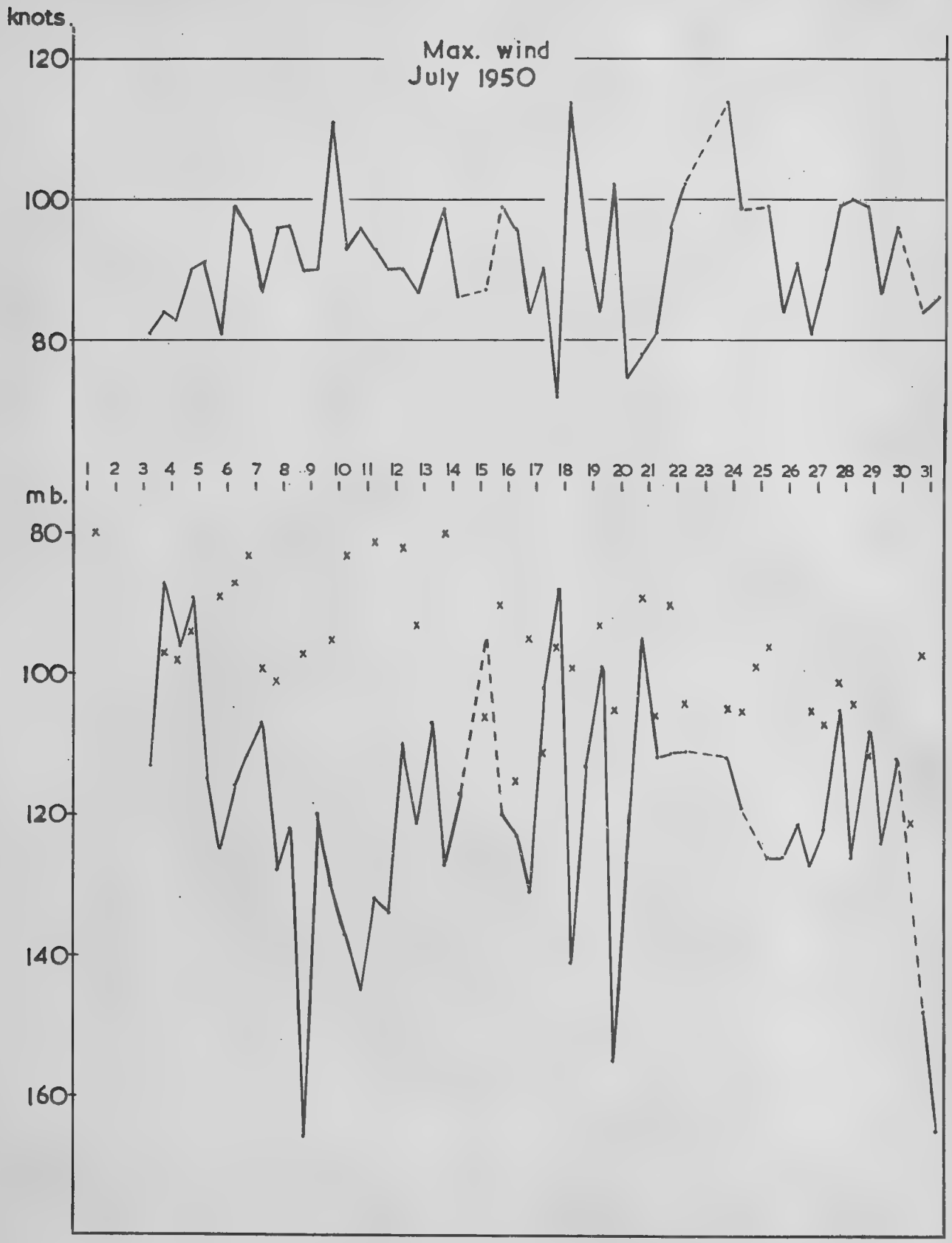
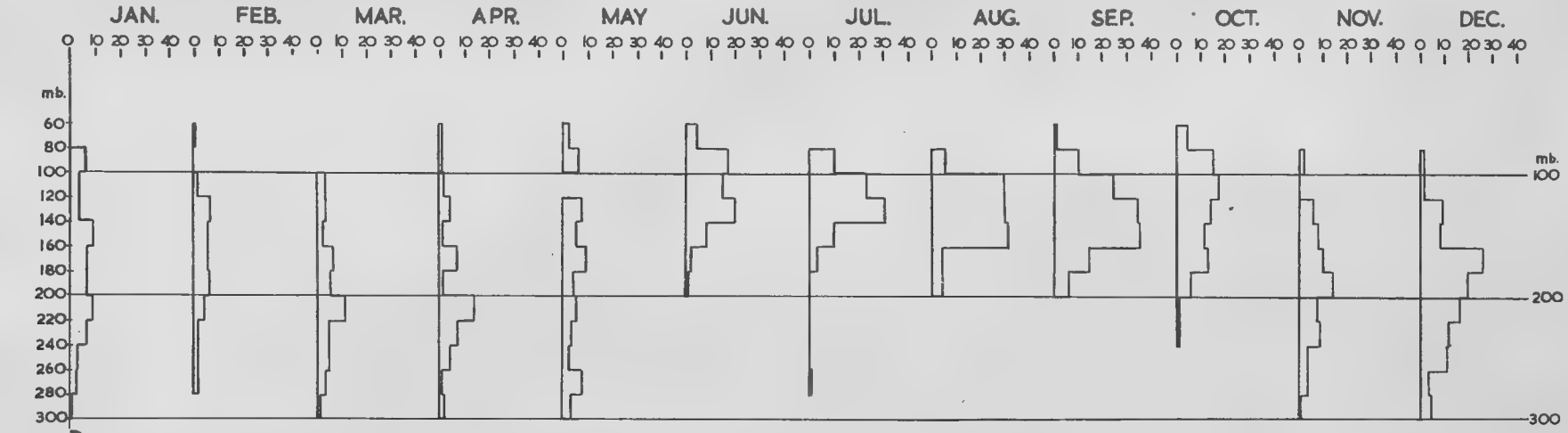


FIGURE 4(a). Daily observations of the speed of the maximum wind, and of the pressure at the level of maximum wind.



The pressure at the tropopause is shown by a ×

FIGURE 5. Pressure at the level of maximum wind for occasions when maximum wind was 30 knots or more at ADEN. Aug. 1948 — Dec. 1950.



	JAN.	FEB.	MAR.	APR.	MAY	JUN.	JUL.	AUG.	SEP.	OCT.	NOV.	DEC.
≥300	5	3	3	3	8					4	4	6
400	1	4		2	5					7	2	5
500												
>500		1		7	12					12	7	1
Total number of occasions of max. wind 30 kts. or more.	65	47	52	58	79	67	79	108	124	105	79	124
Number of occasions of "maximum wind not reached."	5	4	4	4	18	35	24	20	18	11	10	7

Note: The number of occasions when the maximum wind was reached at pressures greater than 300 mb. is shown in figures.

mb.		ascent	terminated	
60	13	2	2	2
100	14	12	11	4
150	1	3	3	3
200	7	7	4	9

SCALE : 1 inch = 50 observations.

0 10 20 30 40 50

1st May, 1952.

AIR MINISTRY

METEOROLOGICAL RESEARCH COMMITTEE

Ice accretion in cirrus cloud

by

A.C. Best. (Meteorological Office)

1. Introduction. Liquid water droplets at temperatures down to -50°C have been observed both in laboratory experiments and in the free air but except for one report of ice accretion in a contrail (Met.Mag.77, p.17) at -58°C no report of icing of aircraft at such low temperatures has been received until recently. It is probable that any such ice accretion would be of little importance compared with the intensity of icing which may be experienced in convective cloud since the rate of accretion in weight per unit area per unit time is likely to be very low. Even a small amount of icing may affect the aircraft performance at very high levels however and if the ice forms on the windscreen it may impair the vision of the pilot and so could have serious consequences in a service aircraft.

Heverly (2) and Dorsch and Hacker (3) have reported experiments on the temperature of spontaneous freezing of water droplets which indicate that the temperature at which a drop freezes depends upon its size, the smaller drops freezing at lower temperatures. The work of Dorsch and Hacker also indicated that for a specified drop size there is a range of temperature at which freezing takes place but with a most favoured temperature for each drop size. Brewer and Palmer (4) have also indicated that the temperature of freezing of water droplets depends upon their size. The results of these three reports are shown plotted in the figure (the most favoured temperature being plotted in the case of the Dorsch and Hacker experiments). It is obviously impossible to extrapolate this curve to a temperature in the neighbourhood of -50°C with any confidence but the available data do indicate that liquid drops at about -50°C must be very small, perhaps with diameter 1μ or less, if the laboratory results are applicable to the atmosphere. Unless the nuclei on which such droplets form are very minute such drops would be smaller than the critical size beyond which the drops grow indefinitely if the relative humidity remains constant (see for example table 5 of M.R.P. 698).

In the following sections we shall discuss some reports which suggest the presence of liquid droplets in cirrus cloud. It seems reasonable to assume that if ice accretion occurs in level flight it must be because the aircraft encounters liquid water drops.

2. Two "Canberra" reports. The following report has been received from 617 Bomber Squadron.

"Date and time - 20.3.52. 1230 hours

Weather - Fair. Cirrus in bands to the west.

During this flight and while flying at 25,000 feet over Liverpool at 0.7 Mach and in clear air Flt.Lt. Hopkinson reported the formation of a crystalline deposit on the outside of the cockpit canopy. This deposit was sufficiently dense to obscure the pilot's view, it persisted on the descent to 10,000 feet.

A previous case of this phenomenon occurred to the same pilot on the 11th March at 25,000 feet over Dover-Wight with Mach 0.71 - cleared at Mach 0.76 and reappeared at 0.70".

Although these reports refer to ice accretion in clear air it is suggested that they actually occurred in cirrus cloud. It is notoriously difficult for a pilot to see cirrus cloud in which he is flying. On March 20th the pilot reported cirrus to the west and on 11th March cirrus was reported at Tangmere at 0600 and 1500 hours (at other S.E. coast stations sky was obscured).

Considering these two reports in chronological order we get the following data.

/11.3.52.

11.3.52.

The Larkhill upper air ascent at 1500 hours gives a temperature of -31.7°C at 400 mb. at 23,670 feet. In these conditions the velocity of sound is 692 m.p.h. and

0.7 Mach = 484 m.p.h.	Full kinetic heating	=	23.4°C
0.76 Mach = 525 m.p.h.	" " "	=	27.6°C

It is likely that there was some transfer of heat through the canopy from the cockpit and that the greater kinetic heating at 0.76 Mach was then just sufficient to melt the ice which had formed on the canopy.

20.3.52.

The Liverpool upper air ascent at 1500 gives a temperature of -32.8°C at 400 mb. at 23,430 feet and the corresponding velocity of sound is 690 m.p.h. The full kinetic heating for 0.7 Mach would be 23.3°C and in this case it seems that the additional heating from the cockpit was not adequate to raise the temperature to 0°C. It is not possible to assess the significance of the persistence of the deposit until the aircraft descended to 10,000 feet since the problem would then be complicated by a cold aircraft descending into warmer air and, moreover, the speed of the aircraft at the lower level is not stated.

3. The "Comet" report. A report of icing on a Comet aircraft by the Captain (with some additional notes by C.S. Durst) is given in the appendix. The full kinetic heating corresponding to a true air speed of 367 kt. is about 18°C and would obviously be inadequate to prevent the formation of ice at -54°C. It is difficult to understand why the ice should have been confined to the area covering the integral fuel tanks.

References

1.	Aanensen, C.J.M.	1948	Met. Mag. 77, p.17.
2.	Heverly, J.R.	1949	Trans. Amer. Geophys. Union, 30, p.205.
3.	Dorsch, R.G. and Hacker, P.T.	1950	N.A.C.A. Tech. Note No. 2142 (1).
4.	Brewer, A.W. and Palmer, H.P.	1951	M.R.P. 637.

APPENDIX

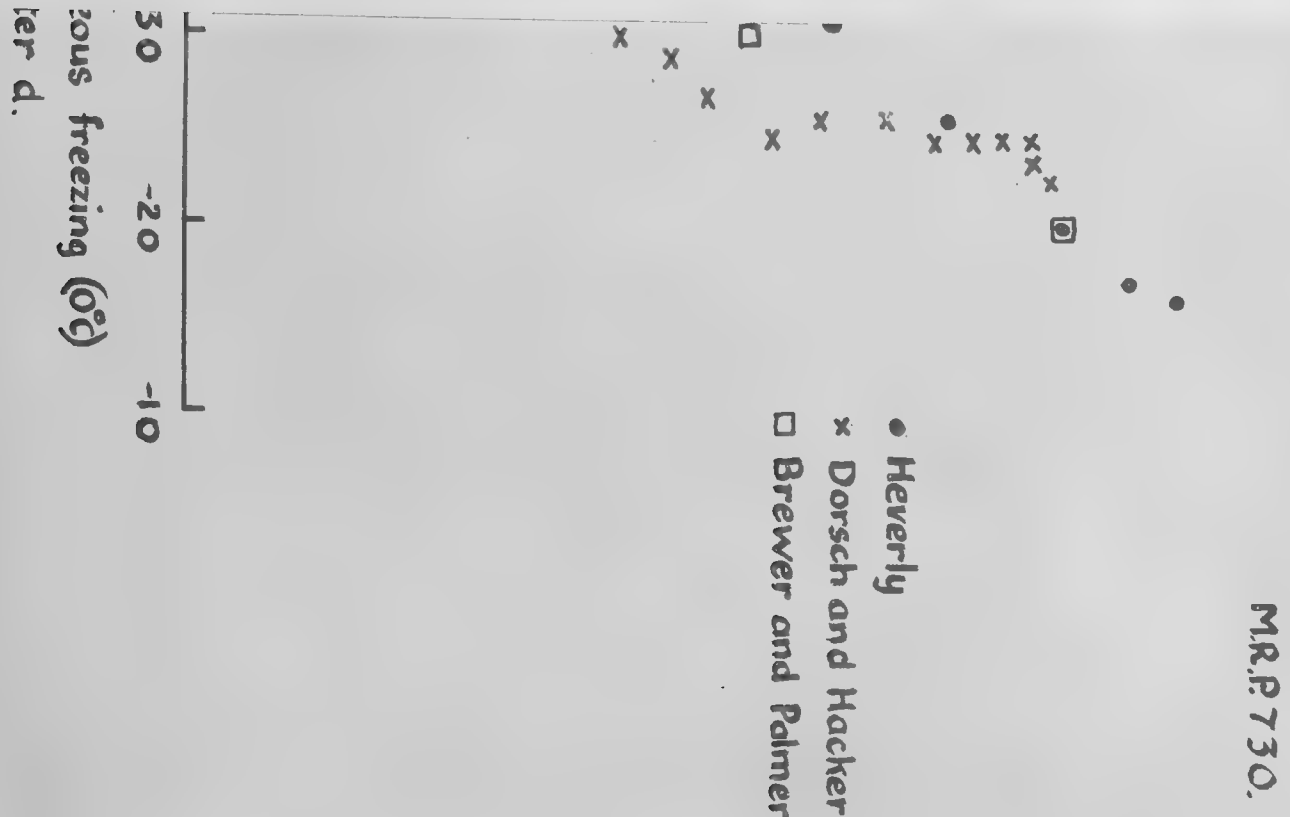
Icing on Comet Flight No. CTT.008

by Capt. A.M.A. Majendie (B.O.A.C.)

Date:	21st March, 1952
Time:	1415 G.M.T.
Height:	39,600 feet Pressure Altitude
Ambient Temperature:	-54°C
True Air Speed:	367 kt.
Indicated Air Speed:	189 kt.
Navigation Wind Velocity:	040° 14 kt.
Position:	04° 40'S 30° 30'E

We had been flying in stable cirrostratus cloud for about an hour, and there had been no indication of turbulence, or cumulonimbus cloud actually on the track of the aircraft. The light deposit of rime icing was observed on the leading edges and upper surfaces of the wings, being almost entirely confined to the areas occupied by the integral fuel tanks; these areas being probably a few degrees warmer in skin temperature than the remainder of the wing. On switching on the thermal de-icing system, the rime was seen to clear, and the stabilised cruising altitude of the aircraft increased almost immediately by about 500 feet to just over 40,000 feet, thereby demonstrating an improvement in aerodynamic performance.

G.332375(a)/AR/4/52/40



Note by C.S. Durst (Meteorological Office)

I was a passenger in the Comet when she was iced at 40,000 feet with a temperature of -54°C. I was sitting in one of the aft seats on the port side looking out over the wing with a fairly clear view for about 90° angle on the beam. I made notes at the time as follows:-

"1314 G.M.T. passing Cb with a cap of false Ci.
1325 G.M.T. still passing under Cs.
1345 G.M.T. flew through Cs. which contained Cb. lumps; flew out of Cs. with Cb. Cs. definitely above us in continuous layer quite separate from Cs. connected with Cb.

1411 G.M.T. have been flying in thick Cs. for about 20 minutes with occasional very slight bumps.

1412 G.M.T. breaking Cs. Veil still above us".

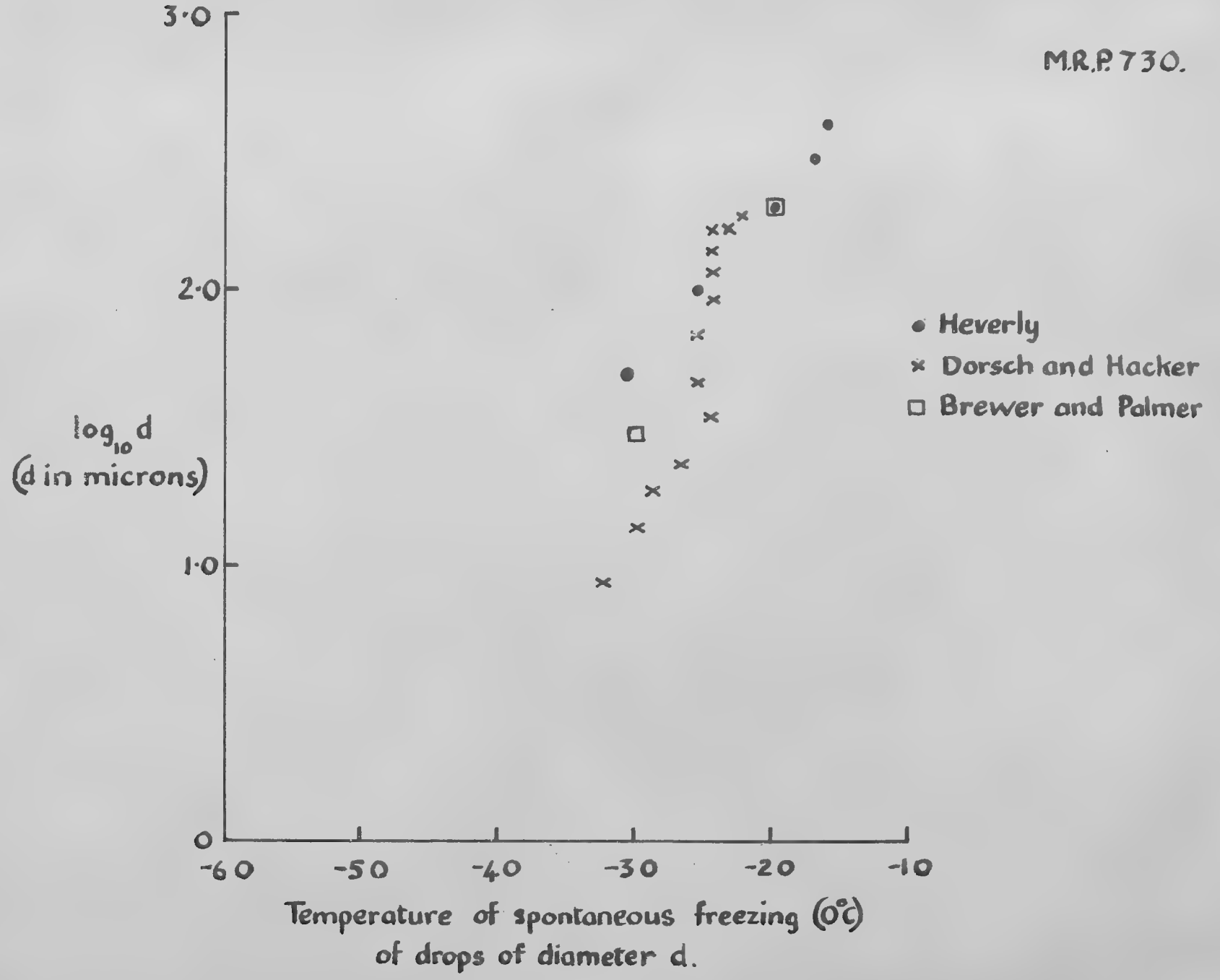
My recollection is that at times when passing through the Ci. and Cs. there were swirls of ice crystals passing over the wing, they had the appearance of the eddies of snow dust blown across a skating pond. They were not sufficiently dense to obscure the wing.

G.332375(a)/AR/4/52/40

References

1. Aanensen, C.J.M. 1948 Met. Mag. 77, p.17.
2. Heverly, J.R. 1949 Trans. Amer. Geophys. Union, 30, p.205.
3. Dorsch, R.G. and Hacker, P.T. 1950 N.A.C.A. Tech. Note No. 2142 (1).
4. Brewer, A.W. and Palmer, H.P. 1951 M.R.P. 637.

M.R.P. 730.


3·0
2·0
1·0
0
log10 d
(d in microns)
-60
-50
-40
-30
-20
-10
Temperature of spontaneous freezing (0°C)
of drops of diameter d.
• Heverly
× Dorsch and Hacker
□ Brewer and Palmer

At the time of issue this report has not been published. Any author desiring to quote this paper should refer to it as "A paper of the Meteorological Research Committee (Lond.)" M.R.P. No..... by Mr..... A copy is available in the Library of the Meteorological Office.

AIR MINISTRY

METEOROLOGICAL RESEARCH COMMITTEE

M.R.P. 731
S.C. II/111
27th May, 1952.

Classification of temperatures in the upper troposphere and lower stratosphere according to tropopause pressure

by

J.K. Bannon (Meteorological Office)

AIR
SEP 3 1952

Summary

Upper air temperatures in the neighbourhood of the tropopause are best classified, for some purposes, according to tropopause pressure. Resulting from such a classification for 20 mb. ranges of tropopause-pressure, mean temperatures at Larkhill are presented for standard pressure levels from 500 to 100 mb. for four months for a period of three years. The use of these mean temperatures for obtaining statistics of lapse rates and for the study of year to year temperature variations is discussed.

1. Introduction

In temperate latitudes the level of the tropopause fluctuates through a large range of heights; over the British Isles it is usually between 20,000 and 50,000 feet. Levels within the range of occurrence of the tropopause will be in the troposphere on some occasions, in the stratosphere on others, and statistics of temperature at such levels will include tropospheric and stratospheric features. For example the temperature frequency distribution at the 200 mb. level at Larkhill (1) is flat and in some months exhibits two maxima.

The level of the tropopause is related to temperature in the troposphere; thus Dines (2) found a correlation coefficient of +0.79 between the mean temperature from 1 to 9 km. and the height of the tropopause (H_c) and Priestley (3) a correlation of +0.65 between the temperature at 500 mb. and the height of the tropopause. Temperatures in the lower stratosphere are also related to the height of the tropopause; though Dines (2) did not compute correlation coefficients for the stratosphere he says regarding the passage of a low pressure area across Britain - "As the barometer falls the temperature of the air column from 1 to 9 km. falls also, the value of H_c (the height of the tropopause) decreases and the temperature of the upper air from 11 to 20 km. rises. As the depression moves away and the barometer rises, the lower air column rises in temperature, H_c increases and the upper air column falls in temperature." It is well known that a low tropopause is associated with a warm lower stratosphere.

The mean temperature at a fixed level in the upper troposphere or lower stratosphere for a given period will then be a function of, among other things, the mean behaviour of the tropopause during the same period. For example, the mean temperature at Larkhill at the 150 mb. level for the month of January, 1948 was 12°F warmer than for the same month in 1949; the corresponding mean tropopause pressures were 265 and 227 mb. respectively.

The variation of tropopause height is governed largely by advection, the horizontal transport of higher or lower tropopauses, and by vertical motion. Sawyer (4) has come to the conclusion that these two factors are equally important in the variation of tropopause height above a fixed point. Formation of a new tropopause at a different level by dynamical means, as opposed to motion of an existing surface, is another source of variability in tropopause height but is less important than the first two. Sawyer's findings are consistent, of course, with the classical correlations of Dines (2). Mean temperature in the troposphere is largely, though not entirely, governed by the source of the air or in other words by the type of air mass; the mean tropopause height in general increases towards the equator and the advection of warmer air from the south will transport a higher tropopause with it. The correlation coefficient of 0.79 between mean temperature from 1 to 9 km. and tropopause height is partly, probably largely, a result of advection. The close relation between pressure at 9 km. and tropopause height (correlation coefficient 0.84) is probably mainly the result of vertical motion, though temperature (advection) effects will also be present; vertical motion in the upper troposphere and lower stratosphere is related to the depressions and anticyclones of temperate latitudes, subsidence occurring over the former and ascent over the latter (5 and 6).

/If upper

The data of Fig.I of Bannon's paper have been used to plot the right-hand side of Fig.I of the present paper. The figures entered under column A are mean temperatures at the various pressure levels when tropopause pressure lies between 140 and 160 mb., those under column B relate to tropopause pressure 160 to 180 mb. and so on. Isopleths of temperature have been drawn in and

If upper air temperatures are classified according to tropopause height so that occasions of similar tropopause height are considered together, not only will statistics of upper air temperature so obtained be for stratosphere or troposphere separately and not for a mixture of the two regimes, but much of the variation associated with the variable height of the tropopause will be eliminated. Classification according to tropopause height was adopted for the discussion of humidity observations in the upper troposphere and lower stratosphere (6). Temperatures at a fixed level and for a particular (small) range of tropopause height (or pressure) will still be affected by the two main causes of tropopause height variation viz. advection and vertical motion but the variations, so arising, will be smaller than the variations of temperature at a fixed level regardless of tropopause height. It may be possible to sift the temperatures further, after classification by tropopause height, so as to eliminate or at any rate to compensate for the effects of vertical motion, arriving at statistics which may be used, for example, for a comparison of year to year differences in temperature in the upper troposphere and lower stratosphere which will be free from the immediate effects of the predominant weather types of the particular years.

This classification and sifting process may be interpreted in the following way. If it is assumed that advection and vertical motion are the two main causes of temperature change in the upper air, then temperatures may be considered a function of these two variables. Thus, symbolically:-

$$T = F(A, V)$$

where T = temperature, A denotes advection and V, vertical motion. Similarly tropopause pressure, P_c, is a function of advection and vertical motion and we may write

$$P_c = f(A,V)$$

If observations of T are classified according to P_c, this is equivalent to picking out all values of T for which $f(A,V)$ = constant, thus reducing the two degrees of freedom to one. If, after this classification, it is possible to classify further, for example by some relation between T and V, then the effects on T of both variables A & V will have been eliminated, remaining variations in T being attributable to other causes. This argument assumes, of course, that advection and vertical motion are the two main causes of variation in temperature and in tropopause pressure.

To demonstrate the possibilities of such a classification of upper air temperature according to tropopause height, the observations made at Larkhill in the years 1948, 1949, 1950 have been so classified for the four months January, April, July and October and the results are displayed and discussed in the following sections. Data for previous years have not been used as they were not recorded on Hollerith tabulating cards.

2. Method of Analysis

The data consist of temperature observations made at Larkhill, four times daily with few exceptions, at approximately 02.30, 08.30, 14.30 and 20.30 G.M.T. for the months January, April, July and October of the years 1948, 1949 and 1950. Only those ascents which attained the level of 150 mb. (approximately 45,000 ft.) or higher were used. By means of Hollerith tabulating machines the observations were assembled in groups, each group having tropopause pressures within a 20 mb. range. (Ranges were 120-139 mb., 140-159mb., 160-179 mb. etc.). Mean temperatures for each of the standard levels 500, 400, 300, 200, 150 and 100 mb. were computed for each group and for each of the four months. The standard deviations about the means were also evaluated.

The statistics assembled in this manner are subject to two sources of error:-

(a) errors in the level of the tropopause as given in the record

(b) errors in the observations arising from instrumental deficiencies.

/Regarding

The data of Fig.1 of [illegible] paper [illegible] side of Fig.I of the present paper. The figures entered under column A are mean temperatures at the various pressure levels when tropopause pressure lies between 140 and 160 mb., those under column B relate to tropopause pressure 160 to 180 mb. and so on. Isopleths of temperature have been drawn in and

Regarding (a) the position of the tropopause is usually in little doubt from the observations, being marked by a sharp change of lapse rate to inversion or isothermal. *

On a minority of occasions, however, the tropopause is difficult to specify unless rigid definitions are followed. Sawyer (4) says that for synoptic purposes it is best to analyse each case taking account of all adjacent observations and thus maintain continuity in space and time. For statistical purposes a rigid definition is, however, essential. The definitions of the tropopause at present in use in the Meteorological Office (see e.g. (4)) are not entirely rigorous but do make determination of the tropopause over the British Isles a simple matter on the great majority of occasions. However in 36 out of the total number of 1436 ascents analysed here the tropopause, as specified, does not obey the present Meteorological Office definitions; in each of these cases the lapse rate above the level named as the tropopause is greater than 1.1°F per 1,000 ft. and the true tropopause should be at a higher level. It is possible that there are other incorrectly placed tropopauses in the data which remain undetected as temperatures are available on the Hollerith cards only at the standard pressure levels which are 6,000 to 8,000 ft. apart in the upper troposphere and lower stratosphere. For the purposes of this analysis, which is regarded as a demonstration only, these obvious errors in tropopause level have not been corrected. For this reason statistics for numbers of observations less than, say, 10, are in some cases liable to even greater errors than the number of observations would imply. It is possible also, especially on occasions of very low tropopause, that the tropopause obeying the definition and chosen for the classification of temperatures here, is not the physically significant level but a spurious tropopause arising from some local peculiarity. Such inaccuracies must be accepted in an analysis of this nature; they provide a further reason for giving little weight to statistics from a small number of observations.

Instrumental inaccuracies, according to Harrison (7) result in random errors distributed with probable errors of about 0.8°F in the stratosphere and about 1.4°F in the upper troposphere. These assessments neglect any errors which may be caused by direct solar radiation on the instrument; it is not known exactly to what extent the observed diurnal variation of temperature in the lower stratosphere is due to such errors or to what extent it is real. However, for the purposes of comparing upper air temperatures observed with the same type of instruments, any such errors may be neglected and the observed diurnal variation considered real, so long as statistics are corrected to allow for differences in numbers of day and night observations.

3. Mean temperature-pressure curves and variation about these curves

After the data had been classified according to tropopause pressure, mean temperatures were calculated for each standard level for each tropopause-pressure range. The easiest way to demonstrate these means is by diagrams showing mean temperature against pressure curves for each tropopause category and for each season. (Figs. 1 to 4). Mean tropopause temperatures are also given. Greater accuracy than can be obtained from the diagrams is probably not justified because of the comparatively limited period covered by the statistics and because of inaccuracies in the data, as discussed in Section 2. The number of observations with very high or very low tropopause pressures was small and some curves for such tropopause-pressure ranges have been omitted. The pressure scale in Figs. 1 to 4 is logarithmic and is approximately proportional to height.

The orderliness of the curves in Figs. 1 to 4 is noteworthy, with few exceptions the mean curves in the stratosphere becoming steadily warmer with decreasing tropopause height, (increasing tropopause pressure), and the reverse in the troposphere. This was to be expected, of course, at any rate in the troposphere where this feature of the curves is merely a pictorial representation of the Dines correlation between tropopause height and mean tropospheric temperature. The mean temperatures shewn at the 100 mb. level are, in general, for fewer observations than at the other pressure levels; the numbers of observations are shewn on the diagrams.

* The tropopause is much more difficult to define in some tropical or sub-tropical regions.

The data of Fig.1 of Barlow's paper have [illegible] side of Fig.I of the present paper. The figures entered under column A are mean temperatures at the various pressure levels when tropopause pressure lies between 140 and 160 mb., those under column B relate to tropopause pressure 160 to 180 mb. and so on. Isopleths of temperature have been drawn in and

The distribution of temperatures at a fixed level is usually far from "normal" (see e.g. (1)). Similarly the temperatures at a fixed level and for a particular range of tropopause pressures show a "flat" distribution about their mean. The diurnal variation of temperature as observed by the British type radio-sonde would be expected, on the average, to account for a spread of the temperatures at 100 mb. level of 5 to 6°F. in summer and 3 to 4°F. in winter (Kay (8) and unpublished means in the Meteorological Office); at 300 mb. the corresponding figures would be of the order 3° and 1°F. respectively. Even allowing for this effect, the distribution of temperatures at a particular level is much flatter than the "normal".

As would be expected, the variation of temperature about the mean at a fixed level is smaller when the observations are classified according to tropopause pressure than when all observations are considered. Table I gives the standard deviations of temperature about the mean for various pressure levels for all observations at Larkhill in the period 1942-50 or 1942 - 51* for January, April, July and October (entries (a)). For comparison with these, means of the standard deviations of temperature for the various tropopause-pressure ranges are also given (entries (b)). The standard deviations vary considerably from tropopause range to tropopause range, in extreme cases approaching the standard deviation for all observations. This is partly the result of small numbers of observations in some tropopause categories. These mean standard deviations given in Table I are the means for all tropopause-pressure ranges containing 10 or more observations and it is thought that they give the correct order of the reduction in variability which can be achieved by this classification. Very similar results are obtained by considering the reduction in the range of temperatures at a particular level by classifying according to tropopause pressure; the relative reduction in range is usually greatest at 200 mb. or 150 mb. and the average ratio of the range in a particular tropopause category to the whole range of temperature at the same level is about 0.7.

In reference (1) it is shown that the standard deviation of temperature varies with pressure in a regular manner, having maxima about 200 and 500 mb. (the latter considerably smaller than the former) and a minimum at 300 mb. A maximum of variability of temperature was also noted by Dines (2) at 6 km. and a minimum at 10 km., the variability increasing again at greater heights. Entries (b) of Table I show that classification with respect to tropopause pressure removes a great part of this variation with pressure of standard deviation of temperature, though there is still a slight tendency for a minimum to occur at 300 mb.

Also given in Table I (entries (c) and (d)) are the standard deviations of temperature at 300 and 200 mb. according as these levels were in the troposphere or stratosphere respectively. The data used to compute these were those for 1948-1950 used in deriving Figs. 1 - 4 and entries (b) of Table I. These standard deviations evolved from the general analysis and are given here for interest, though they are irrelevant to the main argument of this paper. The fact that some of the standard deviations in (d) are greater than the corresponding figures in (a) for all observations, is presumably because of the much shorter period used in computing (c) and (d). It is evident, however, that the standard deviations of temperature at fixed levels in the lower stratosphere are of the same magnitude as the standard deviations at the same levels for all observations, including those occasions when the levels were in the troposphere; this result is, perhaps, surprising.

The grouping of the individual temperature observations about the curves shown in Figs. 1 - 4 is not particularly close (see Table I). Though errors in deciding the tropopause level, errors in observation and the apparent diurnal variation of temperature all help to accentuate this variability, it seems certain that the temperatures are distributed about the means shown in Figs. 1 - 4 with standard deviations of the order of 4 or 5°F. In spite of this, however, forecasters and others may find these mean curves useful for comparison with individual ascent curves.

/4.

* Standard deviations of temperature for the main pressure levels are evaluated for certain stations as a matter of routine in the Upper Air Climatology Branch of the Meteorological Office.

The data of Fig.1 of Barnes's paper [illegible] side of Fig.I of the present paper. The figures entered under column A are mean temperatures at the various pressure levels when tropopause pressure lies between 140 and 160 mb., those under column B relate to tropopause pressure 160 to 180 mb. and so on. Isopleths of temperature have been drawn in and

4. Lapse rates of temperature

The original purpose of this classification of upper air temperatures was to study the changes in mean lapse rate in the upper troposphere and lower stratosphere between the various seasons. Figs. 5(a) to (h) show the mean temperature-pressure curves for the four months January, April, July, October for eight ranges of tropopause pressure and the more important seasonal changes are apparent on these diagrams; the ordinate is again the logarithm of pressure. The number of ascents is given in brackets.

(a) Upper troposphere

There is, perhaps, a slight tendency for greater lapse rates in summer than in winter (e.g. Figs. 5.b, d, e, f, g,) but in some of the tropopause-pressure ranges the reverse is noted, especially just under the tropopause. The mean lapse rates *in the layers just below the tropopause for all the categories in Fig. 5 are, in °F per 1,000 ft:-

Jan.	April	July	Oct.
3.4	3.2	3.4	3.5

Presumably the negligible seasonal change in lapse rate is because in the neighbourhood of southern England penetrative convection is not important in modifying the temperature of the upper troposphere which is largely governed by the horizontal advection of air of varying temperature and by vertical motions associated with the large scale weather features, depressions and anticyclones.

(b) Lower stratosphere

The most striking thing about the curves in Figs. 5 (a) to (h) is that, in each case, the lapse rate just above the tropopause is greatest in October and, in most cases, least in April or January. The means of the lapse rates (°F. per 1,000 ft.) just above the tropopause for the eight tropopause-pressure ranges in Figs. 5 are:-

Jan.	April	July	Oct.
- 1.5.	-1.7	-1.3	-0.3

Considering only the data in Figs. 5(c) to (h) it is seen that the lapse rates in the layer 200-150 mb. which in each case is not immediately above the tropopause but some distance into the stratosphere, again have a maximum in October, except for Fig.5(g) where the maximum is in July; the minimum is usually in January. The mean lapse rates between 200 and 150 mb. for these six tropopause-pressure ranges are, in °F. per 1,000 ft:-

Jan.	April	July	Oct.
-0.3	-0.3	+0.2	+0.3

The explanation of this maximum lapse rate in the lower stratosphere in October is not clear. It may be argued that temperatures in the upper stratosphere follow the calendar with little lag, being greatest at the summer solstice and least at the winter solstice; temperatures in the upper troposphere, however, might be expected to lag behind the seasons as shown by the calendar, as the greater summer temperatures have to be conveyed to the upper levels by turbulence both small scale (convectional and this mainly in the tropics) and large scale (depressions, anticyclones). Figs. 5(a) to (h) do not lend much support to this argument, however, even allowing that April is nearer the summer solstice than October. The difference in temperature in the upper troposphere between October and April is small but the difference in temperature in the lower stratosphere between these months is much larger, the October temperatures being similar to those for January.

/Without

* Mean lapse rates were obtained by dividing mean-temperature differences by mean thicknesses of the layers between the appropriate pressure levels. It can be shewn that a mean lapse rate so obtained is a close approximation to the mean of all the lapse rates for individual occasions.

The data of Fig.1 of Bannon's paper have been used to plot the right-hand side of Fig.I of the present paper. The figures entered under column A are mean temperatures at the various pressure levels when tropopause pressure lies between 140 and 160 mb., those under column B relate to tropopause pressure 160 to 180 mb. and so on. Isopleths of temperature have been drawn in and

Without classification according to tropopause pressure, monthly mean temperatures for Larkhill at the 100 and 150 mb. levels (in the stratosphere on the great majority of occasions) show that the April means are slightly greater than the October means but are considerably less than the annual means. This is also true of Lerwick, Tromso and Aklavik so that this peculiarity of October stratospheric temperatures appears to be true of much of the higher northern latitudes. (For Larkhill the mean temperatures at 100 and 150 mb. for October are less than those for January).

The total amount of ozone in the atmosphere in temperate latitudes of the Northern Hemisphere is at a maximum in the spring (March-April) and a minimum in October (8). As ozone strongly absorbs ultra-violet radiation this is a possible explanation of the maximum lapse rate in the lower stratosphere in October. A good series of temperature observations in the ozone layer (60,000-120,000 ft.) is required to be certain of this explanation. The few observations discussed by Scrase (9) show greater temperatures at 60,000 ft. and above in the Spring than in the Autumn but the differences are small; many more observations are required to confirm this feature.

There is also the possibility that the mean lapse rates in the lower stratosphere for October were influenced by the type of weather in the Octobers analysed. In each month the weather was predominantly undisturbed with surface pressure above the average.

Duperier (10) has pointed out that in the mean over southern England, there is a slight inversion of temperature in the lower stratosphere on occasions of high surface pressure and a lapse on occasions of low surface pressure. In Figs. 1 - 4 it is seen that low tropopauses are associated with comparatively large lapse rates and high tropopauses with inversions in the lower stratosphere. Since surface pressure and tropopause height are related (correlation coefficient +0.68 according to Dines (2)), Duperier's finding is confirmed in the present analysis.

5. Statistics of tropopause pressure

A few statistics of tropopause pressure are presented as they emerged in the analysis.

Fig.6 (a) - (d) shows the frequency of occurrence of tropopause within the various 20 mb. pressure ranges. It will be noted that though the mean tropopause-pressure varied appreciably from season to season, the mode was more nearly constant. Applying the empirical formula (see e.g. reference (11)),

$$\text{Mode} = \text{Mean} - 3(\text{Mean} - \text{Median})$$

the following values are obtained for the modes (in mb.):-

Jan.	April	July	Oct.
195	211	212	206

Monthly mean tropopause-pressures are influenced greatly by the predominating type of weather and a seasonal variation is not apparent from the data for Larkhill for the five years 1946-1950 (1). For example, the highest and lowest monthly mean tropopause-pressures occurred in the following months:-

Year	Monthly mean tropopause-pressure	
	Lowest	Highest
1946	February	September
1947	September	February
1948	September	April & January
1949	February	November
1950	March	April

Priestley (3) found that in 1944 the lowest mean monthly tropopause-height at Larkhill occurred in October and the highest in January. The same lack of consistent seasonal variation in monthly mean tropopause-pressure is apparent in the data for Lerwick for the period 1946-1950; as at Larkhill the values varied

/erratically

The data of Fig.1 of Bannon's paper [illegible] side of Fig.I of the present paper. The figures entered under column A are mean temperatures at the various pressure levels when tropopause pressure lies between 140 and 160 mb., those under column B relate to tropopause pressure 160 to 180 mb. and so on. Isopleths of temperature have been drawn in and

erratically from year to year. It is possible that data for a large number of years may show a tendency for a seasonal variation in tropopause pressure over southern England, though such a seasonal variation will be reversed in individual years.

Priestley (3) gave curves showing the mean temperatures at the tropopause above Larkhill appropriate to different tropopause pressures for the winter and summer halves of the year 1944 and for the whole year and compared these with the mean annual curve for Lerwick. Similar curves are shown in Fig. 7 for each of the months January, April, July and October, the means being computed over the three years 1948-50; as in the rest of the analysis, the tropopause pressures were grouped in ranges of 20 mb. and each point in Fig. 7 represents the mean tropopause temperature for all tropopauses within that particular pressure range for that month. Priestley used 10 mb. ranges of tropopause pressure. The ordinate in Fig. 7 is height on the standard ICAN pressure scale and is adopted as an approximation to true height. Also shown on Fig. 7 is a line whose slope gives the dry adiabatic lapse rate. Sawyer (4) uses potential temperature at the tropopause as a criterion of continuity. Potential temperature might have been taken as abscissa in Fig. 7 but actual temperature was preferred as demonstrating seasonal changes more easily.

There is an indication on the January and July curves in Fig. 7 that for very low tropopause heights, bodily sinking of the air in the layers bordering the tropopause surface may be responsible for much of the variation of tropopause level, as the curves there have slopes similar to the dry adiabatic lapse. The number of observations with very low tropopause height is small, however, and no great reliance can be placed on this argument. For the most part, as Priestley also pointed out for the 1944 data, the tropopause mean temperature-height curves are considerably steeper than the adiabatic lapse rate. Priestley remarked that in his curves the greatest rate of rise of temperature with increasing tropopause pressure was between 180 and 270 mb. This is not shown generally on the curves in Fig. 7, though in January and April the rate of rise does fall off for pressures greater than about 270 mb.

The points in Fig.7 for July and October tropopauses at heights above 45,000 ft. (pressures less than 150 mb.) appear out of harmony with the general run of the curves through the other points. It may be that they represent conditions at a tropical tropopause the appropriate curves for which would not be continuous with those shown which are essentially for a temperate tropopause. There are too few observations to be certain of this.

Tropopause temperatures within the 20 mb. tropopause pressure ranges adopted for the analysis have considerable scatter; the distribution about the mean is flatter than the "normal". Standard deviations of tropopause temperatures about the mean for the tropopause-pressure ranges 180-199 mb., 200-219 mb., 220-239 mb. were calculated for each of the four months discussed in this analysis; they did not vary greatly from category to category or from month to month. Mean values of these standard deviations were:-

January	5.1°F
April	4.9°F
July	4.8°F
October	4.4°F

The greatest and least values were 5.3°F (July, 200-219 mb.) and 3.8°F (October, 180-199 mb.) respectively. These standard deviations are very similar to those given in Table I for the variation of temperature at a fixed level.

6. Variation of temperature in the lower stratosphere

In section 3 it was shown that temperatures at a level usually in the lower stratosphere, have a scatter about their mean value represented by a standard deviation of the order 6 to 12°F. This standard deviation is reduced to the order 4 to 6°F by classifying the temperatures according to tropopause pressure. It is of interest to study some possible causes or associations of the scatter in the temperatures of the lower stratosphere within each tropopause-pressure range. This has been done by evaluating the coefficients of correlation

/between

The data of Fig.1 of Bannon's paper have [illegible] side of Fig.I of the present paper. The figures entered under column A are mean temperatures at the various pressure levels when tropopause pressure lies between 140 and 160 mb., those under column B relate to tropopause pressure 160 to 180 mb. and so on. Isopleths of temperature have been drawn in and

between (a) the temperature at 150 mb. and the height of the 300 mb. level, (b) the temperature at 150 mb. and the surface pressure, (c) the temperature at the tropopause and the height of the 300 mb. level, in each restricting the analysis to observations having tropopause pressures within the same 20 mb. range and for the same month. Table II gives the results of this analysis. Only three ranges of tropopause pressures are considered; the numbers of observations in other ranges are smaller.

It will be remembered that Dines (2) found a high correlation between the pressure at 9 km. and the height of the tropopause (+0.84) and a significant correlation (-0.47) between the pressure at 9 km. and the temperature at the tropopause. The height of the 300 mb. level is approximately equivalent to the pressure at 9 km. Dines found that surface pressure was also related to temperature at the tropopause (correlation coefficient -0.52). These correlations of Dines suggested the pairs of values correlated in Table II.

The figures in Table II (a) may be roughly interpreted as follows; if the temperature at 150 mb. (T_{150}) is assumed to depend on the pressure of the tropopause (P_c) and the height of the 300 mb. level (H_{300}) independently, then the correlation coefficients quoted are equivalent to partial correlation coefficients between T_{150} and H_{300} without first classifying with respect to P_c. Similar meanings may be attached to the coefficients in Table II (b) and (c).

Though the numbers of pairs of observations used in computing Table II are from 35 to 95, these pairs cannot all be considered unrelated. For example, on a day in which little change of weather occurs, it is to be expected that the four upper air soundings will not differ greatly. Assuming a "Persistence length" of about three days (12) it may be surmised, therefore, that the numbers of independent pairs of observations will be about 30 for each of the entries in Table II. (Roughly, 10 independent observations from each specific month for three years). Thus the levels of significance for a single correlation coefficient in Table II will be approximately (13):-

0.1%	1%	10%
0.57	0.46	0.31

The coefficients in Table II (a) may be considered under two hypotheses.

(i) as evidence for or against a general relation, irrespective of season or tropopause pressure, between T_{150} and H_{300} when the data are classified according to tropopause;

(ii) as evidence for or against relations between T_{150} and H_{300} which may or may not change with season or with tropopause-pressure.

Tables II(b) and (c) may be considered similarly.

For Table II(a), hypothesis (i), the fact that the coefficients all have the same sign supports the deduction that there is a general relation between T_{150} and H_{300}. Transforming each coefficient (r) to its equivalent Z, (Fisher's Z transformation: reference (13)), taking the mean and reconverting, a value of $r = -0.4$ is found for the Table II(a) as a whole, the appropriate number of observations for significance test purposes being $12(30-3)+3 = 327$. This mean correlation is thus significant at the 0.1% level. It may also be shewn that the distribution of the 12 values of r in Table II(a) is not inconsistent with the hypothesis that there is no real variation between the values for different months or different tropopause-pressures. The mean coefficient $r = -0.4$ indicates that about 15% of the variance of T_{150} is due to its dependence on H_{300}.

Tables II(b) and (c) do not suggest any general relationship as does Table II(a).

Regarding hypothesis (ii) above, Table II(a) would indicate that the negative correlation between T_{150} and H_{300} is probably significant in July and October but of little importance in January and April. The relationship between surface pressure and T_{150} (Table II (b)), if it is real at all, varies in a similar manner with season, but is much less marked. It was shown in

The data of Fig.1 of [illegible] paper [illegible] side of Fig.I of the present paper. The figures entered under column A are mean temperatures at the various pressure levels when tropopause pressure lies between 140 and 160 mb., those under column B relate to tropopause pressure 160 to 180 mb. and so on. Isopleths of temperature have been drawn in and

reference (6) that the type of circulation, cyclonic, anticyclonic or intermediate, at the 300 mb. level, is of more significance in varying the temperature and humidity of the upper troposphere and lower stratosphere than the pressure type at the surface and this rough conclusion is confirmed by the higher coefficients of Table II (a) as compared with Table II(b). The coefficients in Table II (c) are low and may have no significance. The positive correlations of July and October are surprising, being contrary to the trend shown in Tables II (a) and (b). They may indicate that tropopause temperatures in summer and autumn are more influenced by troposphere phenomena than in the other seasons.

The negative correlation between temperatures of the lower stratosphere and the height of the 300 mb. surface shown in some months in Table II (a) has a rough physical explanation as has been pointed out previously (6). Tropopause height is related to mean troposphere temperature, or, in other words, to the air mass type. Subsidence and ascent associated with depressions and anticyclones of temperate latitudes also influence tropopause height (shown e.g. by the Dines correlation coefficient of 0.84 between pressure at 9 km. and tropopause height). Subsidence and ascent are also roughly related to low and high values respectively of the 300 mb. surface. In a particular tropopause-pressure range one would expect to find temperatures at, say, 150 mb. appropriate to tropical air which had subsided to bring the tropopause within the pressure range considered and also those appropriate to polar air which had ascended in sympathy with a lifting of the tropopause to bring it within the range considered. Thus the tropical air at the 150 mb. level, with increased temperature following subsidence, would be associated with a low value of the height of the 300 mb. level and similarly polar air would be cold following ascent and associated with high values of the height of the 300 mb. level. This argument assumes that the subsidence and ascent necessary to bring our tropical and polar air into the same tropopause-pressure category cause a warming and cooling respectively which are more than enough to off-set the normal temperature differences in the lower stratosphere; for the same state of dynamical development, e.g. for the same surface pressure, for lack of a better index, the tropical air will have a higher tropopause and lower temperature in the lower stratosphere than the polar air. This assumption is reasonable for, as shown in Section 5, Fig.7, the average relation between tropopause pressure and tropopause temperature in the great majority of cases is such that adiabatic descent or ascent will make temperatures in the lower stratosphere higher or lower respectively than the average for the appropriate tropopause pressure.

7. Possible uses of the classification

It is clear that if statistics of any characteristic are to be separated for troposphere and stratosphere then a classification with respect to tropopause height (or pressure) is necessary; temperature lapse rate is an example. It may be possible to assess the utility of such a classification for other purposes from the discussion in previous sections, though the analysis described is only for a short period.

As stated in the Introduction mean temperatures in the upper atmosphere in the neighbourhood of the tropopause are much influenced by the type of weather predominating in the period considered; for some purposes, of course, the reasons for the large variations from period to period (e.g. from a month of one year to the same month of the next year) are unimportant. If, however, it is desired to sift the observations so as to remove part, at least, of what may be called the dynamic influences, then the classification according to tropopause height (or pressure) undoubtedly does this.

(a) Comparison of Seasons.

Figs. 5(a) to (h) allow a comparison of mean temperatures in the various seasons. It may be argued that, since tropopause pressure may have a seasonal variation, the curves in Fig. 5 are not strictly comparable and that, perhaps for example, the curve for January, tropopause pressure range 200-219 mb. should be compared with the July curve for a different tropopause range. It was seen in Section 5, however, that no definite seasonal trend in tropopause-pressure is apparent in the data of the last few years.

/Table III

The data of Fig.1 of Bannon's paper [illegible] side of Fig.I of the present paper. The figures entered under column A are mean temperatures at the various pressure levels when tropopause pressure lies between 140 and 160 mb., those under column B relate to tropopause pressure 160 to 180 mb. and so on. Isopleths of temperature have been drawn in and

Table III gives the mean temperatures for Larkhill at the 150 mb. level for four different months for four tropopause-pressure ranges and also for all observations for the same period 1948-1950. The mean temperatures for a period of 9 or 10 years are given in the last column. It is seen that, on the whole, the seasonal differences for the tropopause-classified data are closer to those for the long period than those for all observations for the short period 1948-1950, especially for those tropopause categories further removed from the 150 mb. level, i.e. for high tropopause-pressures. Summing up, it can be said that there is some evidence that seasonal temperature differences in the lower stratosphere are better shown by data classified according to tropopause pressure when only a few years data are available. There is little doubt, however, that statistics from all observations, regardless of tropopause pressure, should give the best indication of seasonal changes when data for a long period (10 years or more) are available for analysis.

(b) Year to year variations

As remarked previously (Section 1) the mean temperature for a particular month at a level usually in the lower stratosphere may vary widely from year to year and most of the variation may be explained by the character of the weather as reflected in the height (pressure) of the tropopause. To study year to year temperature variations in the lower stratosphere, e.g. to try to relate ozone content with air temperature, it is desirable to eliminate as much as possible of the dynamic effect on temperatures and the classification with respect to tropopause pressure may be used for this purpose.

Table IV is an illustration of how mean temperatures at a fixed level may be compared from year to year; in it are given the mean temperatures at various levels for three different ranges of tropopause-pressure for each of the months January, April, July and October for each year 1948, 1949, 1950 and also the mean temperatures computed from all observations regardless of tropopause-pressure. The number of observations used to form these means was often quite small and unevenly distributed among the four observing hours of the day; the mean temperatures were accordingly corrected for diurnal variation i.e. corrected to the mean which would apply if the numbers of day and night observations were the same. To do this estimates of the apparent diurnal variation at different levels were obtained from mean temperatures for Larkhill for several years for the various hours of observation, after the manner of Kay (8). Maximum correction necessary ranged from 0.4°F. at 400 mb. to 1.2° F. at 100 mb. Also given in Table IV for comparison purposes, are the monthly mean heights of the 300 mb. level for the appropriate tropopause pressures and also for all observations, and mean monthly tropopause pressures.

It will be noted that in the lower stratosphere and to a less extent also in the upper troposphere, the year to year variations are in most cases smaller in the classified mean temperatures than in the means from all observations. Large variations in mean temperatures (all observations) are related to large variations in mean tropopause pressure, as would be expected from the Dines correlations.

As was noted in Section 6 there is a relation in some months, between temperatures of the lower stratosphere in a particular tropopause-pressure category and the corresponding height of the 300 mb. pressure level. From Table IV it is seen that this relation is also apparent in the means, though again, not in a marked degree. Table V gives correlation coefficients between mean temperatures at 150 mb. and corresponding mean heights of the 300 mb. level. Nine of these are not particularly significant, (the coefficients are calculated from three pairs of values only); with two random samples, each distributed normally, the chance of getting such correlations would be greater than 1 in 10. For the other three, the chances would be between 1 in 50 and 1 in 20.

/It is

The data of Fig.1 of Dobson's paper have ... side of Fig.I of the present paper. The figures entered under column A are mean temperatures at the various pressure levels when tropopause pressure lies between 140 and 160 mb., those under column B relate to tropopause pressure 160 to 180 mb. and so on. Isopleths of temperature have been drawn in and

It is concluded that if statistics were available for, say 10 years, it would be possible to compare monthly mean temperatures from year to year within the same tropopause-pressure ranges, after allowing for the small influence of the height of the 300 mb. level by means of a regression equation computed from the long period of observations. There may be other dynamic heating or cooling effects for which it might be possible to allow in this manner but it would appear (Section 6) that the height of the 300 mb. level is the most important. The Dines correlations do not point to any other significant independent parameter.

Temperatures in the upper troposphere might be treated similarly, though within tropopause-pressure categories the relation between the weather type (as reflected, for example, by the height of the 300 mb.level) and the temperature is probably not so great as in the lower stratosphere.

8. Conclusions

From this short analysis of the Larkhill upper air observations for four months in each of the years 1948, 1949, and 1950 it is concluded that:-

(i) For a statistical analysis of any upper air characteristic, classification with respect to tropopause height (pressure) is essential to separate troposphere and stratosphere individualities.

(ii) The lapse rate in the lower stratosphere is notably greater in Autumn than in other seasons. This may be linked with the annual minimum of ozone in that season; available evidence indicates that autumn temperatures in the ozone layer are perhaps lower than corresponding spring temperatures but data are insufficient to demonstrate this conclusively.

(iii) Lapse rate in the upper troposphere has little, if any, seasonal variation.

(iv) To compare monthly mean temperatures in the lower stratosphere from year to year, except for purely climatological purposes, it is necessary to eliminate some, at least, of the dynamical effects associated with the general character of the weather of particular months. Classification with respect to tropopause height (pressure) goes some way towards this and with a longer series of observations than treated here it should be possible to allow for the probably small residual effect on these means arising from vertical motions associated with the predominating weather. Mean monthly temperatures so obtained would then be suitable for correlation with such things as ozone amounts, solar or cosmic influences, though a hemisphere wide distribution of such temperature observations would probably be essential for such analyses.

Not arising directly from the analyses of previous sections but suggested by the general arguments used there, it would seem desirable to proceed with the following investigations:-

(a) Correlation coefficients between such parameters as tropopause height and temperature, mean troposphere temperature, heights of 500 and 300 mb. levels should be evaluated for several stations and for several latitudes including tropical and semi-tropical. This should be done for monthly or at most three-monthly periods.

(b) As an indication of the heights to which the dynamic effects of the tropopause weather systems penetrate, correlation coefficients should be calculated between temperature at various heights in the stratosphere and such things as tropopause height, height of the 300 mb. level and mean tropospheric temperature. Again grouping periods should be monthly or three-monthly at most and data for several stations should be so analysed.

The data of Fig.1 of Barlow's paper ha[illegible] side of Fig.I of the present paper. The figures entered under column A are mean temperatures at the various pressure levels when tropopause pressure lies between 140 and 160 mb., those under column B relate to tropopause pressure 160 to 180 mb. and so on. Isopleths of temperature have been drawn in and

(c) (a) and (b) above should be merely pointers as to which relationships, if any, are important. Rough relationships once established should be further refined, if possible, by direct empirical methods, graphical or otherwise.

(d) Tropopause mean temperature-pressure relations should be studied for a variety of stations. It may be possible to separate the advectional and dynamical effects on tropopause temperature by such a study (Section 5 refers)

Acknowledgment

The classification of the Larkhill data and the evaluation of the resulting mean temperatures were carried out by members of the staff of the Upper Air Climatology Branch of the Meteorological Office. The author has also drawn freely on the advice of other members of the above Branch.

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The data of Fig.1 of Bannon's paper [illegible] side of Fig.I of the present paper. The figures entered under column A are mean temperatures at the various pressure levels when tropopause pressure lies between 140 and 160 mb., those under column B relate to tropopause pressure 160 to 180 mb. and so on. Isopleths of temperature have been drawn in and

Table 1

Standard deviations of temperature about the mean

(a) All observations; period 1942 to 1950 or 1951

(b) Observations according to tropopause pressure (20 mb. ranges) - mean standard deviations for all ranges containing 10 or more observations. Period 1948 - 1950.

(c) Observations in the troposphere; period 1948 - 1950. No. of observations in ( )

(d) Observations in the stratosphere; " " " " " "

			Pressure level (mb.)				
			500	300	200	150	100
January	(a)	Period 1942-1950	9.9	6.3	11.7	11.0	9.5
	(b)	Period 1948-1950	5.1	4.7	5.3	5.4	6.0
	(c)	"	-	5.7 (283)	4.4 (95)	-	-
	(d)	"	-	6.9 (39)	11.0 (227)	-	-
April	(a)	Period 1942-1951	8.8	5.8	12.1	8.8	6.8
	(b)	Period 1948-1950	4.5	4.6	5.1	4.7	5.4
	(c)	"	-	5.3 (245)	3.9 (47)	-	-
	(d)	"	-	6.5 (61)	12.9 (259)	-	-
July	(a)	Period 1942-1951	5.9	5.9	9.5	7.9	6.8
	(b)	Period 1948-1950	4.2	4.2	4.5	4.9	5.1
	(c)	"	-	6.4 (345)	4.3 (73)	-	-
	(d)	"	-	3.2 (9)	8.8 (281)	-	-
October	(a)	Period 1942-1950	8.6	7.2	8.8	9.0	7.6
	(b)	Period 1948-1950	5.6	5.1	4.7	5.0	5.0
	(c)	"	-	8.9 (333)	5.1 (130)	-	-
	(d)	"	-	6.8 (17)	8.2 (223)	-	-

G.332465(a)/MBF/5/52/40.

The data of Fig.1 of Barlow's paper [illegible] side of Fig.I of the present paper. The figures entered under column A are mean temperatures at the various pressure levels when tropopause pressure lies between 140 and 160 mb., those under column B relate to tropopause pressure 160 to 180 mb. and so on. Isopleths of temperature have been drawn in and

Table II

Correlation coefficients: temperature in the lower stratosphere, the height of the 300 mb. level, surface pressure. (numbers of observations given in brackets)

Month	Tropopause pressure range 180–199 mb.	200–219 mb.	220–239 mb.
(a) Temperature at 150 mb. v. height of 300 mb. level.			
January	-0.15 (59)	-0.54 (69)	-0.38 (48)
April	-0.24 (35)	-0.16 (64)	-0.09 (44)
July	-0.50 (47)	-0.45 (95)	-0.46 (89)
October	-0.62 (42)	-0.49 (68)	-0.43 (50)
(b) Temperature at 150 mb. v. surface pressure			
January	-0.07	-0.37	-0.28
April	+0.04	-0.06	-0.15
July	-0.26	-0.50	-0.30
October	-0.23	-0.25	-0.35
(c) Temperature at tropopause v. height of 300 mb. level			
January	-0.27	-0.28	0.00
April	-0.06	+0.09	0.00
July	+0.11	+0.18	+0.44
October	-0.05	+0.12	+0.26

G.332465(b)/MBF/5/52/40.

The data of Fig.1 of Bannon's paper [illegible] side of Fig.I of the present paper. The figures entered under column A are mean temperatures at the various pressure levels when tropopause pressure lies between 140 and 160 mb., those under column B relate to tropopause pressure 160 to 180 mb. and so on. Isopleths of temperature have been drawn in and

Table III

Mean temperatures at 150 mb. (°F.), Larkhill 1948 - 1950

	Tropopause pressure (mb.)				All observations	
	180-199	200-219	220-239	240-259	1948-50	1942-50
January	-78.2	-72.1	-69.9	-65.6	-70.3	-69.3
April	-70.5	-67.2	-63.9	-59.8	-61.3	-64.0 *
July	-67.3	-62.0	-58.8	-55.3	-60.2	-59.7 *
October	-77.4	-72.9	-68.6	-66.9	-72.8	-71.4

* Period 1942 -1951

G.332465(o)/MBF/5/52/40.

Table IV

LARKHILL 1948-1950: Monthly mean temperatures corrected for diurnal v
Monthly mean heights of the 300 mb. level and
Monthly mean tropopause pressures
(Numbers in brackets are the numbers of observations on which the mean

Month	Tropopause-pressure range mb.	Mean temperatures °F. 100 mb. 1948	100 mb. 1949	100 mb. 1950	150 mb. 1948	150 mb. 1949	150 mb. 1950	300 mb. 1948	300 mb. 1949	300 mb. 1950	1948
January	180-199	-72.9 (7)	-79.3 (19)	-76.3 (23)	-75.6 (10)	-80.4 (23)	-77.8 (27)	-52.2	-49.6	-53.2	-26.0
	200-219	-69.6 (8)	-77.7 (21)	-74.3 (30)	-69.4 (14)	-75.8 (21)	-71.5 (34)	-51.0	-52.8	-56.4	-2[illegible].0
	220-239	-69.9 (10)	-74.4 (18)	-71.6 (15)	-67.4 (12)	-71.3 (20)	-70.3 (16)	-54.3	-55.9	-58.4	-28.8
	All	-66.4 (55)	-76.1 (98)	-71.3 (101)	-62.9 (98)	-74.9 (109)	-73.0 (114)	-57.9 (117)	-52.7 (120)	-56.0 (122)	-35.7 (123)
April	180-199	-70.0 (2)	-71.4 (28)	-67.0 (3)	-68.7 (3)	-71.7 (28)	-66.3 (4)	-48.5	-46.5	-49.6	-22.3
	200-219	-65.5 (21)	-70.7 (23)	-66.1 (10)	-65.3 (27)	-70.5 (23)	-66.1 (14)	-50.0	-49.0	-52.0	-22.8
	220-239	-63.7 (13)	-70.4 (14)	-66.0 (11)	-60.2 (16)	-69.0 (17)	-62.4 (12)	-52.1	-51.5	-55.3	-24.3
	All	-61.6 (80)	-68.9 (106)	-62.9 (79)	-58.3 (98)	-67.9 (111)	-57.6 (98)	-51.2 (116)	-49.5 (119)	-53.2 (110)	-29.7 (119)
July	180-199	-65.3 (15)	-66.7 (19)	-60.4 (8)	-69.5 (18)	-67.6 (19)	-64.1 (10)	-34.9	-37.4	-36.6	-6.8
	200-219	-61.9 (25)	-64.7 (32)	-58.3 (31)	-62.8 (29)	-64.3 (32)	-59.0 (34)	-39.9	-39.1	-38.3	-13.0
	220-239	-59.4 (19)	-63.3 (25)	-57.4 (26)	-59.2 (26)	-60.4 (27)	-57.3 (36)	-43.3	-40.0	-42.7	-16.1
	All	-60.8 (94)	-63.8 (116)	-57.3 (93)	-61.6 (119)	-62.2 (120)	-56.8 (116)	-41.5 (123)	-40.7 (122)	-41.6 (122)	-14.9 (124)
October	180-199	-78.2 (16)	-77.1 (9)	-76.4 (13)	-78.9 (19)	-75.9 (10)	-77.9 (13)	-40.4	-44.5	-43.9	-13.3
	200-219	-70.0 (16)	-73.7 (25)	-72.2 (18)	-69.9 (19)	-75.4 (27)	-71.7 (22)	-46.3	-45.7	-47.1	-20.6
	220-239	-69.7 (14)	-70.2 (18)	-70.7 (17)	-68.5 (16)	-68.7 (19)	-69.4 (19)	-47.4	-49.4	-47.9	-20.9
	All	-72.2 (99)	-73.8 (106)	-72.5 (105)	-72.5 (120)	-73.7 (114)	-72.1 (120)	-44.6 (124)	-45.7 (119)	-47.1 (122)	-19.0 (124)

The data of Fig.I of Bannon's paper [illegible]
side of Fig.I of the present paper. The figures entered under column A are mean temperatures at the various pressure levels when tropopause pressure lies between 140 and 160 mb., those under column B relate to tropopause pressure 160 to 180 mb. and so on. Isopleths of temperature have been drawn in and

Table V

Correlation coefficients between monthly mean temperatures at 150 mb. in the various tropopause-pressure categories and corresponding mean heights of the 300 mb. level

	Tropopause pressures mb.		
	180-199	200-219	220-239
January	-0.994	-0.956	-0.978
April	-0.983	-0.744	-0.719
July	-0.739	-0.633	-0.820
October	-0.997	-0.998	+0.610

332465(e)/MBF/5/52/40.

The data of Fig.1 of Damon's paper [illegible] side of Fig.I of the present paper. The figures entered under column A are mean temperatures at the various pressure levels when tropopause pressure lies between 140 and 160 mb., those under column B relate to tropopause pressure 160 to 180 mb. and so on. Isopleths of temperature have been drawn in and

M.R.P. 731

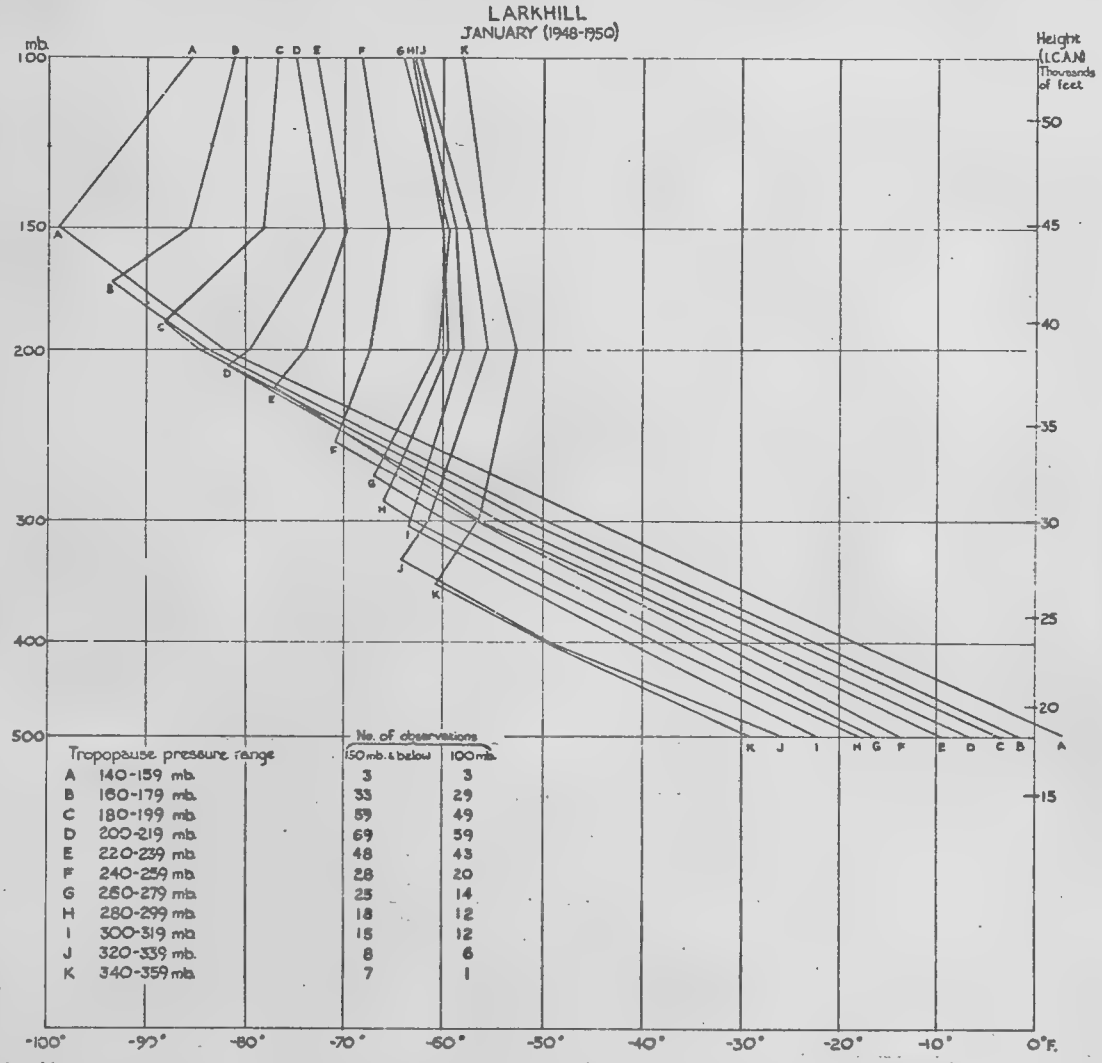
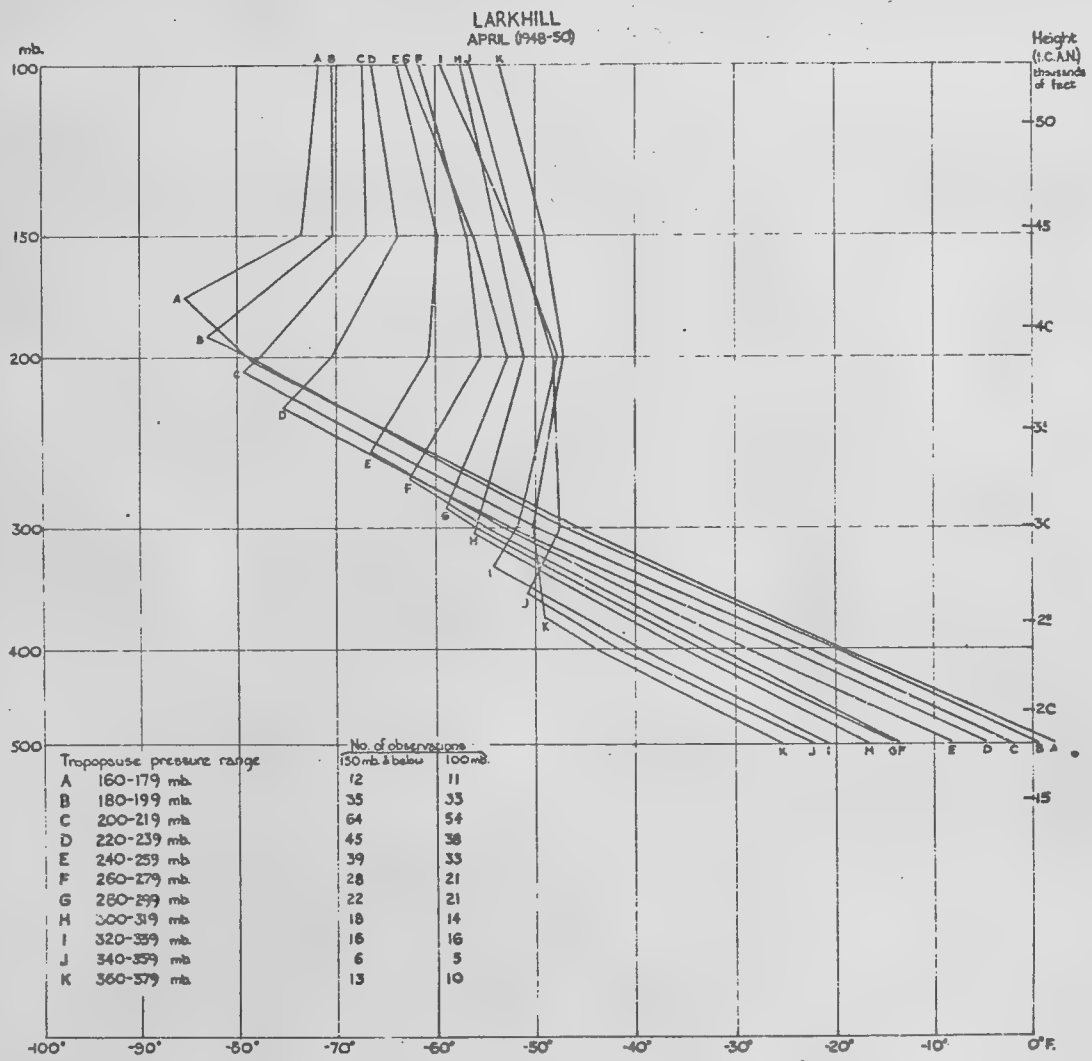


Fig.1 Mean temperature-pressure curves; observations grouped according to tropopause pressure (20 mb. ranges)

The data of Fig.I of Bannon's paper have been used to plot the Fig[illegible] side of Fig.I of the present paper. The figures entered under column A are mean temperatures at the various pressure levels when tropopause pressure lies between 140 and 160 mb., those under column B relate to tropopause pressure 160 to 180 mb. and so on. Isopleths of temperature have been drawn in and

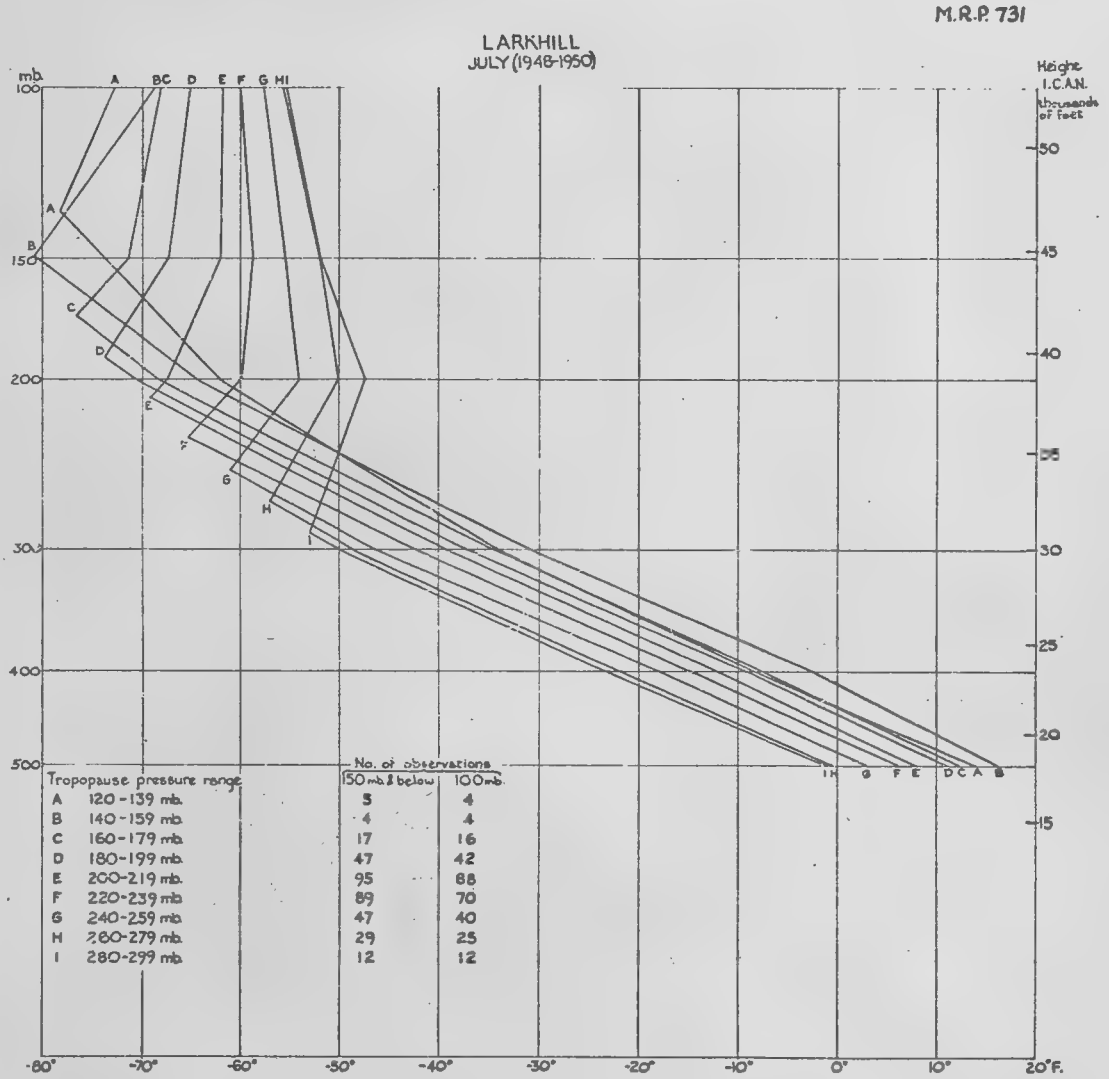
M.R.P. 731



Tropopause pressure range		No. of observations 150 mb. & below	100 mb.
A	160-179 mb.	12	11
B	180-199 mb.	35	33
C	200-219 mb.	64	54
D	220-239 mb.	45	38
E	240-259 mb.	39	33
F	260-279 mb.	28	21
G	280-299 mb.	22	21
H	300-319 mb.	18	14
I	320-339 mb.	16	16
J	340-359 mb.	6	5
K	360-379 mb.	13	10

Fig.2 Mean temperature-pressure curves; observations grouped according to tropopause pressure (20mb. ranges)

The data of Fig.I of Bannon's paper have been used to plot the right-hand side of Fig.I of the present paper. The figures entered under column A are mean temperatures at the various pressure levels when tropopause pressure lies between 140 and 160 mb., those under column B relate to tropopause pressure 160 to 180 mb. and so on. Isopleths of temperature have been drawn in and



Tropopause pressure range		No. of observations 150 mb & below	100 mb.
A	120 - 139 mb.	5	4
B	140 - 159 mb.	4	4
C	160 - 179 mb.	17	16
D	180 - 199 mb.	47	42
E	200 - 219 mb.	95	88
F	220 - 239 mb.	89	70
G	240 - 259 mb.	47	40
H	260 - 279 mb.	29	25
I	280 - 299 mb.	12	12

Fig.3 Mean temperature-pressure curves; observations grouped according to tropopause pressure (20 mb. ranges)

The data of Fig.I of Bannon's paper have been [illegible] side of Fig.I of the present paper. The figures entered under column A are mean temperatures at the various pressure levels when tropopause pressure lies between 140 and 160 mb., those under column B relate to tropopause pressure 160 to 180 mb. and so on. Isopleths of temperature have been drawn in and

M.R.P. 731

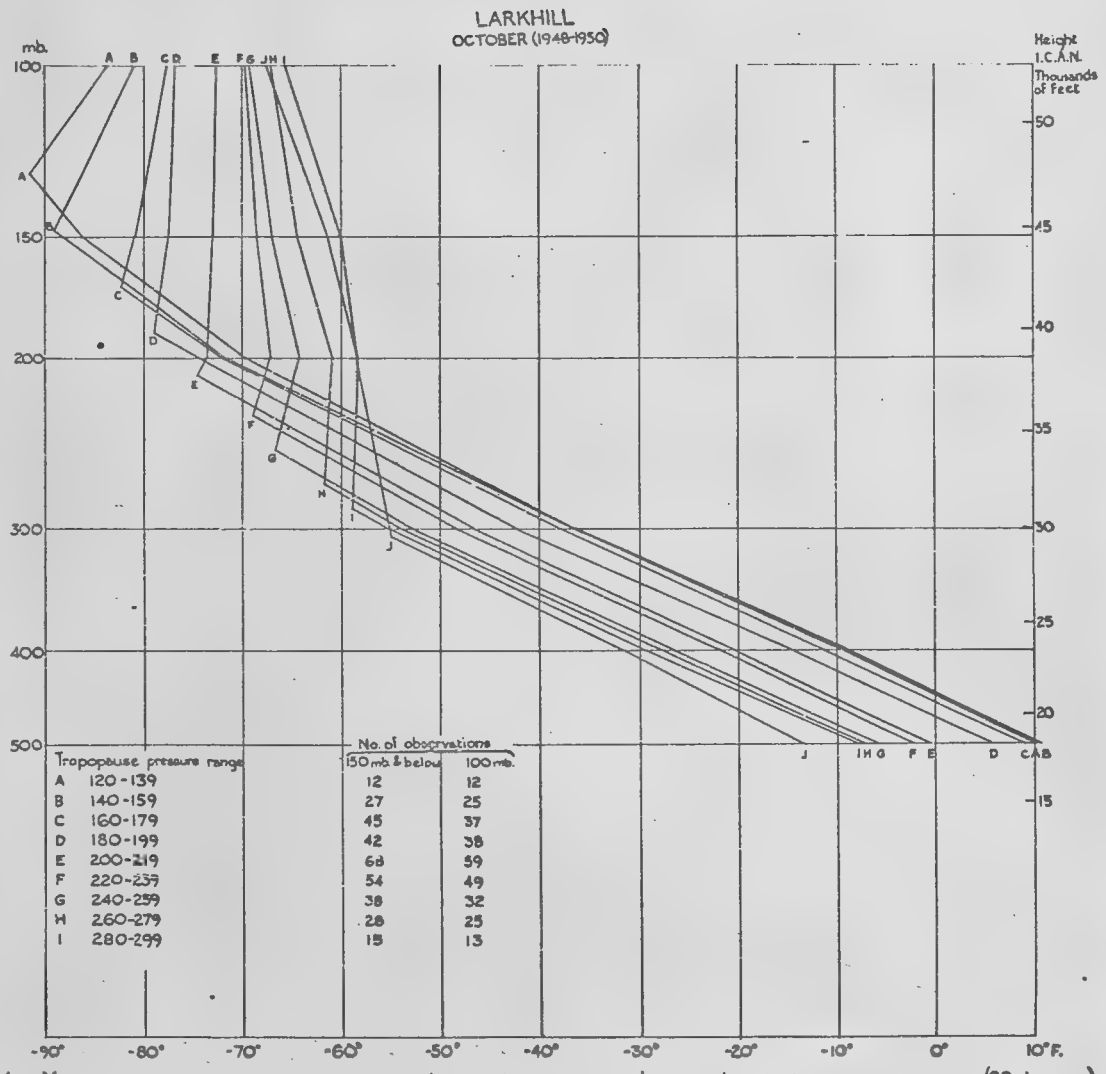


Fig.4 Mean temperature-pressure curves; observations grouped according to tropopause pressure (20 mb. ranges)

The data of Fig.I of Bannon's paper have been used to plot the right-hand side of Fig.I of the present paper. The figures entered under column A are mean temperatures at the various pressure levels when tropopause pressure lies between 140 and 160 mb., those under column B relate to tropopause pressure 160 to 180 mb. and so on. Isopleths of temperature have been drawn in and

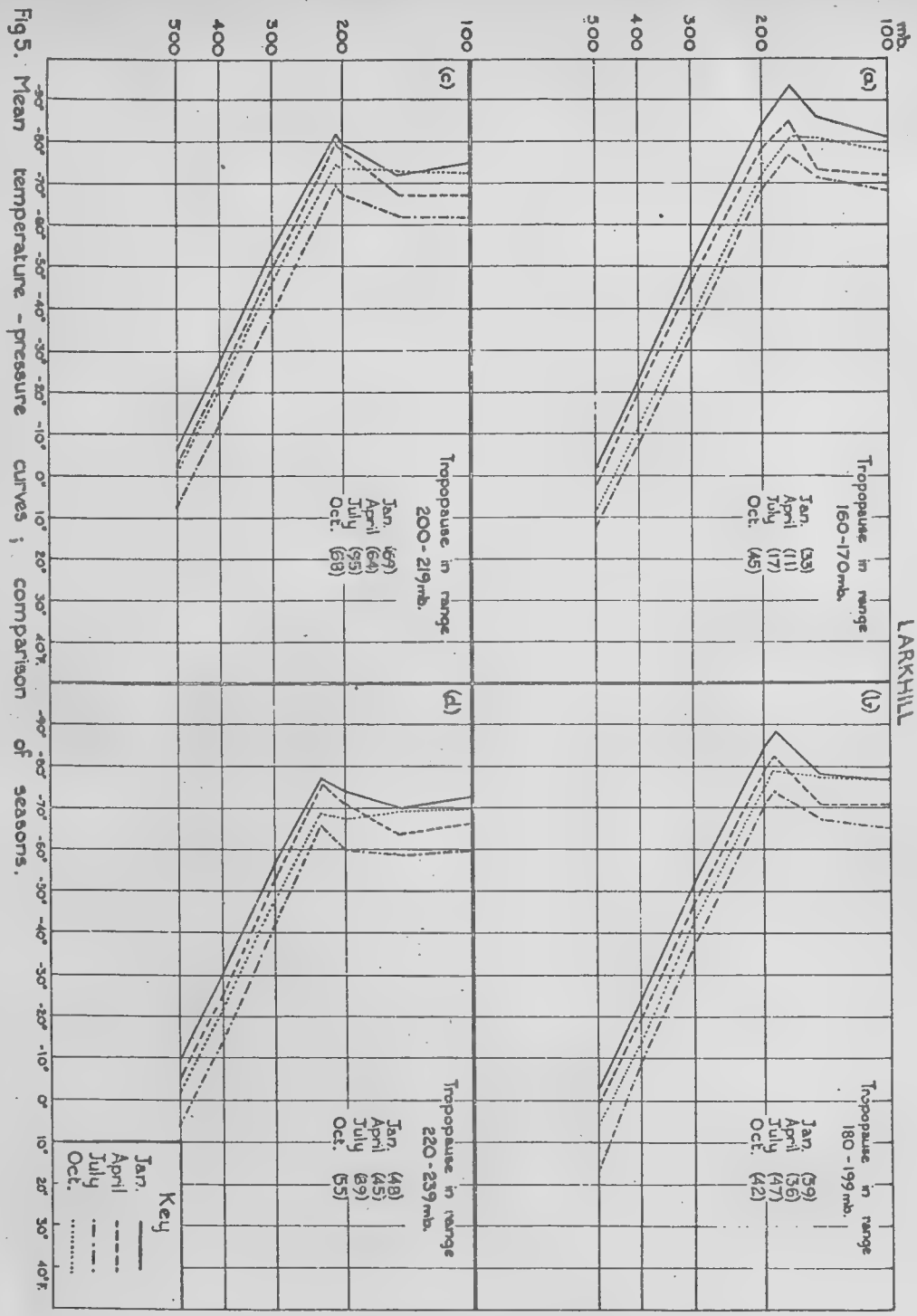


Fig.5. Mean temperature - pressure curves ; comparison of seasons.

The data of Fig.1 of Dalton's paper [illegible] side of Fig.I of the present paper. The figures entered under column A are mean temperatures at the various pressure levels when tropopause pressure lies between 140 and 160 mb., those under column B relate to tropopause pressure 160 to 180 mb. and so on. Isopleths of temperature have been drawn in and

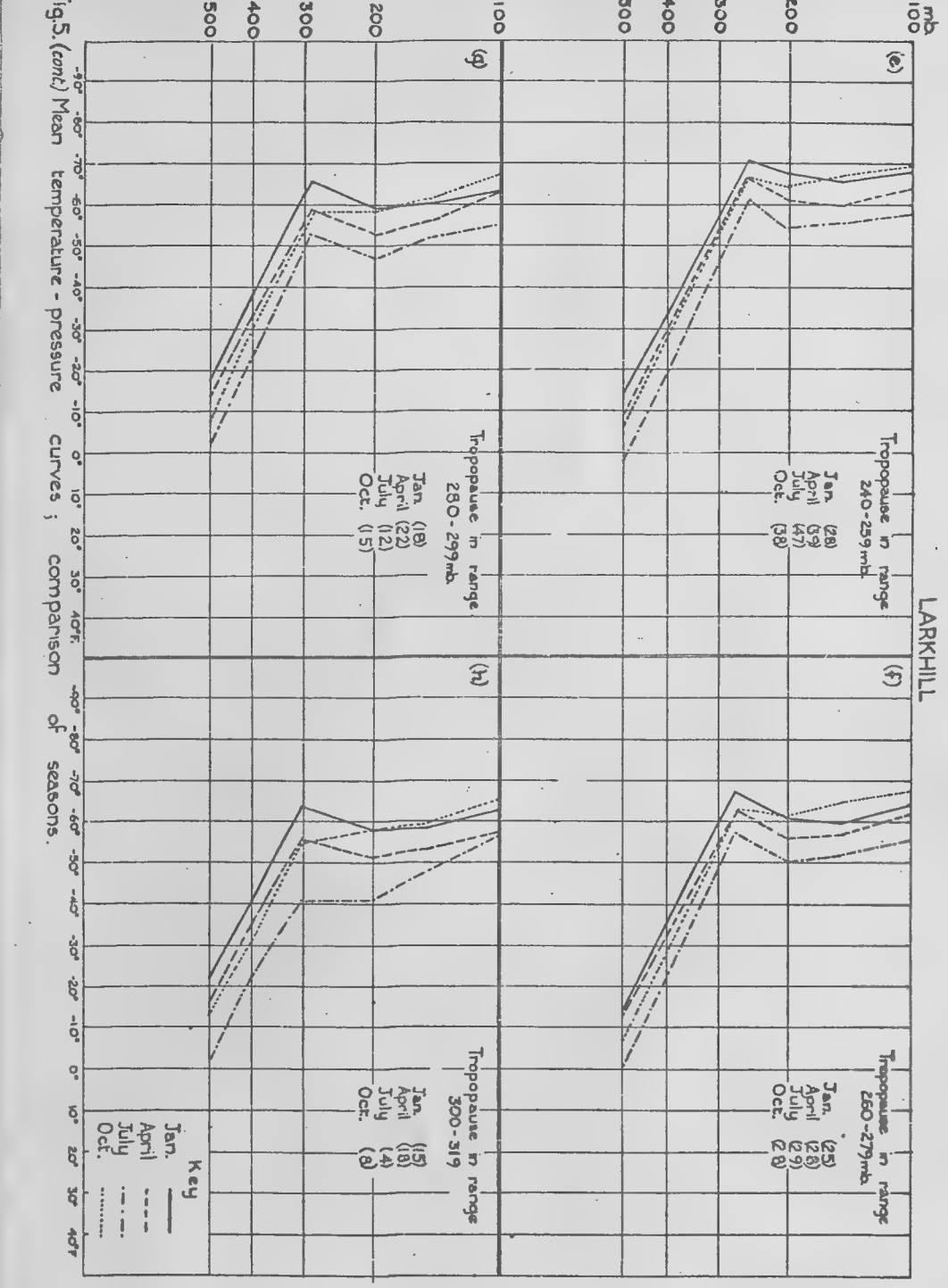


Fig.5.(*cont.*) Mean temperature - pressure curves ; comparison of seasons.

The data of Fig.1 of Barnett's paper [illegible] side of Fig.I of the present paper. The figures entered under column A are mean temperatures at the various pressure levels when tropopause pressure lies between 140 and 160 mb., those under column B relate to tropopause pressure 160 to 180 mb. and so on. Isopleths of temperature have been drawn in and

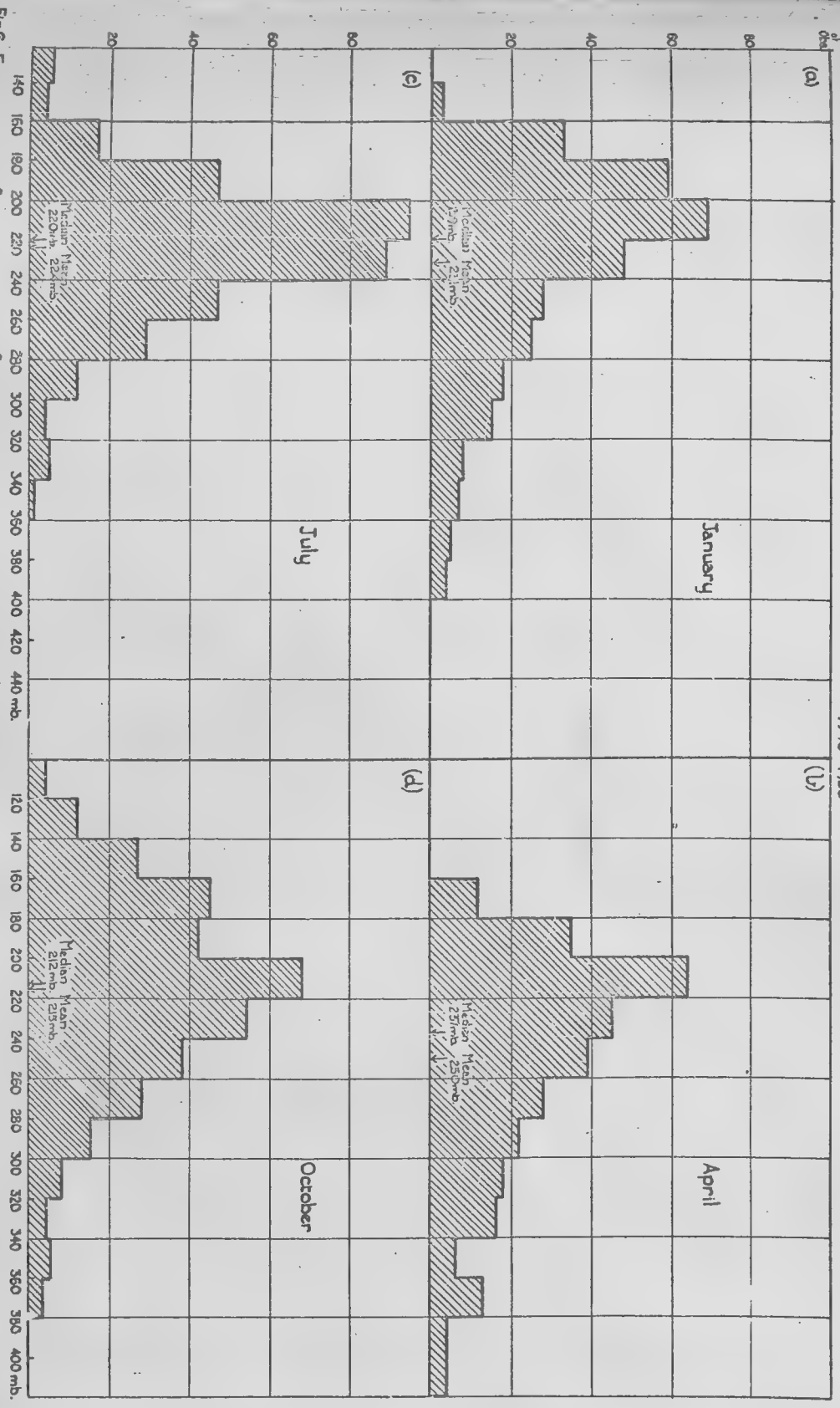


Fig.6 Frequency of occurrence of tropopause pressure within certain ranges.

The data of Fig.I of Bannon's paper have been used to plot the right side of Fig.I of the present paper. The figures entered under column A are mean temperatures at the various pressure levels when tropopause pressure lies between 140 and 160 mb., those under column B relate to tropopause pressure 160 to 180 mb. and so on. Isopleths of temperature have been drawn in and

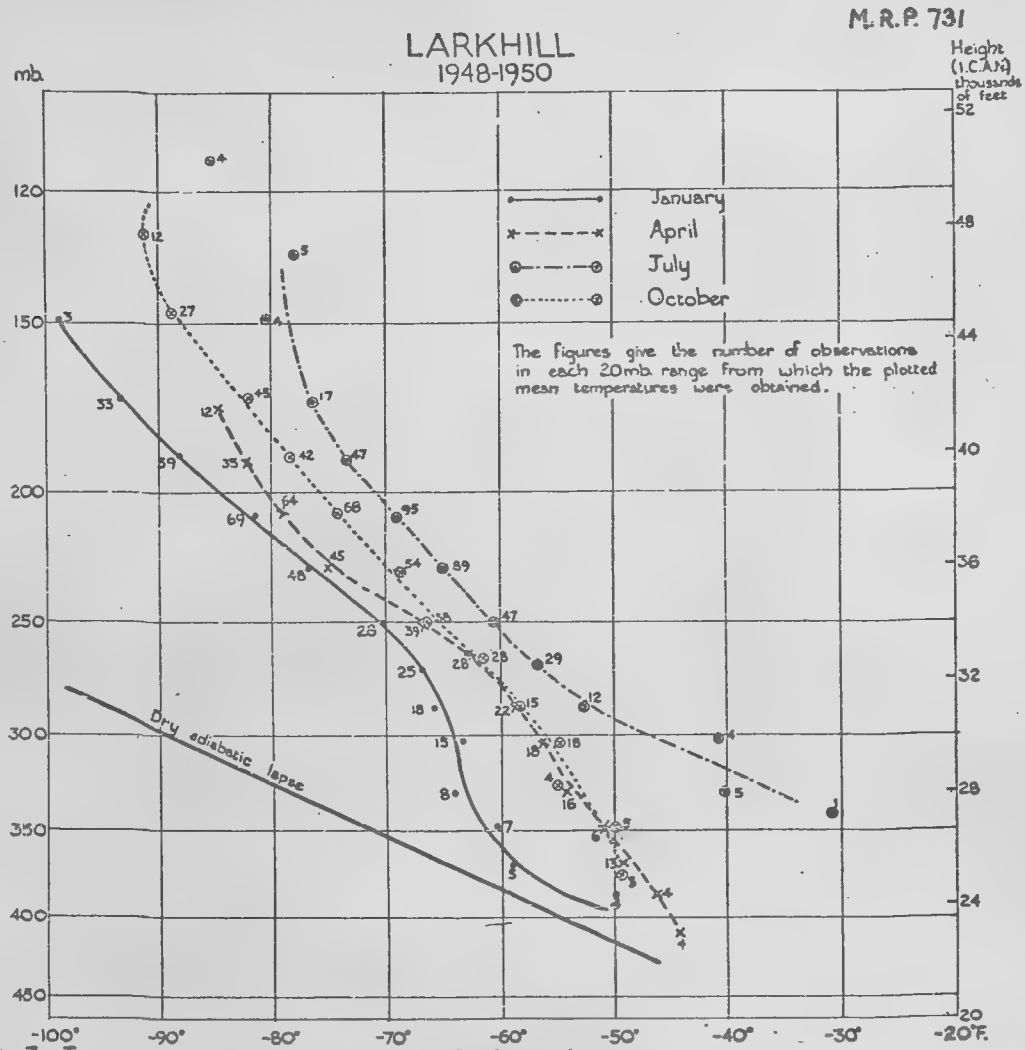


Fig.7. Tropopause mean temperature-height relation.

The data of Fig.1 of Bannon's paper [illegible] side of Fig.I of the present paper. The figures entered under column A are mean temperatures at the various pressure levels when tropopause pressure lies between 140 and 160 mb., those under column B relate to tropopause pressure 160 to 180 mb. and so on. Isopleths of temperature have been drawn in and

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S.C.III/134

Revised Dec. 1952

AIR MINISTRY

METEOROLOGICAL RESEARCH COMMITTEE

The global circulation of stratosphere air and the mechanism of change of tropopause level

by

A.H.R. Goldie (Meteorological Office)

1. Introduction

In earlier papers the writer has referred to the effects on tropopause level and stratosphere temperature of the vertical component of circulation, both as between high and low pressures in specific cases (1) and as between tropical and temperate latitudes in the longer period average global circulation (2).

It is many years since W.H. Dines (3) pointed out that the mean temperature of the atmosphere at the equator differs much less from the mean in England than it would do if temperature were solely a matter of radiative equilibrium and that the factor making for equalisation is, of course, circulation.

The chief purpose of this paper is to discuss the hypothesis that the stratosphere air of middle and high latitudes comes continually from the upper troposphere of the tropics and returns eventually to the troposphere of the polar regions in winter. This is one side of the global circulation. The other side in which air of the lower troposphere drifts equatorwards, is heated and rises, is not discussed in this paper.

The recent classification by Bannon (4) of temperatures in the upper troposphere and lower stratosphere according to tropopause pressure, prompts also some further discussion of the local patterns of air flow above and below the tropopause which might lead to the variety of temperature distributions found in temperate latitudes. Section 4 relates to this aspect.

It is well known (see, for example, Plate 2 of M.O.555 Part I - Larkhill Upper Air Data) that in temperate latitudes the variability of temperature has a maximum at about the 500 mb. level, a more prominent maximum at about the 200 mb. level and a minimum at about the 300 mb. level. The variations with level have generally been regarded as indicating maxima of vertical motion in the mid or upper troposphere and in the lower stratosphere. Amongst other things, Bannon has shown that when temperatures are grouped according to tropopause pressure, the standard deviation of temperature in each group becomes almost uniform with height, the above-mentioned maxima being removed. From this it may be inferred that the excesses of variation in the two levels mentioned are associated with tropopause height.

2. Typical distributions of temperature

The data of Fig.I of Bannon's paper have been used to plot the right-hand side of Fig.I of the present paper. The figures entered under column A are mean temperatures at the various pressure levels when tropopause pressure lies between 140 and 160 mb., those under column B relate to tropopause pressure 160 to 180 mb. and so on. Isopleths of temperature have been drawn in and

/also

also the position of the tropopause. The result may be taken as giving a typical average cross-section from high tropopause to low tropopause in temperate latitudes. On the extreme left of the picture are given mean values for the Equator. The two sets of isopleths are connected by dotted lines.

Fig.2 is a picture of the same data transformed into potential temperatures for a pressure of 400 mb. The conditions from A to K in these two figures are all to be found typically at one time or another in the month of January (or indeed generally in winter) in the southern part of the British Isles. Figs. 3 and 4 represent similarly the typical distributions for July. By what processes of movement and/or radiation can such variety of temperature distributions be brought about in a given month of the year?

It is obvious from all these figures that the tropopause is an important discontinuity or line of demarcation in the pattern of temperature. From the characteristic wind distribution across the tropopause it is known similarly that there is in general a discontinuity in movement. Typically the wind increases with height up to a maximum a short distance below the tropopause. There is then a very steep drop in wind speed on passing into the (relatively) slow moving air of the stratosphere.

There is moreover another feature which establishes not only a discontinuity of conditions between troposphere and stratosphere but a discontinuity of recent past history of the air and which points to an almost complete lack of mixing across the boundary. This feature is the customarily very low frost point in the lower stratosphere, a frost point for which there is no obvious explanation other than that the stratosphere air of temperate latitudes has in its recent (in the sense of weeks) past history come through a temperature equivalent to that in the upper troposphere of the tropics. The coincidence of the range of frost points found in the lower stratosphere over England with the range of temperatures found in the upper troposphere over the equator is too remarkable to be acceptably explained on any other ground we can at present conceive. The further discussion below will therefore be based on the assumption that the air of the lower stratosphere of temperate latitudes has in fact in its recent life history got into the stratosphere via the upper troposphere of the tropics and that the temperatures we find over England in the stratosphere are the combined result of high level tropical conditions, plus circulation including vertical motion, plus loss of heat by radiation in the interval of time since leaving the tropics. Further comment on the causes of the lowering of the tropopause in middle and high latitudes is given in para. 3.

There are several points now to be noted about Figs. 1 and 2. The first is the quasi-constancy of potential temperature with height in the tropics, a condition consistent with global thermal circulation in which the heat is applied to the air principally in the tropics at the ground. (Below the level shown on these diagrams the potential temperature in the tropics actually rises as the ground is approached).

The second point about Figs. 1 and 2 is that, level for level, both actual and potential temperature increase in the troposphere from temperate to low latitudes and in the stratosphere from low to temperate latitudes. The former is understandable enough in terms of the insolation, whether with or without a horizontal movement of air towards the equator. The latter (like the frost points found) can only continue to exist if northward drift with mass subsidence of air of higher potential temperature is continually in progress.

It is further to be noted, however, that the isopleths of potential temperature are not parallel to the tropopause but have only about half the slope of the tropopause. If there were no loss of heat on the journey from tropics to temperate latitudes and in the process of subsidence, the isopleths of potential temperature would be very widely spaced and would be parallel to the tropopause surface. The air at and just under the tropopause is specially cold for its height. Under the tropopause this could be explained by a continual movement of tropospheric air equatorwards in relation to the tropopause surface. Above the tropopause the steepness of the fall of potential temperature with decreasing height suggests a continual loss of heat - the greater the subsidence that has taken place the greater the loss of heat.

/Without

Without knowing the actual trajectories of the air masses (which may be devious) we can see certain consequences. Compare air just above the tropopause in column A with air just above the tropopause in column H. If the stratosphere air all came into the temperate latitude tropopause from the tropics, at or above the tropopause of column A, then the air just above the tropopause at H has subsided roughly 13,000 feet more than that at A since leaving the tropics. With an actual temperature of -98°F at 150 mb. at A, it ought in the course of such subsidence to have risen in temperature to -22°F at 290 mb. Actually the temperature is only about -66°F i.e. subsidence should have gained for it 76°F and the actual gain is 32°F. The loss by radiation at H as compared with A is thus some 58% of the gross adiabatic gain in getting from A to H.

We could also deduce how much heat is lost by stratosphere air in getting from the tropics to temperate latitudes with a tropopause at say 150 mb., but in this case the conditions are less definite in that an assumption is required as to the level at which the air - found eventually near the tropopause in temperate latitudes - leaves the tropics. But since potential temperature varies little with height in the higher part of the tropical troposphere the level selected for the initial state does not make much difference to the result.

If the air leaves at say 90 mb. (at -115°F) originally then the gross adiabatic gain on subsiding to 150 mb. should be 57°. In January the actual gain is 17°F and in July 34°. The losses by radiation are thus 40° or 70% in January and 26° or 46% in July. The total extent of the subsidence in each case is some 11,000 feet. (There cannot have been condensation in the process, but there may have been initially evaporation of some ice crystals. The effect of this is neglected).

A series of examples of subsidence to various levels in January and July gives the following results for the percentage loss by radiation in the lower stratosphere near the tropopause.

Subsidence to Tropopause at mb.		150	170	190	210	230	250	270	290	310	330	350	Mean
Percentage Radiation Loss	Jan.	70	70	68	66	64	64	63	65	66	68	68	67
	July	46	45	52	53	54	55	55	56				52

The comparative consistency and relatively small range of the percentage losses in any one season is to be noted - around 67% in January, and 52% in July.

These examples illustrate what in a broad way is otherwise obvious enough, that estimates of subsidence of stratosphere air in temperate latitudes over periods of weeks, if based on the assumption of purely adiabatic change would in fact be considerably on the low side. It would appear that the actual subsidence in the average case is roughly twice the amount that would have been deduced on the assumption of purely adiabatic change. Since, however, the slope of the isopleths of potential temperature in the stratosphere decreases with height in Fig. 2, it would also appear that the actual amount of subsidence - probably also the actual (but not the percentage) loss of heat by radiation - diminishes with height. As the frost point or other identity label is not available for levels substantially above the tropopause, it does not however seem feasible to make any absolute estimate for the higher levels.

3. The low tropopause of high latitudes

At the thermal equator the tropopause is always high, close indeed to the height computed by the author on the principle of global thermal circulation (2). Even in high latitudes, the tropopause can be high if the pressure distribution is favourable to it; and in middle latitudes it can on occasions be very low. It might be said that the whole or most of the variety of conditions in middle and high latitudes arises from advection and the author was probably the first to give weight to this effect (see Quart. J.R. met. Soc. 49 (1923) pp. 6-20)

/but

but the upper wind observations accumulated since that time show that though average latitude stratosphere characteristics may tend to be advected, the troposphere cannot simply carry the same stratosphere air with it. This does not mean that pressure distribution and circulation of hemispherical dimensions may not earlier have produced the features that were advected, just as pressure features may themselves be advected. Experience suggests that the tropopause cannot become very low or remain very low, unless in a region surrounded by a strong cyclonic circulation. If such a circulation diminishes in intensity the tropopause rises. The evidence would seem to indicate that the primary control on tropopause level in middle and high latitudes is tropospheric circulation, rather than latitude or temperature. If the troposphere becomes very cold in any area, the effect is necessarily to favour a circulation around that area increasing with height up to the tropopause and this is precisely the sort of circulation which should tend dynamically to lower the tropopause (cf. V. Bjerknes - On the dynamics of the circular vortex, Geofisiske Pub. Vol. II, No.4).

Continued loss of heat from the stratosphere in the Arctic winter may in itself tend to obliterate the tropopause, i.e. the diagram of Fig. 2 continued farther to the right would tend to a condition in which the potential temperature at the tropopause is no more than at the ground. It is most probably in this way that in the Arctic and Antarctic air is returned from stratosphere to troposphere. It can be shown that subsidence with radiation losses of the same order would suffice. Consider some examples of air starting with the potential temperature of the upper troposphere of the tropics and travelling to the Arctic or Antarctic with subsidence to various levels. In each case the starting condition will be taken as -115°F at 90 mb. This air if it subsided to the 9 km, 6 km, 4 and 3 km levels with no heat lost in the process would rise to temperatures of +31°F, +97°F, +140°F and +163°F respectively. Actually the average temperatures found in midwinter at some of the coldest stations in high latitudes at these levels are about -61°F, -49°F, -25°F and -20°F. From this it is seen that to arrive at the appropriate temperature the radiation losses accompanying the subsidence require to be 63%, 69%, 65% and 66% respectively. Conversely, radiation losses of slightly higher order would appear to render inevitable the exchange of stratosphere air near the poles of cold with troposphere air of surrounding regions. Considered along its trajectory, the air may be gaining in actual temperature but it is continually losing potential temperature. The extreme temperatures in the upper air imply greater percentage losses and presumably involve loss of heat by radiation over longer periods in the polar winter. For example, the lowest found at Maudheim is given as -90°C (-130°F) at about 11 km. In such a case the radiation loss would exceed the adiabatic gain by 30%.

4. The more rapid changes of tropopause level

What has been said in para. 2 relates to the more gradual processes of tropopause change taking place as air from high levels in the tropics finds its way to positions in temperate latitudes*. We may now consider what changes in stratosphere and troposphere temperature might take place as superposed effects within an air mass in the shorter period changes, such as the deepening or filling up of depressions. Since our scale of time is now in hours or days as against weeks, the extent to which radiation reduces the adiabatic gains or losses of temperature is presumably correspondingly less. Possibly it would be reduced to the order of one tenth of the values estimated in para. 2. This means that the radiation loss would be of the order of 5% of the gross adiabatic temperature change. Where the loss is of this order, 5% (or even up to 20%) the difference in percentage loss at different levels should not seriously affect the conclusions reached below. Probably - in view of the actual temperature distribution - the maximum loss or gain of heat by radiation is close to the tropopause on either side. Thus the immediate region of the tropopause must be regarded as excluded from these calculations.

/Consider

* If we take 1°C per day as a reasonable rate of loss of heat by radiation in the stratosphere, then the losses just computed imply journeys of the order of 3-4 weeks from tropics to a condition of high tropopause in temperate latitudes and a further similar interval on the average to a condition of low tropopause. Other evidence points to a roughly similar time scale.

Consider first curves D and F of Fig. 1 of M.R.P.731, that is the mean temperature-pressure curves for Larkhill for January for tropopause pressures in the ranges 200-219 mb. (D) and 240-259 mb. (F) respectively.

Regarding these temperature distributions as derivable dynamically from one another, we can calculate the amount of ascent or subsidence of air required at each level to convert curve D, with mean tropopause 210 mb., to curve F, mean tropopause 250 mb. For this purpose, and as we are dealing with averages, the stratosphere has been taken as being initially approximately isothermal and the troposphere as having the average lapse rate of 3.5°F. per 1,000 feet. Thus subsidence or ascent of 1,000 ft. in the stratosphere implies a gain or loss of temperature of 5.4°F. (the adiabatic rate), but in the upper troposphere of only 1.9°F. (the difference between the adiabatic and the average lapse). The result is shown in Fig.5. Each vertical arrow indicates the required amount and sense of vertical displacement in feet, at the level at the centre of the arrow. The dotted arrow indicates the vertical movement of the tropopause. From the vertical displacements thus indicated it is possible to calculate the relative convergence or divergence required at different levels in the form of aggregate inward or outward displacement of 50 mb. slices of the atmosphere around the area within which the tropopause change is supposed to take place. These convergences and divergences and their sense are similarly indicated by horizontal arrows on a scale of their own.

Fig. 6 represents the results of a similar calculation applied to a rather extreme example taken from October.

And it has to be remembered that to include the effect of radiation, all the displacements shown in this figure would have to be increased in much the same proportion by a factor of between $\frac{100}{95}$ and possibly $\frac{100}{80}$.

The points of interest attaching to the mechanism thus shown are as follows:-

(a) Each case has maximum vertical displacement downwards in the stratosphere just above the tropopause. In the troposphere from the 500 mb. level up to near the tropopause there is considerable upward motion.

(b) The horizontal divergence in the troposphere and the convergence in the stratosphere each tend to a maximum as the tropopause is approached. Even allowing that the components of convergence and divergence may be small in relation to wind speed, this probably means that appreciable shear of wind exists at the tropopause when tropopause level is changing.

(c) In a downward change of the tropopause, the stratosphere air near the tropopause is rising in temperature by subsidence, but losing some heat by radiation. In the troposphere air just below the effects are reversed. It is here probably therefore that the exchange of heat by radiation reaches its maximum.

5. Summary

On the hypothesis - for the adoption of which reasons are given - that the lower stratosphere air of middle and high latitudes is air which has drifted, with subsidence, from the upper troposphere of the tropics, computations are made of the percentage losses by radiation of the increase of temperature which should have been gained if the subsidence had been under adiabatic conditions. It is found that over 50% (67% in January, 52% in July) of the adiabatic gain of temperature would require to have been offset by radiation losses to produce the average distributions of stratosphere temperature found in temperate latitudes.

/Radiation

Radiation losses equalling or exceeding the adiabatic gains would be required to account only for winter polar extremes of temperature. Losses exceeding around 70% would make inevitable the return of stratosphere air to the troposphere in the polar winter. Assuming corresponding time rates of radiation losses for the shorter period time-scale of the depressions of middle latitudes, estimates are made of the air displacements associated with changing tropopause level.

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2. Goldie, A.H.R. — The average planetary circulation in vertical meridian planes. Quart. J.R. met. Soc., London, Centenary Proc., 1950, pp. 175-180.

3. Dines, W.H. — Free Atmosphere in the Region of the British Isles. Geophys. Mem., London, No. 6, 1914.

4. Bannon, J.K. — Classification of temperatures in the upper troposphere and lower stratosphere according to tropopause pressure. M.R.P. 731.

January Temperature (°F) at Larkhill in relation to tropopause level. Average temperature at Equator.

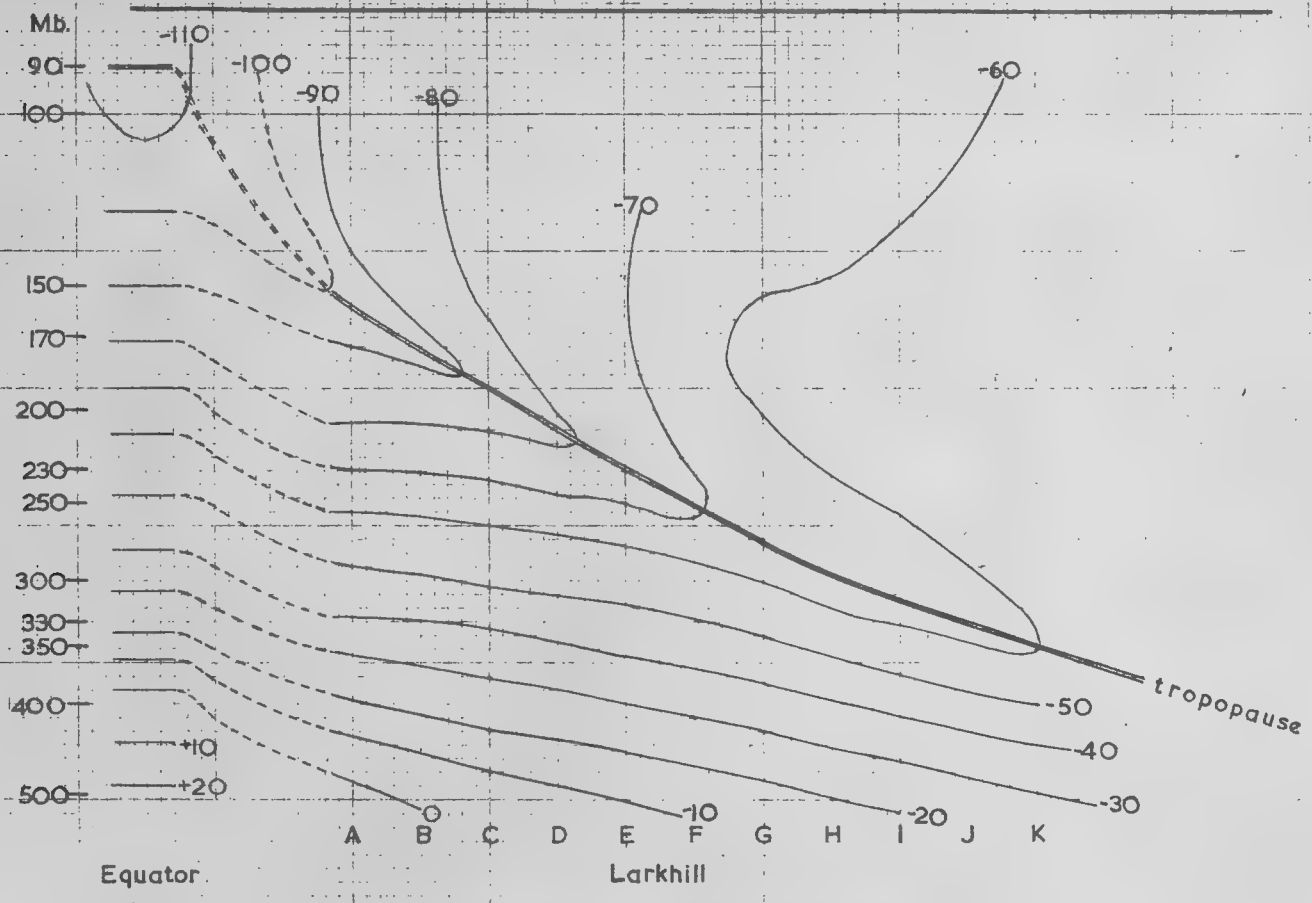

FIG.1 ACTUAL TEMPERATURE

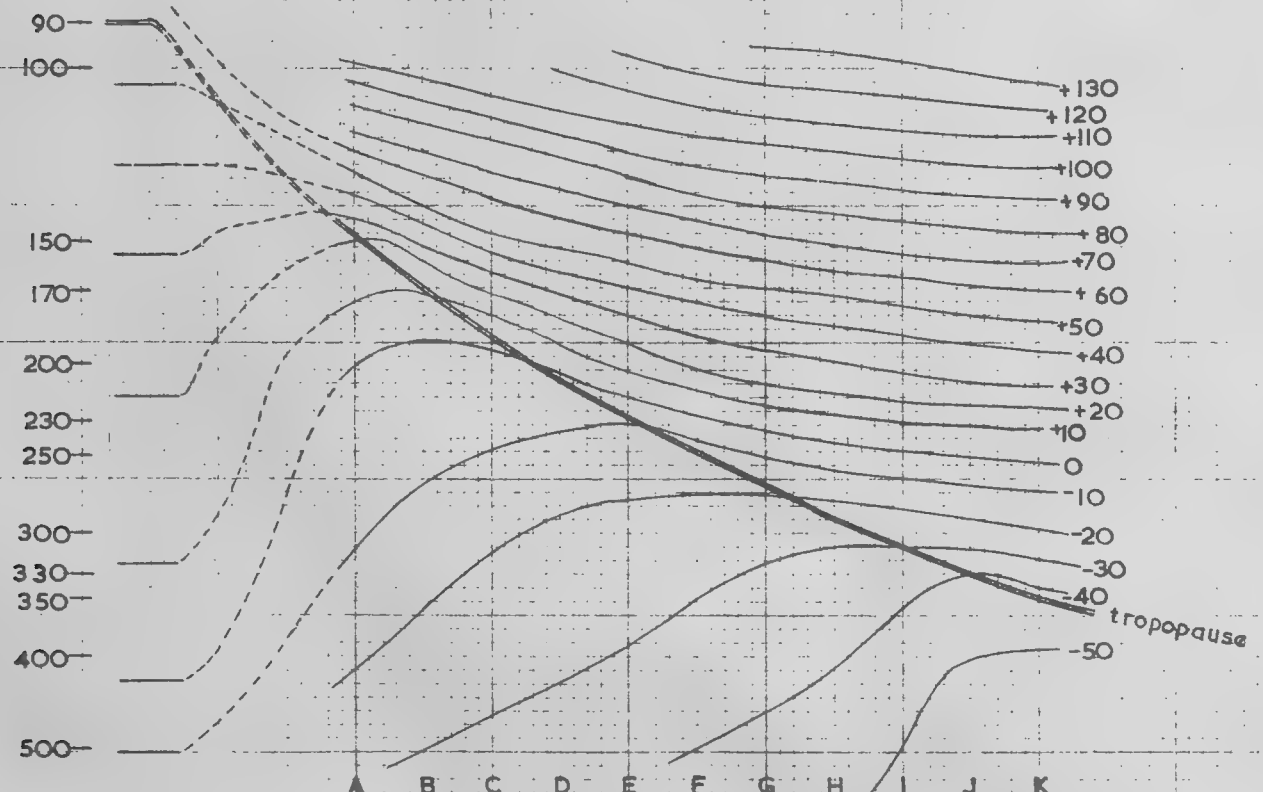

FIG.2 POTENTIAL TEMPERATURE (400 mb.)

July Temperature (°F) at Larkhill in relation to tropopause level. Average temperature at Equator

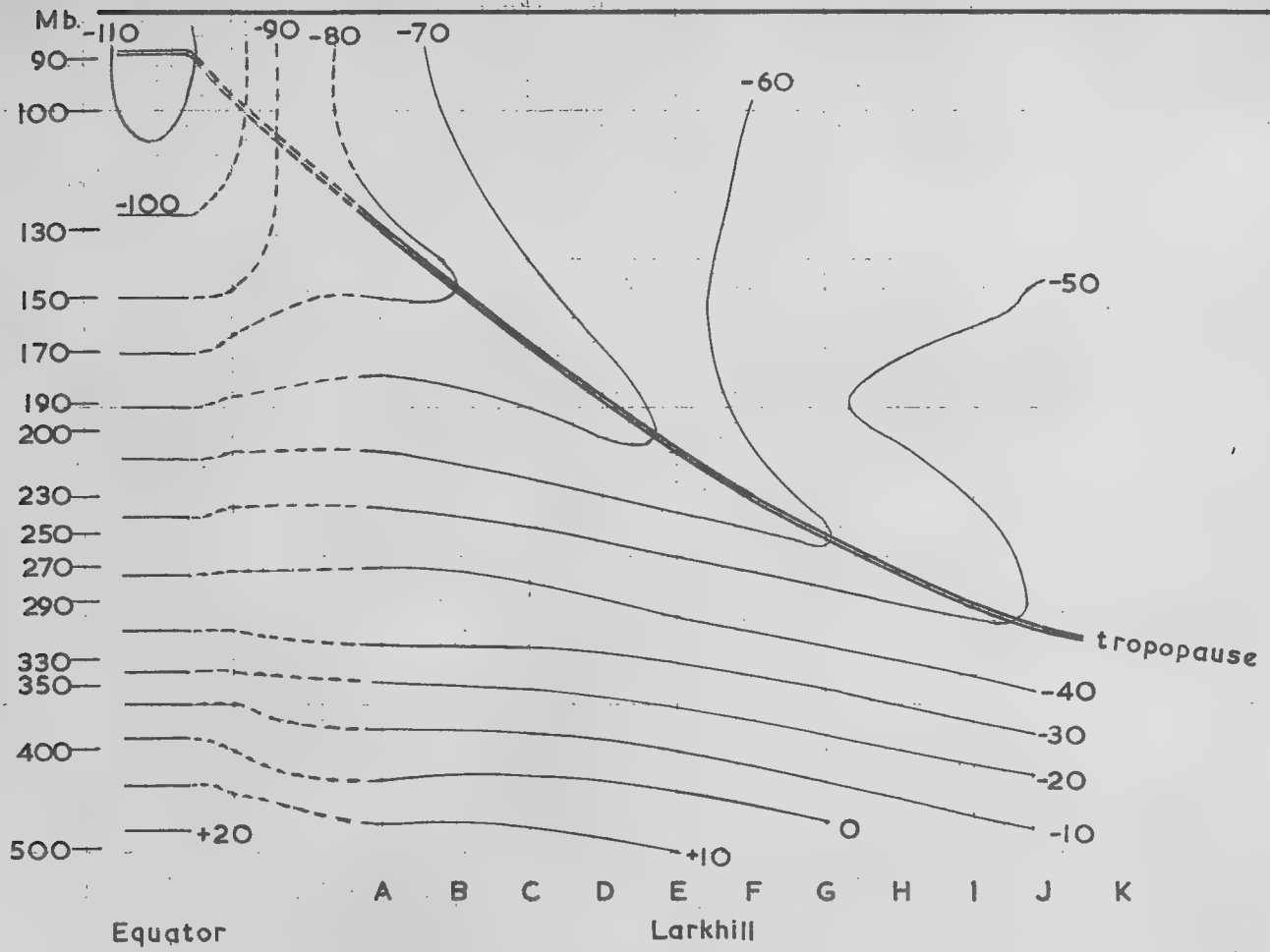

FIG. 3 ACTUAL TEMPERATURE

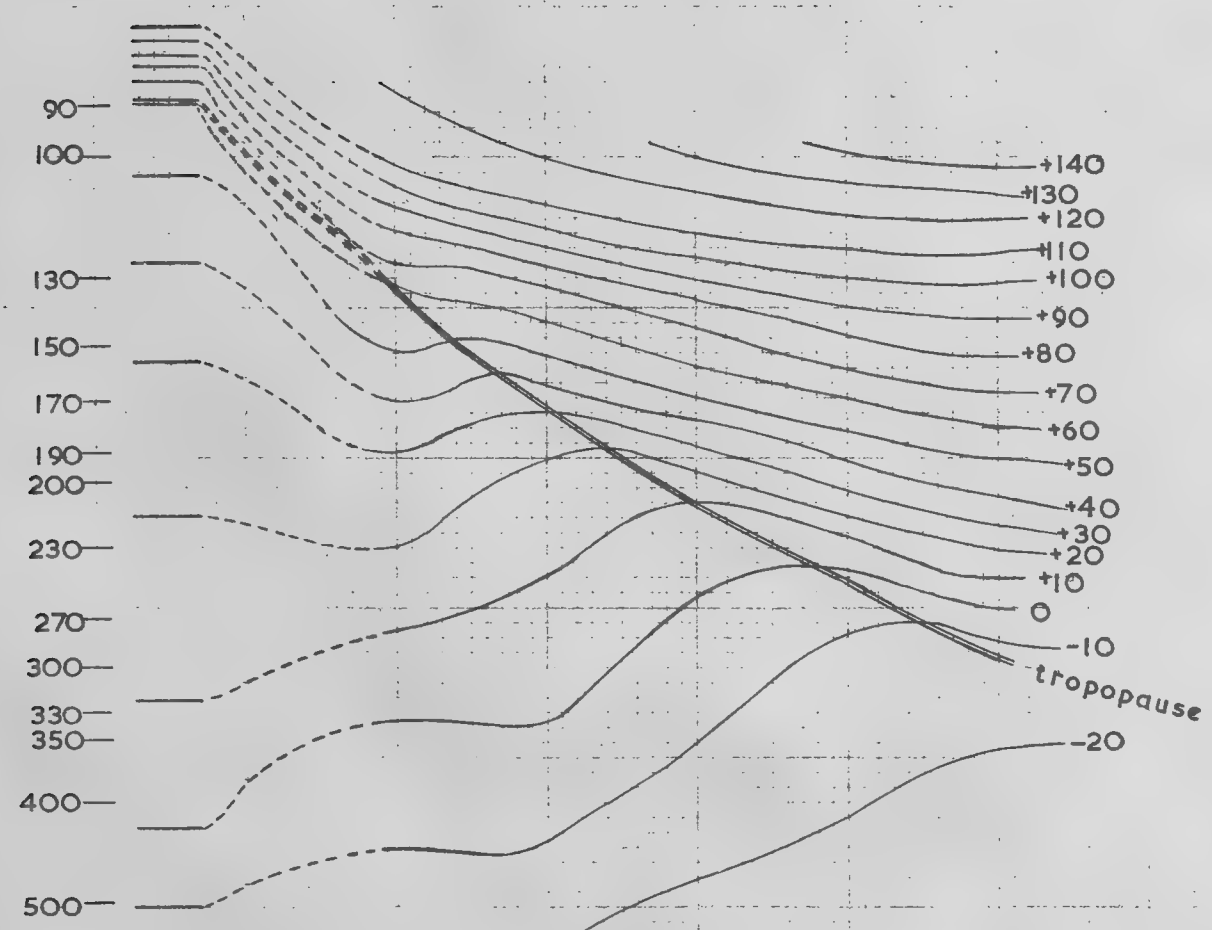

FIG. 4 POTENTIAL TEMPERATURE
(400 mb.)

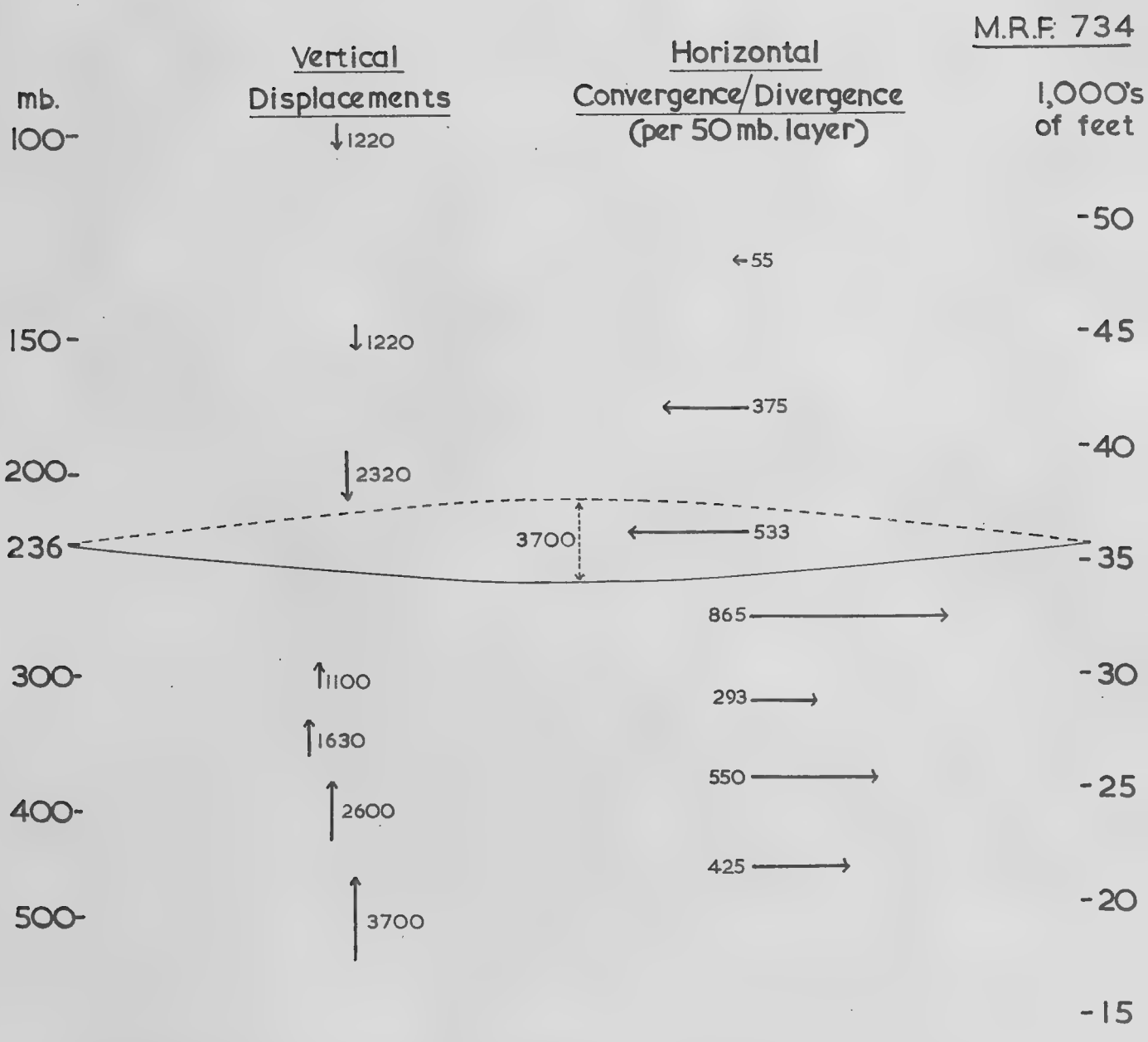

FIG. 5 - Change of tropopause from 210 mb. to 250 mb. (January)

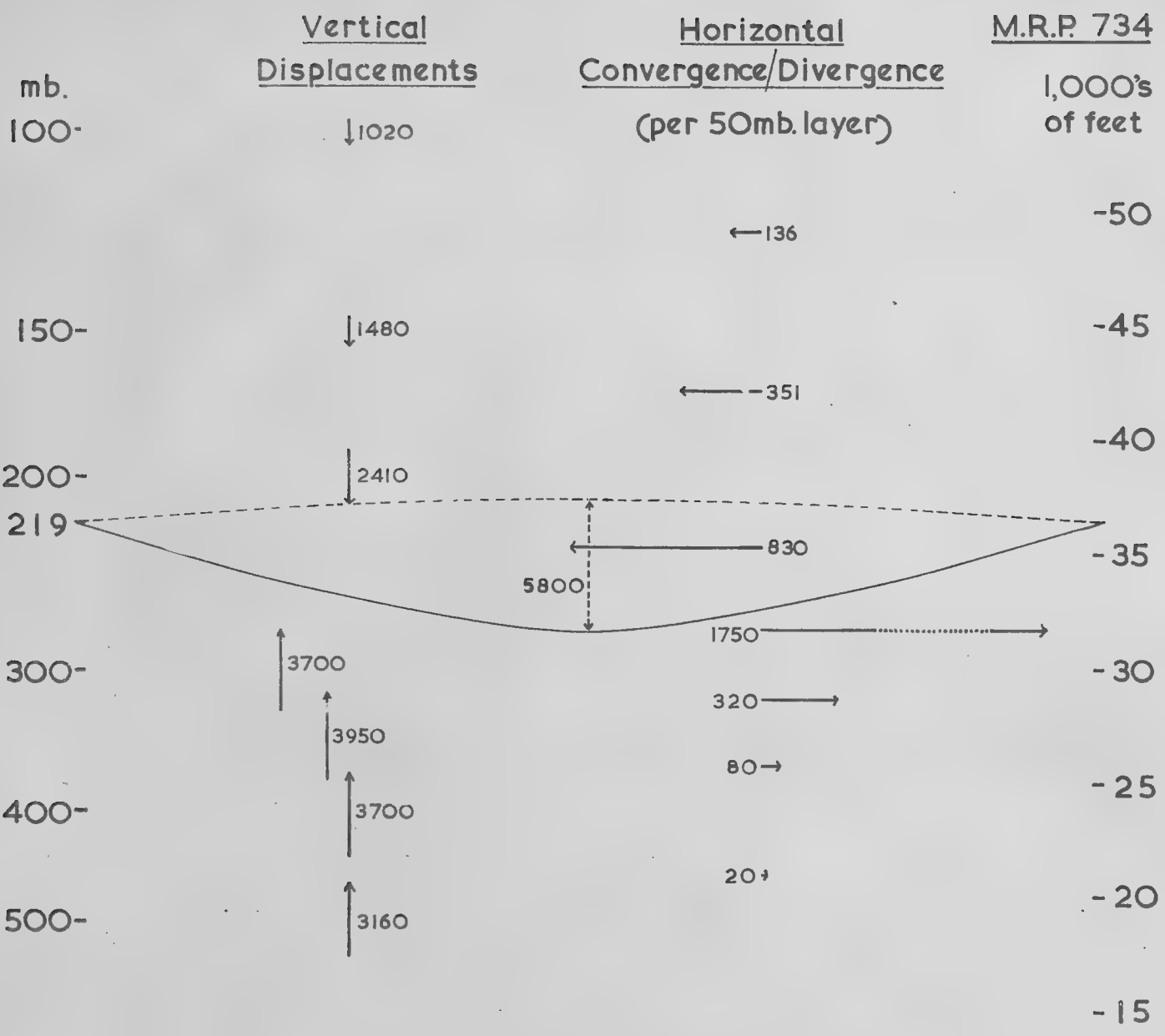

FIG. 6 - Change of tropopause from 210mb. to 270 mb. (October)

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S.C.III/135
30th June, 1952

AD 37936

AIR MINISTRY
METEOROLOGICAL RESEARCH COMMITTEE

Heat Transfer in the Lowest 100 metres of the Atmosphere

by
R. Frost (Meteorological Office)

Summary - Calculations of K_H the coefficient of eddy conductivity of heat and of its mode of variation with height under differing conditions of lapse rate of temperature suggest that it is approximately equal to K_M the coefficient of eddy viscosity.

On the assumption that the flux of heat in the ground is by molecular processes and that during the daylight hours the variation of temperature can be represented by a sine function, the amplitude and phase angle of the diurnal temperature wave are calculated and are shewn to be in good agreement with observations. The phase angle of the annual temperature wave is also discussed.

1. Introduction.- The mean temperature distribution within the lowest 100 metres of the atmosphere has been described in considerable detail in a series of geophysical memoirs by Johnson (1), Best (2), Flower (3), Johnson and Heywood (4) and Best, Knighting, Pedlow & Stormonth (5), but the theoretical conclusions from these observations have been conflicting owing to the complexity of the problem and lack of any clear cut picture of the physical processes involved in the mechanism of heat transfer, and to the different topography of the stations.

In most discussions of heat transfer in the atmosphere c.f. Sutton (6), it is assumed that molecular conduction is negligible except within the lowest cm. or so above the surface, that radiation may be neglected and that at least in the mid period of the day convection is mainly responsible for the transfer of heat in the vertical. There is however disagreement as to the roles played by "free convection" due to buoyancy effects and "forced convection" due to frictional turbulence. Further, Rider and Robinson (7) have stated that in the lowest two metres above the ground the absorption of long wave radiation by the air is at least comparable in effect with convection.

In view of the uncertainty as to the mechanism of heat transfer Richardson's direct approach, which avoids the necessity of making any assumptions as to the nature of eddies, will be adopted.

Richardson (8) defined K the coefficient of eddy diffusivity for any transferable property χ of the atmosphere by

$$\frac{D\chi}{Dt} = \frac{\partial}{\partial z}\left\{K\frac{\partial \chi}{\partial z}\right\} \qquad 1.00$$

where the only limitation imposed upon χ is that χ must represent some property which is unchanged by delay or by transfer to a different level.

Although K as defined by 1.00 was originally assumed to be a coefficient of eddy diffusion this is not essential, and if in heat transfer part of the temperature change at any level is due to factors other than eddy motion, equation 1.00 can still be used to represent the whole of the observed variation where K is a diffusion coefficient which takes into account all the causative factors.

It is evident that K, as defined in this manner, may be a function of χ & z, and hence may vary from property to property if the transferable property under consideration influences the motion. Comparison of values of K for various transferable properties computed by means of this equation thus affords

/some

some evidence as to whether, as suggested by various writers on turbulent motion from Reynolds onwards, the transfer of heat, momentum and water vapour is effected by the same mechanism.

In the case of controlled wind tunnel experiments Pasquill (9) shewed that the coefficients of eddy diffusivity and momentum were practically identical, but in a second paper (10) dealing with the diffusion of water vapour in the lowest two metres above the surface, he concluded that whilst under stable conditions there was equality between the coefficients of eddy diffusivity for water vapour and heat, in unstable conditions the latter was substantially greater.

Rider and Robinson (7) however questioned Pasquill's conclusions, and by a further series of measurements within the lowest two metres above the surface established a strong probability that the coefficients of eddy viscosity, eddy diffusivity and eddy conductivity were approximately equal under all circumstances.

2. The present position of K_H in the lowest atmosphere

It has been assumed by many writers that near the ground the diurnal variation of temperature can be represented by a sine wave of period 24 hours, whilst at great heights in the atmosphere it is negligible. If horizontal advection can be neglected the problem of heat transfer in the lower atmosphere reduces to the problem of obtaining a solution of

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left(K_H \frac{\partial \theta}{\partial z}\right) \qquad 2.00$$

where θ is the potential temperature at a height z above the surface at a time t, subject to the boundary conditions $\theta = \theta_0 + a\sin qt$ when $z \to 0$, and $\theta = \theta_0$ when $z \to \infty$ where $q = \frac{2\pi}{24\times60\times60} = 7.3\times10^{-5}$ and a is a constant. The solution of 2.00 which fits these conditions with K_H a constant was given by G.I. Taylor (11) and is

$$\theta = \theta_0 + a \sin (qt-bz) \exp -bz \qquad 2.01$$

where $b^2 = q/2K_H$

On this assumption therefore, the lag in the time of maximum temperature with height should be a linear function of the height above the surface. Best (2) from a very careful set of measurements between 2.5 cm. and 17 m. above the ground found that the lag varied very nearly as $z^{0.2}$ whilst calculations of K_H which he made, on the assumption that K_H could be treated as a constant over certain shallow layers of the atmosphere, indicated that they increased with height according to the power law

$$K_H = \text{const. } z^{1.8} \qquad 2.02$$

so that the assumption that K_H is a constant is clearly unacceptable. Cowling and White (12) however shewed that methods which assumed that K_H could be regarded as independent of height through even a shallow layer led to erroneous results and could give values of K_H which were not even correct as regards order of magnitude. Values of K_H for Leafield, which they computed by a variety of methods, shewed that K_H increased very rapidly with height above the surface, the rate of increase being greater by day than by night.

In the case of momentum transfer it has been shewn by many writers including the present writer (13) that K_M can be represented by a power law of the form

$$K_M = cz^{1-m} \qquad 2.03$$

where the index m always lies between 0 & 1.

Kohler (14) obtained a solution of 2.00 on the assumption that K_H could be represented by a similar expression in the terms of Bessel Functions of an

/imaginary

imaginary argument, but in this form his solution could not be readily compared with observations. Calder (15) however by employing the asymptotic expansion for this class of Bessel Function found for large values of z that the lag in time of maximum temperature tended to a law

$$\text{Lag} = \text{const. } z^{(1+m)/2} \qquad 2.04$$

and by comparing this with Best's empirical result obtained m=-0.6 which is incompatible with values derived from the momentum theory.

It can be seen that with this value of m = -0.6

$$K_H = \text{const. } z^{1.6} \qquad 2.05$$

which is in reasonably good agreement with 2.02.

Sutton (16), from an examination of the Leafield temperature data, found that in the hours around noon on a clear summer day $\frac{\partial\theta}{\partial z}$ became approximately constant with height and time above 1.2 m. By making certain assumptions as to the amount of heat convected from the surface, he concluded that the variation with height of the vertical flux of heat could be ignored below about 100m. and by plotting log $\frac{\partial\theta}{\partial z}$ against log z found that over the limited range from 7m. to 45 m. $\frac{\partial\theta}{\partial z}$ could be closely represented by a const. $z^{-1.75}$, & hence over the same range

$$K_H = \text{const. } z^{1.75} \qquad 2.06$$

which is in very good agreement with 2.02 and 2.05, but is completely at variance with 2.03.

Sutton therefore argued that as the direct application of 2.00 to the case of heat transfer in the hottest hours of the day led to results which were not in agreement with observations, the mechanism responsible for heat transfer must differ from that for momentum transfer, and suggested that this was due to buoyancy forces.

On the other hand, Knighting and Best (17) in a discussion of the Rye temperature data, whilst making the assumption that $\frac{\partial\theta}{\partial z}$ was approximately constant from 1100 to 1400 G.M.T. on a clear summer day, made no assumptions as to the constancy with height of the vertical flux of heat, and by integrating 2.00 when $K_H = cz^{1-m}$ obtained

$$\theta = \frac{A\, z^{1+m}}{c(1+m)} + \frac{Bz^m}{cm} + C \qquad 2.07$$

where A, B and C where constants. By fitting this equation to a series of potential temperatures at four heights from 1.1 to 106.7 metres they obtained an expression for m which could be solved graphically giving a value of m = 0.45. Unlike the negative value of m derived by Sutton from the Leafield data, this value of m came within the possible range of values of m derived from considerations of momentum transfer, although differing considerably from that appropriate to an unstable lapse rate.

In a later paper Knighting (18) discussed the solution of 2.00 with K_H represented by $K_H = cz^{1-m}$, and shewed that without knowing the values of c and m, it was not possible to assert that the asymptotic expansion of the solution was a good approximation, and that in consequence Calder's conclusions were open to serious objection. In another paper (19) Knighting shewed that if any experimental results were analysed by means of solutions obtained from equation 2.00 and this led to values of m which were negative, then the solutions were not applicable, or alternatively if the experiments resulted in values of m which were negative, no use having been made of any solutions, then equation 2.00 was invalid and could not be used to represent the motion.

In view of these conflicting conclusions it was decided to re-examine the Rye and Leafield data in order to ascertain (a), whether it was possible to reconcile some of the apparent inconsistencies and (b), whether any general

/conclusions

conclusions could be drawn on the problem of heat transfer which, according to Calder, remains the principal unsolved problem of surface meteorology.

3. The mode of variation of K_H with height

In the present section in order to deduce the mode of variation of K_H with height, various independent sets of temperature data are examined, and on occasions when it is possible to analyse these observations by means of solutions obtained from 2.00, it is shewn that $K_H \propto z^{1-m}$ where m is positive & less than unity.

In Table I taken from Sutton's paper (16) the values of $\frac{\partial\theta}{\partial t}$ at Leafield on a clear summer day around noon and at various heights are exhibited. Table II shews the corresponding values of $\frac{\partial\theta}{\partial t}$ at Rye on clear summer days around noon.

Table I Values of $\frac{\partial\theta}{\partial t} \times 10^{-4}$ in °F on clear summer days at Leafield

Time / height in metres	10.30	11.30	12.30	13.30
1.2	3.89	4.15	3.11	2.44
12.4	3.67	3.17	3.00	3.00
30.5	3.39	3.28	3.67	3.22
87.7	3.72	3.22	3.00	2.94

Table II Values of $\frac{\partial\theta}{\partial t} \times 10^{-4}$ in °F on clear summer days at Rye

Time / height in metres	10.30	11.30	12.30	13.30
1.1	4.71	2.50	0.78	0.78
15.2	4.17	2.16	1.25	0.78
47.2	4.28	1.92	1.00	0.78
106.7	3.68	2.11	1.43	1.12

Comparison of Tables I and II shews that at Rye, unlike at Leafield, $\frac{\partial\theta}{\partial t}$ is changing rapidly with time in the hours around noon and hence calculations of m by Knighting and Best from small differences of potential temperature at Rye, based upon the assumption of constancy of $\frac{\partial\theta}{\partial t}$ at Rye, must accordingly be suspect.

Inspection of Table I suggests that the assumption that $\frac{\partial\theta}{\partial t}$ is constant at Leafield at a height of 1.2 m. is only very approximate, and in view of Sutton's other assumption, that for all practical purposes $\frac{\partial\theta}{\partial t}$ could be assumed to be negligible, it was decided to recompute m from the Leafield data using the line of approach adopted by Knighting & Best.

Calculations shewed that the observations at the four heights 12.4 m., 30.5m., 57.4m. and 87.7m. could be fitted very closely by an equation of the form 2.07 with a value of $m = 1/7$.

With this value of m and the value of $A = 3.3 \times 10^{-4}$ °F/sec. from Table I, the values of c and B were 22 and -3.1 respectively.

It will be noted that the value of m is independent of the magnitude of $A = \frac{\partial\theta}{\partial t}$, providing A is a constant, but the values of c and B depend upon the value of A.

/The

The values of the differences of potential temperature over various height intervals on a clear summer day at Leafield and those computed from

$$\theta - \theta_{12.4} = \frac{3.3\times10^{-4}}{22\times^{8}/_{7}}\left\{z^{8/7} - 1240^{8/7}\right\} - \frac{3.1}{22\times\frac{1}{7}}\left\{z^{1/7} - 1240^{1/7}\right\} \qquad 3.00$$

are exhibited in Table III. The potential temperatures used are mean values from 1000 G.M.T. to 1300 G.M.T. and each entry in the second row is the mean of about 30 observations.

Table III

Values of the differences of potential temperature at Leafield around noon

Height interval in m.	30.5 - 12.4	57.4 - 12.4	87.7 - 12.4
Observed differences in °F.	- 0.31	- 0.46	- 0.52
Calculated differences in °F.	- 0.30	- 0.46	- 0.52

Although with small differences of potential temperature such as these the possibility of observational error cannot be excluded, it would appear that, contrary to Sutton's conclusions, the Leafield observations in the hottest hours of a clear summer day are consistent with predictions made from the heat transfer equation using the same value of m as that found from velocity profiles.

Cowling and White (12) noted that at Leafield at sunrise $\partial\theta/\partial t$ was approximately zero at all levels so that from 2.00 the ratio of K_H at any two levels is inversely proportional to the ratio of $\frac{\partial\theta}{\partial z}$ at the same two levels. From this they computed the ratios of K_H at various heights to the value of K_H at 1.2 metres of Table IV, and a logarithmic plot of $\frac{K_H(z)}{K_H(1.2)}$ against log z cf. Fig. I shews that at sunrise at Leafield

$$K_H = \text{const.}\ z^{0.67} \quad \& \text{ hence } m = 1/3 \qquad 3.01$$

Table IV Ratios of the coefficients of eddy diffusivity at sunrise at Leafield

Height in m.	1.2	12.4	30.5	57.4	87.7
Ratio	1	6.3	8.3	11.6	22

At Leafield at sunrise the magnitude of the inversion between 1.2 m. and 87.7 m. on clear summer days is 4°F. whereas at Rye the magnitude of the inversion between 1.1 and 106.7m. is 8.1 °F. Since $\frac{\partial\theta}{\partial z}$ at Rye is also approximately zero at all heights just before sunrise, it follows that the potential temperatures at this time can, from 2.07, be fitted by a simple power law

$$\theta = \frac{Bz^m}{cm} + C \qquad 3.02$$

The values of the potential temperatures at Rye between 0400 and 0500 hours and those calculated from

$$\theta = 50.4 + 1.4\left(\frac{z}{100}\right)^{0.45} \qquad 3.03$$

are exhibited in Table V.

Table V Potential temperatures at Rye at sunrise

Height in m.	1.1	15.2	47.4	106.7
Observed potential temperatures	51.80	55.15	58.43	61.80
Calculated potential temperatures	51.75	55.16	58.34	61.85

/At

At lower levels Rider and Robinson (7) from observations at Kew shewed that when the winds and temperatures measured at several heights between 10 cm. and 200 cm. were superposed, with adjustment for scale so that the profiles as drawn by hand coincided at 25 cm. and 100 cm., the wind and temperature profiles in nearly all cases had the same form, which implied that the ratios of the coefficients of eddy diffusivity and eddy viscosity were constant over this height interval.

Table VI shews the velocities from 25 cm. to 200 cm. for all unstable cases listed for clear summer days at Kew in 1949, at times between 1100 and 1430, reduced to a standard velocity of 200 cm./sec. at a height of 2.02 m. The wind velocities are mean values over a 30 minute period, which Rider & Robinson found was sufficiently long to give smooth mean profiles and which they considered was sufficiently short to exclude the effect of the diurnal variation. For comparison the velocities calculated from the simple power law $U = 200 \left(\frac{z}{202}\right)^{1/9}$ are also tabulated.

Table VI Velocities near the surface at Kew on clear sunny days

Case No	1	2	3	4	5	6	7	8	9	10	-
Date	18/6	20/6	20/6	21/6	21/6	23/6	23/6	24/6	1/7	1/7	$200\left(\frac{z}{202}\right)^{1/9}$
Time	1106-1136	1106-1136	1355-1425	1056-1126	1345-1415	1048-1118	1417-1447	1404-1434	1107-1137	1413-1443	-
Height cm.											
25	159	161	158	152	150	155	153	163	158	156	159
37	162	165	166	163	166	165	163	173	148	161	166
58	173	177	171	174	174	173	175	179	168	173	170
76	176	180	179	178	177	179	180	176	171	175	179
102	187	187	189	188	185	188	186	189	184	185	185
202	200	200	200	200	200	200	200	200	200	200	200

It can be seen that over the height range from 25 cm. to 200 cm. the wind velocities can be fitted reasonably closely by the simple power law

$$U = 200 \left(\frac{z}{202}\right)^{1/9} \qquad 3.04$$

and since the velocity & temperature profiles, when adjusted for scale, coincide it follows that over this height range the temperature data can be fitted by

$$\Theta = \Theta_g - \beta z^{1/9} \qquad 3.05$$

where Θ_g is the value of Θ at the surface, and where

$$\beta = \frac{U_{202}}{(202)^{1/9}} \frac{\theta_{25} - \theta_{202}}{U_{202} - U_{25}} \qquad 3.06$$

The values of β for the above occasions are listed in Table VII and the fluctuations of β are discussed in Section 5.

Table VII Values of β on clear sunny days

Case No.	1	2	3	4	5	6	7	8	9	10
β	6.3	5.8	5.3	5.6	5.0	6.1	4.2	5.8	5.9	4.9

/The

The mean value of β from Table VII is 5.5, & hence

$$\Theta = \Theta_g - 5.5\, z^{1/9} \qquad 3.07$$

so that near the surface

$$K_H = dz^{8/9} \qquad 3.08$$

where d is a const.
The convective flux of heat at the surface is thus

$$C(0) = \lim_{z \to 0} -K_H \rho c_p \frac{\partial \Theta}{\partial z}$$

$$= d \times \tfrac{1}{4} \times \frac{1}{800} \times \frac{5.5}{9} \times 5/9$$

$$= 1.06 \times 10^{-4}\ d \text{ gram cal. cm}^{-2} \text{ sec.}^{-1} \qquad 3.09$$

The mean value of C(0) computed by Rider & Robinson from the heat balance equation (discussed in more detail in the following section) for the occasions listed in Table VI is

$$C(0) = 37 \times 10^{-4} \text{ gram cal. cm}^{-2} \text{ sec.}^{-1} \qquad 3.10$$

& hence $d = 35$ 3.11

The value of d is subject to some uncertainty, as computations of C(0) from the heat balance equation depend upon the fundamental assumption that there is no horizontal flux of heat & Rider & Robinson noted that the temperature fluctuations near the ground at Kew appeared to be inconsistent with this assumption. Values of C(0) computed by Robinson (unpublished) from simultaneous observations of wind & temperature fluctuations were only about 50% of the values of C(0) computed from the heat balance equation. Furthermore Swinbank (20) who computed C(0) from simultaneous observations of temperature & wind fluctuations at low levels at Melbourne on clear sunny days obtained values of $C(0) = 14 \times 10^{-4}$ gram cal. cm^{-2} sec.$^{-1}$ which suggests that the value of C(0) in 3.10 & consequently the value of d in 3.11 are too high by a factor of about two.

The expression derived for K_M by the present writer (13) was

$$K_M = m z_o^{2m} V_h h^{-m} z^{1-m} \qquad 3.12$$

where z_o was a parameter characteristic of the degree of roughness of the surface whilst V_h was the mean velocity measured at a standard height h.

Now the value of V_H at Leafield at a height of 12.7 m, during the hours from 1000 G.M.T. to 1400 G.M.T. was 4.2 m.sec. whilst the mean value of V_h at 2.02 m. at Kew for the above observations was 3.00 m.sec.

The values of z_o at Leafield & Kew are not known, but computations of z_o for the rolling downland of Salisbury Plain & for a moderate sea surface were $z_o = 2.6$ cm. & $z_o = 1$ cm. respectively. Since the effect of z_o in 3.12 is small we may for simplicity write $z_o = 1$ cm., & hence at Leafield with $m = 1/7$, $K_M = 22 z^{6/7}$ whilst at Kew with $m = 1/9$ $K_M = 18.5 z^{8/9}$.

To sum up, in the present section we have shown that the observations are consistent with the assumption that $K_H \propto z^{1-m}$, where m is an index of the stability varying from 1/9 in the hottest hours around noon on a midsummer day at Kew & 1/7 with means taken over several hours around noon at Leafield, to 1/3 under moderate inversion conditions at Leafield & to nearly 1/2 under very stable conditions at Rye.

These values of m are in good agreement with the values of m found by the present writer (13) from wind velocity profiles under similar conditions of lapse rate at Cardington suggesting that the modes of variation of K_M & K_H are the same.

/The

The general agreement between the absolute values of K_H & K_M also suggests that in unstable conditions K_H = a const. K_M where the constant of proportionality is approximately unity.

4. The Diurnal Temperature Wave at the Earth's Surface

From the previous section it appears that K_H = const. z^{1-m} where m is a function of $\frac{\partial\theta}{\partial z}$ varying from a value of about $\frac{1}{2}$ to $\frac{1}{3}$ in the early hours of the morning to a value of about 1/7 or less at midday. This consideration however renders equation 2.00 quite unmanageable, but as the variation of m during the midday hours is slight the solution of 2.00 with m = a const. will be used to discuss the temperature changes during this period. We thus require a solution of

$$\frac{\partial\theta}{\partial t} = c\frac{\partial}{\partial z}\left(z^{1-m}\frac{\partial\theta}{\partial z}\right) \qquad 4.00$$

where $c = m z_0^{2m} V_k^{-m}$

subject to $\theta = \theta_0 + a\sin qt$ when $z \to 0$

and $\theta = \theta_0$ when $z \to \infty$ $\qquad 4.01$

where a is the amplitude of the diurnal temperature wave at the surface and

$$q = \frac{2\pi}{24\times60\times60} = 7.3\times10^{-5}$$

Writing $$\xi = \frac{z^{1+m}\,q}{(1+m)\,c} \qquad 4.02$$

and $$\theta = \theta_0 + \exp(iqt)\, f(\xi) \qquad 4.03$$

in 4.00 gives

$$(1+m)\xi f'' + f' - if = 0 \qquad 4.04$$

The solution of this equation was given by the present writer (13) and substituting this in 4.03 and picking out the real part,

the solution of 4.00 which fits the required boundary conditions is

$$\theta = \theta_0 + a\varphi(\xi)\sin qt - a\psi(\xi)\cos qt \qquad 4.05$$

where

$$\varphi(\xi) = \left\{1 - \frac{\xi^2}{|2(2+m)} + \frac{\xi^4}{|4(1+m)(2+2m)(4+3m)}\right\} - C\xi^{\frac{m}{(2m+1)}}\left\{1 - \frac{\xi^2}{|2(1+2m)(2+3m)}\right\} + D\xi^{\frac{2m}{(m+1)}}\left\{\frac{\xi}{1+2m} - \right\} \qquad 4.06$$

and

$$\psi(\xi) = -\left\{\xi - \frac{\xi^3}{|3(1+m)(3+2m)}\right\} + D\xi^{\frac{2m}{(m+1)}}\left\{1 - \frac{\xi^2}{|2(1+2m)(2+3m)} + \right\} + C\xi^{\frac{m}{m+1}}\left\{\frac{\xi}{1+2m} - \frac{\xi^3}{|3(1+2m)(2-3m)(3+4m}\right\} \qquad 4.07$$

where C and D are functions of m tabulated in (15) and whose values for m = 1/7 are 1.1161 and 0.2221 respectively.

Theoretical expressions for C and D derived by A.F. Crossley are given in an appendix to this paper, and it can be seen from equation 7.24 and 7.25 of the appendix that

$$C = \frac{\Gamma(2-m)}{\Gamma(1+m)}(1-m)^{-(1-m)}\cos\frac{m\pi}{2} \quad \& \quad D = \frac{\Gamma(2-m)}{\Gamma(1+m)}(1-m)^{-(1-m)}\sin\frac{m\pi}{2} \qquad 4.08$$

where $n = \frac{m}{1+m}$ 4.09

so that

$$\sqrt{C^2+D^2} = \frac{\Gamma(2-n)}{\Gamma(1+n)}(1-n)^{-(1-n)} \quad 4.10$$

and

$$\frac{D}{C} = \tan\frac{n\pi}{2} = \tan\left(\frac{m}{m+1}\cdot\frac{\pi}{2}\right) \quad 4.11$$

Equation 4.06 may be written

$$\theta = \theta_0 + a\chi(\xi)\sin(qt-\lambda) \quad 4.12$$

where

$$\tan\lambda = \frac{\psi(\xi)}{\varphi(\xi)} \quad \text{and} \quad \chi(\xi) = \sqrt{\psi^2(\xi)+\varphi^2(\xi)} \quad 4.13$$

& these functions are tabulated & utilised in the following section.

The time of maximum temperature at the surface can be obtained from a consideration of the fluxes of heat in the air & ground as follows. The convective flux of heat C(0) from the surface

$$= Lt_{z\to 0} - c_p\rho cz^{1-n}\frac{\partial\theta}{\partial z} \quad 4.14$$

and hence from 4.02, 4.05, 40.6, 4.07 and 4.08

$$C(0) = mc_p\rho ca\left[\frac{\omega}{c(1+m)}\right]^{\frac{m}{m+1}}\left\{C\sin qt + D\cos qt\right\} \quad 4.15$$

$$= mc_p\rho ca\left[\frac{\omega}{c(1+m)}\right]^{\frac{m}{m+1}}\sqrt{C^2+D^2}\sin(qt+\alpha) \quad 4.16$$

where

$$\alpha = \tan^{-1}\frac{D}{C} \quad 4.17$$

but from 4.11 $D/C = \tan\frac{m}{m+1}\frac{\pi}{2}$

and hence

$$\alpha = \frac{m}{m+1}\frac{\pi}{2} \quad 4.18$$

Thus the convective flux of heat from the surface can be represented by a sine wave and is a maximum at a time t before the time of maximum temperature given by

$$t = \frac{m}{m+1}\cdot\frac{\pi}{2}\cdot\frac{24}{2\pi} \text{ hours} \quad 4.19$$

$$= \frac{6m}{m+1} \text{ hours.}$$

It will be noted that this phase difference is a function of m only and is independent of the wind speed, amplitude of the diurnal wave of temperature at the surface, or nature of the surface.

The flux of heat in the ground may be derived by means of a similar analysis with k_1, the coefficient of thermal diffusivity of the soil, replacing $K_H = cz^{1-m}$ the coefficient of eddy diffusivity of the air. The corresponding solutions may be derived very simply by writing m = 1 in 4.14 and replacing c_p (the specific heat of the air) by c_1 (the specific heat of the soil), ρ (the density of the air) by ρ_1 (the density of the soil) and c by k_1 (the coefficient of thermal diffusivity) whenever they occur in equations 4.15 et sequitur.

Hence the ground flux of heat $G(0)$ is given by

$$G(0) = c_1\rho_1 k_1 a\left(\frac{q_v}{2k_1}\right)^{\frac{1}{2}}\sqrt{C_1^2+D_1^2}\,\sin(q_v t+\alpha_1) \qquad 4.20$$

where from 4.10 and 4.18,

$$\sqrt{C_1^2+D_1^2} = \sqrt{2}, \quad \text{and} \quad \alpha_1 = \frac{\pi}{4} \qquad 4.21$$

and thus

$$G(0) = c_1\rho_1 k_1^{\frac{1}{2}} a q_v^{\frac{1}{2}} \sin\left(q_v t+\frac{\pi}{4}\right) \qquad 4.22$$

and hence the ground flux of heat can be represented by a sine wave and is a maximum at a time t before the time of surface maximum temperature given by

$$t = \frac{\pi}{4} \times \frac{24}{2\pi} \text{ hours}$$

$$= 3 \text{ hours} \qquad 4.23$$

It can be seen that this phase difference is independent of the amplitude of the diurnal temperature wave, nature of the soil, etc.

This result was first obtained by Schmidt (21), who also, on the assumption of a constant coefficient of eddy diffusivity in the air, found that the ground flux of heat was in phase with the convective flux of heat.

Now the heat balance equation on the assumption that there are no horizontal gradients of temperature and water vapour is

$$C(0) + E(0) + R(0) + G(0) = 0 \qquad 4.24$$

where C(0) is the convective flux of sensible heat at the surface, E(0) is the convective flux of latent heat, R(0) is the net radiative flux of heat at the surface and G(0) is the flux of heat into the ground. Fluxes directed away from the surface are considered positive.

Over a desert surface it may be assumed that E(0) is zero, and hence

$$R(0) = -C(0) - G(0) \qquad 4.25$$

and

$$R(0) = -mc_p\rho c a\left[\frac{q_v}{c(1+m)}\right]^{\frac{m}{m+1}}\sqrt{C^2+D^2}\,\sin\left(q_v t+\frac{m}{m+1}\frac{\pi}{2}\right) - c_1\rho_1 k_1^{\frac{1}{2}} a q_v^{\frac{1}{2}}\sin\left(q_v t+\frac{\pi}{4}\right) \qquad 4.26$$

$$\equiv A\sin(q_v t+\gamma) \qquad 4.27$$

where

$$A = a\sqrt{\left\{m^2c_p^2\rho^2c^2\left[\frac{q_v}{c(1+m)}\right]^{\frac{2m}{m+1}}(C^2+D^2) + c_1^2\rho_1^2 k_1 q_v + 2mc_p\rho c\left[\frac{q_v}{c(1+m)}\right]^{\frac{m}{m+1}}\sqrt{C^2+D^2}\,\rho_1 c_1 k_1^{\frac{1}{2}} q_v^{\frac{1}{2}}\right\}} \qquad 4.28$$

$$\times \cos\frac{\pi}{4}\left(\frac{1-m}{1+m}\right)$$

/and

and

$$\tan\gamma = \frac{m c_1 \rho c \left[\frac{q}{c(1+m)}\right]^{\frac{m}{m+1}} \sqrt{C^2+D^2}\,\sin\left(\frac{m}{m+1}\frac{\pi}{2}\right) + c_1 \rho_1 k_1^{\frac{1}{2}} q^{\frac{1}{2}} \sin\frac{\pi}{4}}{m c_1 \rho c \left[\frac{q}{c(1+m)}\right]^{\frac{m}{m+1}} \sqrt{C^2+D^2}\,\cos\left(\frac{m}{m+1}\frac{\pi}{2}\right) + c_1 \rho_1 k_1^{\frac{1}{2}} q^{\frac{1}{2}} \cos\frac{\pi}{4}} \qquad 4.29$$

and hence the net radiative flux of heat at the surface R(0) is also represented by a sine wave which is out of phase with the temperature wave.

The net radiative flux of heat on a clear day is however a maximum at local noon, and hence the time of maximum temperature at the surface occurs at a time 12 + t hours, where

$$t = \gamma \cdot \frac{2\pi}{360} \times \frac{24}{2\pi} = \frac{\gamma}{15} \text{ hours} \qquad 4.30$$

where γ is given by 4.29.

It can be seen from 4.29 that γ must lie between $\frac{m}{m+1}\frac{\pi}{2}$ and $\frac{\pi}{4}$, and hence over the desert the time of maximum must occur not earlier than $\frac{6m}{m+1}$ hours and not later than 3 hours after local noon, the actual times depending upon the relative magnitudes of the convective and ground fluxes of heat, but being independent of the amplitude of the diurnal wave of temperature.

Since in 4.29 $C = m z_0^{2m} V_h h^{-m}$, an increase in wind speed, or to a lesser extent an increase of the surface roughness, results in an earlier time of maximum.

Over a grass covered surface from which evaporation is taking place, the effect of the flux of water into the air is difficult to assess directly. According to Bowen (22) however, the ratio between the flux of latent heat and sensible heat is a constant for any given surface and may be obtained from simultaneous measurements of humidity and temperature at two heights, and although it is not possible to prove this theoretically, Rider and Robinson who calculated values of Bowen's ratio μ near the surface on clear summer days at Kew, found that μ was constant with height and shewed little variation from day to day. If therefore the fluxes of sensible and latent heat are in phase, equations 4.25 to 4.29 can still be used with C(0) over a grass surface being replaced by $(1+\mu)\,C(0)$, and hence if the other variables remain unaltered, the time of maximum will be earlier over a grass surface from which evaporation is taking place, than over a dry surface.

5. Comparison with observations

In order to obtain quantitative results the values of m = 1/7 and $C = m z_0^{2m} V_h h^{-m} = 22$, derived from the Leafield observations, will be used.

The values of $\chi(\xi)$, $\varphi(\xi)$, $\psi(\xi)$ & λ for selected values of ξ for m = 1/7 are given in Table VIII.

Table VIII Values of $\chi(\xi)$ $\varphi(\xi)$, $\psi(\xi)$ & λ for specified values of ξ when m = 1/7

ξ	10^{-5}	10^{-4}	10^{-3}	10^{-2}	5×10^{-2}	10^{-1}
$\chi(\xi)$	.737	.650	.537	.391	.272	.224
$\varphi(\xi)$	.735	.647	.530	.373	.238	.182
$\psi(\xi)$	.050	.067	.089	.119	.132	.131
λ	3°55'	5°55'	9°32'	17°42'	29°01'	35°55'

/The

The lag in the time of maximum temperature in minutes from equation 4.12 is $\lambda \times \frac{24 \times 60}{360} = 4\lambda$, and hence from 4.03 and Table VIII with c = 22 & m = $\frac{1}{7}$, Table IX can be constructed shewing the percentage amplitude and lag in minutes at various heights.

Table IX Values of the amplitude and lag at various heights

Height	22cm.	1.6m.	12.0m.	50.0m.	90.0m.
Percentage Amplitude	65.0	53.7	39.1	27.2	22.4
Lag in minutes	24	38	70	116	144

Fig. 2 which is constructed from Table IX shews the variation in lag for certain wind speeds.

The observations from the desert site at Ismailia, which is singularly free from topographical defects, except in the summer months when it is affected by a slight 'sea breeze' from Lake Timsah, 3 km. to the southeast, can be utilised to test this theory.

From 4.16 the maximum value of C(0) is

$$\frac{1}{7} \times \frac{1}{4} \times \frac{1}{800} \times 22 \times 1.39 \times \left(\frac{7.3 \times 10^{-5}}{22 \times 8/7}\right)^{\frac{1}{2}} \times a \qquad 5.00$$

$$= 2.29a \times 10^{-4} \text{ gram cal. cm.}^{-2} \text{ sec.}^{-1} \qquad 5.01$$

where a is the amplitude of the diurnal temperature wave at the surface, and from 4.19 the maximum value of C(0) occurs 45 minutes before the time of maximum temperature at the surface. Similarly using the values of c_1, ρ_1 & k_1 for Ismailia given by Flower i.e. $c_1 = 0.19$, $\rho_1 = 2.63$ and $k_1 = 10^{-2}$, the maximum value of G(0) is

$$0.19 \times 2.63 \times 0.1 \times (7.3 \times 10^{-5})^{\frac{1}{2}} \times a$$

$$= 4.05a \times 10^{-4} \text{ gram cal. cm.}^{-2} \text{ sec.}^{-1} \qquad 5.02$$

and hence from 4.28 and 4.29, the maximum value of R(0) is

$$R(0) = -6.04a \times 10^{-4} \text{ gram cal. cm.}^{-2} \text{ sec.}^{-1} \qquad 5.03$$

and hence

$$\tan \gamma = \frac{2.29 \sin \frac{\pi}{16} + 4.05 \sin \frac{\pi}{4}}{2.29 \cos \frac{\pi}{16} + 4.05 \cos \frac{\pi}{4}} \qquad 5.04$$

$$= \tan 32^\circ 54' \qquad 5.05$$

Thus the time of maximum temperature at the surface on clear days occurs $\frac{32^\circ 54'}{360} \times 60 \times 24$ minutes after local noon i.e. at

14.12 local time 5.06

Table X shews the times of occurrence of maximum temperature at four heights above the surface as determined by Flower for clear December days and those as derived from Fig. 2 and 5.06, and it can be seen that the agreement between the two sets of times is remarkably close.

/Table X

Table X Times of occurrence of maximum temperature at various heights at Ismailia on clear December days

Height in metres	1.1	16.2	46.4	61.0
Observed times	1447	1524	1605	1623
Calculated times	1447	1530	1605	1616

It is of interest to note that in view of the difficulty and doubtful accuracy in obtaining times of maximum temperature at various heights from smooth curves drawn through mean hourly values, Flower carried out a harmonic analysis of the variation of temperature at each height and determined the time of maximum from the phase angle of the diurnal harmonic.

Flower also calculated the times of occurrence of the maximum temperature at each of the four heights for each month of the year and noted that the time of maximum, especially at a height of 1.1m., varied very closely with the equation of time and suggested that the variation of time of maximum temperature at the surface was equal to the equation of time. It can be seen that providing the mean monthly wind speed varies between small limits, so that the variations in the mean monthly fluxes of convective heat are small, this conclusion follows from the fact that the radiative flux of heat is a maximum at local noon.

In Table XI the ranges of temperature as observed on clear December days are shewn, together with the ratios of these ranges to the range at a height of 1.1m. and the ratio of the amplitudes to the amplitude at 1.1m. as determined from Fig. 2.

Table XI Observed and calculated ratios of the amplitudes at Ismailia - December

Height in m.	1.1	16.2	46.4	61.0
Range in °C	13.9	9.0	6.4	5.6
Observed range ratio	1	.648	.461	.402
Calculated range ratio	1	.658	.498	.430

The ratios of the calculated amplitudes to that at 1.1m. are in somewhat unexpectedly good agreement with the observed ratios, as the calculated values were based on the assumptions that m could be regarded as a constant throughout the 24 hours and that the temperature observations throughout the 24 hours could be represented by a single sine wave, whereas in Section 3 it was shewn that the value of $m = 1/7$ could at most only hold for the daylight hours, whilst Flower's computations shewed that the amplitude of the second harmonic was about 25% that of the first harmonic.

The agreement must therefore be mainly accidental, but it indicates that the amplitude at the surface is given to a good degree of approximation (from Table XI and Fig. 2) by

$$a = \tfrac{1}{2}\left[\frac{13.9}{.562}\right] = 12.4°C \qquad 5.08$$

and hence from 5.01, 5.02 and 5.03, the maximum air, ground and net radiative fluxes are respectively

$$C(0) = 28.4 \times 10^{-4} \text{ gram cal. cm.}^{-2} \text{ sec.}^{-1} \qquad 5.08$$

$$G(0) = 50.3 \times 10^{-4} \text{ gram cal. cm.}^{-2} \text{ sec.}^{-1} \qquad 5.09$$

$$R(0) = -75.0 \times 10^{-4} \text{ gram cal. cm.}^{-2} \text{ sec.}^{-1} \qquad 5.10$$

/There

There are no direct observations available from Ismailia or Egypt which can be used to verify these calculations but the following estimates, for which I am indebted to Dr. G.D. Robinson of Kew Observatory, suggest that they are not unreasonable. The maximum downward flux of short wave radiation on a clear December day at Ismailia may be taken as approximately 140 x 10^{-4} gram cal. $cm.^{-2}$ $sec.^{-1}$ and the albedo of the desert surface as about 20%. The former value is based on American observations in California and is confirmed by observations at Kew Observatory for the same solar elevation as Ismailia in December, whilst the latter is based on measurements by Angstrom (23) and observations from American aircraft at high levels over Death Valley, California. The outward long wave radiation from a desert surface at a temperature of 300°A is 112 x 10^{-4} gram cal. $cm.^{-2}$ $sec.^{-1}$ whilst the incoming long wave radiation of atmospheric origin using Flower's data of temperature and vapour pressure is about 75 gram cal. $cm.^{-2}$ $sec.^{-1}$ Hence the net radiative flux of heat (which Dr. Robinson considers may be in error by as much as 10%) is 75 gram cal. $cm.^{-2}$ $sec.^{-1}$, in very good agreement with the calculated value. Dr. Robinson has also drawn my attention to unpublished measurements by Ashbel of the ground flux of heat in arid soil in Palestine which give $G(0) = 50 \times 10^{-4}$ gram cal. $cm.^{-2}$ $sec.^{-1}$ and by Swinbank in semi desert soil near Melbourne, which give values of G(0) of about 50×10^{-4} gram cal. $cm.^{-2}$ $sec.^{-1}$.

The effect of wind speed upon the time of maximum can be readily determined. The value of c in 4.29 was derived for a wind speed at a height of 12.7 metres of 4.2 metres per second, and hence, if the other factors remain unchanged, the values of γ over a desert surface from 4.29 and 5.04 are given by

$$\tan\gamma = \frac{2.29\left(\frac{V}{4.2}\right)^{\frac{7}{8}}\sin\frac{\pi}{16} + 4.05\sin\frac{\pi}{4}}{2.29\left(\frac{V}{4.2}\right)^{\frac{7}{8}}\cos\frac{\pi}{16} + 4.05\cos\frac{\pi}{4}} \qquad 5.11$$

and the values of γ for various values of V metres per second are shewn in Fig. 3. In Table XII, derived from Fig. 3, times of maximum at various heights are shown for certain wind speeds.

Table XII Calculated times of maximum temperature as a function of wind speed and height

Wind speed (metres per second)	0	$2\frac{1}{2}$	4.2	7.5
Time of maximum at surface	15.00	14.25	14.12	13.55
" " " " 1.1 m.	-	15.04	14.47	14.26
" " " " 16.2 m.	-	15.56	15.29	15.02
" " " " 46.4 m.	-	16.35	16.01	15.28
" " " " 61.0 m.	-	16.46	16.12	15.36

There are no observations available at Ismailia which can be used to check these calculations, but Johnson & Heywood give two tables shewing the times of maximum at various heights at Leafield in summer for the above wind speeds which indicate that the order of magnitude of the wind effect is correct. The two tables are combined and reproduced here as Table XIII.

/Table XIII

Table XIII Observed times of maximum temperature for various wind speeds at Leafield in summer

Wind speed (metres per second)	$2\frac{1}{2}$	4.2	7.5
Time of maximum at 1.2 m.	15.05	14.55	14.30
Time of maximum at 12.4 m.	15.46	15.35	15.00
Time of maximum at 30.5 m.	16.00	15.50	15.12
Time of maximum at 57.4 m.	16.16	16.06	15.22

A curious feature of the Leafield observations which has no counterpart in either the Rye or Ismailia observations is the fact that the time of maximum on clear summer days occurs very nearly an hour later than the time of maximum on a mean June day. The probable explanation of this is that on a clear summer day the horizontal variation of temperature at Leafield, which is situated on a spur of the Cotswold hills, with a rapid fall of 100 m. in 3 km. to the north, and a fall of the same amount in a distance of between 4 and 5 km. to the south, is not negligible. During the hottest hours of the day it is probable that the surface air at Leafield mainly consists of air which has come from a relatively great height above the lower ground to the south, or north, and which therefore has the characteristics, including the retarded time of maximum temperature, of relatively high level air.

The non-representative nature of the meteorological elements at Leafield on a clear summer day had previously been noted by Johnson & Heywood, who shewed, inter alia, that the maximum day temperatures at Leafield were about 5°F. lower than the day maximum temperatures at Porton, that the diurnal range of temperature at Leafield was only 70% that at Porton and that the lapse rate at Leafield between 1.2 and 12.4 metres during the daytime was only 50% of the lapse rate at Porton. Johnson & Heywood pointed out that these differences could be due to differences in the topography or to differences in the thermal conductivities of the soils at Leafield and at Porton, both of which would operate to produce similar results, but concluded that the meteorological observations did not enable us to decide how much of the observed differences were due to each cause. As the time of maximum occurs outside the theoretical time of maximum based on the assumption of very light winds and a very high value of the thermal conductivity, it would appear that the topographical influences are the main cause of these anomalies.

The station at Rye which is situated on flat marshland is only 5 km. from the sea and is, unfortunately from the point of view of checking this theory, affected by the sea breeze. The irregularities in the mean temperature curve on clear June days make itimpossible to determine the time of maximum with any degree of confidence, and shew clearly that the simple propagation of a sine wave upwards from the surface does not afford an adequate explanation of the vertical thermal structure at Rye on a clear summer day.

From 4.05, 4.06, 4.07 & 4.08, Θ is given to a good degree of approximation at small heights above the surface by

$$\Theta = \Theta_0 + a \sin qt - a \left\{ \frac{m}{m+1} \right. (C \sin qt + D \cos qt) \qquad 5.12$$

and from 4.11 and 4.02

$$\Theta = \Theta_0 + a \sin qt - az^m \left[\frac{q}{(m+1)\epsilon} \right]^{\frac{m}{m+1}} \sqrt{C^2+D^2} \sin \left(qt + \frac{m}{m+1} \frac{\pi}{2} \right) \qquad 5.13$$

so that $\Theta = \Theta_g - \beta z^m$

/where

where ϵ_g is the value of ϵ at the surface and

$$\beta = a \left(\frac{q}{(m+1)c}\right)^{\frac{m}{m+1}} \sqrt{C^2+D^2} \sin\left(qt + \frac{m\pi}{m+1}\frac{}{2}\right) \qquad 5.14$$

hence with the values of c and m derived from the Kew observations in Section 3
i.e. $c = 18.5$ & $m = 1/9$

$$\beta = 0.31\, a \sin\left(qt + \frac{\pi}{20}\right) \qquad 5.15$$

The values of the soil constants at Kew given by Rider and Robinson are $c_1 = 0.30 \pm 0.2$, $\rho_1 = 1.1$ and $k_1 \rho_1 c_1$ varying between 2×10^{-4} and 5×10^{-4} with a mean value of 3.2×10^{-4}.

Hence the ground flux of heat from 4.20 is

$$G(0) = 8.86a \times 10^{-5} \text{ gram cal. cm.}^{-2} \text{ sec.}^{-1} \qquad 5.16$$

and identifying this with the value of the ground flux of heat at Kew found by Rider and Robinson

i.e. $G(0) = 10 \times 10^{-4}$ gram cal. cm.$^{-2}$ sec.$^{-1}$

$$a = 11.3°C, \qquad 5.17$$

and hence $\beta = 3.5 \sin\left(qt + \frac{\pi}{20}\right)$ with temperatures measured in °C. 5.18

or $\beta = 6.3 \sin\left(qt + \frac{\pi}{20}\right)$ with temperatures measured in °F, 5.19

An estimate of the time of maximum temperature at the surface can be obtained if, in order to allow for the convective flux of latent heat, we replace C(0) in equation 4.29 by $-R(0) - G(0)$ so that

$$\tan\gamma = \frac{80 \sin\frac{\pi}{20} + 10 \sin\frac{\pi}{4}}{80 \cos\frac{\pi}{20} + 10 \cos\frac{\pi}{4}}$$

or $\gamma = 12° 50'$ 5.20

hence the time of maximum temperature at the surface at Kew occurs $4 \times 12\frac{2}{20}$ minutes after local noon, i.e. at 12 hours 51 mins.

The values of β at times corresponding to those at which Rider and Robinson measured their profiles together with the observed values of β from Table VII are given in Table XIV.

Table XIV Observed and calculated values of β at Kew

Case No.	1	2	3	4	5	6	7	8	9	10	Mean
theory	6.1	6.1	5.5	6.0	5.6	6.0	5.2	5.4	6.1	5.3	5.7
observed	6.3	5.8	5.3	5.6	5.0	6.1	4.2	5.8	5.9	4.9	5.5

In view of the uncertainties of the soil constants, the agreement between the theoretical and the observed values is remarkably good indicating that the main factors have been taken into account in dealing with the problem of heat transfer. In other words the theory suggests that the mechanism of heat transfer, in the lowest 100 metres, is the same as that of momentum transfer and that the coefficients of eddy viscosity and eddy conductivity are approximately equal.

/6.

6. The annual temperature wave

The results of the previous sections can be used with slight modification to discuss the annual variations of temperature. For example during the summer months at Ismailia the annual variation of temperature can be closely represented by a single sine wave of period one year. Similarly during the period May to September the average monthly lapse rate of potential temperature between 1.1m.& 61m. is positive i.e. the potential temperature decreases with height, so that during this period the index m can be assumed to be approximately constant and equal to about 1/7.

The solution of 2.00 with m = 1/7 will therefore be utilised to discuss the variation in the date of occurrence of the maximum air temperature at Ismailia.

When dealing with the annual variation of temperature, the phase lag of the temperature against the annual convective flux is from 4.18

$$\alpha = \frac{m}{m+1} \pi/2 \qquad 6.00$$

and since for annual values $q_a = \frac{q}{365} = \frac{2\pi}{365 \times 24 \times 3600} = 2 \times 10^{-7}$

the convective flux of heat from the surface is a maximum at a time t_1 before the time of maximum temperature at the surface given by

$$t_1 = \frac{m}{m+1} \frac{\pi}{2} \times \frac{12}{2\pi} \text{ months}$$

$$= \tfrac{3}{8} \text{ months}$$

$$= 11\tfrac{1}{4} \text{ days approximately} \qquad 6.01$$

The phase lag of the air temperature against the annual ground flux is from 4.21 $\alpha = \pi/4$ and hence

$$t_2 = \frac{\pi}{4} \times \frac{12}{2\pi} \text{ months}$$

$$= 1\tfrac{1}{2} \text{ months} \qquad 6.02$$

and it will be noted that these times are independent of the amplitude of the temperature wave and of the physical properties of the air and soil etc.

The phase lag of the temperature against the net radiative flux is given by 4.29 with q_a replacing q.

Using the same values of the soil constants at Ismailia as in 5.00, the time interval t_3 is from 5.04

$$t_3 = 32^\circ 54' \times \frac{2\pi}{360} \times \frac{12}{2\pi} \text{ months}$$

$$= 33 \text{ days} \qquad 6.03$$

Now from 4.12 the lag in days at various heights $= \frac{\lambda . 365}{360} = \lambda$ approximately, whilst from 4.02 assuming that c does not increase with the increased time over which the means are now taken, the heights corresponding to specified values of ξ in Table VIII have to be multiplied by $(365)^{\frac{1}{2}}$, so that the height corresponding to $\xi = 10^{-4}$ is 44m. and to $\xi = 10^{-3}$ is 320m. Hence at a height of 1.1m, the date of the maximum air temperature should occur 33 days after the date of the summer solstice i.e. on July 24th; at 44m. it should occur on July 30th and at 320m. on August 2nd.

The dates of maximum air temperatures from a harmonic analysis by Flower of the monthly mean temperatures at four heights during the period October, 1931 to October, 1932 are as follows:-

/at

at 1.1m. July 10th; at 16.2m. July 29th; at 46m. July 30th; and at 61m. July 31st.

The agreement is very good except at 1.1m. where Flower's date of maximum, in the light of his values at greater heights, appears to be very early, suggesting that c is independent of the period over which means are taken. It is possible that this discrepancy is due to the limited period over which Flower's observations extended, or it may be that the present theory has over-simplified the problem of the annual temperature wave.

7. Appendix by A.F. Crossley

Frost's equation 13 (Q.J.R.M.S. Vol. 74 p.317) is

$$(1+m)\xi\frac{\partial^2 W}{\partial\xi^2}+\frac{\partial W}{\partial\xi}-iW=0 \qquad 7.00$$

substituting $W=y\xi^{\frac{1}{2}}$ and $x=2(1-n)^{\frac{1}{2}}e^{-\frac{i\pi}{4}}\xi^{\frac{1}{2}}$ 7.01

where

$$n=\frac{m}{1+m} \qquad 7.02$$

gives

$$\frac{\partial^2 y}{\partial x^2}+\frac{1}{x}\frac{\partial y}{\partial x}+\left(1-\frac{n^2}{x^2}\right)y=0 \qquad 7.03$$

which is Bessel's equation of order n

Since $0\leq m\leq 1$

therefore $0\leq n\leq \frac{1}{2}$

so that (except for $n=0$) $y=J_n(x)$ & $y=J_{-n}(x)$ are independent

solutions of 7.03 and hence

$$W_1=\xi^{n/2}J_{-n}(x),\ \&\ W_2=\xi^{n/2}J_n(x) \qquad 7.06$$

are independent solutions of 7.00.

Now

$$J_n(x)=\frac{x^n}{2^n\Gamma(1+n)}\left\{1-\frac{x^2}{2^2(1+n)}+\frac{x^4}{2^4\lfloor 2\,(1+n)(2+n)}-\cdots\right\} \qquad 7.07$$

and hence the two solutions of 7.00 are

$$W_1=\xi^{n/2}J_{-n}(x)=\frac{(1-n)^{-n/2}\,e^{\frac{in\pi}{4}}}{\Gamma(1-n)}\left\{1+i\xi+\frac{i^2\xi^2}{\lfloor 2\,(2+m)}+\cdots\right\} \qquad 7.08$$

and

$$W_2=\xi^{n/2}J_n(x)=\frac{(1-n)^{n/2}\,e^{-\frac{in\pi}{4}}\,\xi^n}{\Gamma(1+n)}\left\{1+\frac{i\xi}{1+2n}+\frac{i^2\xi^2}{\lfloor 2\,(1+2n)(2+3n)}\right\} \qquad 7.09$$

Except for the constant factors these solutions coincide with Frost's (17) and (18). We require asymptotic expressions for the two series which (following Frost) we separate into their real and imaginary parts. Thus

$$f_1(\xi)+if_2(\xi)=\Gamma(1-n)e^{-\frac{in\pi}{4}}(1-n)^{n/2}\xi^{n/2}J_{-n}(x) \qquad 7.10$$

/and

and $$f_3(\xi) + i f_4(\xi) = \Gamma(1+m)\, e^{\frac{i m\pi}{4}} (1-m)^{-m/2}\, \xi^{m/2}\, J_m(\alpha) \qquad 7.11$$

Now from 7.01 $\arg p\alpha = -\frac{\pi}{4}$, and the corresponding asymptotic expression for $J_m(\alpha)$ is (Gray, Matthews and MacRobert, Bessel Functions, p.61)

$$J_m(x) \to \left(\frac{2}{\pi x}\right)^{\frac{1}{2}} \cos\left(x - \frac{\pi}{4} - \frac{m\pi}{2}\right) \qquad 7.12$$

$$= \frac{e^{\frac{i\pi}{8}}\, \xi^{-\frac{1}{4}}}{\pi^{\frac{1}{2}}(1-m)^{\frac{1}{4}}} \cos\left\{2^{\frac{1}{2}}(1-m)^{\frac{1}{2}}(1-i)\xi^{\frac{1}{2}} - \frac{\pi}{4} - \frac{m\pi}{2}\right\} \qquad 7.13$$

or putting $$\alpha = \frac{\pi}{4} + \frac{m\pi}{2}, \text{ and } u = \left\{2(1-m)\xi\right\}^{\frac{1}{2}} \qquad 7.14$$

$$J_m(\alpha) \to \frac{e^{\frac{i\pi}{8}}\, \xi^{-\frac{1}{4}}}{\pi^{\frac{1}{2}}(1-m)^{\frac{1}{4}}} \left\{\cos(u-\alpha)\cosh u + i \sin(u-\alpha)\sinh u\right\} \qquad 7.15$$

$$\to \frac{\xi^{-\frac{1}{4}}\, e^{u}}{2\pi^{\frac{1}{2}}(1-m)^{\frac{1}{4}}}\, e^{i\left(u - \frac{\pi}{8} - \frac{m\pi}{2}\right)} \qquad 7.16$$

Similarly

$$J_{-m}(\alpha) \to \frac{\xi^{-\frac{1}{4}}\, e^{u}}{2\pi^{\frac{1}{2}}(1-m)^{\frac{1}{4}}}\, e^{i\left(u - \frac{\pi}{8} + \frac{m\pi}{2}\right)} \qquad 7.17$$

Therefore

$$f_1(\xi) + i f_2(\xi) \to \frac{\Gamma(1-m)(1-m)^{-\frac{(1-2m)}{4}}\, \xi^{\frac{(2m-1)}{4}}\, e^{i\left(u - \frac{\pi}{8} + \frac{m\pi}{4}\right)}}{2\pi^{1/2}} \qquad 7.18$$

and

$$f_3(\xi) - i f_4(\xi) \to \frac{\Gamma(1+m)(1-m)^{-\frac{(1+2m)}{4}}\, \xi^{\frac{(2m-1)}{4}}\, e^{i\left(u - \frac{\pi}{8} - \frac{m\pi}{4}\right)}}{2\pi^{1/2}} \qquad 7.19$$

Frost's equations (26) and (27), therefore become

$$0 = -\rho_0 G_{10} \Gamma(1-m)(1-m)^{-\frac{(1-2m)}{4}} \cos\left(u - \frac{\pi}{8} + \frac{m\pi}{4}\right) + \Gamma(1+m)(1-m)^{-\frac{(1+2m)}{4}} \left\{C\cos\left(u - \frac{\pi}{8} - \frac{m\pi}{4}\right) - D\sin\left(u - \frac{\pi}{8} - \frac{m\pi}{4}\right)\right\} \qquad 7.20$$

and

$$0 = -\rho_0 G_{10} \Gamma(1-m)(1-m)^{-\frac{(1-2m)}{4}} \sin\left(u - \frac{\pi}{8} - \frac{m\pi}{4}\right) + \Gamma(1+m)(1-m)^{-\frac{(1+2m)}{4}} \left\{D\cos\left(u - \frac{\pi}{8} - \frac{m\pi}{4}\right) + C\sin\left(u - \frac{\pi}{8} - \frac{m\pi}{4}\right)\right\} \qquad 7.21$$

/whence

whence

$$C\cos\left(\upsilon-\frac{\pi}{8}-n\frac{\pi}{4}\right)-D\sin\left(\upsilon-\frac{\pi}{8}-n\frac{\pi}{4}\right)=\rho_0 G_0\frac{\Gamma(1-n)}{\Gamma(1+n)}(1-n)^n\cos\left(\upsilon-\frac{\pi}{8}+n\frac{\pi}{4}\right) \qquad 7.22$$

&

$$C\sin\left(\upsilon-\frac{\pi}{8}-n\frac{\pi}{4}\right)+D\cos\left(\upsilon-\frac{\pi}{8}-n\frac{\pi}{4}\right)=\rho_0 G_0\frac{\Gamma(1-n)}{\Gamma(1+n)}(1-n)^n\sin\left(\upsilon-\frac{\pi}{8}+n\frac{\pi}{4}\right) \qquad 7.23$$

whence

$$C=\rho_0 G_0\frac{\Gamma(1-n)}{\Gamma(1+n)}(1-n)^n\cos\frac{n\pi}{2} \qquad 7.24$$

$$D=\rho_0 G_0\frac{\Gamma(1-n)}{\Gamma(1+n)}(1-n)^n\sin\frac{n\pi}{2} \qquad 7.25$$

These forms are suitable for computation. The figures in Frost's Table II are confirmed (apart from some minor differences in the fourth place of decimals).

Also

$$\tan\psi_0=\frac{D}{C}=\tan\frac{n\pi}{2} \qquad 7.26$$

$$\psi_0=\frac{n\pi}{2}=\frac{m}{m+1}\frac{\pi}{2} \qquad 7.27$$

which confirms Frost's equation (30).

References.

1. Johnson, N.K. 1929, Geophys. Mem. London. No.46.
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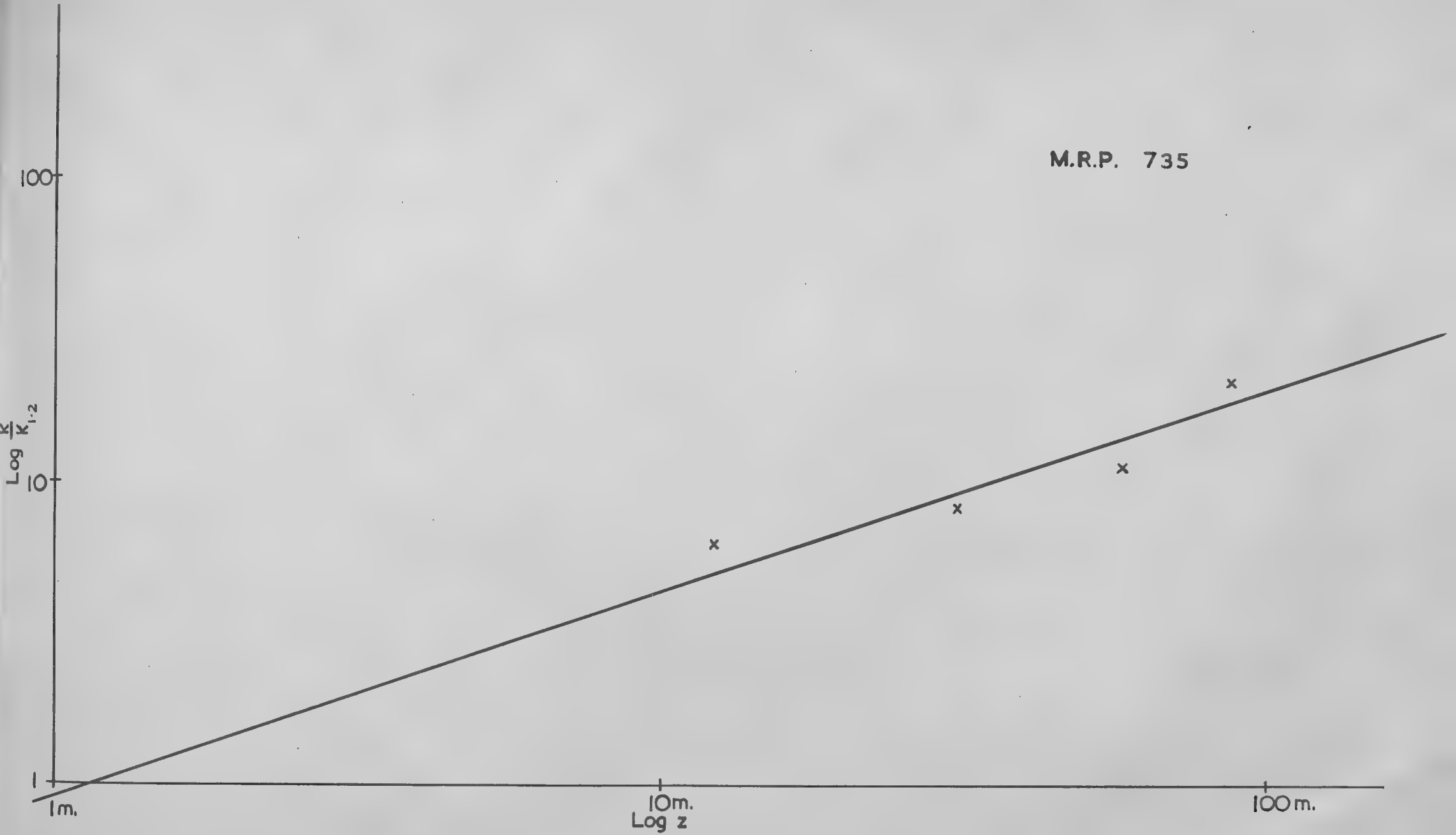

Fig. 1. VARIATION OF $\frac{K}{K_{1\cdot2}}$ WITH HEIGHT — CLEAR JUNE DAYS — LEAFIELD.

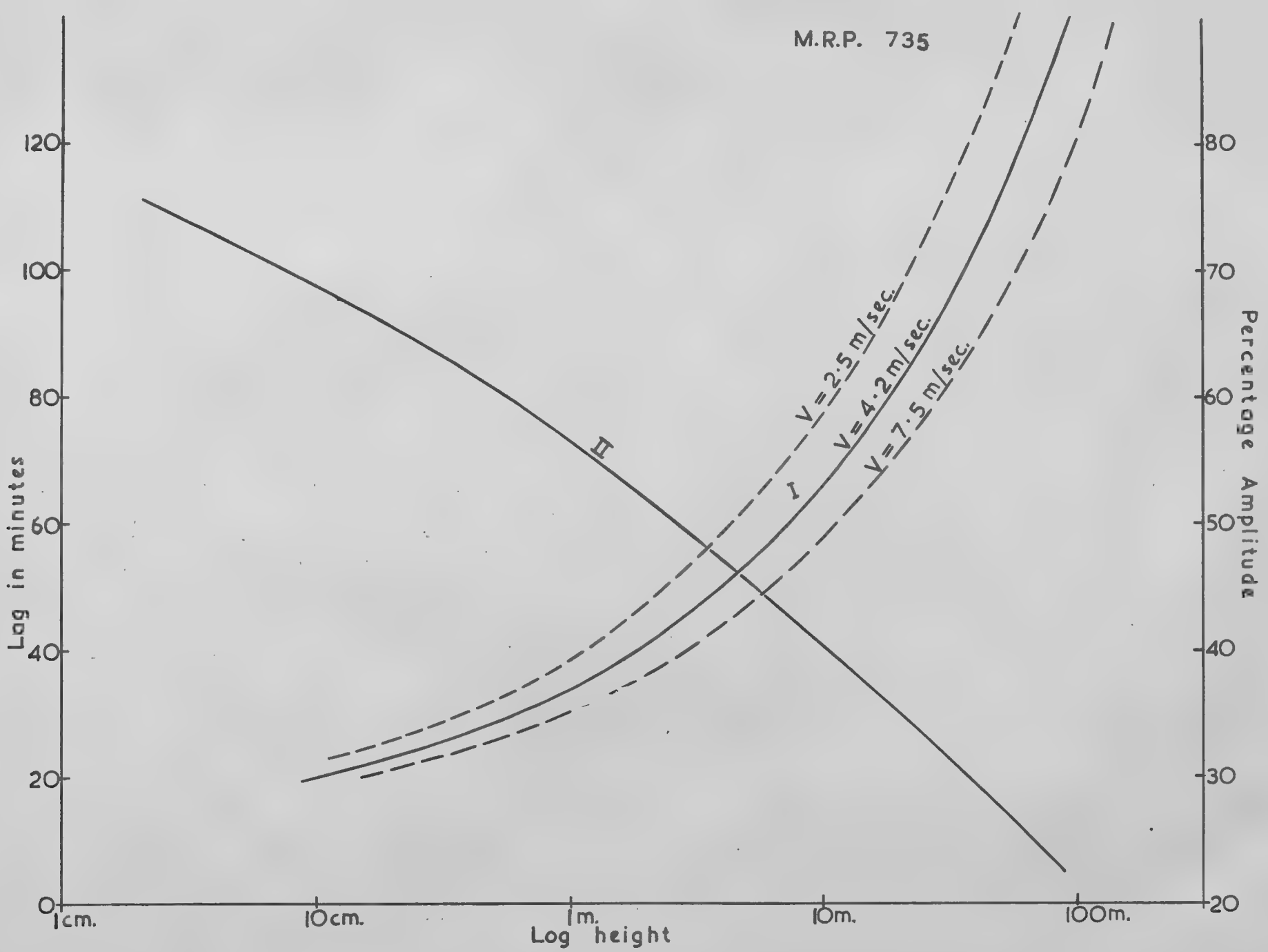

Fig.2. CURVE I VARIATION OF LAG WITH HEIGHT FOR SELECTED WIND SPEEDS
CURVE II VARIATION OF DIURNAL AMPLITUDE WITH HEIGHT

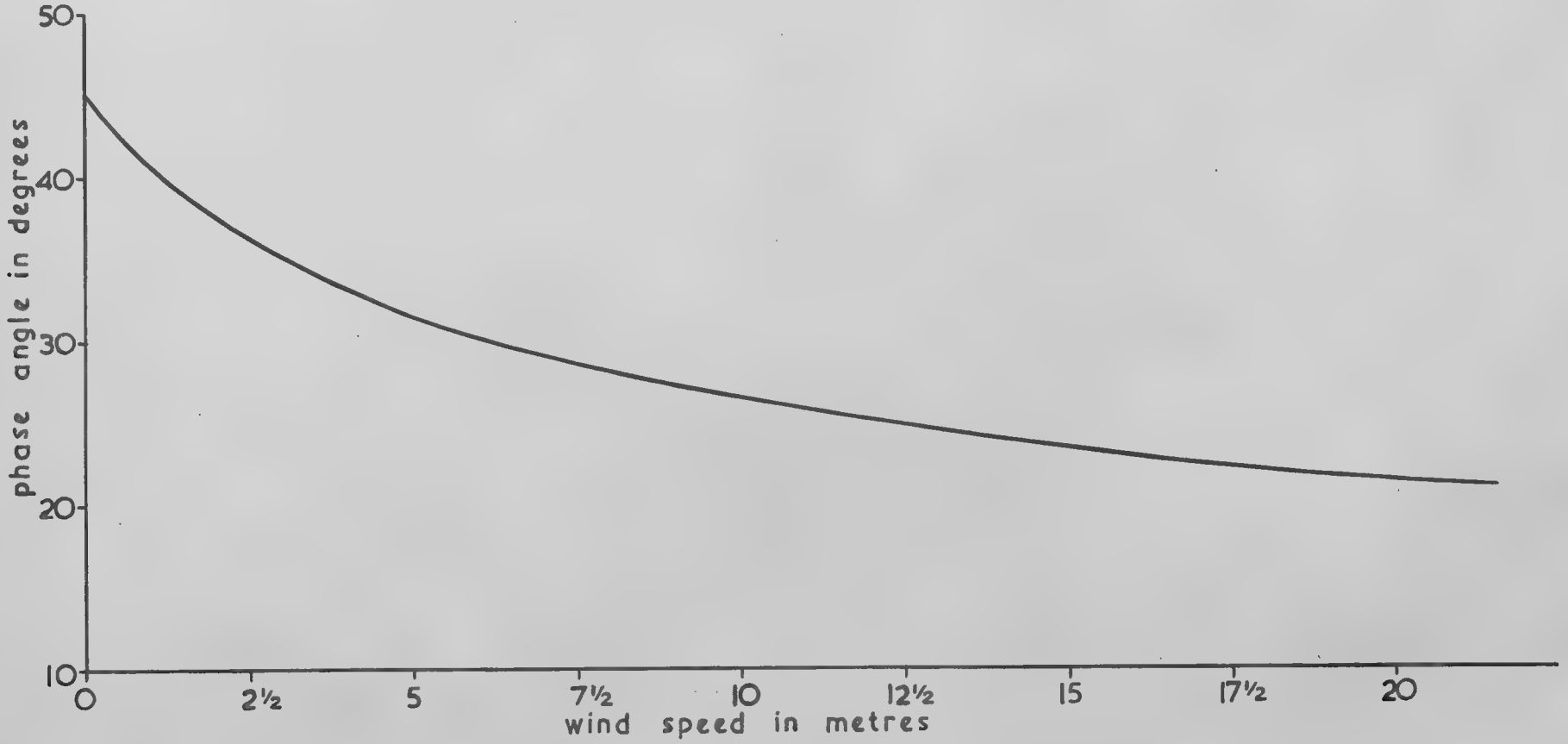

Fig. 3. VARIATION OF PHASE ANGLE WITH WIND SPEED

A copy is available in the Library of the Meteorological Office."
AIR MINISTRY

M.R.P. 737
S.C.I/66
S.C.III/136
13th June, 1952.

AD 23035

METEOROLOGICAL RESEARCH COMMITTEE

Methods of Measuring Liquid Water Content in Cloud

by
R.J. Murgatroyd (Meteorological Office)

Introduction.

This report is confined to measurements made from aircraft. At present there is no universal method of measuring liquid water content in cloud and the system used has to be varied with the physical state of the cloud i.e. whether it consists of water drops at temperatures above freezing point, supercooled water drops or ice crystals. The two latter may sometimes coexist. In addition snow or raindrops may be present in the cloud being investigated. The useful range of the instrument should be from about 0.05 gm/cu metre to about 5 gm/cu metre for most practical purposes and it should be able to deal with cloud water or ice particles ranging from a few microns to about a hundred microns in diameter. This latter range will not be adequate if precipitation elements are present when the maximum size may be a centimetre or more. In addition to overcoming the difficulties imposed by the different physical states of the water and the wide ranges to be covered the designer of any instrument must consider the following factors:-

(a) Speed of response. Liquid water content varies rapidly from zero to high values in convective clouds and at aircraft speed this means that the lag of the instrument for research purposes at least must be very small, i.e. a few seconds at the most.

(b) Robustness and reliability. At aircraft speeds and particularly in icing conditions a fragile element would have a very short life or may easily lose its calibration.

(c) Accuracy.

(d) Good sampling characteristics. The sample measured must be a good representation of conditions within the cloud being measured. This also imposes the requirement that the instrument itself must disturb the environment as little as possible.

(e) Whether the instrument is an absolute one or, alternatively, can be readily calibrated. The accuracy of the calibration process usually requires careful consideration. It is relatively simple to produce given temperature, velocity, water content and even drop sizes in a wind tunnel but the effect of altitude is not so easily produced.

(f) Whether it is required for research or statistical investigation. The specification for the former is more stringent but if the instrument is to be used by untrained personnel, e.g. in routine airline operation, it must be simple and easy to use.

(g) Ease of manufacture and

(h) Whether continuous recording can be obtained. With these factors in mind the various methods of measurement used in this country and abroad are now considered. A bibliography of the literature on this subject is appended.

/1.

1. Impaction Methods. (including the Cloud Droplet Camera)

An Impactor is primarily used for measuring drop sizes of cloud particles. A surface is momentarily (say for 0.01 sec.) exposed to the air stream and the cloud droplets are then captured in an oil or alternatively make craters related to their size in a sooted surface. By counting the number of droplets under a microscope or after photographic enlargement the total water content in the air sample captured may be evaluated.

The method is absolute and accurate, and the instrument robust. Sampling is very poor as the operation cannot be repeated quickly. The counting of the droplets is extremely tedious and this makes it impossible to evaluate a long series of records without excessive effort. The system usually fails when precipitation elements are present, as they destroy the prepared surface, and it does not work very well when ice crystals are present. It is not however invalidated by the water droplets being supercooled. Some disturbance of the air stream during the measurement is unavoidable but this can be minimised by suitable design. A camera to photograph cloud droplets directly instead of using an Impactor has been developed with some success in Canada and this gets over the latter difficulty.

Summarising: although this is a useful instrument for particular purposes it is not in general a practical method of measuring water contents in clouds. In some form or other it is used regularly in M.R.F. and also in Canada and the U.S.A.

2. Absorbent Cylinders.

Fine grained porous ceramic cylinders of high porosity or other absorbent materials have been used at temperatures above freezing to absorb cloud particles. The capacity to absorb is up to 70% of the bulk volume in some cases. The same principle has been used in wind tunnel calibrations as a standard against which other instruments have been tested. The increase in weight of the cylinder over a given period of time is a measure of the liquid water content.

The method is not an absolute one as the cylinders have a collection efficiency which should be determined beforehand. It can only be used for obtaining means over a period and its use is limited to clouds above freezing with no precipitation present.

The instrument can be made robust and reasonably accurate although losses by evaporation are difficult to assess. It is inconvenient to use in the aircraft.

Summarising: this instrument is not regarded as a suitable working instrument in the air although it is useful in calibration work in wind tunnels. As far as is known it is not in general flight use anywhere.

3. The Capillary Collector.

The third instrument which measures directly the volume or weight of cloud droplets collected is Vonnegut's Capillary Collector. A small cup of some porous material such as sintered metal or glass to the back of which is attached a glass tube of fine bore is exposed to the airstream. The water from the impinging cloud droplets under the action of capillary forces is sucked into the glass tube. The rate of accumulation of water in an attached collector tube is a measure of the instantaneous water content in the cloud, and in a recent modification this is measured by the change in electrical capacity between two plates of a condenser as the air in the tube is replaced by water.

The instrument is quick in response, reasonably accurate (although evaporation losses may occur at the collector head), and has good sampling characteristics. Its use is effectively limited to water clouds above freezing as supercooled drops tend to ice-up the collector head. Heating the head to obviate this increases evaporation errors. It is of little use in mixed or ice crystal clouds or when precipitation is present.

/Summarising:

Summarising: this instrument has good potentialities as a research tool within a somewhat limited field and has been used successfully in America. It has little application in statistical work.

4. Measurement of Electrical Resistance of Absorbent Paper.

In this method a strip of paper is driven behind a slit exposed to the air stream. The paper becomes wetted by an amount dependent on the liquid water content as it passes across the slit. The amount of water it absorbs varies its electrical resistance and this is measured across two contacts resting lightly on the paper behind the slit.

The method is simple and the instrument robust. The accuracy claimed is ± 20% which appears to be conservative. The speed of response is very good. Calibration against a standard is necessary beforehand.

Its use is limited to temperatures above freezing and probably rain would produce spurious though perhaps recognisable results.

As described in the original Australian Paper there are certain disadvantages in its arrangements, chief of which is the necessity for measuring resistances in the order of hundreds of megohms. A modification recently suggested at M.R.F. is to measure the change of electrical capacity of the paper rather than its resistance with the water absorbed but no work has yet been done on this possibility.

Summarising: this appears to be a very useful and simple instrument within its limited field. It can only be used as a research tool at present.

5. Rotating Cylinders.

This method and those in the following two sections are essentially measurements of accumulated ice and therefore only applicable to supercooled water droplets.

In this method a series of cylinders are rotated slowly as the aircraft passes through the cloud so that ice forms evenly on the periphery of the cylinders. After several minutes flying the cylinders are retracted into the aircraft and the gain in weight of the cylinders and/or their increase in diameter due to icing determined. The amount of icing is a function of aircraft speed, collection efficiency of the cylinders and liquid water content of the air.

The water content computed is a mean over the run of usually 10 miles or so. The collection efficiency is determined from theoretical work due to Langmuir and Blodgett. According to this theory use of several cylinders of different diameter will enable the drop size distribution also to be found. In practice this has not been so and also it appears that the results are affected considerably by the distance at which the cylinders are exposed from the aircraft. There also appears to be a theoretical upper limit to the water content that can be dealt with by this system. More experimental work appears necessary to substantiate the theoretical basis of this method and its accuracy in flight.

The instrument is robust, but its accuracy somewhat doubtful and the speed of response very slow. Its field of use is limited and it is cumbersome to operate in the air.

Summarising: for some time this has been a standard method in icing conditions particularly in the U.S.A. Its use now however is mainly as a statistical tool and better instruments exist for research work. An installation exists at M.R.F. but all practical work there in this field utilises the Rotating Disc Icing Meter instead.

/6.

6. The Rotating Disc Icing Meter.

In principle this instrument is similar to the rotating cylinders. A thin disc rotates slowly with its edge exposed to the air stream. As ice accumulates the diameter of the disc increases and moves a feeler which through a transducer provides an electrical indication. A scraper removes the ice as the disc rotates and the process repeats, a change of ice being obtained every 10 seconds.

Calibration is required before use and the instrument is open to many of the objections to the rotating cylinders. It is however easy to operate and reasonably rapid. It is suitable for continuous recording, robust, and quite small.

Summarising: although this instrument is open to some objections as regards accuracy, experience with it in M.R.F. up to date has been wholly good and in practice it has been very dependable and suitable for research purposes. Design and manufacture are somewhat complicated but most of these difficulties have now been solved. This instrument can also be adapted for statistical use. It has also been used in Canada and the U.S.A.

7. Use of Orifice Type Icing-Detectors.

The operation of this instrument is based on the creation of a differential pressure between an ice-free total pressure system and a total pressure system in which small holes are allowed to be blocked by ice accretion. At a given differential pressure an electrical heater switches itself on and de-ices the holes. The resulting cyclic process varies with the intensity of the icing and serves as a measure of the icing rate and hence can be used as a means of measuring liquid water content.

This instrument is robust and simple. It requires calibration before use and of course can only be used in supercooled water conditions. It is rather slow and has a somewhat coarse scale.

It has been developed with some success in Canada and the U.S.A. as a statistical instrument for measuring icing in airline work. It probably could be made to work almost as well as the icing disc meter in research work.

Summarising: this appears to be reliable and robust instrument. In view of the comparative success at M.R.F., with the icing disc-meter, however, no effort has been made to test it here.

8. Heated Wires or Cylinders.

A wire or cylinder heated to a high temperature is exposed to the air stream. Any droplets falling on it are evaporated and cool its surface. The amount of this cooling, which is measured by a resistance thermometer or thermocouple, is a measure of the water content. Calibration is required beforehand and air temperature, altitude and airspeed must also be known. The collection efficiency of the cylinder must be measured or determined from the Langmuir-Blodgett theory. The instrument will operate satisfactorily in all water droplet cloud but ice crystals appear to bounce off before evaporation can take place. The speed of operation is high - lags of less than 5 sec. being easily obtainable. The equipment is easy to construct and robust although about 200 watts or more of electrical power is needed in most designs. The cylinder can be made small and the interference with the air stream correspondingly small. It is adaptable to continuous recording and has received considerable attention both in U.S.A., Canada and this country. The experience with this instrument in M.R.F. has been very promising and the difficulties in the way of producing a research or statistical type of instrument based on this principle are mainly practical and not insurmountable.

An American instrument known as the Electric Water Collection Rate Meter measures the rate at which water impinges on a small surface by measuring the electrical power necessary to evaporate the water as fast as it collects. The electric current used for heating is controlled by the resistance between two electrodes which varies with the amount of water on the collector. It appears to have no special advantages over the ordinary heated cylinder method.

/Summarising

Summarising: the heated cylinder is a very promising instrument which is being developed at the moment in several countries.

9. Heated Intake and Hygrometer Method.

A method which at first sight is obvious is to pass a sample of the cloud through a heater, evaporate all the liquid water and then measure the dew point with a hygrometer. Theoretically then, knowing the air temperature and pressure, the liquid water content is determinable. In fact, practical difficulties are considerable but as this appears to be almost the only method of making measurements in ice crystal or mixed clouds a certain amount of work at M.R.F. is being done with this instrument.

The wattage used must be high (500 watts say) if a reasonable sample is to be investigated. Lag in the heater and pipes must be minimised as far as possible. The system is inherently inaccurate as small differences in temperature between hygrometer and air temperature readings are equivalent to rather large changes of water content. The measurement of the air temperature in cloud is difficult because evaporation effects on the thermometer bulb affect the magnitude of the airspeed correction to be applied, although this effect may not be important in pure ice crystal cloud. Lags are large because the hygrometer is slow to operate but it may be possible to reduce this by another system of humidity measurement. The instrument is robust and reasonably easy to operate but its sampling power and speed of response are poor.

Summarising: this instrument has been considered in Canada and the U.S.A. as well as in England but results so far have been disappointing. In view of its general potentialities in ice and mixed clouds however it deserves further study and development as a research instrument.

10. Methods using (a) Transmitted Light (b) Scattered Light

(a) Here a source of light distant from the observer - usually on the aircraft wing tip - is viewed by a photocell. The source of light is pulsed and a filter is used in the photocell circuit to separate this frequency and so the aircraft source from the background of natural illumination. The amplitude of the signal from the photocell in cloud is compared with that in clear air and is related to both water content and median drop size. The latter has therefore to be known if the water content is to be determined uniquely. This has usually meant that the chief value of an instrument of this type is in checking the results of observations with those from other instruments. The great advantage here is the avoidance of interference with the air-flow and the very rapid speed of response.

If measurements are made at two wavelengths of light it is possible to make measurements which will allow the median size and concentration to be determined separately. The only work so far reported with this feature however has been in wind tunnels with particles of very small size.

(b) Methods using scattered light are basically similar to those using transmitted light, and there seems to be no theoretical reason why both should not be used simultaneously. The angular distribution of intensity and degree of polarisation of the light scattered by a cloud is related to the size of the particles and the index of refraction (which must be known).

The purity and brightness of the colour spectra produced by the scattered light increase with uniformity of particle size.

Any apparatus based on these principles would be fairly complex. Accuracy may suffer because of the inhomogeneity of the cloud particles. Results would be difficult to interpret in ice-crystal cloud as Mie's theory on which the interpretation would be based assumes that the particles are spherical.

Various Canadian and American reports on water-content meters depending on the above principles have been published but there appears to have been considerable difficulty in making these instruments operate satisfactorily in practice. No work has been done with them in M.R.F.

/Summarising:

Summarising: these instruments are rather complex and as other methods have progressed sufficiently to give reasonable results in water droplet clouds the effort in developing these instruments has been comparatively small in spite of their promising nature.

11. Miscellaneous other methods.

Various other methods have been tried or proposed and short notes are given on these:-

(a) The "corona" method.

In an American extension of the rotating cylinder methods to water clouds above freezing point the droplets pass through a corona discharge and acquire electric charges. The droplets then encounter a series of cylinders of different radii which accumulate charges depending on the size and number of droplets. From the charges acquired by the cylinders the water contents and drop sizes are computed. The method appears to be unduly complicated and its accuracy is doubtful. The use of very high voltages on the aircraft is also a disadvantage.

(b) Measurement of dielectric constant.

If the droplets pass between the plates of a condenser the effective dielectric constant and hence capacity of the condenser changes. A method on these lines is attractive as regards speed of response, automatic recording and sampling. It is not at all certain to what extent drop size distribution would affect the results and no practical instruments on these lines appear to have been constructed. A further possibility is to use this system in conjunction with a heated intake.

(c) The use of interferometric measurements of the variation of the refractive index of air.

A proposal has been made that with a heated intake followed by an interferometer the refractive index of the air could be measured. This could then be related to vapour pressure and hence the water content. This, however, has most of the disadvantages of other methods using heated intakes and also introduces difficulties due to aircraft vibrations and it is very doubtful if any worthwhile accuracy could be achieved.

(d) A raindrop microphone in which precipitation striking a microphone produces a signal which is then amplified to produce meter readings or a cathode ray tube display may be useful for water content determinations in rain. This apparatus exists at M.R.F. but has not yet been made to work satisfactorily in the air.

Discussion

The design of a water content meter may take many forms and it is certain that the above list is not exhaustive. The most promising methods have, however, been discussed and the following suggestions are made for the instrumentation of a Flight making these measurements in cloud:-

1. At temperatures above freezing there should be available either a capillary collector, a heated cylinder or an absorbent paper type of instrument. M.R.F. have used the heated cylinder in previous work and have another model in construction. Laboratory experiments are also at present being made with absorbent papers.

2. For supercooled droplets either a rotating disc or orifice type of instrument should also be available. The former is in everyday use at present in M.R.F.

3. For mixed and ice crystal clouds there seems at present to be no alternative to a heated intake method and attempts are now being considered to improve its accuracy in use.

/4.

4. When precipitation elements are present it is likely that most methods will fail. It may be that this aspect can best be tackled by using in addition an impactor method with a sooted screen or gauze as the surface of impingement and some work is now being done along these lines at M.R.F.

5. The idea of using scattered or transmitted light is attractive and might be considered on a long term basis in this work.

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To members of Meteorological
Research Committee and of S.C.II.

Will you please make the following amendment to your copy of M.R.P.739:-

page 3, line 3, delete the words "compared with the wind speeds".

R Frost
for Secretary,
Meteorological Research Committee

M.O.15
Air Ministry,
London, W.C.2

23rd October, 1952.

G.341834/SEF/10/52/40

Summary

The results of a statistical test applied directly to the wind reports show that for British land stations the standard vector errors in reported winds at the 100 mb. level are generally less than 6 kt.

(1) Introduction

A notable feature of the wind field at the 100 mb. level is its steadiness in space and time. This provides an opportunity to establish directly an upper limit to the observational error in the wind reports. Bannon (1) has already estimated from the performance of the radar equipment that high level wind errors should be small. This is confirmed in the present paper.

(2) Estimates of Wind Errors from instrumental performance

Estimates of the errors in winds measured with the G.L.III radar equipment in use at British land stations have been made by J.K. Bannon (1). These apply to measurements of wind over one minute intervals and were deduced from the basic errors of the radar instrument as functions of height and of the mean wind speed up to the level in question. Bannon's estimates of the root mean square vector error in the measurement of winds at 50,000 ft, are reproduced in the following table.

Root mean square vector errors in winds at 50,000 ft. to the nearest knot (due to Bannon)

Mean Wind (kt.)	10	20	30	40	50	60	70
r.m.s. error (kt.)	2	2	3	4	5	6	7

These values are expected to vary slightly with the skill of the operators and the state of efficiency of the instrument.

(3) Errors at 100 mb.

The radio sonde observations of pressure and the radar observations are related through time. Winds are calculated from radar observations made at one minute intervals but it is the practice in the British Meteorological Office to ascribe to a given pressure level a wind meaned over a period of either two or three minutes. At 100 mb. the random errors in the measurement of pressure are known to be large (Harrison has given the probable error at 200 mb. as 5 mb. - Ref. 2). It follows that winds at 100 mb. can be measured less accurately than winds at 50,000 ft. Additional errors may arise in the computation and transmission of the data.

/A feature

M.R.P. 739
S.C. II/112
18th July, 1952.

At the time of issue this report has not been published. Any author

AIR MINISTRY

METEOROLOGICAL RESEARCH COMMITTEE

A Note on the Accuracy of 100 mb. Winds

by
D.H. Johnson (Meteorological Office)

Summary

The results of a statistical test applied directly to the wind reports show that for British land stations the standard vector errors in reported winds at the 100 mb. level are generally less than 6 kt.

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/A feature

A feature of the 100 mb. wind reports is the general coherence of the observations both in space and time with the suggestion of a steadiness in the wind exceeding that of lower levels. An example of the general coherence in space is shown in the diagram. Such smooth flow patterns are regularly to be seen on the working charts which itself implies that the total effect of the vector errors discussed above must be small. It was decided to utilise this property of the flow to estimate statistically an upper limit to the total standard vector error of the reported winds rather than to attempt to estimate the magnitude of the component errors individually.

(4) The statistical test

We require three stations A, B, C spaced at equal intervals along a line.

Let the true (vector) winds at A, B, C be V_1, V_2, V_3

Let the measured winds at A, B, C be V_1', V_2', V_3'

Let the (vector) errors in winds at A, B, C be $\varepsilon_1, \varepsilon_2, \varepsilon_3$

Then we may estimate the wind at B from the winds at A and C by linear interpolation with an error η

Thus $$V_2 = \tfrac{1}{2}(V_1 + V_3) + \eta$$

or $$V_2' + \varepsilon_2 = \tfrac{1}{2}(V_1' + V_3') + \frac{\varepsilon_1}{2} + \frac{\varepsilon_3}{2} + \eta$$

Thus $$\left\{V_2' - \tfrac{1}{2}(V_1' + V_3')\right\} = \frac{\varepsilon_1}{2} + \frac{\varepsilon_3}{2} - \varepsilon_2 + \eta$$

Now $\varepsilon_1, \varepsilon_2, \varepsilon_3$ and η are independent and $\overline{\varepsilon_1}^2 = \overline{\varepsilon_2}^2 = \overline{\varepsilon_3}^2 = \sigma^2$ (say)

Therefore $$\overline{\left\{V_2' - \tfrac{1}{2}(V_1' + V_3')\right\}}^2 = \frac{3}{2}\sigma^2 + \eta^2$$

$$\therefore \sqrt{\frac{3}{2}}\,\sigma < \text{r.m.s.}\left\{V_2' - \tfrac{1}{2}(V_1' + V_3')\right\}$$

The three British upper air stations at Aldergrove, Liverpool and Downham Market are approximately collinear and equally spaced so occasions were chosen from the first half of 1951 when reports of wind at 100 mb. were given in the Daily Aerological Report for all three stations. For 74 cases the r.m.s. $\left\{V_2' - \tfrac{1}{2}(V_1' + V_3')\right\}$ was 6 kt, implying $\sigma < 5$ kt. The mean Liverpool wind speed for this sample was 13 kt. As might be expected, there was a preponderance of summer observations, so this result is probably most representative of the summer season (M.O. 555 (part 1) gives the average of 100 mb. reported wind speeds at Larkhill as 13 kt. in June and 27 kt. in December).

In order to obtain a result more applicable to winter observations, occasions were chosen from the years 1950 - 51 when the three winds were reported and the wind at Liverpool was greater than 19 kt. For 78 cases the r.m.s. $\left\{V_2' - \tfrac{1}{2}(V_1' + V_3')\right\}$ was 7 kt., implying $\sigma < 6$ kts. The average wind speed at Liverpool was 26 kt.

It is believed that the standard vector errors in the winds were, in fact, appreciably less than these upper limits since it was clear on a number of occasions that the wind field was not linear. However, these occasions were included in order to make the test completely objective.

/(5)

(5) Conclusions

These tests, made directly on the reported winds, indicate that the standard vector error of 100 mb. wind reports from British land stations must be small both in summer and winter. They confirm the order of magnitude of the errors in winds at 50,000 ft. deduced by Bannon from the basic errors of radar.

References

(1) J.K. Bannon — Errors in winds measured with G.L. III Radar Equipment. M.R.P. 406. London 1948.

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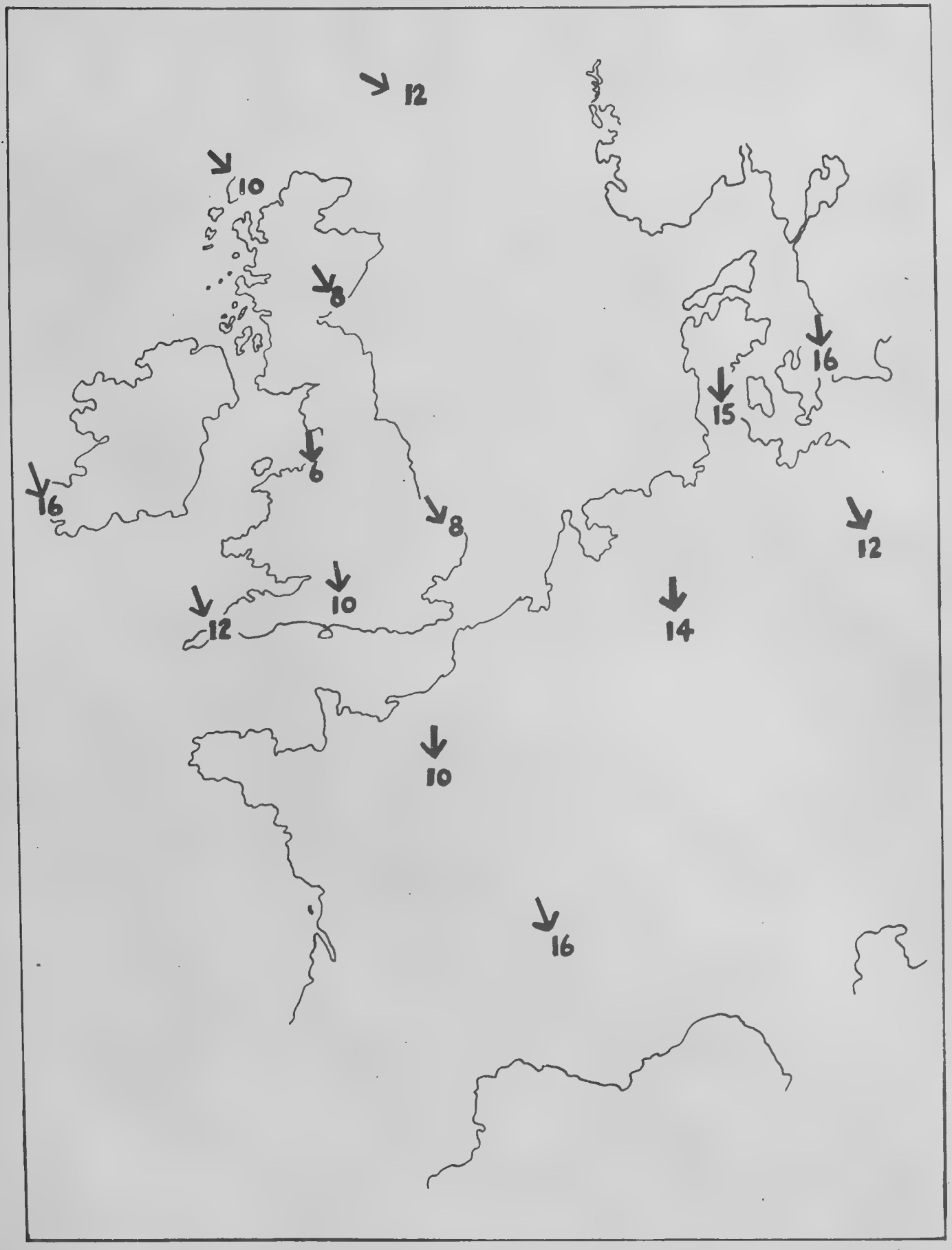
G.336524(r)/IW/10/52/165

M.R.P. 737

FIG 1 - WINDS AT THE 100 mb LEVEL

1500 G.M.T. 17th MARCH 1952.



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M.R.P. No. [illegible]
A copy is available in the Library of the Meteorological Office.

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S.C.III/138
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METEOROLOGICAL RESEARCH COMMITTEE

Some observations of the small scale fluctuations of wind and temperature near the ground

by

A.J. Lander and G.D. Robinson (Meteorolgoical Office)

1. Summary.

Simultaneous measurements of the horizontal and vertical components of wind and the temperature have been made at a height of 25 cm above a short grass surface, using an apparatus of linear dimensions about 1cm and time constant about 0.1 sec. The measurements have been combined to give the vertical convective flux of heat and an approximation to the vertical flux of linear momentum. The assumptions involved in these computations are discussed. Estimates of the vertical convective heat flux based on measurements of the radiative flux of heat and the heat flux in the soil, together with the profiles of temperature, humidity, and wind, have been made, and are compared with the results derived from the fluctuation measurements. The latter are found to be 30% to 50% of the former. Possible reasons for the discrepancy are discussed.

Some statistics illustrating the amplitude and periodicity of the fluctuations of wind and temperature - frequency diagrams and various correlation coefficients - are presented.

2. Introduction.

This report deals with an attempt to measure temperature and wind at a height of 25 cm above a surface of short grass in a volume of about 2cm x 2cm x 1cm, with instruments of time constant of the order 0.1 sec. Observations were limited to cloudless sky conditions, with mean wind speeds at the instrument about 100 to 200 cm sec^{-1}. Simultaneous measurements of mean temperature, humidity, and wind speed at heights from 5 cm to 200cm were made, and results obtained in lapse, isothermal, and inversion conditions. The primary purpose was to derive direct estimates of the vertical heat flux, and to compare these with estimates obtained from the heat balance and mean profiles by methods similar to those advocated by Rider and Robinson (1951) and Pasquill (1950). It was hoped that discrepancies between these two investigations would be resolved. The investigation has not yet succeeded in this purpose, but the lack of success raises questions of some interest, and some material descriptive of the air motion has been derived. It seems useful to put this information on record, and to take the opportunity to invite criticism and suggestions for future improvement of the work, which we hope to continue.

Previous investigations of small-scale short-period fluctuations in the lower atmosphere have not been lacking, for example those of Scrase (1930) and Best (1935) on the components of wind, and Lander (unpublished 1939) on temperature. However the first simultaneous measurements of horizontal wind, vertical wind and temperature known to us were made by Swinbank (1948) in cooperation with Mr. L.F.G. Simmons at the N.P.L., Teddington, in 1947. Our apparatus is a slight modification of that used by Simmons and Swinbank. Swinbank has continued his work in Australia and has reported results (1951 a and b); some of the interest of the present report arises from a comparison with these. The technique, variously modified, has become popular and similar work is known to be in progress in America, Germany, Japan and Russia, though

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details are not available to us. Apart from this small-scale work the estimation of heat flux on a world wide scale by correlation of wind and temperature is at present attracting very great interest and effort; we suggest that experience with the small scale observations may not be without application in the large scale problem.

3. Method of Observation.

(i) The detecting elements.

Two components, vertical and horizontal, of wind were measured by means of a V hot wire anemometer of the type described by Simmons and Bailey (1927). The arrangement and dimensions are shown in Fig. 1. The wire used was Pt, 0.0005 in. diameter and the operating temperatures between 170°C and 430°C. The supports were of minalpha wire, and soft soldered connections were (surprisingly) found satisfactory. The anemometer was connected in the double out-of-balance-bridge circuit shown in Fig. 2. The reading of galvanometer GΘ depends on the difference of resistance of the two arms of the heated platinum V, which difference is controlled to a large extent by the direction of the component of wind in the plane of the V, and to a much lesser extent by the wind speed. The reading of galvanometer GV depends on the resistance of the whole of the bridge on the left of Fig. 2. The only varying resistance in this bridge is the heated platinum V, whose total resistance varies rapidly with wind speed, but only slowly with wind direction.

Temperature was measured by means of a resistance thermometer of similar 0.0005 in. diameter Pt., arranged as shown in Fig. 1. These wires were welded to 1cm lengths of heavy Pt wire, which in turn were soldered to minalpha supports. The thermometer and its compensating leads were arranged in the out-of-balance-bridge circuit shown in Fig. 2. The current in the thermometer wire was 2ma. Tests were made in the wind tunnel, over a wide range of air speeds, to confirm that the heating current in the anemometer wires had no detectable effect on the thermometer. It was also confirmed that solar radiation had a negligible effect on the temperature of the wires in the range of air speeds met with, the most convincing demonstration being obtained in the wind tunnel using the radiation from a 2000 watt tungsten filament lamp - the normal fluctuations of temperature make field tests of this point unconvincing. The anemometer and thermometer were mounted on small ball bearings with a very light vane to hold them into wind, as shown in Fig. 3, electrical connections being made to the terminal board shown in the photograph by loose leads of braided fine gauge minalpha. The compensating leads were terminated on this board.

(ii) The recording elements.

The derivation of heat flux depends on the multiplication of vertical wind and temperature; the vertical wind is itself obtained by multiplication of results derived from the two anemometer records. It is therefore essential for each record to contain the minimum of amplitude distortion, and for the phase distortion on all three records to be the same. This condition was met by making the recording element (i.e. the galvanometer) rather than the sensing element (the wire) control the distortion. Galvanometers of period 0.1 sec and identical properties were used on each circuit. By observation of the decay curves the damping constant of each was set to about 0.7, giving a nearly linear phase lag - frequency relation over the range of frequencies recorded with appreciable amplitude (see e.g. Ferguson Hall 1950). Rough calculation shows the distortion introduced by the wire to be negligible at frequencies less than $10 c.sec^{-1}$; we hope to confirm this by experiment shortly. There was ample sensitivity in the anemometer circuits, and low resistance shunts were required. The high damping resistance and low self heating current resulted in a sensitivity (3°C per cm) rather less than is desirable in the thermometer circuit. Simultaneity in reading the photographic records was ensured so far as is conveniently possible by using one drum for the three records and recording a light flashing $\frac{1}{2}$ seconds at the two edges of the chart. The possibility of errors due to distortion of the paper in processing remains, and any such error would result in too low a value for the derived flux. It is however very unlikely that errors approaching the size of the discrepancies to be discussed later could be introduced.

(iii) Calibration.

Calibration of the anemometer was necessary, and was carried out in a 9in. square section wind tunnel in which the air speed was variable from 20 to 1350 cm sec^{-1}. The flow as determined by the hot wire and standard anemometers was smooth, and was checked for uniformity across the section. Three standard anemometers were available, a pitot tube, an Ower-type vane, and a Simmons shielded hot wire. There were some slight discrepancies between these standards at low velocities (probably connected with their temperature coefficients - see below). The shielded hot wire was normally used as the standard - the velocities most used were below the convenient range of the pitot, and the vane, otherwise ideal except at the very lowest speeds, was too large to be used simultaneously with the hot wire under calibration. For calibration the anemometer - thermometer head was removed from its mounting, the ball bearing clamped, and the whole set up so that it could be rotated in a vertical plane from outside the tunnel. It was set horizontal by means of the spirit level, the wind speed adjusted to a convenient value, and both galvanometers read. The instrument was then tilted through a series of known angles relative to the air flow, and readings taken. This was repeated at different air speeds. The instrument provides a unique resolution of wind into two components by virtue of the very slow variation of reading of the "total wind" galvanometer with change of tilt at constant wind. (The same is true if the instrument is yawed out of wind. A yaw of 20° has negligible effect on the readings in the ranges of total wind and tilt of interest in the present observations.) Fig. 4 shows an example of calibration. It was necessary to keep a constant check on calibration and in particular to check the curves immediately before and after a run in the open air. On one occasion (out of eleven) the two calibrations did not agree and the observation was rejected.

Temperatures were obtained by calibrating the thermometer bridge in terms of resistance changes in the thermometer arm, and using the known temperature coefficient of Pt. This procedure was checked whenever possible by readings taken in the wind tunnel which was in an unheated room subject only to slow temperature changes.

(iv) Temperature coefficient of the anemometer.

The question "precisely what is measured by the anemometer?" is of some theoretical importance, as will appear later. Anemometers may measure velocity V, momentum ρV, or kinetic energy $\frac{1}{2}\rho V^2$. Swinbank (1951a) claims that the reading of a hot wire anemometer can be reduced directly in terms of ρV. This is true when hot wire is operated at constant temperature in air of constant temperature, but it can be seen from King's equation (see e.g. Ower 1949 p.244) that it does not hold in other circumstances. The temperature coefficient of our instrument, obtained both by experiment and theory, was of the order of 1.5% per $^{\circ}C$ change of air temperature in our working conditions. The change of wire temperature with air speed at constant air temperature is of course taken care of in calibration, and results in a considerable divergence of the velocity calibration from the square root law. At the calibration temperature the readings of the instrument are identified with those of the standard anemometer; the instrument measures whatever the standard measures i.e. ρV for the hot wire standard, $\frac{1}{2}\rho V^2$ for the pitot, and a rather doubtful quantity, approximately V, for the vane. At other temperatures - and the fluctuations imply operation at other temperatures - it is not clear what is measured, and in view of the discrepancies between the standards further discussion is fruitless. It was possible on all occasions to obtain calibrations at a temperature close to the mean air temperature during the field observations. The results were then reduced on the assumption that the readings are a measure of velocity (V). In Section 5(ii) possible consequences of this assumption will be examined.

(v) Interpretation of records of a fluctuating wind.

The anemometer is calibrated in terms of a wind uniform over the volume occupied by the instrument, and uniform in time. In the field neither of these conditions is fulfilled, but the only practicable method of reduction of the records is to assume that both hold. There is therefore a spurious element in the derived wind components. A horizontal wind of different speed over the two halves of the V will be recorded as having a vertical component, and it is assumed that these spurious components are not correlated with either mean wind speed or

temperature. Similarly if the wind is not uniform over the volume of the anemometer during a time comparable with the time constant of the recording circuit, the record will indicate some kind of mean value and, since the anemometer is not a linear detector, this mean will not be the arithmetic mean. The circuit approximates to a square-root law detector, so that the mean value of rapidly fluctuating winds will be underestimated.

The position may be summarised in terms of the effects of eddies of various sizes. Only a very loose definition of the term "eddy" is possible: we may say it is a region of the atmosphere, in which coherent velocities persist for a time, the velocities being supposed to be measured by a relatively small, quick acting instrument. If the region is large compared with a 2cm cube, and the time longer than $^1/_{10}$ sec, the velocities will be recorded adequately by our instrument. If the volume is appreciably smaller, or the time appreciably shorter, or both, the effect on the instrument will be negligible. Intermediate sizes and periods will produce the spurious effects noted above.

It is clear that the size and time constant of the instrument cannot be settled independently; for example there is no reason, a priori, to expect useful results from operating a very large instrument with a very small time constant. The reverse, small instrument with large time constant, is often used (e.g. by Swinbank) and it is not clear why this arrangement gives satisfactory results, as it appears to do. In the present experiments the time constant is reasonably well matched with the vertical space scale, but is still an order of magnitude higher than that appropriate to the horizontal space scale (i.e. the average distance travelled by the air in $^1/_{10}$ sec. is of order 2cm in the vertical but of order 20cm in the horizontal).

4. The observations.

(i) Site and exposure.

The site has been described by Rider and Robinson (1951), and an idea of the nature of the surface can be obtained from Fig. 3. The anemometer and thermometer were exposed at a height of 25 cm at a convenient point within about 5m. of the mast carrying the wet and dry bulb thermometers. A set of sensitive cup anemometers was arranged nearby. The hot wire instrument was placed on a thin brass disc to facilitate levelling, which was done using the two spirit levels, shown in Fig. 3.

(ii) Procedure.

For half hour periods during cloudless or nearly cloudless conditions the following observations were taken, using the apparatus and methods described by Rider and Robinson (1951).

Mean temperature and humidity profile 5 cm to 200 cm.
Mean wind profile 5cm to 100 cm.
Earth temperature profiles 1cm to 32cm before and after period.
Solar radiation on a horizontal surface.
Downward flux of atmospheric radiation.
Upward flux of terrestrial radiation with surface temperature as check.

During the half hour period as many separate records as possible were made of the wind and temperature fluctuations - usually six or seven if $^1/_{10}$ sec. galvanometers were used with a 1 minute recording drum, and three when 2 sec. galvanometers were used with a 5 minute drum.

(iii) Material available.

The results obtained from the various field observations, and the extent to which they have been reduced, are summarised in Table I. The only persistently disappointing aspect was the poor quality of the humidity profiles. No really good humidity curve was obtained; several were almost useless. The faults arose in a misguided attempt to make the apparatus more readily portable and it is hoped they have now been corrected. An attempt to obtain results in

/a

a period of zero gradient of mean humidity was successful (23rd October, 1951); the run obtained in isothermal conditions (24th April, 1951) was more a matter of good luck.

(iv) Method of reduction and computation.

It was found convenient to draw pencil lines across the chart, at intervals of 1/5 or 1/4 second when using the 0.1 sec. galvanometers, and to read the deflections at the intersection of the records with these lines by means of a glass scale. These readings were converted to total wind, angle of incidence in the vertical and temperature by tables constructed from the calibration curves. Resolution into horizontal and vertical wind components was then carried out using a pilot balloon slide rule. Multiplication of horizontal component by vertical component, and vertical component by temperature, summation, and derivation of mean value of all elements were carried out in various ways, according to the facilities available. Using a keyboard calculating machine it was found that with practice the measurement and complete reduction of a single chart (about 40 sec. run) occupied two men rather more than one day. These details are noted here since the time and labour involved must be a major factor in any consideration of future application of the method.

5. Determination of the fluxes of heat and momentum.

(i) Theory of the method.

Consider in the first place a conserved quantity (e.g. matter) of mass concentration s in an element of air, density ρ, moving relative to a surface in which it is instantaneously situated with a normal component of relative velocity w. In time t the amount of this quantity passing through an area A is

$$\int_A \int_t \rho s w \, dA \, dt$$

The flux is

$$\frac{1}{At} \int_A \int_t \rho s w \, dA \, dt \equiv \overline{\rho s w}$$

defining the mean value of $\rho s w$. (N.B. We are not concerned with the Reynolds mean, or with mean values appropriate for use in equations of motion). The mass flux of air through the surface is

$$\frac{1}{At} \int_A \int_t \rho w \, dA \, dt \equiv \overline{\rho w}$$

The treatment of the flux of a non-conservative quantity is not simple. The quantities with which we are concerned are heat and linear momentum. "Heat", however defined, or any other particular manifestation of energy, is not necessarily conserved in the motion, nor is linear momentum if a system limited in space or time is considered, since the motion may be affected by non-uniform pressure fields maintained by processes external to the system (e.g. the whole investigation might be carried out within the pressure field of a large eddy). The flux of heat has been discussed by various authors (See e.g. Montgomery, Robinson 1951); one aspect of the problem is of particular interest in connection with the present results. We are specifically concerned with "sensible heat", and a formal definition of this quantity, such as that due to Born (1921), refers to its transfer to and from a closed system. It may be valid to extend this definition to an open system of strictly constant mass and composition, but it cannot be extended to the general open system. A suitable system in the atmosphere may be defined, ignoring changes of humidity, by the earth's surface, material walls, and an upper surface in which the pressure is strictly constant. If A is the area of this upper surface the heat flux through it is

$$\frac{1}{At} \int_A \int_t \rho c_p T' w \, dA \, dt \equiv c_p \overline{\rho T' w} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

/where

where T' is the excess of temperature of the element over its surroundings, with the conditions

$$\left.\begin{aligned} \frac{1}{At}\int_A\int_t \rho w \, dA \, dt \equiv \overline{\rho w} = 0 \\ \text{and } p = \text{constant in A and t} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

The point of interest is that if Born's definition is adopted it is legitimate to speak of an eddy flux of heat, associated with no mass transport, but not of a "mass flux of heat" associated with a change of mass of the system. If the latter phrase is used, as it frequently is, "heat" has a meaning different from that adopted here. Certain alternative forms of (1) are of interest. We have

$$c_p \overline{\rho T' w} = c_p \overline{\rho T w} \quad \text{since } \overline{\rho w} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

$$= c_p \frac{p}{R} \overline{w} \quad \text{since } p = \text{constant} \quad \ldots\ldots\ldots(4)$$

where R is the gas constant per gram. It may also be seen that

$$c_p \overline{\rho T' w} \simeq c_p \bar{\rho}\, \overline{T' w} = c_p \bar{\rho}\, \overline{T' w'} \quad \ldots\ldots\ldots\ldots\ldots(5)$$

but

$$c_p \overline{\rho T' w} \not\simeq c_p \bar{\rho}\, \overline{T w} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

Expression (4) seems at first sight a promising way of determining the heat flux, but substitution of numerical values shows that $\overline{w}$ is of order 10^{-2} cm sec^{-1} which is beyond the possibility of instrumental detection. Expression (3) may be used if ρw is measured, but if w is measured it is necessary to use the approximate form (5).

(ii) Limitations of the method.

We are now able to discuss the limitations of the method used to determine the flux of heat. These are

(a) The "element" of space (i.e. the size of the instrument) is comparatively large.

(b) The "element" of time (roughly the free period of the galvanometers) is comparatively long.

(c) The sample is taken at only one point in space, so that the integration over area cannot be performed.

(d) Constant pressure is assumed, whereas it is known that the pressure fluctuates.

(e) Velocity rather than momentum is measured, so that an approximation must be used.

The consequence of (a) and (b) is that if the scale of motion responsible for any appreciable part of the heat flux is comparable with or less than the element, this part of the heat flux will not be detected. By reason (c) if there is any heat flux associated with a stationary spatial pattern of motion, or if for any other reason time and space integrations are not equivalent, the flux may not be correctly determined. The effect of fluctuating pressure (d)

cannot readily be determined. Processes of heat exchange not restricted to constant pressure have been suggested but would hardly change the heat flux by a factor greater than the ratio c_p/c_v . In the examples to be presented the Reynolds pressures ($\frac{1}{2}\rho V'^2$) are very small in comparison with the temperature fluctuations, but it has never been demonstrated that they are the only pressure fluctuations present. On balance it seems unlikely that in the present observations pressure fluctuations can affect the heat flux by more than a few parts per cent. A more direct estimate can be obtained of the effects of (e). Assuming that velocity is measured, and that p is constant one set of results was worked up using expressions (3) and (5). The heat fluxes were found to differ by about 2 per cent in rough agreement with estimates of this error which can be made from the observed ratio T'/T and the observed correlation of w' and T', constant pressure being assumed.

(iii) The vertical flux of linear momentum.

The limitations just discussed apply with equal force to the determination of the momentum flux. There is an added difficulty. We will denote the total (instantaneous) wind vector by $\underline{V}$, its components by u, v, w, and mutually perpendicular unit vectors by $\underline{i}$, $\underline{j}$, $\underline{k}$. Here u is the instantaneous component of $\underline{V}$ in the direction of the horizontal component of the mean wind, magnitude $\bar{u}$; $\underline{i}$ is the unit vector in this direction. The apparatus determines the magnitude, but not the direction, of the horizontal component of $\underline{V}$, and the magnitude and direction of its vertical component i.e. $|\underline{i}u + \underline{j}v|$ and w . To indicate the deviation of $|\underline{i}u + \underline{j}v|$ from the mean value $\overline{|\underline{i}u + \underline{j}v|}$ we write $|\underline{i}u + \underline{j}v|'$ The notation is clumsy, but it preserves the distinction between the measured horizontal component, and the components u and v, which are not determined separately (and which can only be specified in terms of an artificial post facto definition of "mean wind"). Since u is not determined it is possible to make only an estimate of the vertical flux of the component of momentum in the direction of the mean wind. This flux is usually identified with the shearing stress τ in the surface considered, and τ with the frictional drag at the ground, though, as mentioned above, if the determination is over a limited area and time, the shear may be affected by fluctuating external pressure fields. It seemed worth while to compute the momentum flux, although no simultaneous measurement of drag had been attempted for comparison. The quantity actually determined was the approximation $\overline{\rho\,|\underline{i}u + \underline{j}v|'\,w'}$.

(iv) The vertical flux of kinetic energy.

An approximation to this quantity may readily be determined from the observations either directly in the form $\frac{1}{2}\overline{\rho\,|V|^2 w'}$ or in the approximately equivalent form $\overline{\rho.|\underline{i}u + \underline{j}v|.|\underline{i}u + \underline{j}v|'w'}$ which contains the already computed momentum flux. Similarly the flux of kinetic energy of either the vertical or horizontal component of the turbulence can be computed. The order of magnitude of the total vertical kinetic energy flux near the ground is well known in the form $u\tau$ (e.g. Calder 1949) and computation from the present observations adds nothing useful. A few computations of the vertical flux of the kinetic energy of turbulence were attempted. The fluxes derived were extremely small, and in view of the large discrepancies in the heat flux (see below) the question was not pursued. Since these computations were made some more detailed work has been published by Taylor (1952) on the fluxes of total and turbulent kinetic energy. As pointed out by Calder (1949) the flux of kinetic energy of turbulence is a quantity of some importance in the study of the maintenance of turbulence in atmospheric motion. Our observations confirm Taylor's in suggesting that it is negligible close to the surface.

(v) The results for heat and momentum flux.

Table II, to be read in conjunction with Table I, shows the results obtained. The computed fluxes are in columns 5 and 6, the uncertainties quoted being the mean deviation of the results of separate runs. In column 7 the heat fluxes derived from the heat balance and profile measurements by the Bowen ratio

/method

method (Rider and Robinson 1951) are listed. In columns 8 and 9 are given values of heat fluxes derived from the formula $\rho k^2 z^2 (\partial u/\partial z)(\partial T/\partial z + \Gamma)$ applied at heights z = 25cm and z = 10cm respectively.* Column 10 contains values of the momentum flux derived from the formula $\rho k^2 z^2 (\partial u/\partial z)^2$ for z = 25cm. The latter formula may be regarded as empirically established in neutral stability for a smooth surface by the work of Sheppard (1947). Pasquill (1950) has suggested modifications which make it applicable in neutral stability over a rough surface, and Pasquill (1949), Rider (1951), and Rider and Robinson (1951) have discussed the application of both formulae in lapse and inversion conditions. The various investigations do not yield concordant results, but there is certain common ground.

(a) In inversion and neutral stability the Bowen ratio method is reliable.

(b) In superadiabatic lapse the Bowen ratio method does not overestimate the heat flux.

(c) In superadiabatic lapse the aerodynamic formulae quoted above usually underestimate the fluxes of heat and momentum by an amount which increases with height.

This common ground is sufficient for our present purpose without further discussion of the respective merits of the methods of estimating fluxes from the mean profiles.

The limits of error shown in column 7 are estimated from a review of all the circumstances of each observation. The humidity profiles were poor, and most of the estimated error arises from this cause. The readings of a recording evaporimeter, and the use of an empirical formula previously derived (Rider and Robinson 1950) helped to ensure that there were no gross errors in the heat flux term. Comment on certain individual cases is necessary.

11th December, 1950. The Bowen's ratio method failed because freezing was observed on the grass during the course of the observation, introducing an indeterminate source of heat. The lower limit given in column 7 assumes this source to be negligible.

24th April, 1951. The conditions were isothermal within the limits of observation. It is likely that there was a large advective heat flux.

23rd October, 1951. This was an attempt to secure readings on an occasion when humidity was constant with height. Unfortunately the wet and dry bulb circuits for heights below 25cm failed and the observation would have been useless had it not been observed that the bulb of the grass thermometer was acting as a dew point hygrometer, and later been found that the humidities at heights above 25cm were equal, and equal to the humidity observed for the surface level.

/The

Footnote

* In the Bowen ratio method all components of the energy balance at the surface are measured with the exception of the convective exchange of heat with the air, and the loss (or gain) of heat consequent on evaporation (or condensation). The sum of these two terms is therefore known. It is then assumed that the vertical fluxes of heat and water vapour above the surface are related to the gradients of potential temperature and mass concentration of water vapour through the coefficients of eddy conductivity and eddy diffusivity for water vapour respectively, and that these coefficients are equal. Neglecting the divergence of the fluxes, the ratio of the gradients is therefore the same as the ratio of the fluxes - the "Bowen ratio". The essence of the method is the assumption of the existence and equality of the coefficients of eddy conductivity and eddy diffusivity. The essence of the "aerodynamic" formula quoted is the assumption of the existence and equality of the coefficients of eddy viscosity and eddy conductivity.

The vertical flux of water vapour was therefore assumed to be zero.

In Table II no attempt has been made to assess accuracy in columns 8, 9 and 10. Apart from any theoretical difficulties (e.g. the question of surface roughness) and from the observed large departures of individual wind profiles from the mean of several observations in similar conditions (see particularly Rider (1951)), these estimates depend on the product of two gradients, each determined subjectively. From this cause alone the uncertainties are greater than those ascribed to the estimates in column 7.

When all the evidence is combined there seems little doubt that with the possible exception of those on 11th December, 1950 all the fluxes computed from the fluctuations are too low, lying between 30% and 60% of the true result. All possible errors in the fluctuation measurements would combine to reduce the correlation between w, $|\underline{i}u + \underline{j}v|$ and T, and so to reduce the computed fluxes, but it can be said confidently that these errors could not approach the observed discrepancy. The cause of this must be sought in the limitations discussed in paragraph 5(ii).

Columns 11 and 12 of Table II are respectively the "eddy conductivity" and "eddy viscosity" computed by dividing the flux of heat or momentum, determined from the fluctuations, by the gradient of the mean profile at $z = 25$cm. In the four cases where all the available fluctuation observations were used these two coefficients are approximately equal - the differences are certainly well within the limits set by the uncertainties of the flux and gradient measurements. This agreement is the more remarkable since we have concluded that the flux measurements suffer from a large systematic error.

(vi) Results for the kinetic energy flux.

The total kinetic energy flux, and the flux of kinetic energy of turbulence were computed for two runs, both on 25th October, 1950, using departures of individual readings from means of 25.

The results obtained were

Run 3.

Flux of kinetic energy of turbulence

$$\simeq \frac{1}{2}\overline{\rho}\,\overline{|\underline{i}u + \underline{j}v|'^2 w'} = +2 \text{ erg cm}^{-2} \text{ sec}^{-1}$$

Total kinetic energy flux

$$\simeq \overline{\rho}\,\overline{|\underline{i}u + \underline{j}v|.|\underline{i}u + \underline{j}v|'w'} = -18 \text{ erg cm}^{-2} \text{ sec}^{-1}$$

Run 6.

$$\frac{1}{2}\overline{\rho}\,\overline{|\underline{i}u + \underline{j}v|'^2 w'} = +4 \text{ erg cm}^{-2} \text{ sec}^{-1}$$

$$\overline{\rho}\,\overline{|\underline{i}u + \underline{j}v|.|\underline{i}u + \underline{j}v|'w'} = -180 \text{ erg cm}^{-2} \text{ sec}^{-1}$$

6. Possible causes of error in the flux measurement.

In addition to the low values of the fluxes Table II contains another feature which was, to us, unexpected. The mean vertical velocities are given in column 4, and it will be seen that they are comparatively large. It is thought that these are not due to instrumental error, which could arise in the following

/three

three ways.

(a) Incorrect levelling.

(b) The vertical gradient of mean velocity results in a higher horizontal velocity over the upper arm of the V than over the lower; with the necessary assumption of uniform velocity over the whole anemometer this would be interpreted as a mean negative vertical velocity.

(c) A shift of zero with change of ambient temperature, due to the lack of exact balance in the two arms of the V anemometer. It is impossible to match these two arms exactly during construction, and a precise balance is made at calibration temperature by means of the slide wire R1. (Fig. 2). The shift of zero between curves A, B, C., Fig. 4, is an effect of this lack of balance. A similar but smaller shift of the whole family of these curves results from a change in ambient temperature, and if no allowance is made when computing the results a spurious mean vertical velocity is found. This cannot be automatically compensated, since the fluctuating velocities measured are accompanied by, and correlated with, fluctuating temperatures. It is not possible to investigate these defects of the instrument in any detail, but experiments in the wind tunnel and rough calculations show that they cannot be responsible for spurious vertical velocities of the size of those observed. It must be concluded that mean velocities of this order persist for periods of 30 minutes or more at a height of 25cm. and that they indicate some form of flow with a spatial pattern. It is not profitable to speculate on the cause of this pattern; it might be due to surrounding obstacles to the flow or to minor undulations in the surface.

We are now in a position to relate the observations to the limitations listed in paragraph 5(ii). We have already dismissed (d) and (e) as incapable of explaining errors of the magnitude with which we are concerned. The existence of appreciable mean vertical velocities shows that (c) may be important. The heat fluxes tabulated are obtained from the formula $\overline{\rho}\, c_p\, \overline{w'T'}$ the assumption being that no heat flux is associated with the pattern of mean vertical velocity, an assumption which, though necessary, seems inherently improbable. However, the results obtained by Swinbank (1951 a and b) give some support. Working at about 200 cm above a rather rougher but much more extensive surface he found heat fluxes from the fluctuations in good accord with heat balance measurements, in spite of large mean vertical velocities*, the only possible conclusion being that the spatial pattern of flow had no part in the heat transport.

There remains the limitation imposed by the size and time constant of the instrument. It is difficult to separate the space and time effects. The following type of fluctuation would remain undetected -

(a) movements over a volume small compared with that of the instrument in a time small compared with its time constant

/(b)

* It is interesting, though not relevant to the present point, to note that the heat fluxes determined by Swinbank in conditions of weather, season, and L.A.T. similar to ours, are if anything smaller than those determined by us from the fluctuations, and much smaller than our heat balance and aerodynamic estimates; this in spite of the lower latitude in which Swinbank worked.

(b) movements small compared with the dimensions of the instrument, but persisting for a relatively long time

(c) movements over comparatively large distances, but too rapid for detection

(d) very slow movements i.e. slow in comparison with the total period of recording.

We can dispose first of (d). Certain of the observations were analysed in several ways. The 200 readings spaced at $^1/_5$ or $^1/_4$ sec. intervals were split into groups of n consecutive observations, the means of these groups found, the deviation from the group mean of $|\underline{i}u + \underline{j}v|$, w and T found for each reading, the mean products found for each group, and the mean over all the groups taken. Decreasing n has the effect of filtering out the contribution of the fluxes of the longer period components. Similarly the shorter period components not already suppressed by the instrument may be filtered by regarding the mean of n readings as a single observation forming products of corresponding values of $\sum_n |\underline{i}u + \underline{j}v|/n$, $\sum_n w/n$ and $\sum_n T/n$, and meaning these products over the 200 readings. An observation made on 14th September, 1950 for which no heat balance measurements were available was treated systematically in this way with the results shown in Table III. These suggest that the observed flux of heat and momentum was carried by fluctuations of period between the instrumental limit of $^1/_{10}$ sec. and about 2.5 sec., and that longer period fluctuations produced a flux which in some degree opposed the main flux. No other case was analysed so completely but several were worked out for n = 200 and n = 25, and it was found that the differences were in all cases quite small. They are shown in Table III. It seems that the effect of fluctuations of periods of order 10 seconds or more is unimportant. The results of filtering out the short period components (Table III, 14th September, 1950 and 6th June, 1951) suggest indeed that the main effect is due to fluctuations of period less than 2 sec., which is hardly in accord with the results obtained on 2nd July, 1951 using 2 sec. galvanometers. The filtering technique is however very crude and the material limited, so that it would be unwise to stress the point. Swinbank's results, obtained with 2 sec. galvanometers over a 5 minute period, suggest that small eddies have little influence on the heat flux at heights between 1 and 2 metres, the horizontal movement being of the order of 10 metres in the 2 secs. The vertical gradient of mean wind and temperature at 25cm may be 100 times that at 2m. This fact alone suggests that much smaller eddies must exist at the lower heights to support the gradients. It is not therefore safe to apply to our problem arguments based on this aspect of Swinbank's experience.

Summarising the position, we may say that 50% to 70% of the heat flux is not accounted for. The "patterned flow" explanation cannot be excluded on our observations alone, but those of Swinbank suggest that it is not valid. In our observations, fluctuations of period longer than about 10 sec. appear to carry little of the heat flux. There is some evidence that the proportion detected does not increase rapidly as the lower limit of eddy period examined is reduced from about 2 sec. to 1/10 sec. This last fact is difficult of explanation unless the fault lies in the dimensions of our detecting instrument rather than in the time constant of the recorder: if this is the case it is of course likely to be the vertical dimension which is too great. The suggestion is that the undetected heat flux may be carried by movements of type (b) above, but movements of type (a) seem inherently more probable, and are certainly not ruled out by our results.

This discussion has concentrated on the heat flux, since the independent estimates of heat flux are better than those of momentum flux and the observations have been more completely analysed in terms of heat flux. The effective equality of "eddy heat conductivity" and "eddy viscosity" disclosed by Table II suggests that the motions responsible for heat and momentum flux are the same in some degree of detail, but this is not well supported by an examination of individual readings.

/7.

7. Results descriptive of the air motion.

The material used in computing the vertical fluxes throws some light on the nature of the air motion. The extent to which the velocities and temperatures derived from the records depend on the detecting instrument rather than on natural phenomena has been discussed, but can hardly be settled without experiments using different sizes and types of detector. The results will be presented with the minimum of further discussion of this question.

(i) Distribution of velocities and temperature.

For 25th October, 1950 and 2nd June, 1951 the frequency distributions of $|\underline{i}u + \underline{j}v|$, w and T were extracted, the results being shown in Figs. 5 and 6. The frequency distribution of w only was extracted for 23rd October, 1951, and is shown in Fig. 7. The numbers of individual observations involved are 1375 (Fig. 5), 1400 (Fig. 6), and 1200 (Fig. 7), grouped in sets of approximately 200 in 40 to 50 secs., spaced at fairly regular intervals covering a total period of 30 minutes. The sampling is therefore rather inadequate. The large variations between adjacent classes shown in all the histograms is partly due to the use of systematic methods of rounding off when reducing the records. The vertical wind distributions are symmetrical but not Gaussian. On 25th October, 1950 and 2nd June, 1951 the central parts of the distribution can be fitted closely by a normal curve, but there is a considerable excess of large values of $|w|$. A large part of the heat flux is associated with these large values of $|w|$; on 25th October, 1950 25% of the heat flux was carried by vertical wind components greater than 45cm sec^{-1}. On 23rd October, 1951 the departures from normal distribution are more obvious, with excess of both small and large values of $|w|$.

The distribution of $|\underline{i}u + \underline{j}v|$ is not symmetrical. (In both cases examined the distribution of $\log |\underline{i}u + \underline{j}v|$ is approximately symmetrical but not Gaussian). Frenkiel (1951) has discussed the distribution of $|\underline{i}u + \underline{j}v|$, on the basis of normal distributions of u and v, and shown that the assymmetry of the horizontal wind distribution increases with the ratio σ_v/σ_u. Distributions such as those shown in Figs. 5 and 6 occur only with large values (> 2) of this ratio, but in view of Frenkiel's assumption of normal distributions for the components of velocity and our observation that the distribution of w is not normal further discussion is not profitable.

The distribution of T is not symmetrical. The general shape is what might be expected from a combination of a symmetrical distribution of w and a non linear temperature profile. The large dispersion of T is noteworthy: it is of course far greater than that which would be caused by vertical motion associated with the mean temperature profile.

(ii) Correlations between the variables.

Correlation coefficients were derived for two groups of 200 readings. The first occasion (run 6, 25th October, 1950) had been analysed in subgroups of 25 readings (see paragraph 6), and the variables correlated were the departures of each element from the mean of the group of 25 in which it occurred. The readings were spaced at approximately 1/5 sec. The values obtained were

$$R_{|\underline{i}u + \underline{j}v|', w'} = - 0.35$$

$$R_{w',T'} = - 0.22$$

$$R_{|\underline{i}u + \underline{j}v|',T'} = + 0.25$$

/and

and for the partial correlation of $|\underline{i}u + \underline{j}v|'$ and T'

$$R_{|\underline{i}u + \underline{j}v|',T';w'} = + 0.18$$

The second example, run 3, 9th June, 1951, was analysed in one group of 200 readings spaced at about 1/4 sec.; the variables correlated are the departures of the individual readings from the means of 200. The results were

$$R_{|\underline{i}u + \underline{j}v|',w'} = - 0.43$$

$$R_{w',T'} = + 0.34$$

$$R_{|\underline{i}u + \underline{j}v|',T'} = - 0.31$$

$$R_{|\underline{i}u + \underline{j}v|',T';w'} = - 0.22$$

The partial correlations must be treated with great caution in view of the spurious element in the readings of w. Nevertheless it is clear that the dispersion of T cannot be explained by vertical motion only, and whilst there is no obvious reason for a correlation between horizontal velocity component and temperature the possibility should not be dismissed.

(iii) Anisotropy of the turbulence (intensity).

It is clear from the distributions that the turbulent energy is concentrated mainly in the fluctuations of the horizontal component. R.M.S. values of the components have been extracted for the two runs for which the correlation coefficients were worked, and for the whole series on 25th October, 1950. In the case of 25th October, 1950, the deviations are from 5 sec. means, the results being:-

All runs (1375 obs.) $\overline{|V|} = 305$ cm sec^{-1}, $\sigma_{|\underline{i}u+\underline{j}v|} = 48$ cm sec^{-1}, $\sigma_w = 20$ cm sec^{-1}

Run 6 (200 obs.) $\overline{|V|} = 270$ cm sec^{-1}, $\sigma_{|\underline{i}u+\underline{j}v|} = 60$ cm sec^{-1}, $\sigma_w = 21$ cm sec^{-1}

For 9th June, 1951, run 3, the deviations are from the mean of 200 readings in 50 secs. and

$$\overline{|V|} = 164 \text{ cm sec}^{-1}, \sigma_{|\underline{i}u+\underline{j}v|} = 49 \text{ cm sec}^{-1}, \sigma_w = 16.5 \text{ cm sec}^{-1}$$

In each case the energy carried by the fluctuations of the combined horizontal components is five to ten times that carried by the vertical component.

(iv) Anisotropy of the turbulence (periodicity).

An attempt was made to detect any notable periodicity in the vertical component of wind. If the nth of a series of readings spaced at time interval T be designated w_n, and the correlations $R_{w_n,w_{n+t/T}}$ $(=R(t))$ be formed for $t/T = 1, 2$, etc., then the intensity, i.e. (amplitude)^2, of the component of period $\varkappa$ is proportional to $\int R(t) \cos 2\pi\varkappa t \, dt$.
The range of $\varkappa$ which can be covered is clearly limited by the spacing and duration of the observations. Qualitative information on periodicities can in certain cases be obtained by inspection of the R(t) curve, without computing the Fourier integral. The serial correlations have been computed in two cases for the vertical wind and in one case for the horizontal component of turbulence. Fig. 8 shows the serial correlation of w and its Fourier transform for run 3, 25th October, 1950. Fig. 9 shows the serial correlation of w and its Fourier transform and the start of the serial correlation curve for $|\underline{i}u + \underline{j}v|$ for run 3, 9th June, 1951. The peaks in the "spectra" of w^2 are probably greatly exaggerated by the exclusion of higher frequencies, partly by the instrumental cut off and partly by the choice of a longer separation of readings than the instrument would allow. The most striking feature is the difference in the

serial correlations of the horizontal and vertical components, showing, as is indeed clear from the original records, that the fluctuations of vertical component are much more rapid than those of horizontal component. It is nevertheless equally clear, from both original records and Fig. 9, that there is a high frequency variation of horizontal component, of amplitude not greatly different from that of the vertical component, and further investigation may well show that the motion is composed of a small scale, high frequency, approximately isotropic component, superposed on slower larger scale fluctuations whose energy is concentrated in the horizontal components.

8. Future work.

We propose to concentrate on establishing the cause of the errors in the heat fluxes obtained from the fluctuation measurements. In the first place the present apparatus will be used at a height of 100-200cm, to seek confirmation of Swinbank's observation that the true value of the heat flux is obtained at this height. We can only hope to do this very approximately on the small site at Kew. Secondly work is in hand to produce an instrument of time constant about 0.01 sec., and subsequently to reduce the dimensions of the anemometer and thermometer. This apparatus will be used at a height of 25cm.

At present no effort is being made to mechanise the heat flux computations, a development necessary to make the method suitable for routine use. The difficulties in the way of mechanisation are such that it seems more profitable to make the computations "by hand" in a few cases, and examine whether the results can be used to justify the "Bowen ratio", "hydrodynamic", or other comparatively simple semi-empirical method of flux determination.

Acknowledgement.

We have received help from a number of staff members and visitors at Kew Observatory, notably Mr. R.H. Eldridge who made most of the profile measurements, and Miss E. Greaves who did much of the numerical work in the early stages.

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G.336578/PN/7/52/40

T A

Summary

Date	Time	Observations at 25 cm. Temp. Taken	Temp. Reduced	Wind Taken	Wind Reduced
30 Aug. 50	1445	0	0	1	1
14 Sept 50	1430	1	1	1	1
25 Oct. 50	1645	7	7	7	7
11 Dec. 50	1715	5	5	5	5
24 Apr. 51	1530	6	2	6	2
2 June 51	1115	7	7	7	7
6 June 51	1130	7	1	7	1
9 June 51	1030	8	1	8	1
2 July 51	1545	3	3	3	3
23 Oct. 51	1645	7	6	7	6

* Zero gradient

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TABLE II

Heat and momentum fluxes and eddy-diffusion coefficients.

1	2	3	4	5	6	7	8	9	10	11	12
						Estimated heat flux $mw.cm^{-2}$			Estimated momentum flux. $dyne\ cm^{-2}$		
							$-\rho c_p\left(\frac{\partial T}{\partial z}+\Gamma\right)\left(k^2z^2\frac{\partial u}{\partial z}\right)$		$-k^2z^2\left(\frac{\partial u}{\partial z}\right)^2$	$\frac{K}{\overline{w'T'}/\partial T/\partial z+\Gamma}$	$\frac{N}{\overline{w'\lvert iu+jv\rvert'}/\partial u/\partial z}$
Date	Type	No of runs analysed	$\overline{w}$ $cm.sec^{-1}$	$\overline{\rho c_p w'T'}$ $mw.cm^{-2}$	$\overline{\rho.w'\lvert iu+jv\rvert'}$ $dyne\ cm^{-2}$	By Bowen Ratio	z=25cm	z=10cm	z = 25 cm	cm^2sec^{-1}	cm^2sec^{-1}
25th Oct. 50	Inversion	7	-1	-4.2 ± 1	$-42\pm28\times10^{-2}$	-9.2 ± 3	-6	-	-120×10^{-2}	200	140
11th Dec. 50	Inversion	5	-5	-1.7 ± 0.3	$-5\pm3\times10^{-2}$	-5.5	-2		-4×10^{-2}	60	70
24th Apr. 51	Isothermal	2	-2.5	-0.2 ± 1	-5×10^{-2}	0	-1×10^{-2}		-20×10^{-2}	-	40
2nd June 51	Lapse	7	-2	$+7.1\pm2$	$-25\pm18\times10^{-2}$	$+21\pm4$	+11	+11	-34×10^{-2}	120	120
6th June 51	Lapse	1	+3	+7.1	-10×10^{-2}	$+13.5\pm3$	+5	+11	-26×10^{-2}	170	60
9th June 51	Lapse	1	+3	+7.1	-42×10^{-2}	$+12.5\pm3$	+4	+10	-16×10^{-2}	190	310
23rd Oct. 51	Inversion	6	-0.2	-1.7 ± 0.3	$-2\pm2\times10^{-2}$	-5 ± 1	-4		-10×10^{-2}	30	20
2nd July 51	Lapse	3	-3	$+2.5\pm0.6$	$-6\pm3\times10^{-2}$	$+8\pm2$	+7	+8	$-29\times10^{+2}$	60	30

TABLE III

Analysis in terms of mean values over different periods. Each run comprises 200 observations at approx. 0.2 sec. intervals, combined in groups of n consecutive readings.

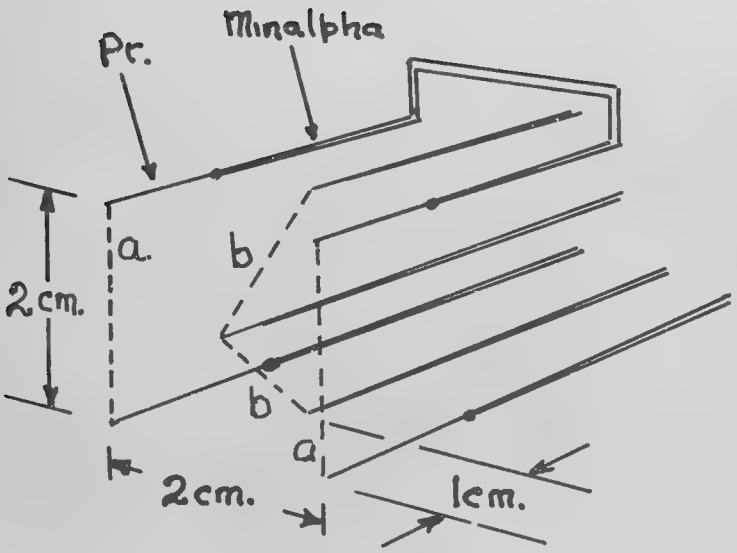
Date	No. of runs analysed	n	$\frac{1}{n^2}\sum_n w \sum_n T$	$\overline{(w - \frac{1}{n}\sum_n w)(T - \frac{1}{n}\sum_n T)}$	$\overline{(\lvert \underline{iu}+\underline{jv}\rvert - \frac{1}{n}\sum_n \lvert \underline{iu}+\underline{jv}\rvert)(w - \frac{1}{n}\sum_n w)}$
14 Sept 50*	1	200	-	+0.34	-0.07
		100	-	+0.35	-0.07
		50	-	+0.54	-0.17
		25	-	+0.46	-0.23
		10	-0.15	+0.58	-0.22
		5	-0.06	+0.46	-0.15
25 Oct. 50/	7	200	-	-0.11	-0.38
		25	-	-0.08	-0.30
11 Dec. 50**	5	200	-	-1.4	-40
		25	-	-0.9	-34
24 Apr. 51**	1	200	-	-1.1	-
		25	-	-1.2	-
6 June 51**	1	200	-	+5.6	-
		5	+2.0	-	-
23 Oct. 51**	1	200	-	-1.5	-
		25	-0.2	-1.3	-

* Units - Temperature arbitrary. wind ft. sec-1

/ Temperature °C wind ft. sec-1

** Temperature °C wind cm sec-1

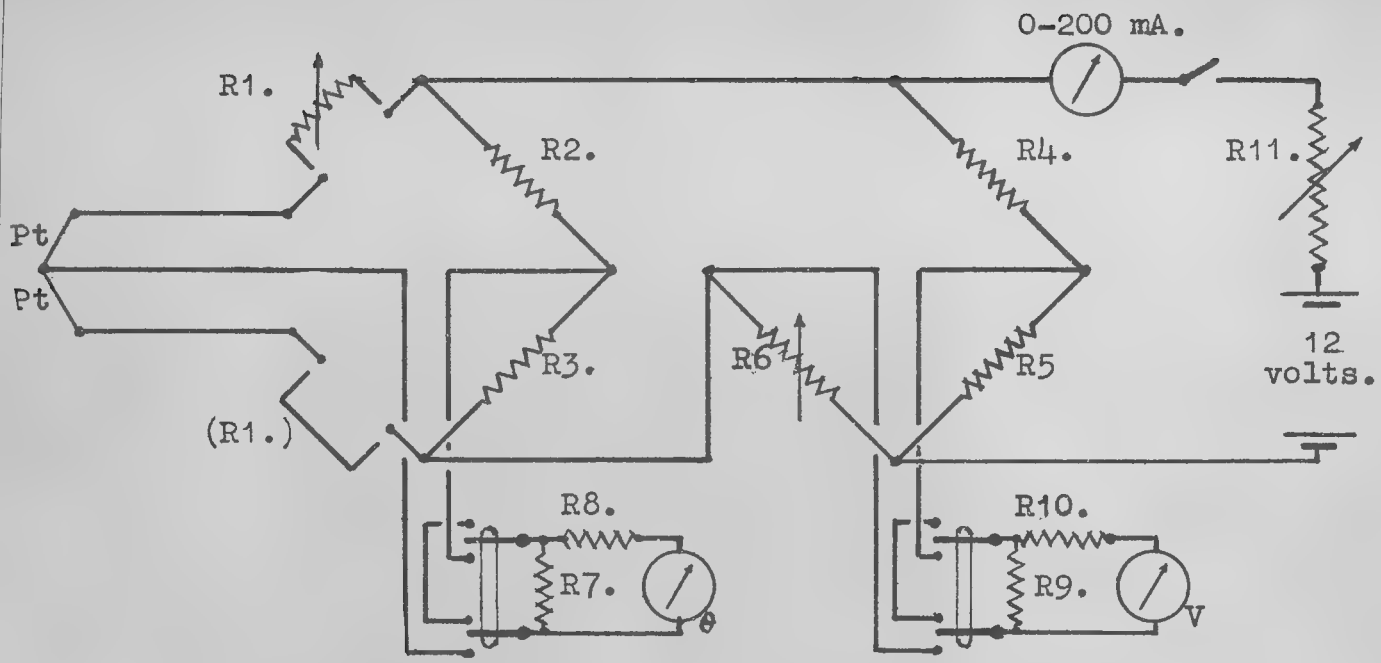
M.R.P. 740

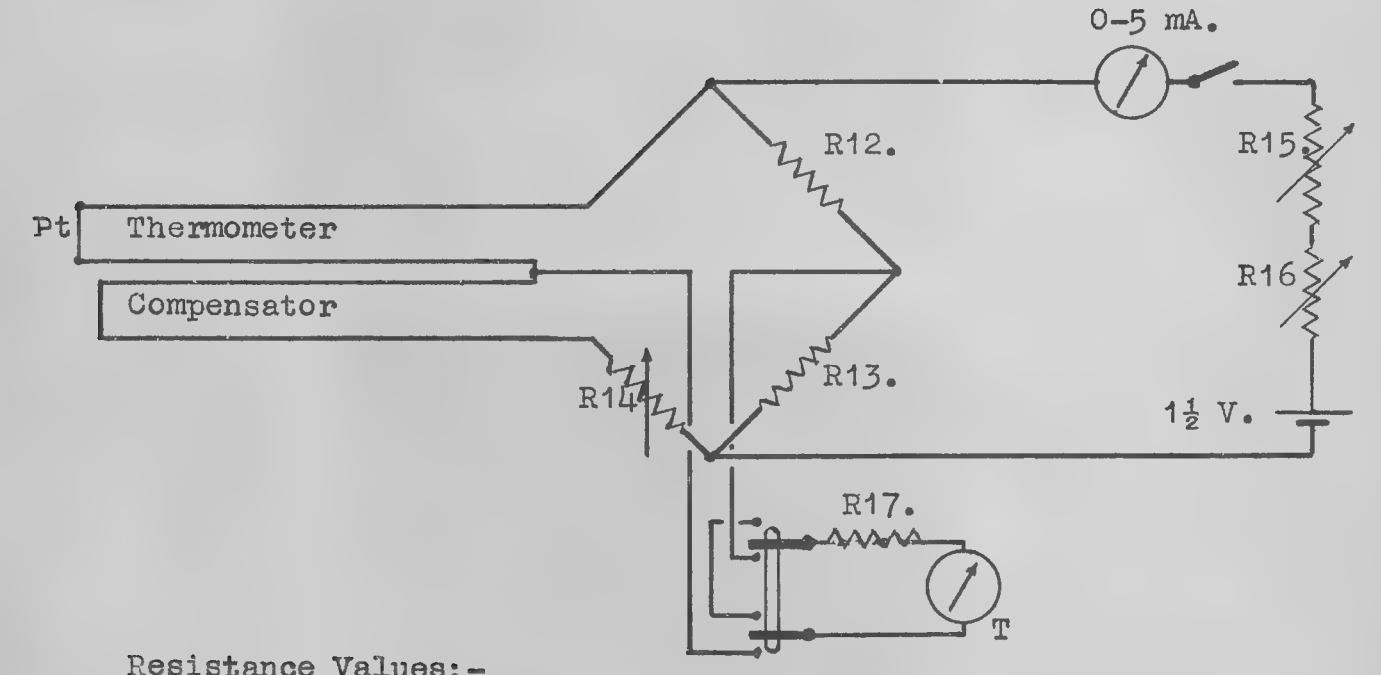


aa Thermometer wires
bb Anemometer wire
(Pt. 0·0005 in. diam)

Fig. 1. Head of anemometer - thermometer.

Fig; 2., Circuit. M.R.P. 740





Resistance Values:-

R1. Slide-wire	0-1 ohm.	R12,13	50 ohms.
R2,3,4,5	500 ohms.	R14. Decade	10-100
R6 Decade	10-100	..	1- 10
..	1- 10	..	0.1- 1.0
..	0.1- 1.0	R15	5000
Slide-wire	0.01- 0.1	R16	50
R7	5	R17	2070
R8	2650		
R9	1	Galvanometers:-	
R10	2300	●	158 ohms.
R11	0-115	"V"	150
		"T" (no shunt)	147

Fig. 3. Apparatus on site

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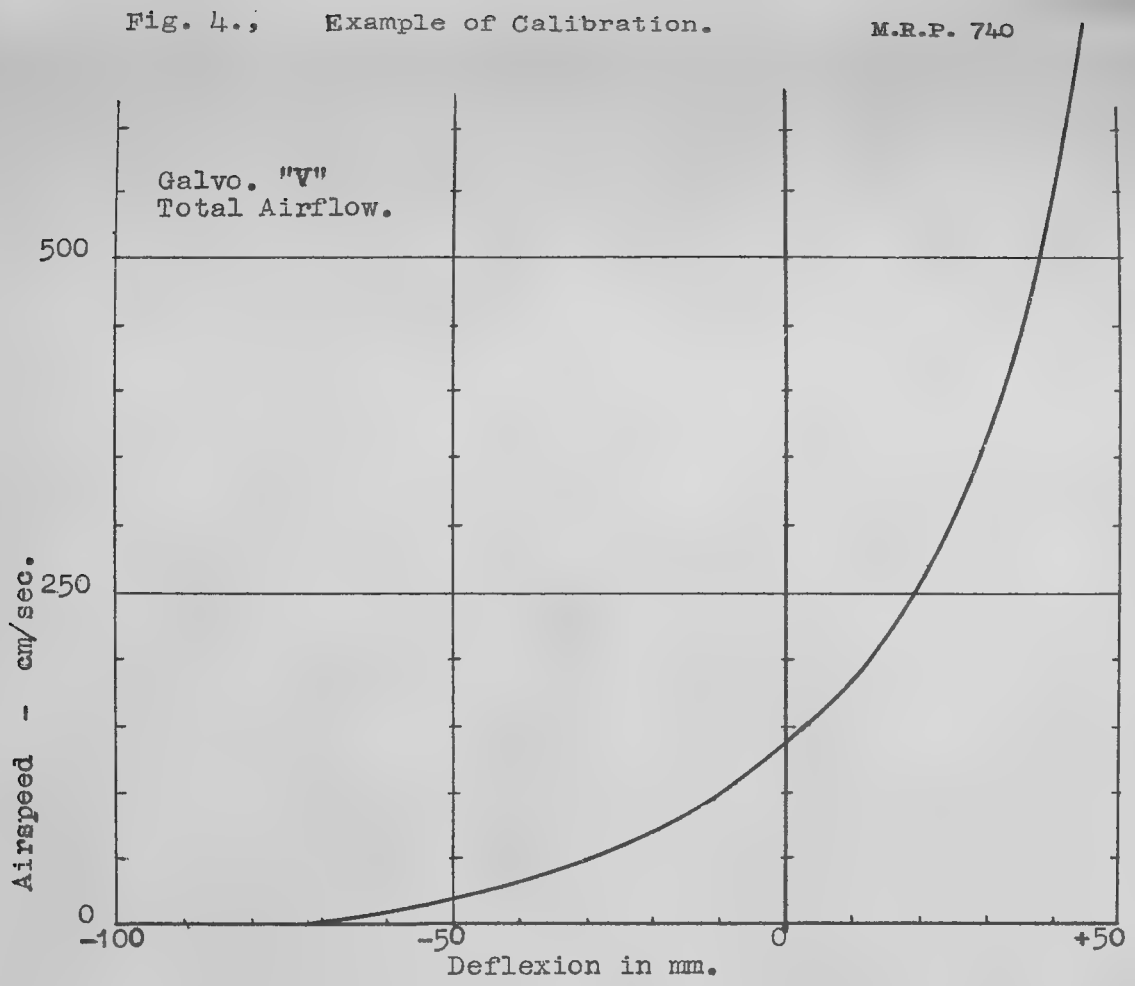
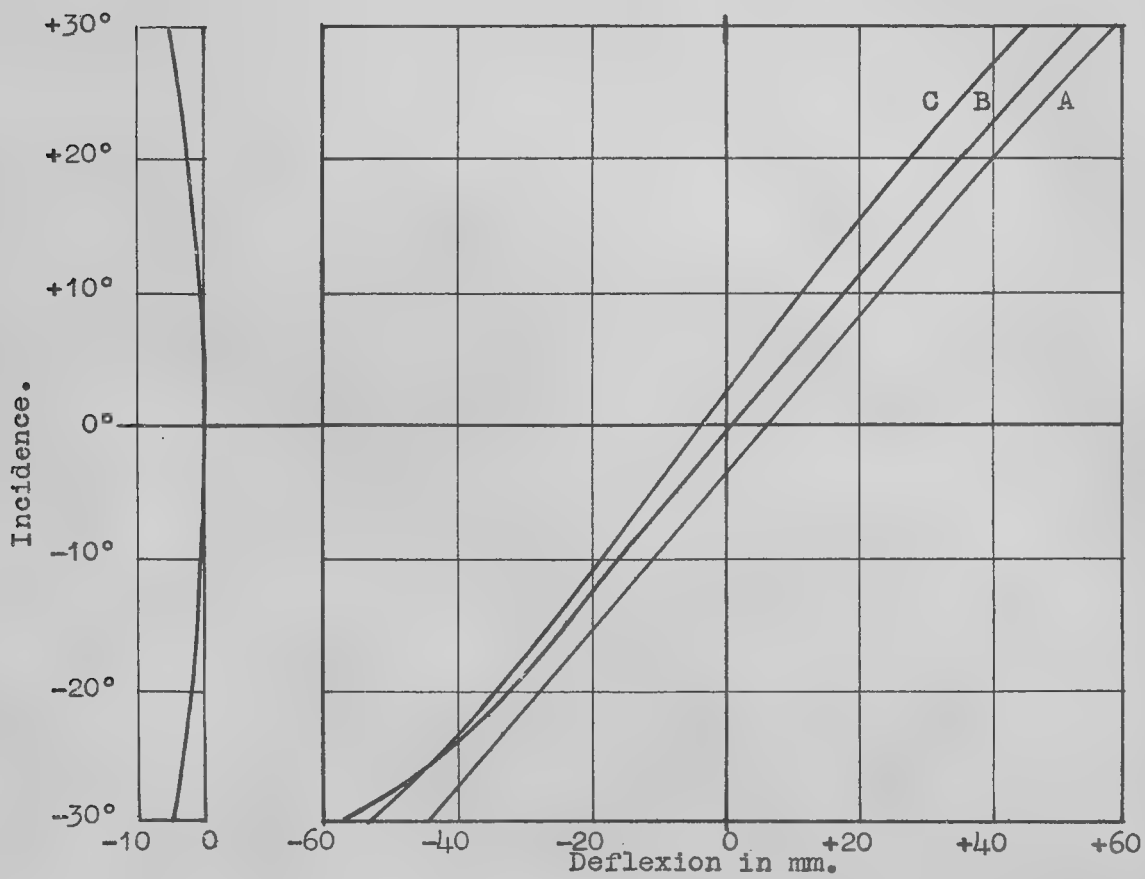
a. Thermometers for mean profiles

b. H.W.A. and resistance thermometer on mounting

Fig. 4., Example of Calibration. M.R.P. 740

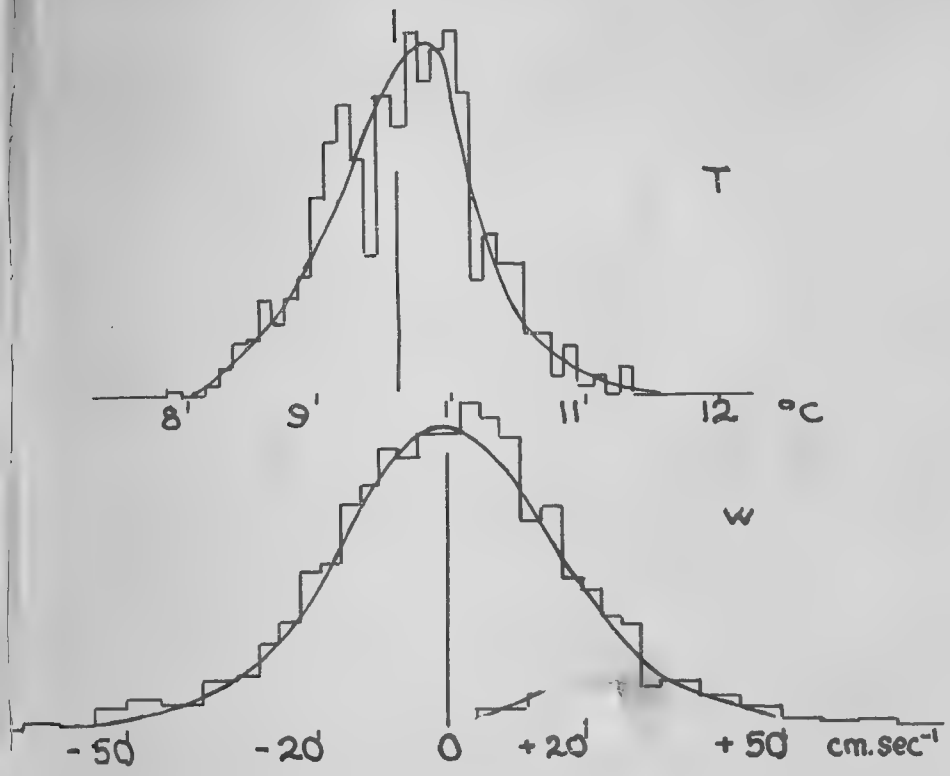




Left - Galvo "V" - Variation of indicated total airflow with incidence (virtually constant over the full range of airspeeds)

Right - Galvo "θ" - Incidence of airflow; selection of curves:-

A at 63 cm/sec.
B 150 .. ..
C 460 .. ..



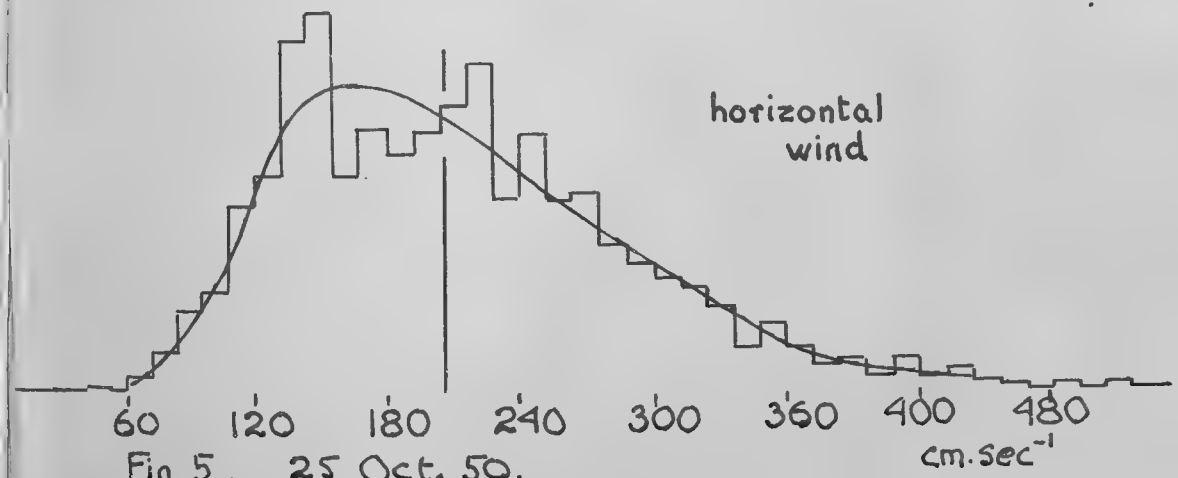
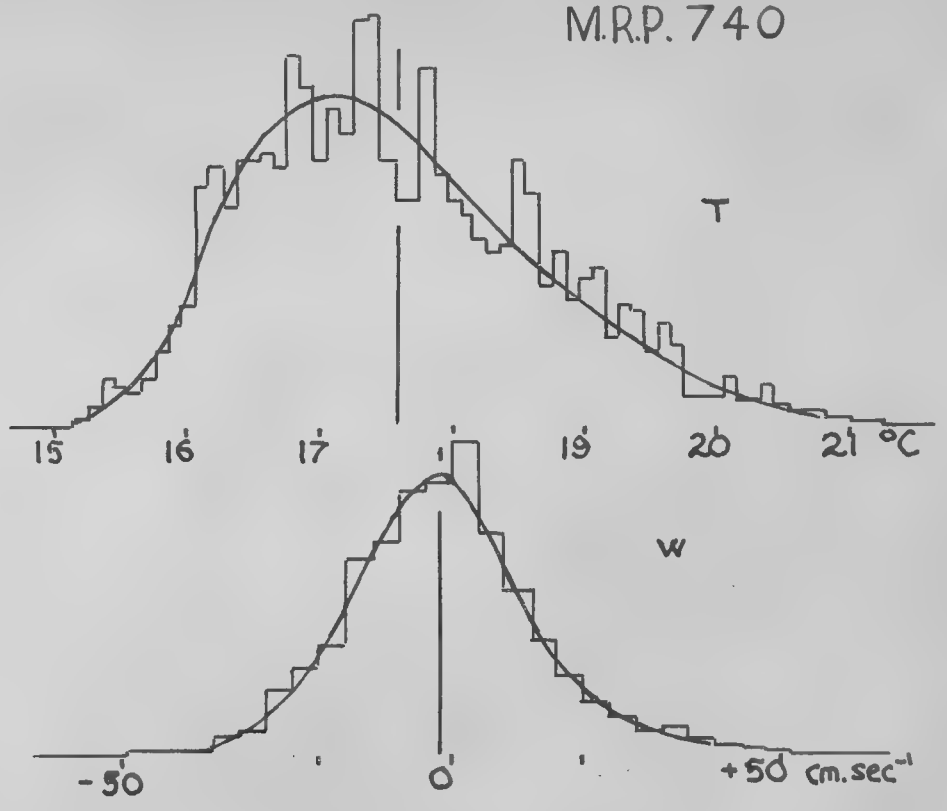


Fig 5. 25 Oct. 50.



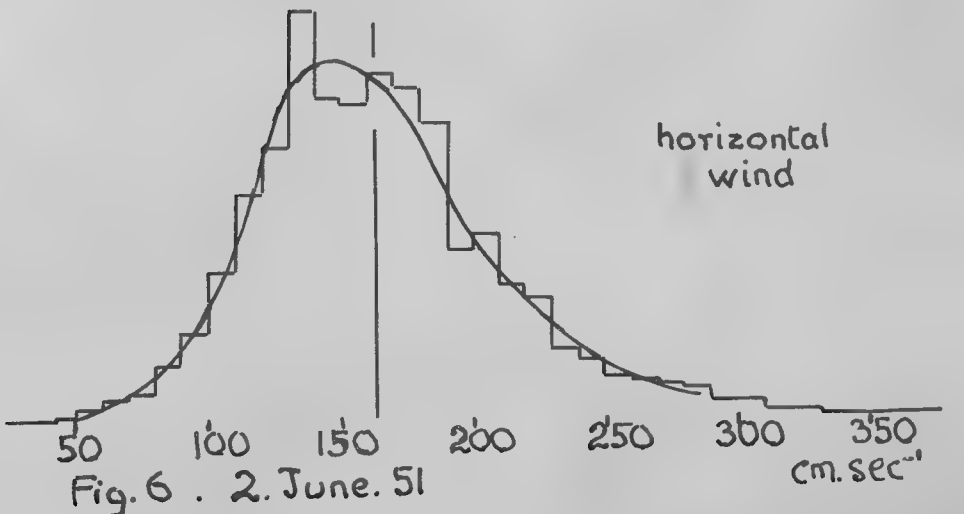


Fig. 6 . 2. June. 51

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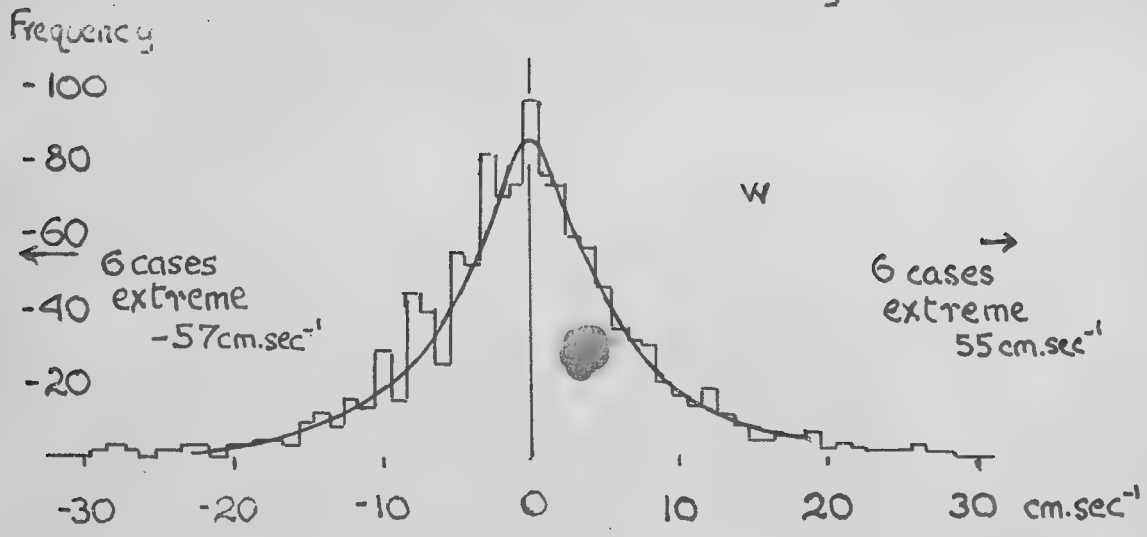
Fig 7 . 23 Oct. 51



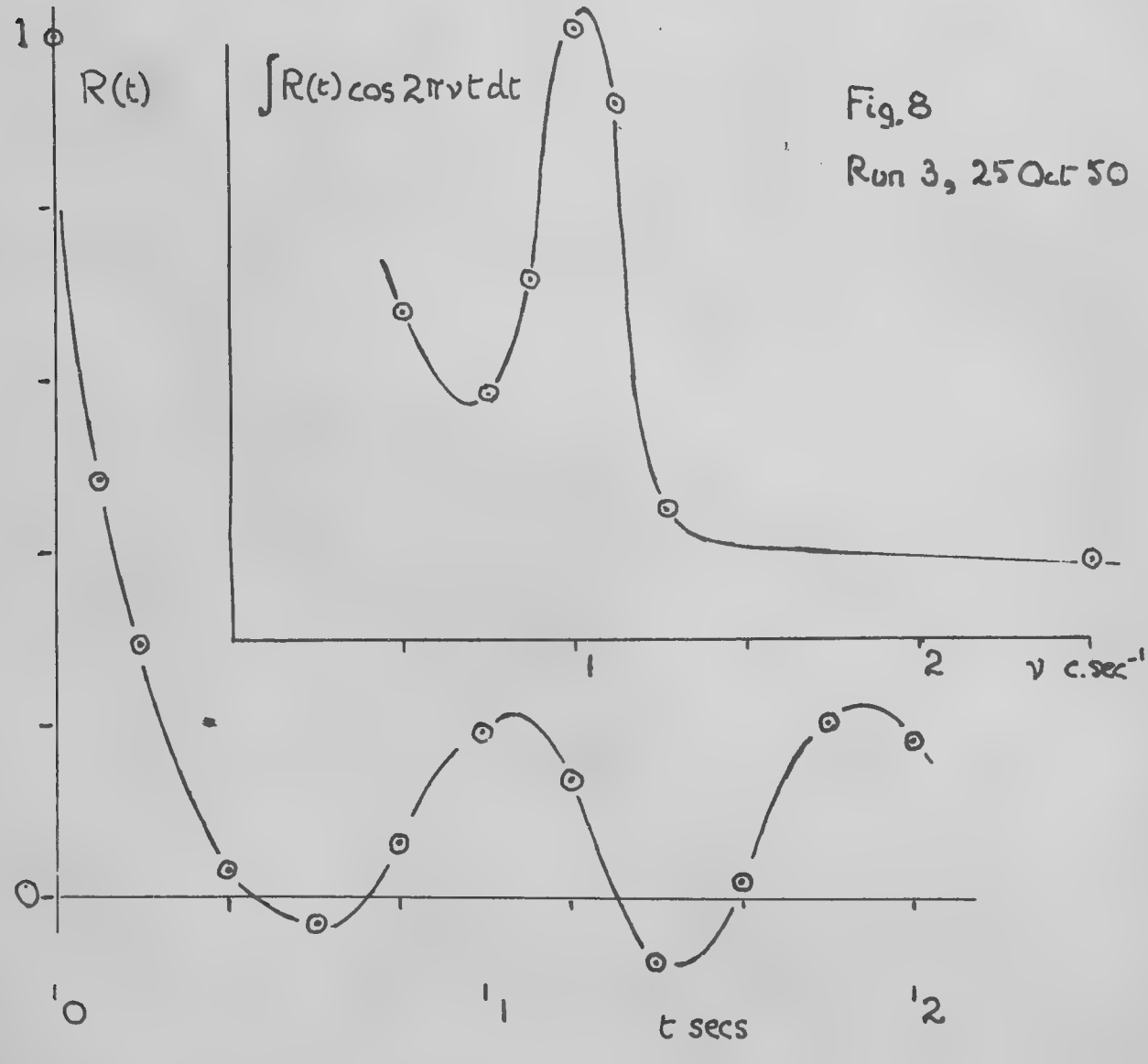


Fig. 8

Run 3, 25 Oct 50

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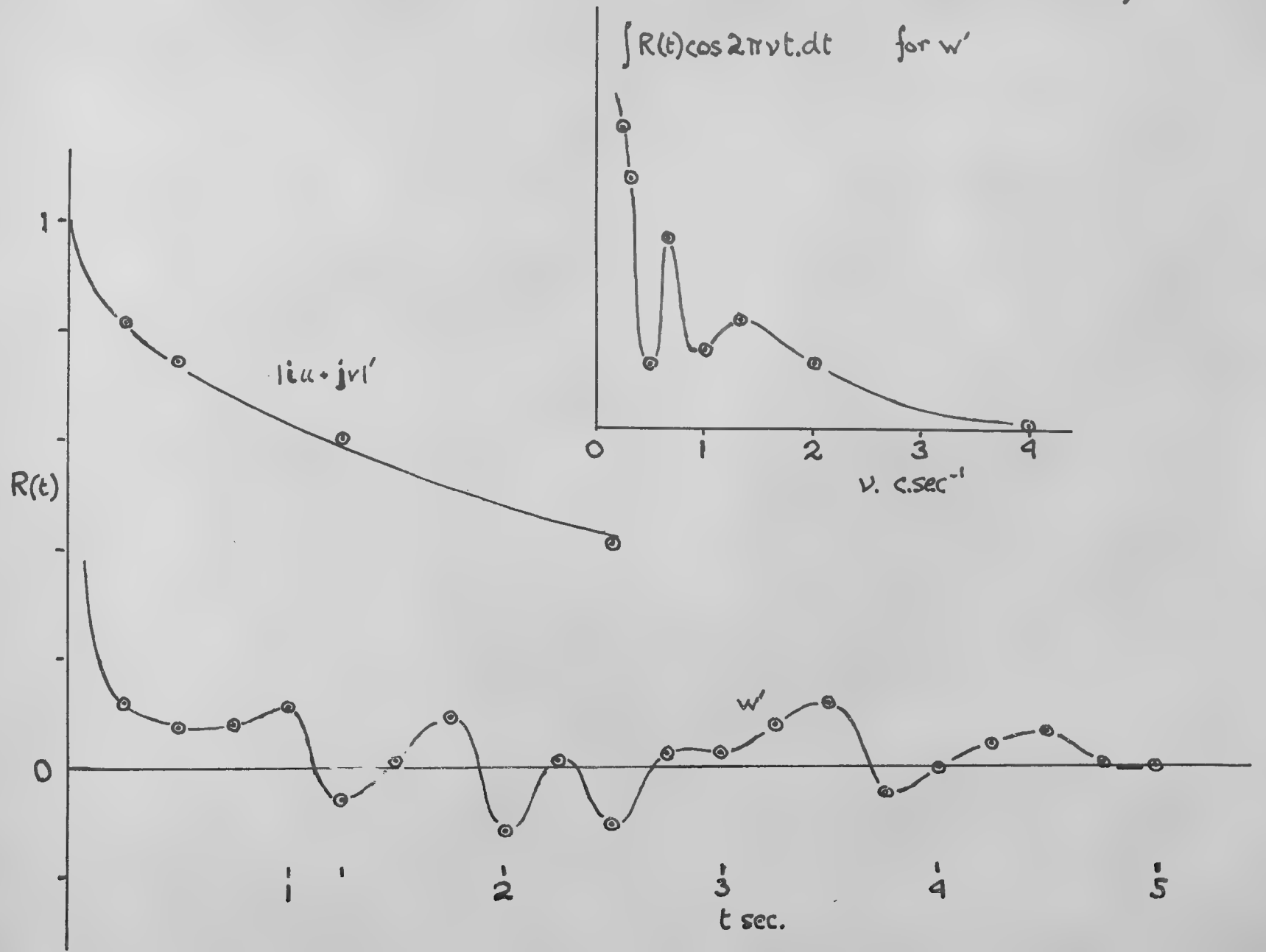
Fig 9.
Run 3, 9 June 51.



M.R.P. 742 (Revised)
S.C.II/113
8th August, 1952.

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A note on Night Cooling under Clear Skies

by

R.H. Pedlow (Meteorological Office)

1. Summary

It is shown that the discontinuity in the rate of fall of temperature on clear nights noted by Saunders (1) extends to at least 107 m. and that it is accompanied by discontinuities in the rate of change of vapour density up to 107 m. and in temperature lapse rate up to at least 47 m. Within the accuracy of the observations (at 20-minute intervals) the changes are simultaneous at all heights and it is thought that these are related to the breakdown of the laminar flow near the surface soon after sunset. The night minimum temperature at Rye is found to be higher, compared with the discontinuity temperature, than in the observations used by Saunders (2), and the discontinuity occurs earlier. It is suggested that these discrepancies may be due to the proximity of the sea.

2. Material used

Details have been given elsewhere (3) of the measurement near Rye, Sussex, of temperature and humidity at heights of 1.1, 15.2, 47.2 and 106.7 m. above the ground. Occasions during the period July, 1945 to June, 1948 were selected when there was not more than 3/10 cloud at 1500 G.M.T. and when there was no interruption of sunshine or night-sky camera records between then and midnight. Only 25 such occasions could be found when the temperature and humidity traces at all heights were complete; these fell, six in March, five in April, six in July, three in August and one each in May, June, September, October and November. Mean temperatures and humidities were extracted over 20-minute periods centred at 20-minute intervals between 1300 and 2300 G.M.T.

21 occasions showed a discontinuity of the Saunders type in the 1.1 m. temperature record. Of the remainder, three curves were too smooth for any discontinuity to be found, and the fourth showed a rapid succession of discontinuities. Some discontinuities were secondary minima followed by a temporary rise in temperature - as much as 2°F. in two cases. Examination of humidity curves gave further indications of advective effects, but it was decided that the only course was to retain all cases showing 1.1 m. temperature discontinuities. On the majority of occasions a temperature discontinuity could be seen at higher levels at the same time ±20 minutes, and often a humidity discontinuity too, though the greater irregularity of the humidity curves made recognition more difficult and less confident.

3. Numerical results

Saunders (2) gives for Northolt the temperature (T_r) at which the screen-level discontinuity occurs as:

/T_r

$$T_r = \tfrac{1}{2}(T_{max} + T_d) - T_e$$

where T_e = 4.0 (S.D. 0.88°F.) with an air-mass inversion below 900 mb.

and T_e = 0.6 (S.D. 1.04°F.) with no such inversion;

T_d = dew point at time of maximum temperature (T_{max}).

Of the 21 cases at Rye, eight showed the inversion and 13 did not. For these two classes, the mean values of T_e were 6.3 (S.D. 3.3°F.) and 5.7 (S.D. 2.6°F.) respectively. The marked discrepancy from Saunders' values and the wider scatter may well be due to the fact that in 16 of the cases the air was drawn from a seaward or coastwise direction during the cooling period.

Part of the curve given by Saunders (2) for the time of discontinuity is shown at Fig. 1, with the times of discontinuities observed at Rye inserted in the appropriate places. It will be seen that the general shape of the curve is followed, though at a rather lower level - i.e., earlier hour. The few occasions do not warrant a more quantitative statement on this point.

A curve for the prediction of the night minimum temperature from T_r (similar to Saunders (2), Fig. 3) was prepared from the data for those nights which fell within the dates laid down by Saunders. Nights from the midsummer period were used after 'correction' of the minimum temperature from Saunders (2), Fig. 5. T_r was calculated from the previously quoted formula using the appropriate mean value of T_e = 6.3 or 5.7. It was not possible to separate lines equivalent to AB and CD in the original. The resultant curve, with Saunders' AB and CD for comparison, is at Fig. 2. It will be seen that the minimum temperature at Rye is 5° to 7°F. higher than would be indicated by AB: this may well be due to the proximity of the sea.

4. Vertical structure

When several occasions fell in one month, mean temperature and humidity curves were drawn. Fig. 3 shows as examples the curves for six cases in March and four cases in July: the mean times of the 1.1m. temperature discontinuities are indicated. Similar mean curves of temperature and humidity differences between levels were prepared, but none of these led to any clear conclusions. It was noticed, however, that for example the mean vapour density at 1.1 m. in the July cases rose by only 10 mgm/cu.m. during the 20 minutes following the mean time of discontinuity; whereas the average rise during the 20-minute periods following each individual discontinuity was 220 mgm/cu.m. Instead of mean values of the various elements at the same times G.M.T., mean values were therefore found at times equi-distant from the individual times of discontinuity in the 1.1.m. temperature curve (Table I). These figures are shown graphically in the very significant curves of Fig. 4.

It will be seen that the rate of fall of temperature is suddenly decreased, not only near the ground but at all heights up to 107 m., and the rate of increase of the temperature inversion is suddenly checked up to 47 m. In the range 107 - 47 m. there is no significant alteration in the growth of the inversion.

The sudden rise in vapour density at all heights is of particular interest. It would appear that at a time when the ground is cooling, this rise could be brought about by downward transport of water vapour from aloft which might in turn occur with a humidity inversion, and Saunders considers that this humidity inversion could not be set up without the previous deposition of dew. Fig. 4D, however, shows an unequivocal inversion only in the 15 - 1 m. range; at higher levels the decrease in hydrolapse ceases just before inversion is established (except momentarily in the 107 - 47 m. range). This may be ascribable to uncancelled advection effects and possibly small instrumental errors. There is inevitably some uncertainty in the measurement of the small

/humidity

humidity differences involved: there is no doubt about the rise in vapour density at each level, and this certainly occurs within a few minutes of the discontinuity in the 1.1 m. temperature. Inspection of Fig. 4 will convince one that the margin of ±20 minutes is too generous an allowance; the 1, 15 and 47 m. discontinuities in the change of vapour density probably occur during the 10 minutes preceding the time selected as that of the temperature-change discontinuity, and that at 107 m. during the period 10 minutes on either side of that time. Fig. 4A indicates that, if anything, the 1.1 m. temperature discontinuity occurs, on the average, a few minutes before the selected observation-time, and it is probably that all the discontinuities occur within 10 minutes.

In view of the nearly simultaneous changes of water vapour and temperature at all heights however it is probable that the downward transport of water vapour could be more readily accounted for by the onset of turbulence soon after sunset due to the breakdown of the laminar flow when the vertical gradient of wind speed exceeds the limit of stability set by the temperature lapse rate.

It is unfortunate that the wind observations at Rye do not enable the Richardson number to be evaluated for these occasions in order to confirm this hypothesis, but on 14 out of 21 occasions the pressure tube anemograms gave strong support to the supposition of breakdown of laminar flow within 20 minutes of the time of discontinuity. On a further 6 occasions the breakdown was possible but not marked, and on one occasion only did it appear quite unlikely.

The irregularity in timing noted above (para. 3 and Fig. 1) does much to smooth out the discontinuities from the mean daily curves for each month and for clear summer and winter days previously prepared at Rye (3). Nevertheless, a sharp reduction in the rate of growth of temperature inversions is noticeable shortly after sunset in almost every month (sometimes even in the 107 - 47 m. layer) and a minimum of vapour density occurs at some or all levels up to 107 m. soon after sunset in eight of the twelve months and in the curves for mean clear days in both summer and winter.

References

(1) Saunders W.E. — Night cooling under clear skies. Quart. J.R. met. Soc., London, 75, 1949, p. 154.

(2) Saunders W.E. — Some further aspects of night cooling under clear skies. M.R.P. 687, 1951.

(3) Best A.C., Knighting E., Pedlow R.H. and Stormonth K. — Temperature and humidity gradients in the first hundred metres at Rye. M.R.P. 546, 1950.

/Table I.

Table I: Mean variation of temperature and vapour density at four levels as recorded at regular intervals before and after the various observed times (D-hr.) of the 1.1 m. temperature discontinuities

	Time in minutes relative to D-hr.															
	-180	-160	-140	-120	-100	-80	-60	-40	-20	0	+20	+40	+60	+80	+100	+120
1.1m. Temperature (°F)	58.5	57.7	56.9	55.8	54.1	52.5	50.9	49.5	47.9	46.7	46.6	46.2	45.9	45.5	45.1	44.9
15.2m. "	58.3	57.6	57.0	56.2	54.9	53.7	52.6	51.9	50.7	49.9	49.9	49.5	49.3	48.8	48.6	48.5
47.2m. "	57.8	57.1	56.7	56.0	54.9	54.1	53.4	52.8	52.0	51.5	51.5	51.0	50.9	50.6	50.3	50.3
106.7m. "	57.9	57.3	56.9	56.3	55.3	54.4	53.8	53.1	52.4	51.9	52.0	51.6	51.6	51.4	51.3	51.3
1.1m. vapour density (mgm/m^3)	7550	7490	7490	7410	7340	7220	7050	6920	6740	6640	6810	6840	6860	6830	6820	6860
15.2m " "	7190	7160	7150	7090	7060	6990	6880	6820	6770	6770	7000	7070	7110	7100	7160	7230
47.2m " "	7030	6970	6980	6950	6900	6830	6720	6730	6700	6700	6920	6990	7070	7100	7130	7220
106.7m. " "	6850	6730	6770	6730	6660	6650	6600	6660	6680	6720	6880	6900	6980	7020	6990	7080

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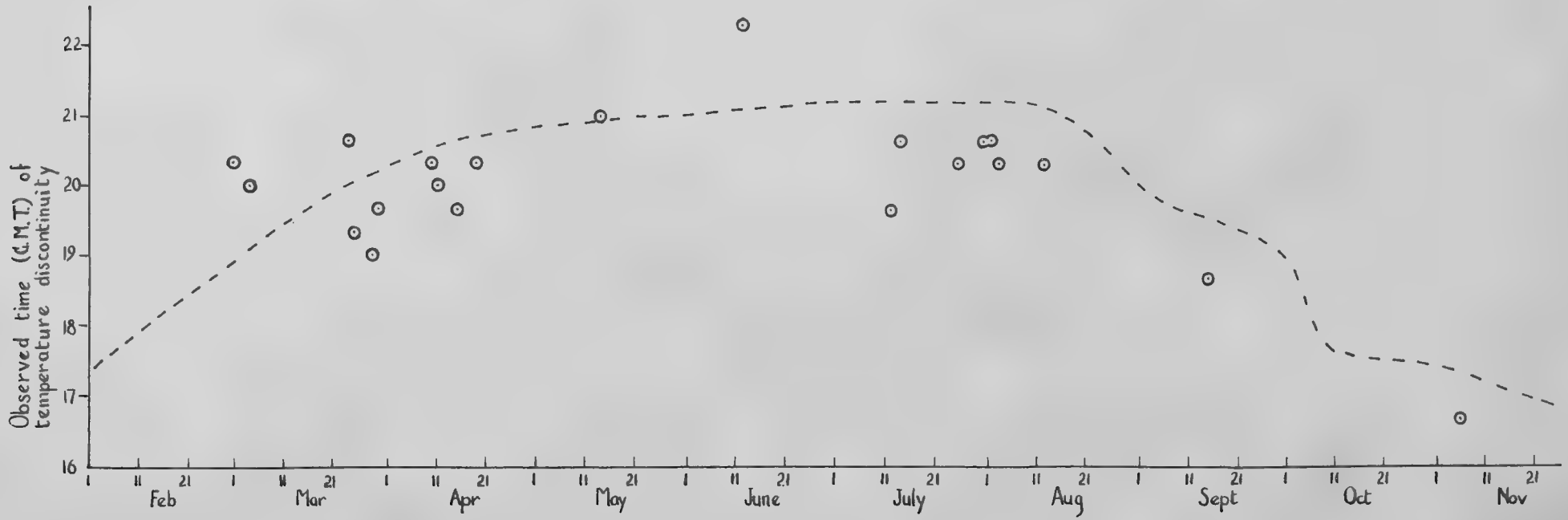
Fig 1: Observed times of temperature discontinuity compared with curve of mean times shown in Saunders (2).



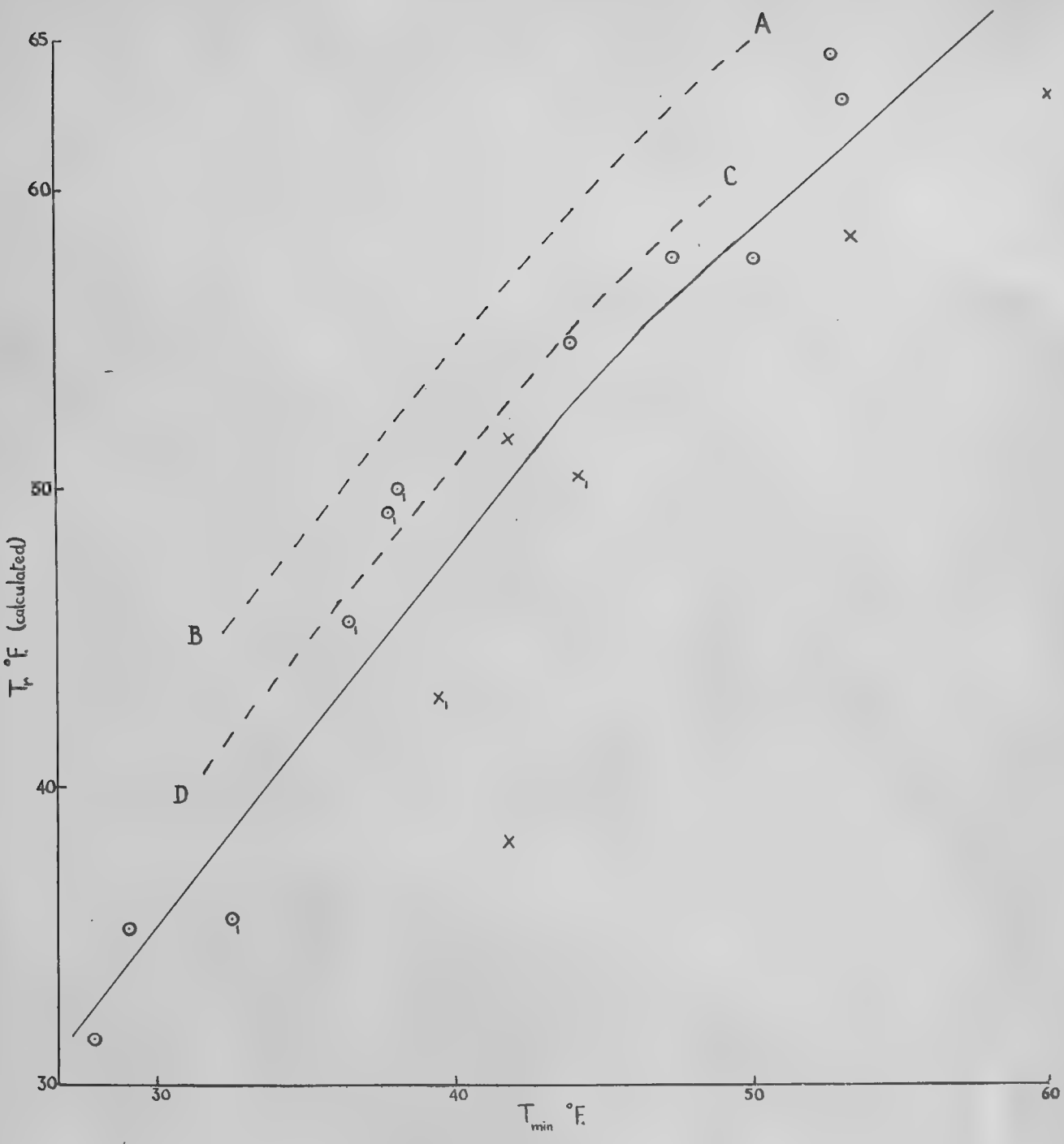


Fig. 2: Relation between discontinuity temperature and night minimum temperature

Spring and summer occasions are shown: midsummer values of T_{min} are 'corrected' from Fig. 5 of Saunders (2)

× = calm
$×_1$ = light surface wind } with air-mass inversion below 900 mb.

⊙ = calm
$⊙_1$ = light surface wind } with no such inversion.

Lines AB and CD are from Fig. 3 of Saunders (2) where they are the best 'calm' and 'light wind' curves respectively.

Fig. 3A: Variation of temperature and vapour density at four levels — mean of six occasions in MARCH

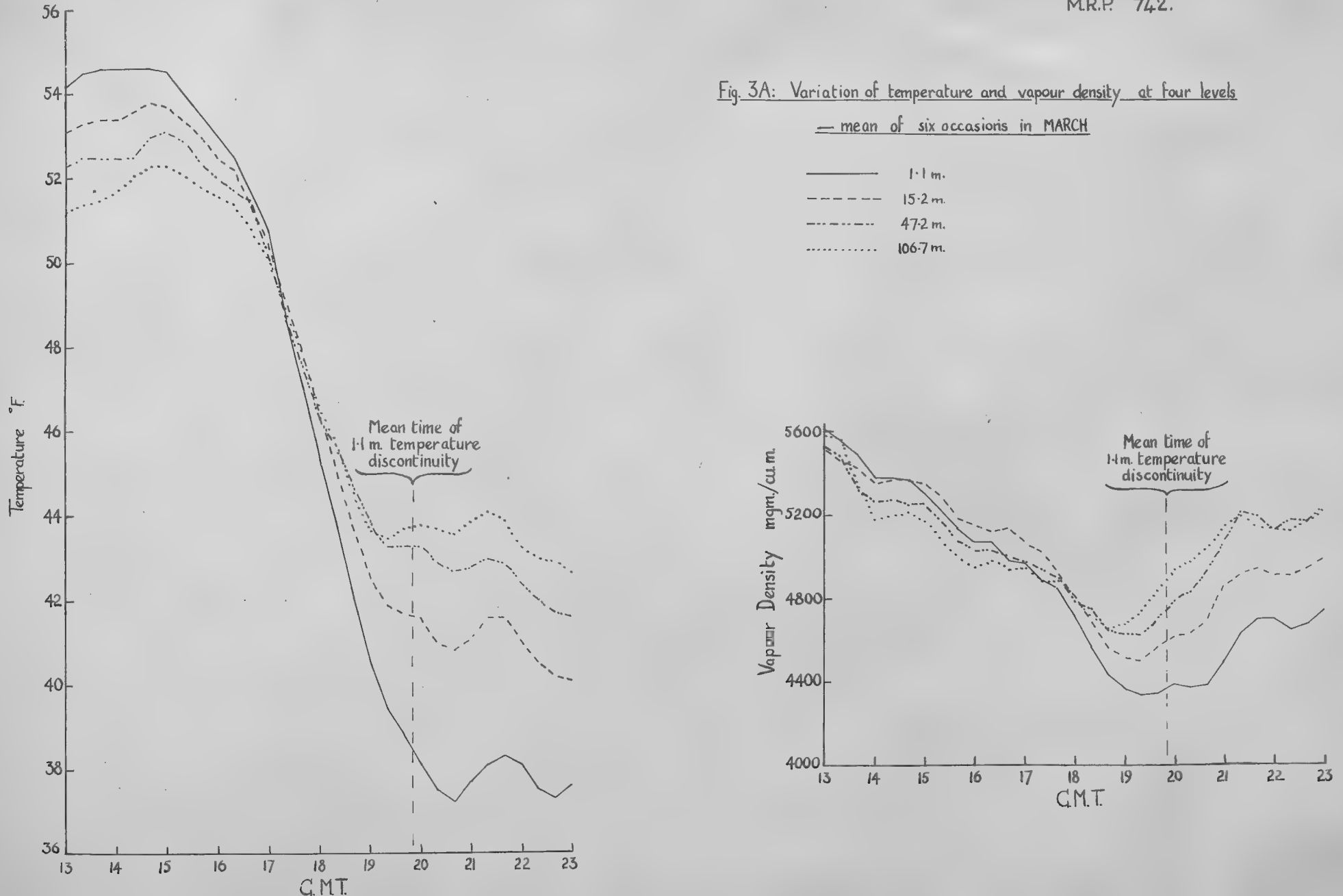
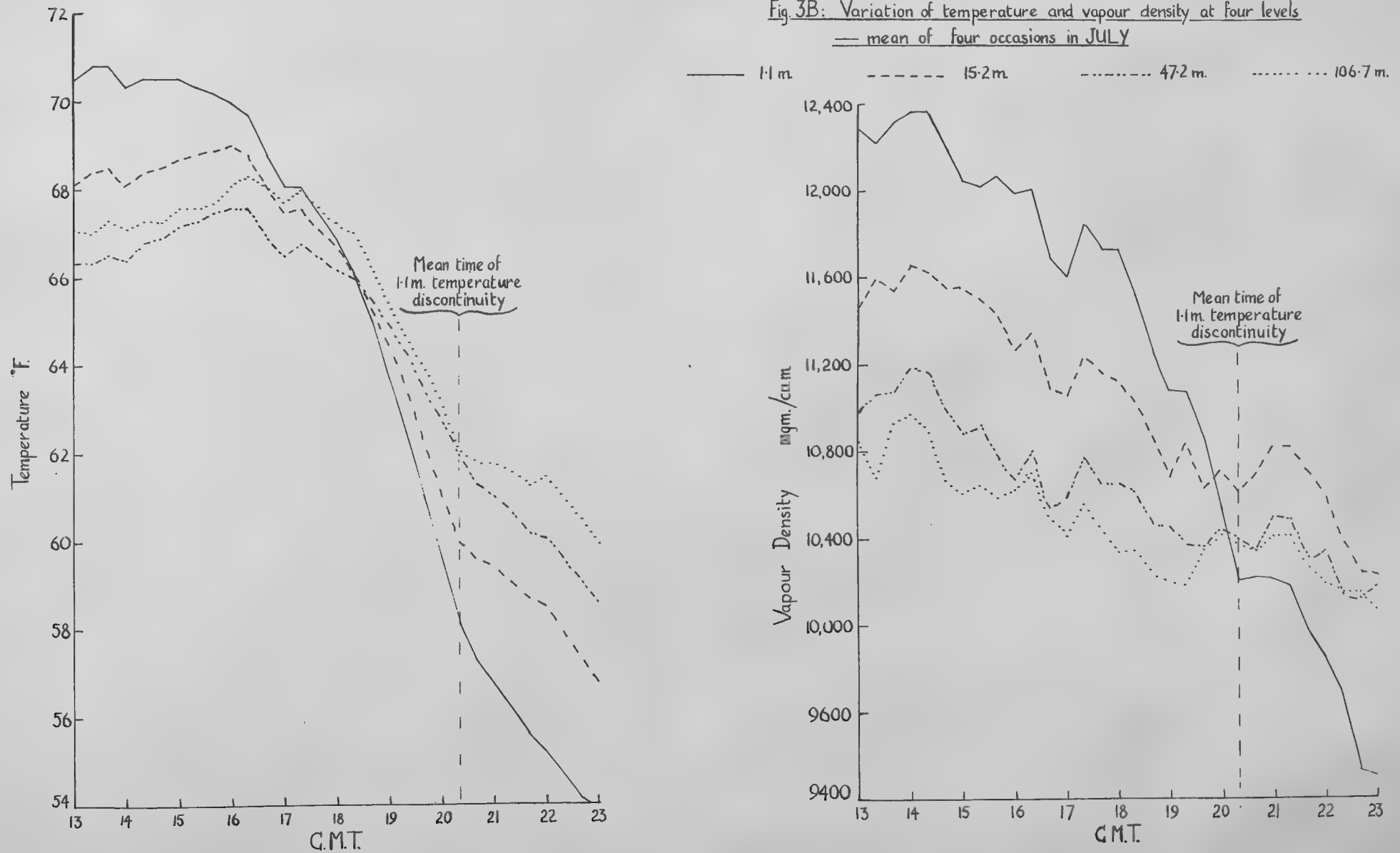


Fig. 3B: Variation of temperature and vapour density at four levels — mean of four occasions in JULY



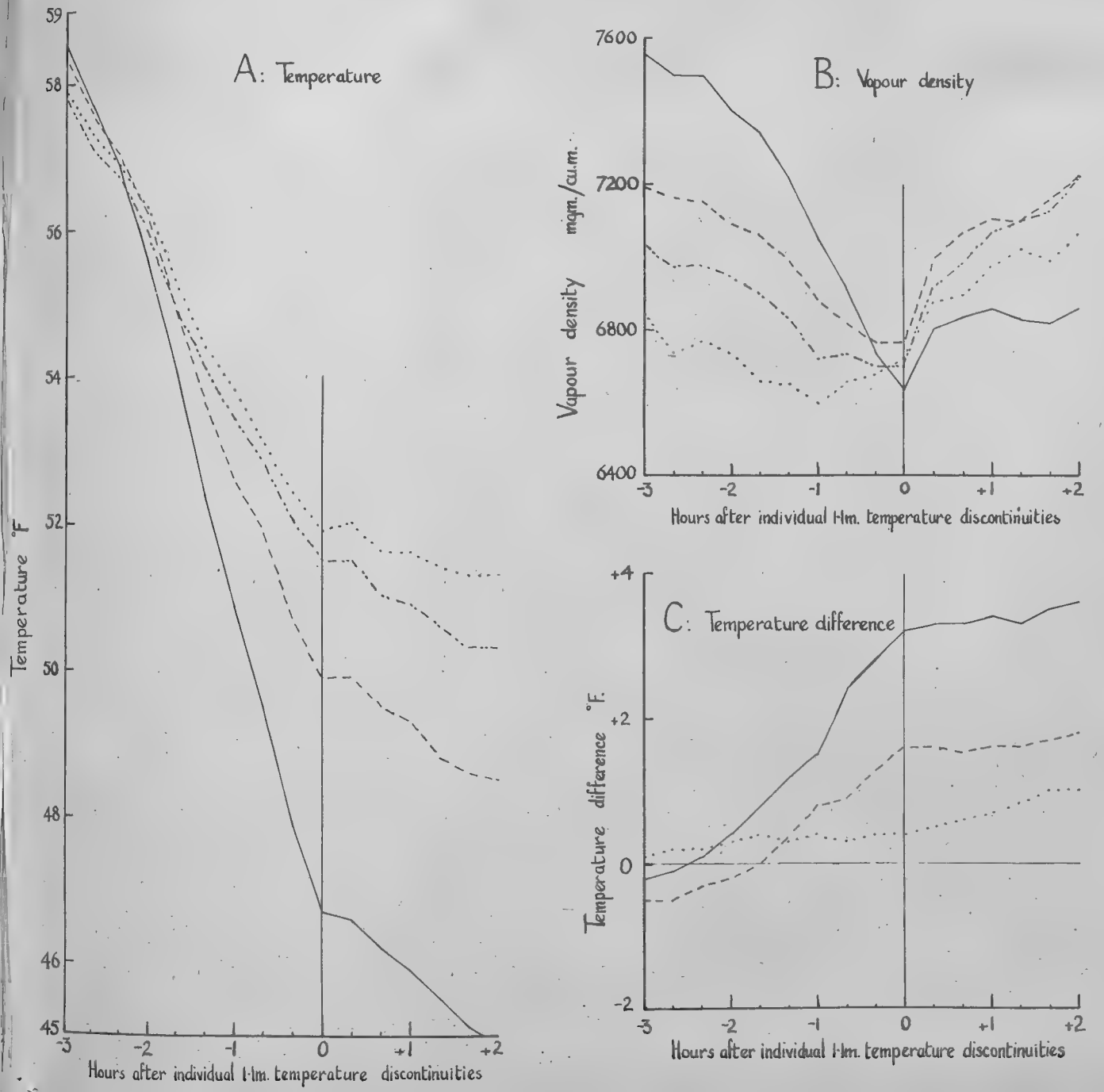


Fig. 4: Mean variation of temperature and vapour density at four levels, and of differences between levels, over periods measured from times of individual discontinuities in the cooling rate at 1·1m.

(Mean of 21 cases).

In A and B:

- 1·1 m.
- 15·2 m.
- 47·2 m.
- 106·7 m.

In C and D:

- 15·2 – 1·1 m.
- 47·2 – 15·2 m.
- 106·7 – 47·2 m.

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The jet streams over the British Isles during 14-18 June, 1951

by
R. Murray (Meteorological Office).

Summary.

It is shown that the strong upper flow over the British Isles during the period 14-18 June, 1951, which occurred on the forward side of a major long-wave trough, was associated with two distinct jet streams, although both existed together for only a short time. Both jet streams occurred in association with the same generalised frontal zone of a cold front, but it was not possible to analyse it in accordance with a simple frontal model of the Bjerknes type.

The lateral movement of the jet streams is shown to be roughly related to the surface geostrophic advection perpendicular to the jet stream axis: dynamical explanations for deviations from the geostrophic advection are suggested.

The nature of the tropopause and variations in the height and intensity of the jet stream centres are discussed.

1. Introduction.

The favourable location, orientation and persistence of the strong SW air flow in the upper troposphere over the British Isles during 14-18 June, 1951 made this period particularly suitable for a detailed space and time analysis of the wind field and its association with other synoptic features. The SW flow occurred on the forward side of an upper slow-moving cold trough (in the rear of a cold front). Mere inspection of the upper charts shows that a single wind maximum existed on 14, 16, 17 and 18 June, but the position was confusing on 15 June when considerable variations in wind speed were shown by several of the aerological stations. This report describes the pattern of behaviour of the wind field and shows that two distinct jet streams actually existed for a time on 15 June, 1951.

The period analysed offered a good opportunity for examination of certain specific questions connected with jet streams. One of these is concerned with relating the lateral movement of a jet stream to other factors in the synoptic field. Another is concerned with the nature of the tropopause near and especially on the low pressure side of a jet stream. The variation of height of a jet stream centre along its axis is yet another question requiring clarification. This study attempts to throw some light on these problems.

2. Synoptic situation.

The general synoptic developments can be followed from Figs.1 to 5 which show the surface and 500 mb maps daily at 15 G.M.T. from 14 to 18 June, 1951. The main features are summarised below.

A depression moved NE some 200 miles off NW Scotland on 14 June. Its cold front crossed England as a weak katafront (Sansom, 1951), giving only drizzle here and there, and reached North France about midnight of 14 June; for the next four days the surface front over France and Germany was rather weak and illdefined but broadly associated with a belt of shallow lows and thundery outbreaks rather typical of summer weather over the Continent. A minor trough swept east across Scotland some 200-300 miles behind the primary front on 14 June, but this trough had little or no obvious effect on either the pressure or the weather of England. Meanwhile, during 14 June, cyclogenesis occurred some 200 miles south of Iceland

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variation and the increase of speed; for instance, of the 36 cases of winds below 40 knots, 18 have a change less than 2.9 knots in 10 minutes; of the 31 of winds between 41 and 60, 15 have a change less than 3.1 knots in 10 minutes; whereas of the 19 cases with winds over 60 knots, 9 have a change less than 4.5 knots in 10 minutes.

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in the elongated trough extension from the depression off NW Scotland. This new centre, after a period of about 24 hrs of consolidation in situ, moved 300 miles southwards, then turned east and reached about NW Ireland at midnight on 16 June; thereafter it travelled steadily NE across North Scotland. A cyclonic vortex also formed at 300 mb on 14 June, and subsequently followed much the same track. The cold trough to the south of the upper low moved east at the average rate of 5 degrees longitude per day, reaching the British Isles on 18 June in a form sharper than existed previously. This cold trough is essentially a 'long-wave' feature in the westerlies.

3. Movement and behaviour of the jet streams.

The wind and temperature fields were examined by means of sections drawn at six-hourly intervals from 09 G.M.T. 14 June to 15 G.M.T. 18 June, 1951. For most of the period a fixed section - shown in Fig. 1 to 4 - was employed; this section utilised upper-air observations from some or all of the stations Stornoway, Aldergrove, Liverpool, Larkhill, Trappes and Lyons. The main flow backed somewhat during 18 June so that a rather differently orientated section was used at 09 and 15 G.M.T. of 18 June (see Fig. 5). The sectional analysis ceased with the movement of the jet stream centre away from England into the Continent later on 18 June, 1951.

The jet stream picture can be seen by means of Fig. 6 to 13, which are reproductions of relevant cross-sections. A single wind maximum (A) just under the tropopause is shown in Fig. 6 (15 G.M.T. 14 June). Wind maximum A moved south and is shown just north of Larkhill in Fig. 7 (09 G.M.T. 15 June). However, a small new centre (B) appears for the first time north of Aldergrove. This new wind maximum is not shown by direct wind observation, but is based on geostrophic wind computations made by means of a scale similar to that described by Matthewman (1950) in the Meteorological Magazine. Fig. 8 (15 G.M.T. 15 June) shows wind maximum B as a major feature, well substantiated by wind reports as well as by geostrophic computations. In Fig. 8 wind maximum A is shown as a weakening system south of Larkhill, based on geostrophic computations (Note: the thermal field over North France has not been drawn to fit the reported Trappes temperatures which were clearly in error, being too warm above 400 mb: if the Trappes observations were to be accepted the location of wind maximum A would not be affected but its intensity would be greater). At 21 G.M.T. 15 June (Fig.9) wind maximum B is shown south of Aldergrove; the Larkhill winds at high levels have now moderated, indicating that the relative wind minimum between maxima A and B of Fig. 8 has also moved farther south into France, where the lack of observations made the construction of the section impossible. However, at 21 G.M.T. on 15 June a section over Germany, shown as Fig. 10 (Jever to Munich), locates one wind maximum (identifiable with A) south of Jever with indications of another to the north of Jever. At 03 G.M.T. on 16 June wind maximum B is situated north of Liverpool and dominates the section, with only a weak residual centre south of Trappes. Henceforth wind maximum B is the only one shown on the sections. Fig. 11, 12 and 13, referring to 15 G.M.T. on 16, 17 and 18 June respectively, clearly indicate the slow southwards movement of wind maximum B which was, however, temporarily stationary or slightly retrograde between 09 and 15 G.M.T. on 17 June.

Thus the sequence of events appears to have been as follows. Jet stream A moved southeast into Europe, weakening rapidly during 15 June, and disappearing completely as a synoptic entity early on 16 June. Meanwhile jet stream B extended across the British Isles and moved slowly southeast in a direction perpendicular to its axis, rapidly becoming the dominant system on 15 June. The sequence is shown in Fig. 1 (b) to 5 (b), where the broad broken lines give the estimated positions of the jet stream axes.

Additional evidence for the existence and behaviour of the two jet streams is contained in Fig. 14, which shows the time variation of the maximum wind speed on the section Aldergrove-Liverpool-Larkhill, based on reported winds at six-hourly intervals: the movement and change in intensity of wind maxima A and B are fairly obvious on this diagram.

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variation and the increase of speed; for instance, of the 36 cases of winds below 40 knots, 18 have a change less than 2.9 knots in 10 minutes; of the 31 of winds between 41 and 60, 15 have a change less than 3.1 knots in 10 minutes; whereas of the 19 cases with winds over 60 knots, 9 have a change less than 4.5 knots in 10 minutes.

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4. The tropospheric thermal field.

The thermal field during the period 14-18 June, 1951 was fairly typical of that associated with jet streams - the main feature being the broad baroclinic zone, sloping in the lower troposphere but almost vertical in the upper troposphere. The thermal distribution at 850 mb (representation of the lower troposphere) and at 400 mb (representation of the upper troposphere) can readily be seen in Fig. 15. The soundings on the section Aldergrove to Lyons at 03 G.M.T. on 16, 17 and 18 June and at 15 G.M.T. on 16 and 17 June, 1951 were employed in constructing Fig. 15: during this two-day period the jet stream B was very slow-moving over England (the surface front was also quasi-stationary over France). Fig.15 shows, in particular, that the baroclinity over France is marked at 850 mb but quite slight at 400 mb; whereas the baroclinity over the British Isles is more pronounced at the upper level and is spread over a horizontal zone several hundred miles broad.

The broad baroclinic zone - which may be regarded as a sort of generalised frontal zone - was a persistent feature of the entire period, but a great deal of complexity occurred within the zone itself. Sometimes, as in Fig. 8 (15 G.M.T. 15 June, 1951), several subsidiary zones (based on temperature lapse rate discontinuities) could be drawn within the main baroclinic zone, although it was rarely possible to maintain the continuity of these subsidiary zones for long. Even at the same time neighbouring soundings through the broad baroclinic zone showed markedly different detailed structure on occasion: for instance at 15 G.M.T. on 16 June, 1951 (Fig. 11) the Larkhill ascent gave a single major temperature inversion at 820-800 mb, whilst the Trappes sounding showed two distinct isothermal layers within the region 870-750 mb. On the other hand, a narrow sloping zone of the conventional frontal type is shown in Fig. 13 (15 G.M.T. 18 June, 1951), which is fairly well established by the soundings at Brussels and Downham Market and also supported by the 0900 hours observations: however it is noteworthy that the 'front' as drawn accounts for only a rather small fraction of the baroclinity that exists within the broad baroclinic zone.

The transient nature of the temperature lapse rate discontinuities within the broad baroclinic zone suggests that the analysis of the fine details is of dubious value: the significant reality on the synoptic scale appears to be the broad baroclinic zone itself.

The behaviour of the generalised frontal zone during the period 14-18 June, 1951 is interesting and suggestive. On 14 June it was associated both with the surface front which moved into France and with the jet stream A over the British Isles. On 15 June the part of the generalised frontal zone in the lower troposphere remained closely associated with the surface front over France, but the upper tropospheric part acquired a double structure with the encroachment of jet stream B (the wavy character of the isotherms in the upper troposphere in Fig. 8 shows this feature). The rapidly weakening jet stream A and its associated baroclinic zone in the upper troposphere moved south and appeared to become detached from the almost stationary part of the baroclinic zone in the lower troposphere over France. Meanwhile as jet stream B developed and moved south its associated baroclinic zone in the upper half of the troposphere also intensified and moved south, so that soon there was re-established a broad, and more or less sloping, baroclinic zone throughout the entire troposphere. At this stage the broad baroclinic zone assumed much the same position and general shape relative to the surface front [illegible] had done. Thereafter, on 16, 17 and 18 June only the single generalised baroclinic zone frontal zone and jet stream (B) were in existence. It seems [illegible] and necessary features of the forward side of the upper trough in the long-wave pattern during the period were a generalised frontal zone and associated jet stream: the complexity that occurred on 15 June (whatever the reason for it) quickly became damped out by these over-riding long-wave requirements.

5. Lateral movement of the jet streams related to geostrophic advection.

The rate of lateral movement of the jet streams can be estimated from the movement of the wind maxima along the Stornoway to Lyons section. Mean speeds have been computed over 12 hour intervals overlapping by 6 hours, and these are shown (continuous lines) in Fig. 16.

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variation and the increase of speed; for instance, of the 36 cases of winds below 40 knots, 18 have a change less than 2.9 knots in 10 minutes; of the 31 of winds between 41 and 60, 15 have a change less than 3.1 knots in 10 minutes; whereas of the 19 cases with winds over 60 knots, 9 have a change less than 4.5 knots in 10 minutes.

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The surface pressure difference between points 200 miles on either side of the line of section Stornoway to Lyons along the jet stream axes has been measured for each occasion when the wind maxima A and B would be determined on the cross-sections. Mean pressure differences over 12 hour intervals overlapping by 6 hours are also graphed in Fig. 16 (broken lines). The pressure difference scale on the right of Fig. 16 is roughly equivalent, geostrophically, to the scale of speeds given on the left of the diagram. Pressure differences obtained in this way give a rough estimate of the surface geostrophic advective component operating in a direction normal to the jet stream axis, that is, along the direction Stornoway to Lyons.

The average speed of lateral movement of the jet streams (A and B together) is 7.9 kt which may be compared with the average geostrophic component of 6.5 kt. The relationship between these two quantities at different times during the period is shown in Fig. 16. The lateral speed and geostrophic component do not differ by more than 15 kt at any time and are generally in agreement to 5/10 kt. However there is clearly little correlation between the day to day variations of the two quantities.

It is interesting to note the acceleration in the movement of jet stream A early on 15 June and its subsequent motion at a speed greater than indicated by the geostrophic advection during the phase of rapid decay of the system. A qualitative explanation of the behaviour of jet stream A is that marked tropospheric subsidence occurred to the south of the new and dominant jet stream B, as is suggested by the low humidities reported. Empirical evidence for descending motion in the upper troposphere near the right exit of jet streams has also been presented by Murray and Daniels (1951). Such subsidence would be expected to decrease the baroclinity and hence the intensity of jet stream A - as indeed happened, according to the evidence of the cross-sections. Moreover, theory (e.g. Sutcliffe, 1938) indicates that descending motion in a region of vertical wind shear produces an ageostrophic component from the cold to the warm air. In this case the ageostrophic component would point towards the southeast - in the correct sense for causing the (thermal) jet stream A to move rather more quickly than suggested by the geostrophic advection.

Fig. 16 shows that jet stream B moved slowly southeast, at about the same speed as the geostrophic advective component, for nearly two days up to the morning of 17 June; thereafter it became quasi-stationary although the geostrophic advective component increased. During the period of steady motion the intensity of jet stream B changed little, but it is noteworthy that a marked intensification occurred on the morning of 17 June, at about the same time as the geostrophic advective component increased and the jet stream slowed down. An ageostrophic component pointing northwest against the strengthening surface geostrophic advection would be consistent both with the slowing down and the intensification of the jet stream: such an ageostrophic component might indeed be expected in the present case with the approach of the confluent jet stream entrance (see Fig. 4 (b)).

Summing up, it appears that a rough first approximation to the lateral movement of the jet streams is given by the surface geostrophic advection, and that the apparently anomalous movements are broadly consistent with dynamical ideas.

6. Variations in intensity of the wind maxima.

The intensity variations of the centres of jet streams A and B are shown in Fig. 17 (constructed from the information contained in the series of cross-sections).

As was expected, the changes in intensity of the wind maxima were closely related to changes in intensity of the mean thermal gradient of the troposphere: inspection of the cross-sections and 1000-300 mb thickness charts confirms this point.

The rapid weakening of jet stream A on 15 June occurred as jet stream B extended across the British Isles to the north of the first jet stream. General subsidence to the south of jet stream B can be invoked to account for the weakening of jet stream A (by weakening its associated baroclinity), as has been suggested in section 5.

variation and the increase of speed; for instance, of the 36 cases of winds below 40 knots, 18 have a change less than 2.9 knots in 10 minutes; of the 31 of winds between 41 and 60, 15 have a change less than 3.1 knots in 10 minutes; whereas of the 19 cases with winds over 60 knots, 9 have a change less than 4.5 knots in 10 minutes.

Jet stream B, after its establishment as the dominant system, remained more or less constant in intensity until early on 17 June when a noticeable increase in wind speed occurred, followed by a slight decrease and then by another increase on 18 June.

The first significant intensification on 17 June occurred in a limited region at the base of an upper trough as it travelled northeast across England as a sort of perturbation on the basic jet stream current. The 300 mb charts show the sequence of events quite clearly: as the centre of gravity of the upper vortex was transferred northeast across north Scotland, a trough extension also moved northeastwards along, and as if steered by, the basic upper current. The trough flattened out and lost its individuality east of the British Isles later, although it appeared to be partly instrumental in extending the jet stream farther eastwards. Such non-developing wave-like perturbations on jet streams are not uncommon of course, but the cause of their formation is obscure. It may be significant however that, in this particular case, the perturbation originated at the confluent jet stream entrance (off SW Ireland on 16 June).

After the passage of the trough disturbance, the jet stream intensity in the cross-section region weakened temporarily, then increased again with the approach of the marked confluent jet stream entrance.

7. Variations in height of the wind maxima.

The variation with time of the height of the centres of jet streams A and B can be seen from Fig. 17.

First, it will be noted that the height of the centre of jet stream A changed little until during the period of decay on 15 June when it increased rapidly (pressure decreased and potential temperature increased). Secondly, as regards jet stream B, the height remained fairly constant, the mean pressure at the centre of the jet stream throughout the period being 272 mb with standard deviation about 9 mb. It is considered that the probable error in estimating pressure at the jet centre is between 5 and 10 mb so that the apparent variability of the level of jet stream B may be unreal. The time scale of Fig. 17 may also be regarded as a distance scale along the axes of the jet streams: the right and left of the time scale refer to the entrance and exit regions respectively of the jet streams. As regards jet stream B there is no evidence of any material change in the level of the centre of the jet stream along the length of the axis. The height of the axis of jet stream A near the entrance region is greater than elsewhere at earlier times, but the section over Germany at 21 G.M.T. on 15 June (see Fig. 10) shows the jet centre to be also just below 200 mb or at about the same level as at 15 G.M.T. on the same day over France. There is thus no evidence of any variation of the level of jet stream A along its length at a particular time: the higher level reported later on 15 June appears to indicate lifting of the whole of the decadent jet stream.

In a previous report Murray and Daniels (1951) have shown that there is a statistical tendency for the height of jet stream centres to be slightly greater in the entrance than in the exit region. The statistics were based on jet streams near the British Isles but the entrance and exit regions did not refer to the same jet stream, and the interpretation of the statistical evidence was uncertain. However, one possible interpretation was that there might be a tendency for the jet stream axis to slope down from the entrance to the exit region; but the evidence of the present investigation, summarised in the previous paragraph, does not support this interpretation.

8. Tropopause structure.

It is well known that the tropopause is at a higher elevation on the high than on the low pressure side of jet streams, but several writers (e.g. Palmén, 1948; Durst and Davis, 1949) imply that the tropopause on one side is not continuous with that on the other. Murray and Johnson (1952) have suggested however that the tropopause is sometimes continuous, at other times folded or discontinuous - the nature of the tropopause depending on the life history of the jet stream.

/No

variation and the increase of speed; for instance, of the 36 cases of winds below 40 knots, 18 have a change less than 2.9 knots in 10 minutes; of the 31 of winds between 41 and 60, 15 have a change less than 3.1 knots in 10 minutes; whereas of the 19 cases with winds over 60 knots, 9 have a change less than 4.5 knots in 10 minutes.

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No difficulty was encountered in drawing the tropopause surface as continuous on the high pressure side of the jet stream axis. On the temperature soundings the tropopause was frequently indicated by an abrupt temperature lapse rate discontinuity, with an inversion above. Sometimes the change from tropospheric to stratospheric conditions occurred in two abrupt steps - above the lower discontinuity existed a decreased lapse rate or near isothermal layer, followed at some distance usually varying between 5 and 30 mb by a sharp inversion. It is conventional to place the tropopause at the lower discontinuity unless it is insignificant relative to the upper one, but the tropopause can also be visualised as a rather narrow zone in such cases. It remains to be explained why a finite transition zone should merge into a single tropopause surface at different places or times; certainly no systematic relationship was noted in the jet stream cases examined.

What then are the observational facts as regards the tropopause on the low pressure side of the jet stream axis in this case? Most of the cross-sections were readily constructed with the tropopause as a continuous surface, using an abrupt change of temperature lapse rate as the criterion to fix the tropopause and not quasi-constancy of potential temperature. 36 temperature soundings at distances between 10 and 345 miles from the positions of the jet stream axes given on the series of cross-sections have been examined. Of these soundings 26 gave a definite type I tropopause (abrupt change from lapse rate to inversion); 5 ascents showed a little complexity above the first abrupt lapse rate change, of the type that could quite easily be explained by small-scale fluctuations and instrumental errors, so that no difficulty arose in deciding the most probable position of the tropopause; the remaining 5 cases showed some complexity of the type that occurs with a folded tropopause (see, e.g. Sawyer, 1951). These last five cases are shown in Fig. 18. One occurred at Aldergrove at 09 G.M.T. on 16 June at 180 miles from the jet axis; the complexity was not reproduced at Aldergrove on later ascents, nor downwind on any of the four soundings made at Leuchars on that day. The four other cases shown in Fig. 18 occurred at Liverpool between 09 G.M.T. 17 June and 03 G.M.T. 18 June, at distances between 95 and 125 miles from the jet axis, at about the time that the jet stream had intensified to 120/130 knots and was very slow moving: moreover the Liverpool soundings were situated in that half of the jet stream nearer to the entrance (see Fig. 4 (b)). No soundings were available downwind at about the same distance from the jet axis. However, five soundings (Stornoway 09 G.M.T. 15th, Aldergrove 21 G.M.T. 15th and 03 G.M.T. 16th, Liverpool 21 G.M.T. 16th and 03 G.M.T. 17th) were located along the lines of section on 15 and 16 June, in the half sector near the jet stream exit at distances varying from 70 to 120 miles; these five ascents all showed simple type I tropopauses.

The evidence of the previous paragraph suggests that the tropopause was in effect continuous at one stage, at least in the exit half sector of jet stream B; but that at a later stage in the entrance half sector, when the jet stream was in the process of intensifying or had just intensified, it was folded or disrupted over a narrow zone at a distance of about 100 miles from the jet stream axis.

A model for the tropopause structure and behaviour to the left (looking downwind) of jet streams, suggested by this study and other work in the literature, may be put forward. Early in the life history of the jet stream the tropopause is continuous and in some degree sloping. As the jet stream intensifies (perhaps by confluence of cold and warm air masses as suggested by Namias and Clapp, 1949) the slope of the tropopause increases; then the stage may be reached when the tropopause actually folds. As Sawyer (1951) has pointed out, the seat of the folding process is near the entrance region, but the folded part of the tropopause will be advected downstream, where it will eventually be disrupted (presumably mainly by turbulence). Late in the jet stream's life history, when the mechanism which produces or maintains the jet stream ceases to operate, the tropopause must become continuous once more; this may come about by advection of the continuous tropopause in existence upwind or by some reformation process.

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variation and the increase of speed; for instance, of the 36 cases of winds below 40 knots, 18 have a change less than 2.9 knots in 10 minutes; of the 31 of winds between 41 and 60, 15 have a change less than 3.1 knots in 10 minutes; whereas of the 19 cases with winds over 60 knots, 9 have a change less than 4.5 knots in 10 minutes.

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The mean tropopause configuration in relation to the jet stream centre is shown in Fig. 19, based on 14 cross-sections through jet streams A and B. Sections showing distinct double wind maxima on 15 June and a few others of limited extent because of observational shortage were not utilised in the construction of the mean picture. Also, on those sections which showed some indication of a fold or a break in the tropopause to the left of the jet stream, the tropopause was regarded as simple and continuous (the exact level taken as the tropopause on a temperature sounding showing complexity depended mainly on continuity considerations). Thus the mean picture is somewhat idealised on the low pressure side, where a fold or a discontinuity in the tropopause may occur at about 100 miles (or a little more) from the jet stream centre. Incidentally, at some distance greater than 200 miles from the jet stream centre, on the low pressure side, the tropopause usually slopes upwards again. However, in Fig.19 the main point of note, which may be regarded as fairly typical of jet streams of moderate intensity (say, 90 to 130 knots), is that the tropopause surface is about 3000 ft above the jet centre (the standard deviation about the mean in the present example was roughly 1000 ft).

9. Concluding remarks.

This report has dealt in the main with specific attributes of the wind and temperature fields during the five-day period under review: the aim has been to give an account of events and to clarify in some degree certain aspects of the jet stream. An attempt must now be made to indicate how some of the parts are related to the whole synoptic picture.

The most persistent feature appears to have been the slow-moving long-wave trough, however complex in detail, dominating the entire troposphere. The broad baroclinic zone on the forward side of the long-wave trough was the region analysed by means of the series of cross-sections: such a baroclinic zone is of course an essential concommitant of a major cold trough. It is noteworthy that, in the present instance, the baroclinic zone was favourably located for persistence, with the cold air over the NE Atlantic and the warm air over the warm Continent. Numerous studies in recent years have confirmed that a jet stream and a tropospheric baroclinic zone occur together, in other words that the wind is a "thermal wind": this particular case was no exception. Thus, on the largest scale, there existed together, apparently as essential parts of the same long-wave complex, a major cold trough, a broad baroclinic zone and a jet stream.

The distinctive feature of the period requiring explanation was the formation of two separate wind maxima in association with the broad generalised frontal zone. In this connection it is significant that the surface counterpart of the upper-wave trough consisted for a time of a double-centred depression: the original surface low (of which the cold front and jet stream affected the British Isles on 14 June) separated into two centres about midday of 14 June - one centre moved slowly into the southern Norwegian Sea where it filled up later, whilst the other centre developed south of Iceland, then swung southwards and later eastwards towards the British Isles in the manner described in section 2. It appears that the second jet stream (i.e. B) came into existence as a distinct entity in sympathy with the plunging southwards of the Icelandic low which was clearly instrumental in concentrating the thermal gradient and thus intensifying the upper current in the region south and southeast of the centre of the surface depression. All the same it is far from clear why two separate wind maxima should have occurred. It would also have seemed reasonable if one continuous jet stream had existed, with the wind maximum being transferred temporarily northwards again early on 15 June owing to the northwards advection of the main thermal belt in a limited region to the east of the surface depression. However, accepting the existence of the second and dominant jet stream (i.e.B), the subsequent motion and behaviour of the first jet stream (i.e.A) appears to have been a logical consequence on dynamical grounds (see section 5).

Some of the points concerned with the variations in intensity of the individual jet streams A and B are worth reiterating; synoptic experience suggests that they may be of fairly general validity. First, the intensity fluctuations of the jet streams were closely related to fluctuations of the

/tropospheric

variation and the increase of speed; for instance, of the 36 cases of winds below 40 knots, 18 have a change less than 2.9 knots in 10 minutes; of the 31 of winds between 41 and 60, 15 have a change less than 3.1 knots in 10 minutes; whereas of the 19 cases with winds over 60 knots, 9 have a change less than 4.5 knots in 10 minutes.

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tropospheric mean thermal gradient. Second, the rapid destruction of jet stream A occurred when it was located within the region where subsidence was to be expected on dynamical grounds, that is, within the region near the right exit of the developing and overtaking jet stream B. Third, fluctuations in intensity did not usually occur simultaneously along the entire length of jet stream B, but, appearing to originate in, and to be associated with variations in intensity of, the confluent entrance region, moved as perturbations along the axis of the jet stream.

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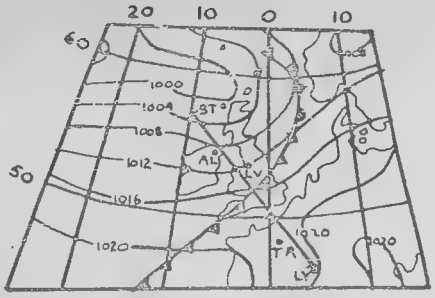

FIG. 1(a) Surface chart for 15 GMT
14 June 1951
Isobars at 4 mb intervals.

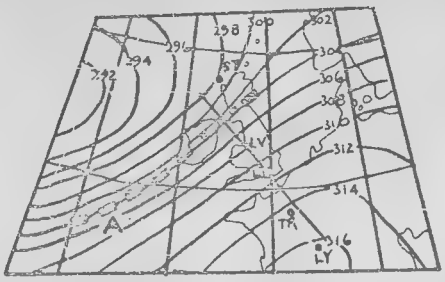

FIG. 1(b) 300 mb chart for 15 GMT
14 June 1951
Contours in hundreds of feet
at 200 ft intervals.

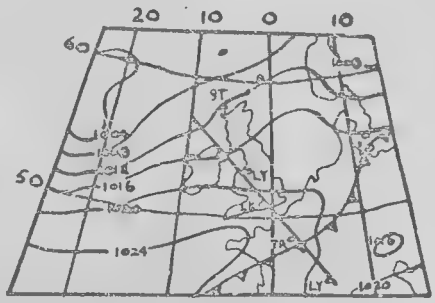

FIG. 2(a) Surface chart for 15 GMT
15 June 1951
Isobars at 4 mb intervals.

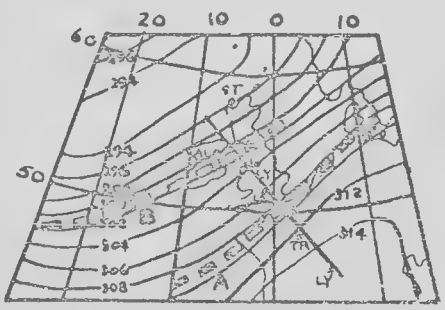

FIG. 2(b) 300 mb chart for 15 GMT
15 June 1951
Contours in hundreds of feet
at 200 ft intervals.

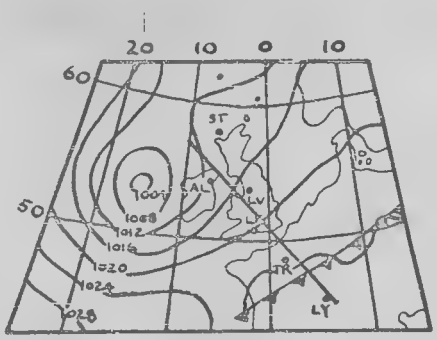

FIG. 3(a) Surface chart for 15 GMT
16 June 1951
Isobars at 4 mb intervals.

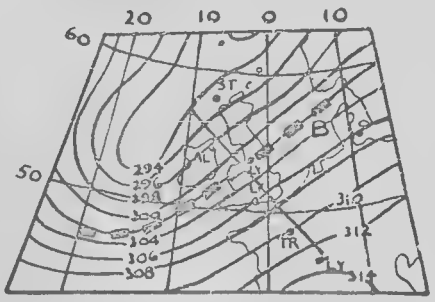

FIG 3(b) 300 mb chart for 15 GMT
16 June 1951
Contours in hundreds of feet
at 200 ft intervals.

variation and the increase of speed; for instance, of the 36 cases of winds below 40 knots, 18 have a change less than 2.9 knots in 10 minutes; of the 31 of winds between 41 and 60, 15 have a change less than 3.1 knots in 10 minutes; whereas of the 19 cases with winds over 60 knots, 9 have a change less than 4.5 knots in 10 minutes.

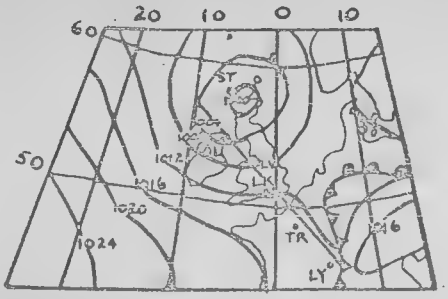

FIG. 4(a) Surface Chart for 15 GMT
17 June 1951
Isobars at 4 mb intervals.

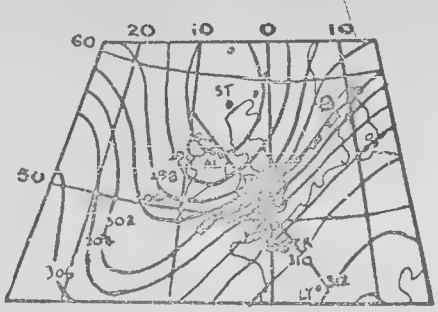

FIG 4(b) 300 mb Chart for 15 GMT
17 June 1951
Contours in hundreds of feet
at 200 ft intervals.

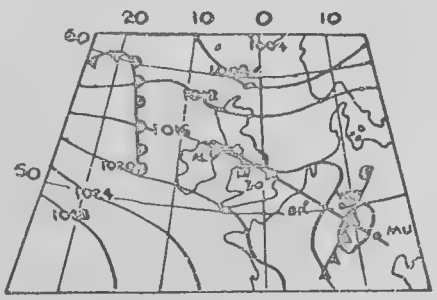

FIG. 5(a) Surface Chart for 15 GMT
18 June 1951
Isobars at 4 mb intervals.

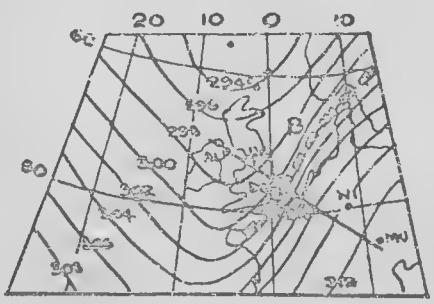

FIG 5(b) 300 mb Chart for 15 GMT
18 June 1951
Contours in hundreds of feet
at 200 ft intervals.

FIG. 1 to 5 Abbreviations for stations employed in constructing lines of section :-

ST = Stornoway.	AL = Aldergrove.	LV = Liverpool.
LK = Larkhill.	TR = Trappes.	LY = Lyons.
DO = Downham.	WI = Weisbaden.	BR = Brussels.
MU = Munich.		

Jet Streams A and B indicated thus ▬▬▬ on 300mb charts.

variation and the increase of speed; for instance, of the 36 cases of winds below 40 knots, 18 have a change less than 2.9 knots in 10 minutes; of the 31 of winds between 41 and 60, 15 have a change less than 3.1 knots in 10 minutes; whereas of the 19 cases with winds over 60 knots, 9 have a change less than 4.5 knots in 10 minutes.

FIGURES 6 to 13.

Abbreviations employed in the sections are:—
Isotherms as thin broken lines at 10°F intervals.
Isokinetics as thin continuous lines at 10kt intervals.
Tropopauses as thick lines (broken lines when uncertain or possibly discontinuous)
Fronts or temperature lapse rate discontinuities as sloping continuous lines.

ST = Stornoway	AL = Aldergrove	LV = Liverpool
LA = Larkhill	TR = Trappes	LY = Lyons
JE = Jever	HA = Hanover	IS = Iserlohn
WI = Wiesbaden	BR = Brussels	MU = Munich

DO = Downham Market

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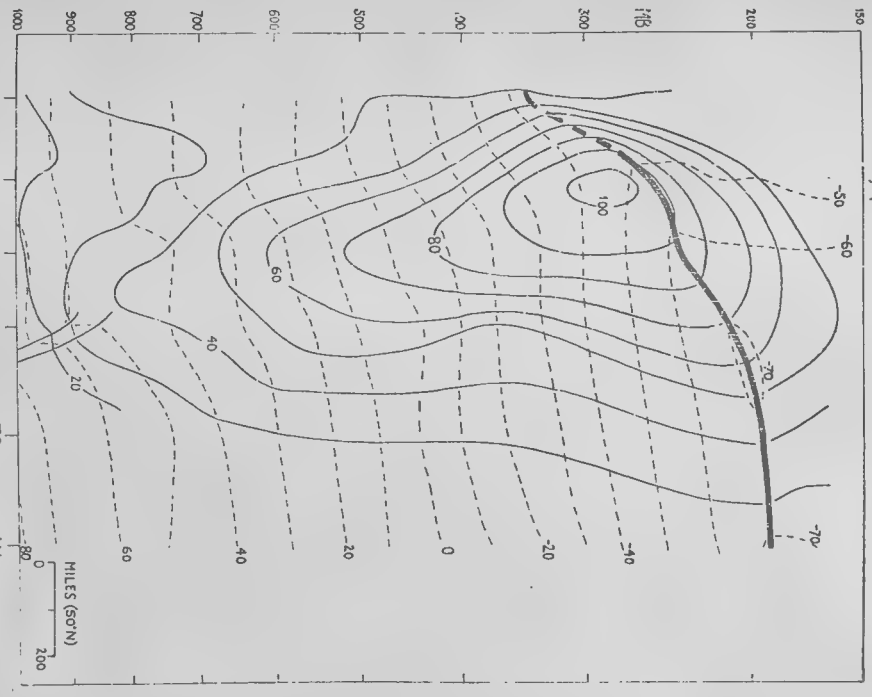

FIG.6. SECTION STORNOWAY - LYONS 15 G.M.T. 14 JUNE 1951.

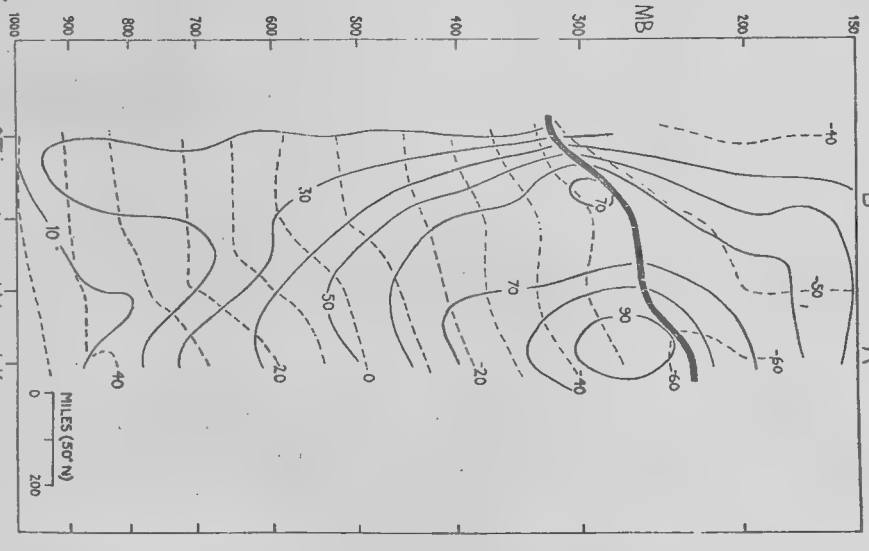

FIG.7. SECTION STORNOWAY - LARKHILL 09 G.M.T. 15 JUNE 1951

variation and the increase of speed; for instance, of the 36 cases of winds below 40 knots, 18 have a change less than 2.9 knots in 10 minutes; of the 31 of winds between 41 and 60, 15 have a change less than 3.1 knots in 10 minutes, whereas of the 19 cases with winds over 60 knots, 9 have a change less than 4.5 knots in 10 minutes.

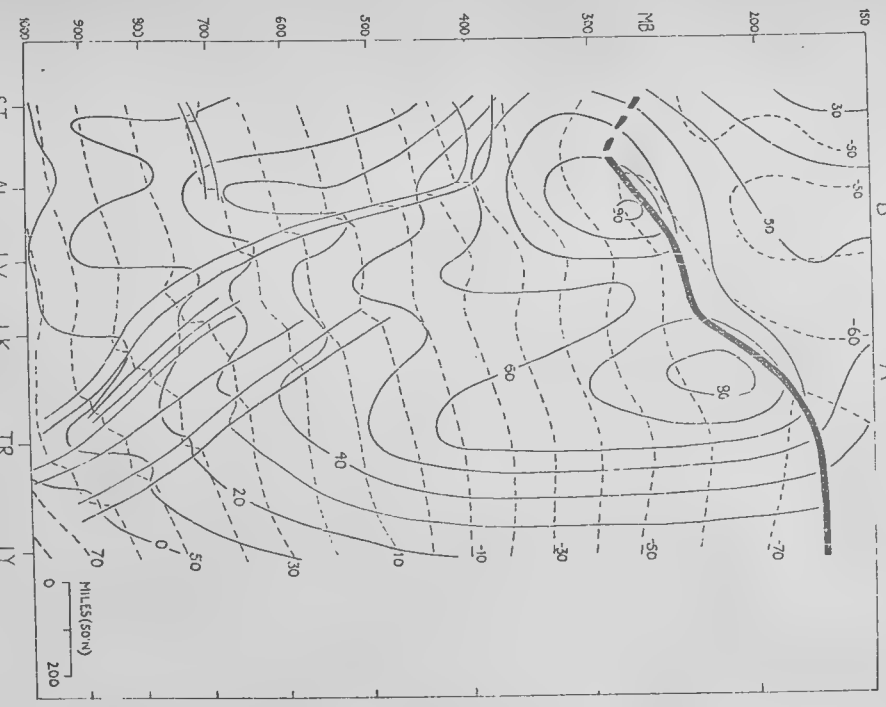

FIG 8. SECTION STORNOWAY - LYONS 15 GMT 15 JUNE 1951.

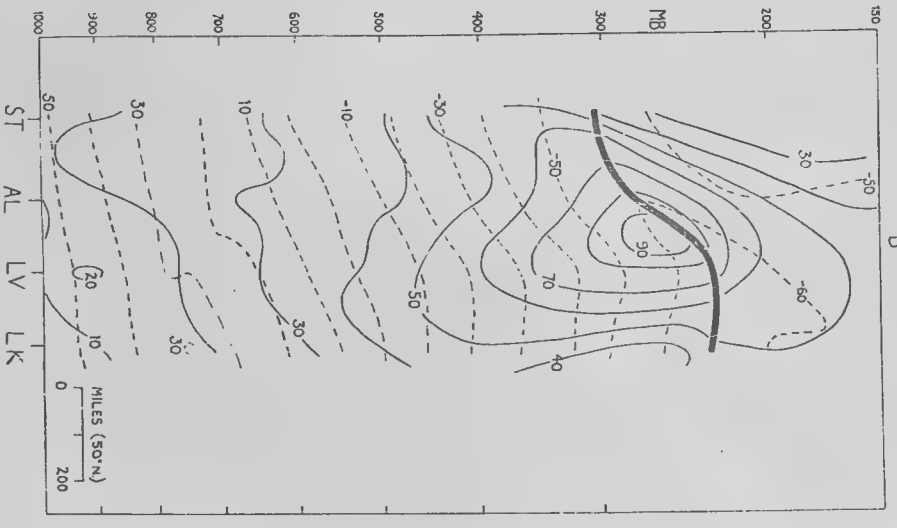

FIG 9. SECTION STORNOWAY - LARKHILL 21 GMT 15 JUNE 1951

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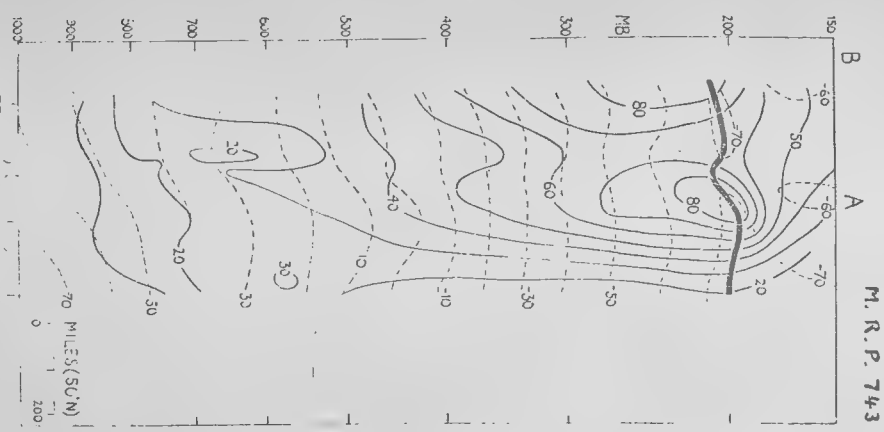


FIG 10. SECTION OVER NORTH ATLANTIC 15 JUNE 1951

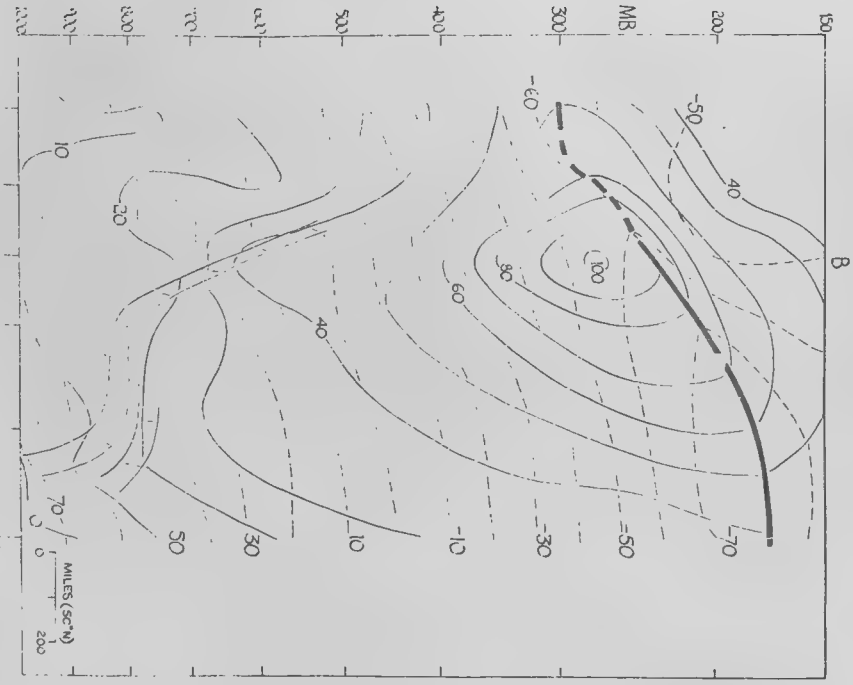


FIG. 11 SECTION STORNOWAY - LYONS 15 GMT 16 JUNE 1951.

variation and the increase of speed; for instance, of the 36 cases of winds below 40 knots, 18 have a change less than 2.9 knots in 10 minutes; of the 31 of winds between 41 and 60, 15 have a change less than 3.1 knots in 10 minutes; whereas of the 19 cases with winds over 60 knots, 9 have a change less than 4.5 knots in 10 minutes.

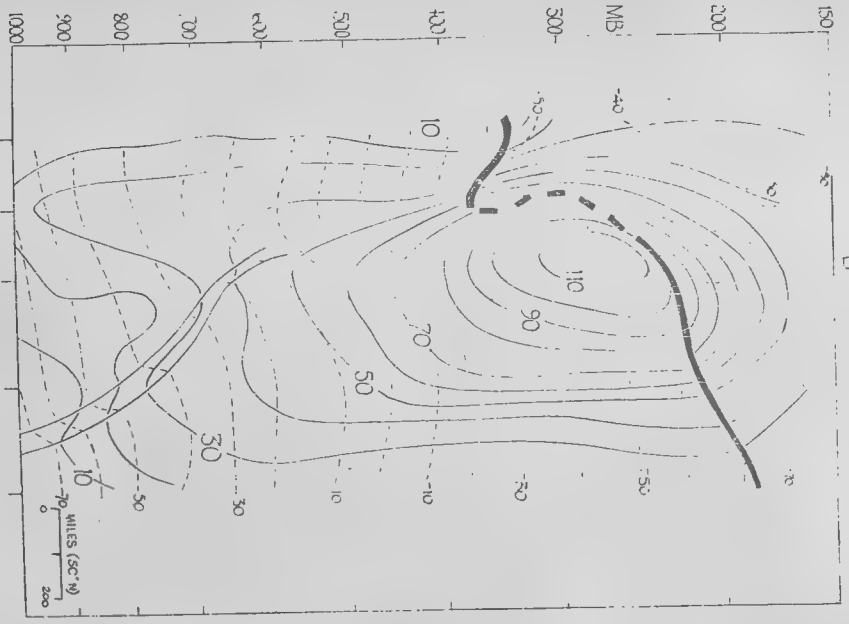


FIG. 12 SECTION STORNOWAY - LYONS 15 GMT 17 JUNE 1951.

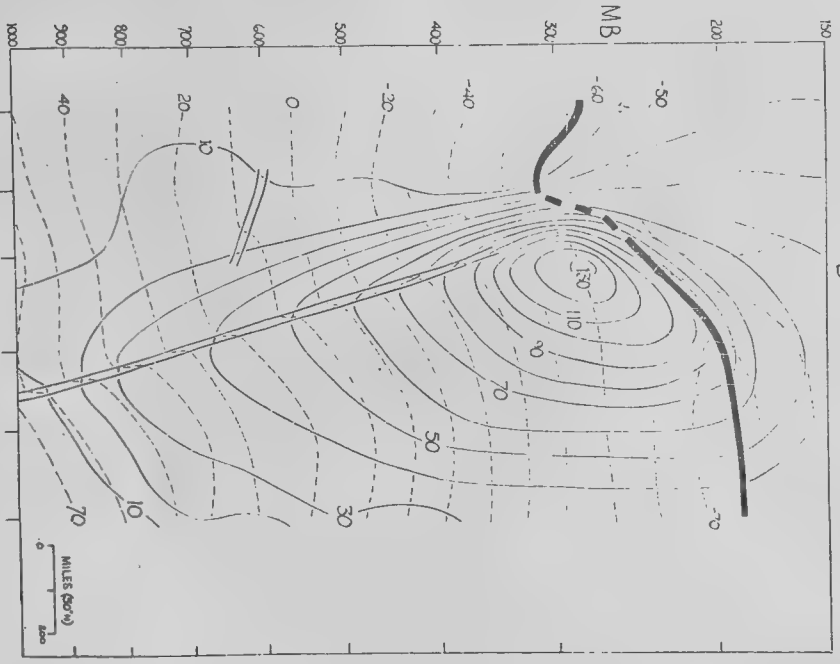
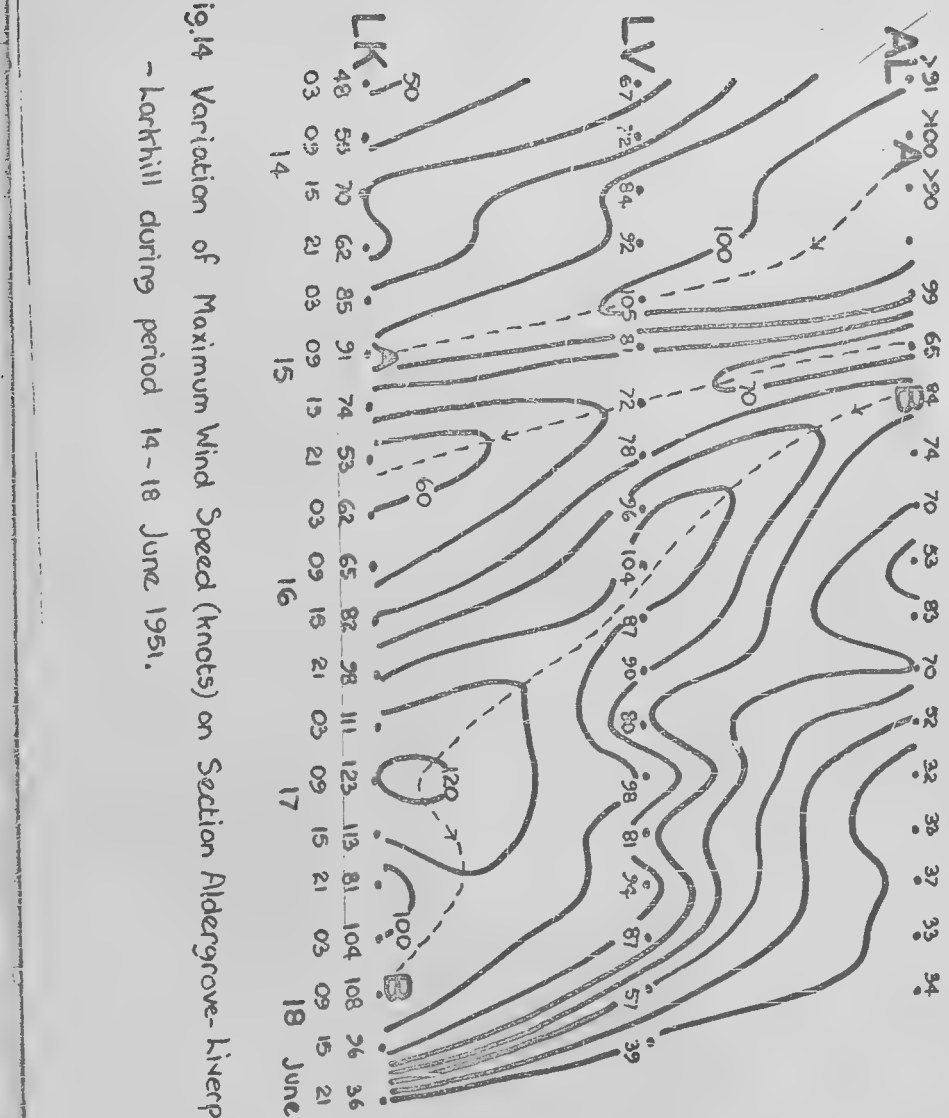


FIG. 13 SECTION ALDERGROVE - MUNICH 15GMT 18 JUNE 1951.

variation and the increase of speed; for instance, of the 36 cases of winds below 40 knots, 18 have a change less than 2.9 knots in 10 minutes; of the 31 of winds between 41 and 60, 15 have a change less than 3.1 knots in 10 minutes, whereas of the 19 cases with winds over 60 knots, 9 have a change less than 4.5 knots in 10 minutes.



M.R.P. 743

Fig.14 Variation of Maximum Wind Speed (knots) on Section Aldergrove-Liverpool- Larkhill during period 14-18 June 1951.

variation and the increase of speed; for instance, of the 36 cases of winds below 40 knots, 18 have a change less than 2.9 knots in 10 minutes; of the 31 of winds between 41 and 60, 15 have a change less than 3.1 knots in 10 minutes; whereas of the 19 cases with winds over 60 knots, 9 have a change less than 4.5 knots in 10 minutes.

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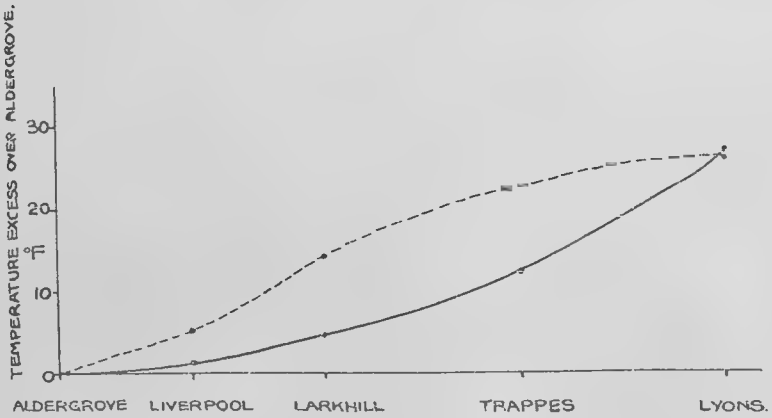

Fig.15. MEAN TEMPERATURE DISTRIBUTION AT 850mb (——) AND AT 400mb(- - - -) ON SECTION ALDERGROVE TO LYONS, BASED ON SOUNDINGS AT 03 GMT ON 16, 17, AND 18 JUNE AND AT 15 GMT ON 16 AND 17 JUNE 1951, WHEN JET STREAM B WAS VERY SLOW-MOVING OVER ENGLAND. MEAN TEMPERATURE AT ALDERGROVE AT 850mb = 34·4°F AND AT 400mb = -36·8°F.

1951.

variation and the increase of speed; for instance, of the 36 cases of winds below 40 knots, 18 have a change less than 2.9 knots in 10 minutes; of the 31 of winds between 41 and 60, 15 have a change less than 3.1 knots in 10 minutes; whereas of the 19 cases with winds over 60 knots, 9 have a change less than 4.5 knots in 10 minutes.

M.R.P. 743

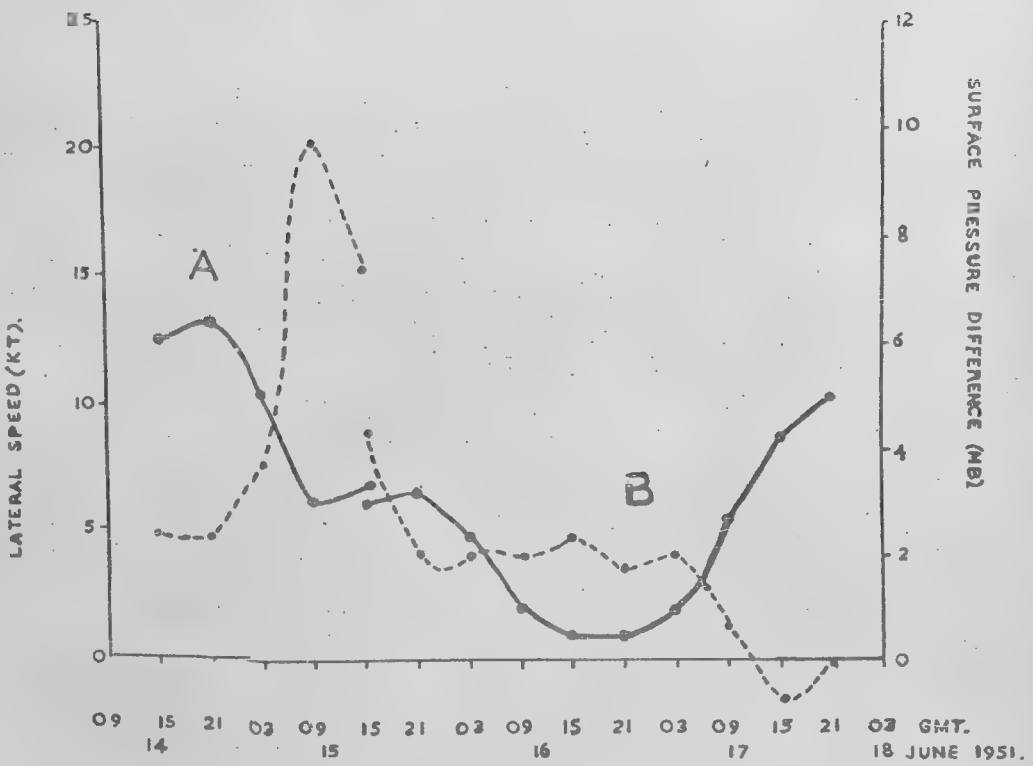

FIG.10. SPEED OF LATERAL MOVEMENT OF JET STREAM (----) COMPARED WITH SURFACE PRESSURE DIFFERENCE OVER 400 MILES ALONG JET STREAM AXIS (——) FROM 09 GMT 14 JUNE 1951 TO 03 GMT 18 JUNE 1951.

variation and the increase of speed; for instance, of the 36 cases of winds below 40 knots, 18 have a change less than 2.9 knots in 10 minutes; of the 31 of winds between 41 and 60, 15 have a change less than 3.1 knots in 10 minutes; whereas of the 19 cases with winds over 60 knots, 9 have a change less than 4.5 knots in 10 minutes.

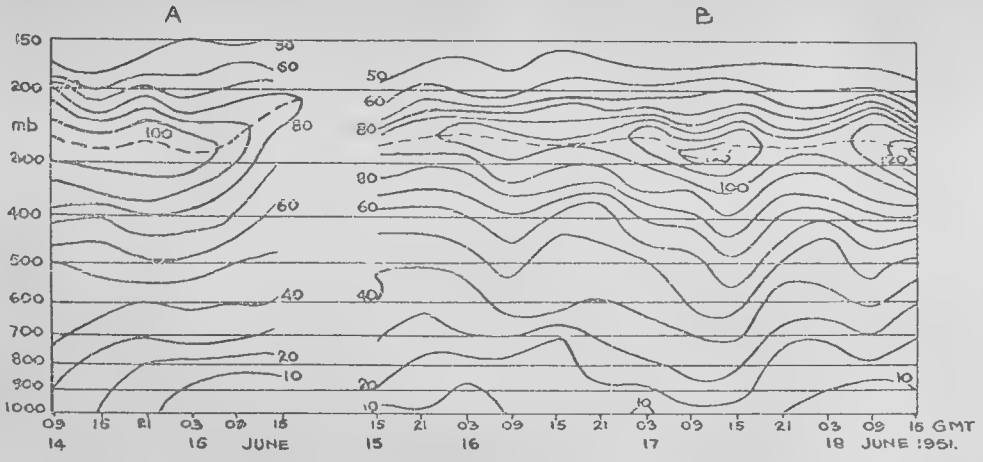

FIG.17 Time variation of the vertical distribution of the wind components perpendicular to the cross-sections at the axes of jet streams A and B. Isokinetics (every 10 kt) ——. Level of jet centres – – –.

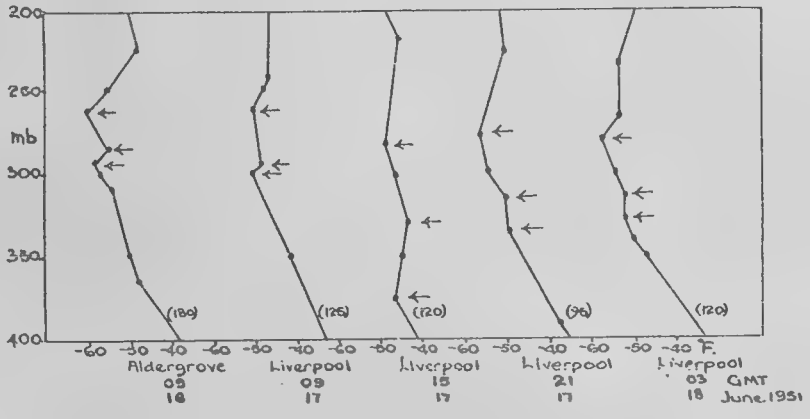

FIG. 18. Temperature soundings showing possible fold in tropopause (note small arrows) on the low pressure side of jet stream B. Distance in miles of ascent from jet axis shown in brackets.

variation and the increase of speed; for instance, of the 36 cases of winds below 40 knots, 18 have a change less than 2.9 knots in 10 minutes; of the 31 of winds between 41 and 60, 15 have a change less than 3.1 knots in 10 minutes; whereas of the 19 cases with winds over 60 knots, 9 have a change less than 4.5 knots in 10 minutes.

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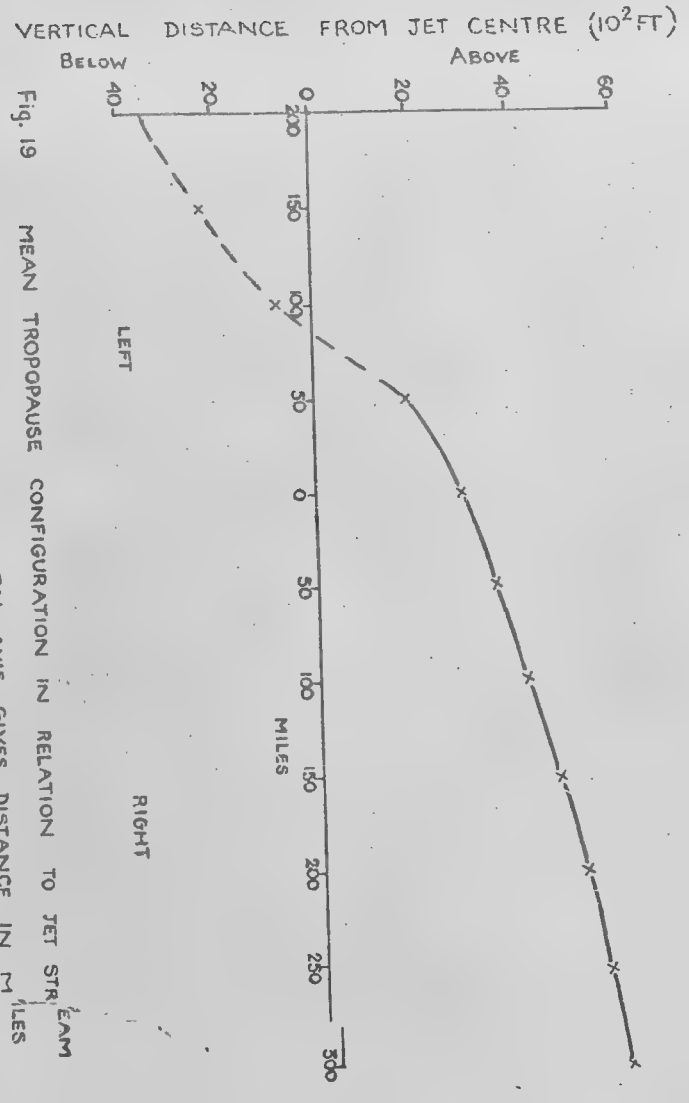

Fig. 19 MEAN TROPOPAUSE CONFIGURATION IN RELATION TO JET STREAM CENTRE AS ORIGIN; HORIZONTAL AXIS GIVES DISTANCE IN MILES FROM JET CENTRE, VERTICAL AXIS GIVES HEIGHT IN 100'S OF FEET ABOVE OR BELOW LEVEL OF JET CENTRE.

variation and the increase of speed; for instance, of the 36 cases of winds below 40 knots, 18 have a change less than 2.9 knots in 10 minutes; of the 31 of winds between 41 and 60, 15 have a change less than 3.1 knots in 10 minutes; whereas of the 19 cases with winds over 60 knots, 9 have a change less than 4.5 knots in 10 minutes.

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METEOROLOGICAL RESEARCH COMMITTEE

The variation of wind in short periods of time between 30,000 and 35,000 feet, January, 1950 to January, 1952.

by

C.S. Durst (Meteorological Office)

1. At the end of 1948 the Quarterly Journal of the Royal Meteorological Society printed a paper on the Fine Structure of Wind in the Free Air (1) which was based on observations made in a camera obscura at Orfordness of smoke puffs emitted from aircraft. In that series of observations most of the runs were made at 5,000 feet or less but there were four at 19,000, 27,000, 15,000 and 10,000.

2. As a result of the interest aroused by these observations Orfordness were asked to made further measurements at 30,000 feet or more whenever practicable in the course of other operations and this was put on the programme of the station in January, 1950. A considerable body of observations was assembled but much of it referred to times when the aircraft was ascending or descending and so was inappropriate to the discussion of the variation of wind with time. There were however 86 pairs of winds observed at the same height (between 30,000 and 35,000 feet), mainly with a 10 minute interval of time between the pair; in some cases where the interval was slightly longer or shorter than 10 minutes a proportional decrease or increase could be reasonably made in the wind change. Each pair of observations was classified according to the general direction and speed of the wind, the vector variation of wind in 10 minutes, the height of observation and the height above or below the tropopause. Details of these observations are shown in the Appendix.

3. In Table I is shown the number of occasions on which vector speed changes of various amounts occurred when the general wind at flying height was of specified speeds.

Table I. Relationship between wind speed at a height of 30,000 to 35,000 feet and the vector change in wind in 10 minutes.

Vector change of wind speed (Knots per 10 minutes)	Wind Speed (kts.) 0-20	21-40	41-60	61-80	81-100	101-120	Total
0-2.0	1	11	10	1	1	1	25
2.1-4.0	3	11	13	2	1	0	30
4.1-6.0	2	6	2	2	2	5	19
6.1-8.0	0	1	2	0	1	1	5
8.1-10.0	0	0	3	0	0	0	3
Over 10.0	0	1	1	1	0	1	4
Total	6	30	31	6	5	8	86
Root mean square of change (kts)	3.3	4.9	5.1	9.1	4.5	6.6	

There appears to be a certain degree of relationship between the increase of the variation and the increase of speed; for instance, of the 36 cases of winds below 40 knots, 18 have a change less than 2.9 knots in 10 minutes; of the 31 of winds between 41 and 60, 15 have a change less than 3.1 knots in 10 minutes; whereas of the 19 cases with winds over 60 knots, 9 have a change less than 4.5 knots in 10 minutes.

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4. The interval of 10 minutes between observations was much greater than that aimed at in the first series of observations referred to in paragraph 1. However, from the larger amount of data at the levels of 6,000 feet or less, 82 cases were picked out on which observations were made at about 10 minute intervals and estimates were made of the change of wind in 10 minutes. The changes were found to be as shown in Table II.

Table II. Vector change of wind (in knots) in 10 minutes at heights between 2,000 and 6,000 feet.

Change of wind	0-1.0	1.1-2.0	2.1-3.0	3.1-4.0	4.1-5.0	5.1-6.0
Number of occasions	18	22	22	11	6	3

At first sight it appears that the changes are much less at the lower heights than those shown in Table I but the wind speeds at the lower heights were very considerably smaller than in the new set of observations, indeed no wind was above 26 knots in the series represented by Table II and on the whole it would seem that if some rough allowance were made for the rate of air flow there would be little difference in the changes between the lower and the upper level.

5. In Table III is shown the frequency of various wind changes in 10 minutes grouped according as the smoke puffs were fired above or below the tropopause or within 1,500 feet of it.

Table III. Frequency of changes in wind in 10 minutes in the Stratosphere, Troposphere or Tropopause at heights of 30,000 to 35,000 feet.

	Vector changes in wind in 10 minutes (kts.)						
	0-2.0	2.1-4.0	4.1-6.0	6.1-8.0	8.1-10.0	10.1 or more	Total
In Stratosphere	9	7	7	3	0	2	28
In Tropopause	9	9	7	1	3	2	31
In Troposphere	7	14	5	1	0	0	27
Total	25	30	19	5	3	4	86

The biggest changes (6 kts. or more) seem to occur near the tropopause or in the stratosphere, rather than in the troposphere.

1 Durst, C.S. The Fine Structure of Wind in the Free Air.

Q.J.R. Met. S. Vol. 74, No. 321-2, July-October, 1948.

Appendix

Changes of wind speed observed in short periods at 30,000 to 35,000 feet over Orfordness

Date	Time	Height	Wind		Adjusted vector change in 10 minutes		Time interval between measurement minutes	Height of tropopause at 1500 G.M.T.	
			Dir.	Speed kts.	Dir.	Speed kts.			
28. 1.50	11.40	32,000	040°	35	190°	6.8	10	34,000	
28. 1.50	12.00	34,000	040°	29	170°	4.9	15	34,000	
16. 3.50	15.55	33,500	270°	53	250°	3.1	10	35,000	
"	16.05	33,500	270°	53	195°	1.7	10	"	
"	16.15	33,500	270°	53	070°	3.5	10	"	
24. 3.50	15.40	35,000	325°	55	285°	2.2	15	38,000	
20. 4.50	15.30	35,000	035°	55	125°	7.1	10	35,000	
2. 6.50	08.50	35,000	240°	15	330°	4.3	10	36,000	
"	09.00	35,000	240°	17	060°	1.7	10	"	
27. 7.50	16.20	35,000	260°	61	010°	4.0	10	39,000	
"	16.30	35,000	260°	61	360°	2.5	15	"	
"	16.45	35,000	260°	61	265°	4.5	10	"	
"	16.55	35,000	260°	61	205°	1.4	12	"	
24. 8.50	12.50	35,000	200°	96	090°	6.7	10	32,000	
25. 8.50	14.05	35,000	230°	53	300°	3.1	10	35,000	
"	16.45	35,000	235°	55	230°	9.4	5	"	
7. 9.50	15.40	35,000	270°	114	050°	4.4	8	35,000	
"	15.58	28,000	275°	102	090°	4.8	10	"	
3.10.50	10.45	35,000	310°	90	240°	5.2	10	40,000	
	10.55	35,000	310°	90	080°	2.7	15	"	
14.11.50	11.03	30,000	285°	43	085°	3.5	18	29,000	
4.12.50	14.55	30,000	280°	42	080°	16.2	10	26,000	
11.12.50	11.38	30,000	230°	27	060°	0.9	10	27,000	
	11.48	30,000	230°	27	070°	1.4	10	"	
	11.58	30,000	230°	27	030°	1.9	10	"	
12.12.50	10.35	30,000	320°	42	340°	1.4	10	30,000	
"	14.30	30,000	330°	56	170°	10.0	12	"	
"	14.42	30,000	330°	56	210°	0.9	14	"	
28.12.50	13.00	30,000	040°	26	220°	1.0	10	34,000	
	13.10	30,000	040°	26	230°	0.7	10	"	
9. 1.51	10.55	30,000	275°	61	120°	21.0	10	29,000	
"	11.05	30,000	275°	61	260°	4.1	10	"	
"	11.30	35,000	265°	47	145°	0.9	15	"	
"	11.45	35,000	265°	47	295°	6.4	15	"	
"	11.55	35,000	265°	47	225°	4.2	10	"	
12. 1.51	10.50	35,000	220°	40	310°	3.1	10	27,000	
	10.50	35,000	220°	40	300°	3.5	10	"	Double Puffs
	11.00	35,000	220°	40	270°	1.1	10	"	
	11.00	35,000	220°	40	120°	2.0	10	"	
24. 1.51	14.15	33,000	230°	31	050°	20.5	10	32,000	
	14.25	33,000	230°	31	205°	2.9	10	"	
	14.35	33,000	230°	31	190°	3.9	10	"	
	14.45	33,000	230°	31	010°	6.0	10	"	
6. 2.51	15.30	31,000	230°	25	015°	3.0	10	28,000	
	15.40	31,000	230°	25	270°	3.1	10	"	
	15.50	31,000	230°	25	160°	0.8	10	"	
7. 2.51	11.20	30,000	280°	32	090°	5.0	10	28,000	
	11.30	30,000	280°	32	250°	2.8	10	"	
	11.40	30,000	280°	32	100°	1.2	10	"	
21. 2.51	11.15	35,000	265°	35	190°	4.8	10	27,000	
27. 2.51	15.20	35,000	005°	102	160°	13.3	10	32,000	
	15.30	35,000	005°	102	165°	5.2	10	"	
	15.40	35,000	005°	102	360°	4.7	10	"	
	15.55	35,000	005°	102	230°	5.5	15	"	
	16.05	35,000	005°	102	235°	7.1	10	"	
15. 3.51	11.25	34,000	285°	49	205°	2.5	10	36,000	
	11.35	34,000	285°	49	110°	3.0	10	"	

/15. 3.51

Date	Time	Height	Wind		Adjusted vector change in 10 minutes		Time interval between measurement minutes	Height of tropopause at 1500 G.M.T.
			Dir.	Speed kts.	Dir.	Speed kts.		
15. 3.51	15.50	33,400	280°	50	270°	2.8	15	36,000
17. 4.51	10.50	33,000	280°	86	255°	4.7	15	38,000
17. 4.51	15.55	35,000	280°	83	145°	1.8	15	38,000
19. 4.51	14.50	33,500	350°	37	260°	3.1	15	38,000
	15.05	33,500	350°	37	240°	5.2	10	"
29. 5.51	08.35	35,000	040°	24	330°	2.9	15	33,000
	08.50	35,000	040°	24	030°	0.7	10	"
	09.00	35,000	040°	24	230°	2.4	10	"
29. 5.51	12.40	35,000	360°	22	010°	4.3	15	33,000
31. 5.51	08.30	35,000	090°	55	080°	3.0	10	36,000
	08.40	35,000	090°	55	240°	1.6	10	"
	08.50	35,000	090°	55	215°	1.3	10	"
	09.00	35,000	090°	55	100°	1.4	5	"
31. 5.51	15.30	35,000	100°	51	163°	5.9	10	36,000
	15.40	35,000	100°	51	300°	8.9	7	"
	15.47	35,000	100°	51	020°	3.2	8	"
	15.55	35,000	100°	51	220°	0.8	10	"
6. 6.51	15.55	34,000	270°	5.2	290°	2.3	10	35,000
	16.05	34,000	270°	5.2	105°	5.0	5	"
8. 6.51	11.10	34,000	265°	40	155°	2.8	10	37,000
18. 6.51	12.40	34,000	210°	114	015°	0.7	15	34,000
20. 6.51	10.50	35,000	255°	48	025°	2.2	10	37,000
	11.00	35,000	255°	48	305°	1.5	15	"
9. 10.51	15.30	30,000	085°	7.7	345°	3.1	15	40,000
11.10.51	11.25	34,000	155°	28	345°	1.1	15	41,000
25.10.51	11.00	32,000	035°	20	330°	2.2	10	42,000
29. 1.52	14.45	30,000	285°	60	260°	3.9	10	33,000
	14.55	30,000	285°	60	125°	1.6	10	"
	15.05	30,000	285°	60	230°	3.5	10	"

31st August, 1952

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ON THE THREE DIMENSIONAL STRUCTURE OF FRONTS

PART II

Cloud in relation to warm and quasi-stationary fronts near Bircham Newton in winter

by

A.G. Matthewman (Meteorological Office).

Summary

The cloud coexisting in the vertical with frontal zones of warm, warm occlusion and quasi-stationary types as reported by aircraft over Bircham Newton between January 1 and March 31, 1942 is classified and studied in relation to a number of parameters which might suggest themselves to the forecaster. Moderate correlations are found between the vertical extent of cloud and the forward component of wind relative to the front, but other parameters give negligible correlation coefficients.

1. Scope of the investigation

The present investigation consists of a statistical examination of the relation of cloud to frontal zones of warm, including warm occlusion, and quasi-stationary types near Bircham Newton, the cloud data being derived from aircraft reports over the winter period January 1 to March 31, 1942.

Cloud is not an easy element to deal with statistically, but it was here considered that all the cloud co-existing in the same vertical column of air with one or more frontal zones could be usefully divided into five or six classes, with the class-number increasing, roughly, with the cloudiness. Details of the classification adopted are given below in section 4. It did not appear profitable to attempt greater detail in relating amount and location of the cloud to the frontal zones. Various parameters, which are usually or sometimes available to the forecaster, have been examined in relation to this cloud classification.

2. The data

The British Daily Weather Reports (International Section) over the period January 1 to March 31 were examined for all analysed surface warm fronts, warm occlusions and quasi-stationary fronts near Bircham Newton.

All cases were admitted for which the following conditions were satisfied:-

(i) the cold or transitional air could be identified near the ground over Bircham Newton;

(ii) the boundary of the warm air at Bircham Newton was below 400 mb;

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(iii) a series of soundings was available roughly in a straight line perpendicular to the surface front.

The German Daily Weather Reports provided much data from aircraft ascents over Occupied Europe. The times of the vertical cross-sections thus available were around 07.00 and 15.00 hours. Wind soundings from Downham Market were also used.

3. Technique of the investigation

Simplified vertical cross-sections were prepared with I.C.A.N. height as ordinate, showing

(i) the saturated-potential temperature (i.e. the temperature of the dry-bulb measured on the scale of saturated adiabatics on the T-ø gram, labelled according to the value at 1000 mb.);

(ii) the relative humidity, if known;

(iii) the location and amount of cloud whenever reported on aircraft ascents.

The cross-sections were analysed, except in one or two very doubtful or very peculiar cases, by indicating the boundaries of the frontal zones. The classical view of a frontal zone was adopted, namely a sloping zone of lesser lapse-rate bounded by a sudden change to the larger and more or less steady lapse-rate in the "air-mass" on either side.

The wind soundings reported from Downham Market were plotted in the form of a time section with I.C.A.N. height as ordinate, and time as abscissa. Isopleths of speed and direction were drawn to obtain interpolated values of the wind at the levels of the boundaries of the frontal zone at the time of the relevant Bircham Newton aircraft soundings.

Measurements of the geostrophic wind shear at the fronts at the time, and the changes over about 12 hours, were made on the surface charts as published in the Daily Weather Reports. The speed of the fronts averaged over about 12 hours were also derived from measurements on these charts.

4. Uncertainties of the frontal analysis

The atmosphere is not made up of ideal homogeneous air masses interacting only at frontal zones of mathematical simplicity. Even in the interior of well-defined air masses there are random fluctuations of temperature of 1 or 2°F on a scale of 10 to 50 miles (Frith, 1950). Fluctuations of frost point are much greater and wind varies by 2 or 3 kt. Such irregular fluctuations are likely to be greater in the neighbourhood of fronts where the gradients of wind and temperature are larger and complications are introduced by condensation and precipitation. It is not surprising, therefore, that it should be difficult to identify with certainty the boundaries of the airmasses and frontal zone on the vertical sounding even if there is a tendency to establish a sharp discontinuity in lapse rate at these boundaries.

The analysis of the frontal cross-sections cannot therefore be regarded as entirely objective. Of the 55 vertical cross-sections 49 could be analysed in reasonable accord with the classical frontal model. Confidence in the analysis was assessed as follows:-

	Certain	Probable	Doubtful
No of Cases	10	25	14

In 1942 reports of humidity were irregular and of doubtful accuracy, but in general it was possible to avoid mis-identifying subsidence inversions as frontal zones.

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The frontal analyses which were adopted deliberately avoided any complicated analyses into several frontal sub-zones unless the evidence for them was overwhelming. Nevertheless in 12 cases, two frontal zones were deemed probable. In some cases two or more surface fronts were analysed on the surface charts while not more than one frontal zone could be identified on the vertical soundings.

Modification of the warm, transitional and cold air masses particularly near the ground or over the sea gave some difficulty in deciding the positions of the frontal zone on the surface chart; it was considered that in eight cases (five being of warm occlusion type) the warm air did not reach the ground along the line of the cross-section; in an additional 18 cases there was also some uncertainty.

5. Classification of frontal cloud

It was assumed that all cloud in the vertical column from the ground to 400 mb (the limit of the Bircham Newton aircraft ascents) co-existing with one or more frontal zones was "frontal cloud". The cloud was then classified into the following six classes:

Class 1: No cloud.

Class 2: Neglecting cirrus, cirrostratus or cirrocumulus, cloud only below or near the base of the frontal zone(s).

Class 3: One or more thinnish layers of cloud above or near top of frontal zone(s). Here thinnish implies not more than 150 mb deep.

Class 4: Thick cloud extending above the frontal zone, but with one or more layers or lanes of clear air at least 150 mb deep, between the ground and 400 mb. Here thick implies more than 150 mb deep.

Class 5: Continuous or almost continuous cloud extending from near the ground to near or above 400 mb but with one or more layers or lanes of clear air not more than 150 mb deep.

Class 6: Continuous cloud extending from near the ground to near or above 400 mb.

Since the number of cases in each of the classes 5 and 6 was small, for certain calculations of correlation coefficients, classes 5 and 6 were combined.

The frontal cloud having been classified according to the above definitions, relationships were sought between the class-numbers and various synoptic parameters associated with the fronts.

6. Results

(i) Relation of the cloud at Bircham Newton to the cloud at a sounding towards (a) the warmer side (b) the colder side of the frontal zone.

(a) Taking the comparison with the sounding towards the warm side first, six cases were included in which the sounding on the warmer side, though near Bircham Newton, was entirely in warm air. A marked relation existed between the class-numbers at the two soundings. Of the 23 cases for which the necessary data were available, only four cases showed a larger class-number at Bircham Newton than the class-number on the warmer side, and two of these four were with the surface front between the soundings. Moreover in these four cases the class-number at the warmer sounding was only greater by one than the class-number at Bircham Newton.

(b) Taking the comparison with the sounding on the colder side, eight cases, the class-number at Bircham Newton was in no case less than the class-number on the cold side.

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These relationships suggest both that the classification is meaningful, and that within the definitions of the various classes given in section 5, the frontal cloud rarely decreased, and then only slightly in the direction of the surface front.

These relationships are of course in harmony with accepted ideas, but it is perhaps surprising that on these data the relationships were so good.

(ii) Relationship of the cloud at Bircham Newton with the distance from Bircham Newton of the mid-point of the frontal zone at the ground.

Associated with the relationship in (i) above, and thus confirmatory of them, the mean value for each class of the distance of Bircham Newton from the mid-point (in the cross-section) of the frontal zone at the ground was found to decrease almost steadily with increasing class-number, from about 320 mi in class 1 to 80 mi in class 6. There was considerable scatter about the mean in each class: in particular in class 2, the distance varied from 620 mi to - 95 mi.

The tendency for the cloud class-number to increase as the front is approached is also clear from Table 1 in which the cases are analysed according to both class-number and distance from the front.

Table 1. No.of cases of specified cloud structures within stated ranges of distance from the surface front.

Distance of Bircham Newton from surface front	Cloud Class-Number						
	1	2	3	4	5	6	All
	(No. of cases)						
Less than 200 mi	1	4	7	8	2	2	24
200 to 400 mi	2	7	7	3	1	0	20
More than 400 mi	1	2	1	2	0	0	6

(iii) Relationship of the cloud at Bircham Newton with the pressure-height of (a) the upper (warm) boundary of the frontal zone and (b) the lower (cold) boundary of the frontal zone.

Since the pressure-height of the boundaries of the frontal zone was related to the distance of the surface front from Bircham Newton, it was to be expected that the relationship of cloud with the pressure-height of the boundaries of the frontal zone would be similar to the relationship in (ii) above.

(a) Taking the upper (warm) boundary first, and averaging for each class, the means of the pressure-height decreased with the class-number, from 680 mb in class 2 to 825 mb in class 6, but 720 mb in class 1.

(b) Taking next the lower (cold) boundary, and again averaging for each class, the means of the pressure-height decreased with increasing class-number, from 830 mb in class 2 to 1000 mb in class 6 (the ground was taken as the I.C.A.N. zero height corresponding to 1013 mb); the mean in class 1 was 885 mb.

The scatter in (a) and (b) was again large, e.g. in class 3 the individual pressure-heights ranged from 920 to 500 mb.

/(iv)

(iv) Relationship between the cloud at Bircham Newton and the slope of the warm boundary of the frontal zone.

The tangent of the angle of slope of the warm boundary of the frontal zone was measured in the vertical cross-sections, and multiplied by the secant of the angle between the direction of the surface front and the direction of the cross-section. This was considered to be the best practicable measure of the tangent of the angle of greatest slope of the warm boundary.

The mean value over all cases for the tangent of the angle of slope was 0.005 = 1/200.

The mean values of the slope in each class showed no clear trend with class-number but varied between 0.006 (in class 3) and 0.003 (in class 1).

(v) Relationship of cloud at Bircham Newton with the pressure-thickness of the frontal zone.

The average over all cases of the thickness of the frontal zone was about 160 mb. The averages for each class showed no apparent relationship with class-number, varying from 175 mb in class 6 to 150 mb in class 4 - a small range. But the scatter within each class was again considerable, e.g. in class 3 from 45 mb to 450 mb.

(vi) Relationship of location in the vertical of the base of the lowest significant medium cloud at Bircham Newton with the location of (a) the upper (warm) boundary, (b) the middle of the frontal zone and (c) the lower (cold) boundary.

The elementary picture of a warm frontal zone with the medium cloud extending upwards from the (warm) boundary of the frontal zone was not well verified. Table 2 gives the percentage frequency with which cloud was present at various levels below, in and above the frontal zone. For this purpose the presence or absence of cloud has been tabulated at intervals of 2,000 feet above the upper limit of the frontal zone, at 2,000 ft. intervals below the lower limit of the frontal zone and at three equally spaced points within the zone.

Table 2 - Percentage frequency of occurrence of cloud at levels specified in relation to the height of the frontal zone.

Level		Percentage frequency of Cloud
Above frontal zone	16,000) feet above upper boundary of frontal zone	42
	14,000)	46
	12,000)	36
	10,000)	38
	8,000)	40
	6,000)	36
	4,000)	34
	2,000)	36
Frontal zone	Upper boundary of frontal zone	28
	$\frac{3}{4}$ of depth of frontal zone above lower boundary	33
	Mid-point of frontal zone	28
	$\frac{1}{4}$ of depth of frontal zone above lower boundary	41
	Lower boundary of frontal zone	33
Below frontal zone	2,000) feet below lower boundary of frontal zone	32
	4,000)	12
	6,000)	0

It appears that cloud is equally likely at any level in or above the frontal zone and down to about 4,000 feet below it. The chance of finding cloud at a specified level is about 1 in 3.

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(vii) Relationship of cloud at Bircham Newton with the temperature difference across the frontal zone (measured on the saturated-adiabatic scale).

The saturated-potential temperature of the warm air above the frontal zone was generally nearly constant or increased only slightly with height (in 40% of the cases the lapse-rate of the warm air was equal to the saturated-adiabatic, in 37% the lapse-rate was slightly less than the saturated-adiabatic). The difference of saturated-potential temperature from the cold to the warm air vertically across the frontal zone provided a measure of the thermal strength of the frontal zone.

On average (over the 58 frontal zones) the saturated-potential temperature difference was 11.3°F.

The means for each class separately ranged from 10.7°F in class 4 to 13.3°F in class 6. The mean in class 5, like the mean of class 6, was high, 13.0°F, but not much practical significance can be attached to the differences. The scatter in each class was large, e.g. in class 3, the values ranged from 3°F to 25°F. The most that can be suggested is that a large temperature difference across a frontal zone may or may not imply much cloud, but that extensive cloud as in class 5 or 6 is unlikely to occur with small temperature contrasts. It must be recalled however that the only season studied has been winter and the conclusion may not necessarily be valid in summer when vertical instability may be a factor in producing cloud at otherwise weak fronts.

(viii) Relationship between cloud at Bircham Newton and the geostrophic wind-shear in the horizontal across the front at the ground.

The geostrophic wind-shear, i.e. the change in geostrophic component parallel to the front, was measured by a geostrophic wind-scale on opposite sides of the front near Bircham Newton. The measurements were made on the surface chart as drawn at C.F.O. nearest the time of the Bircham Newton sounding.

The results are shown in Table 3.

Table 3 - Relation between frontal cloud and shear of geostrophic wind at fronts.

Class-No.		1	2	3	4	5	6	ALL
Horizontal geostrophic wind-shear at m.s.l. mi/hr.	Mean	24	16	20	38	43	40	25.9
	Standard Deviation	26	19	17	27	18	14	
No. of cases		3	10	19	10	3	3	48

There is an increase in the means from class 2 to class 5, but the scatter is large in all classes. Part of this scatter is due to observational errors, to inaccuracies in the drawing and copying of the m.s.l. isobars and in the estimation of the geostrophic speeds. It was considered that the errors might be reduced by taking in each case the average of the geostrophic shear on three charts, the chart nearest the time, the chart before, and the chart following. Time changes may be to some extent involved. The results showed however a distribution similar to that in Table 3.

(ix) Relationship of cloud at Bircham Newton to the time rate of increase of geostrophic wind-shear in the horizontal across the front at the ground.

It might be considered that the null relationship in (vii) and the uncertain relationship in (viii) were largely due to the property of a fluid to maintain equilibrium with a sloping discontinuity of density provided that the wind-shear is appropriate. The next parameter to be tested was accordingly the time rate of change of geostrophic wind-shear in the horizontal across the front at the ground. Forecasters often regard "troughing", in the sense of increasing geostrophic shear, as related to the activity of the front.

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The mode of measurement was first to draw the normal from Bircham Newton to the front, at the time nearest to the ascent at Bircham Newton, next to find the intersections of this normal with the front on the preceding and following charts, and to measure the geostrophic wind-shear across the front in each case. Dividing the difference by the time gave a measure of the troughing on the front. The Daily Weather Report was used for this purpose and the results are shown in Table 4 below:

Table 4. Relation between frontal cloud and rate of change of geostrophic wind shear.

Class-No.		1	2	3	4	5	6	ALL
Time-rate of change in mi/hr^{-2} of the geostrophic wind-shear in the horizontal across the front at the ground	Mean	-2.0	+0.2	-0.1	-0.4	-0.9	+1.0	-0.2
	Standard Deviation	0.3	1.0	2.4	1.2	1.5	0.2	
No. of cases		2	11	19	12	3	2	49

There is evidently no useful relationship. Partly this may be due to errors of observation, but it is considered that one difficulty not overcome in this mode of measurement is that as the front advances along the normal, the measurements are made at points nearer to the tip of the warm sector: the "troughing" may be masked by a weakening of the shear towards this tip of the warm sector. Another mode of measurement was tried, following the mean direction of the warm-sector isobars. This appeared to eliminate the largish negative value in class 5. However, further consideration of a satisfactory parameter to express the forecaster's experience that "troughing" of a front implies dynamical activity is held over for future study.

(x) Relationship of cloud at Bircham Newton to the actual wind-shear along the vertical across the frontal zone.

The classical view of a frontal zone as a sloping zone of lesser lapse-rate of (virtual) temperature implies that there is a larger horizontal gradient of (virtual) temperature within the zone than in the air-masses on either side. This in turn implies within the zone a larger rate of change in the vertical of geostrophic component parallel to the zone.

The actual wind is not geostrophic, but usually a well-marked frontal zone is associated with a conspicuous increase with height of the actual wind component parallel to the zone.

The components of wind-shear parallel to the front were computed from the serial graphs of wind-speed and direction at Downham Market; the orientation of the front was taken to be that of the surface front. The shear was taken from the lower to the upper boundary of the frontal zone as analysed on the cross-section.

The results are shown in Table 5:

Table 5. Relation between frontal cloud and vertical wind-shear.

Class-No.		1	2	3	4	5	6	ALL
Actual wind-shear (mi/hr) in the vertical from the lower to the upper boundary of the frontal zone	Mean	4	11	18	17	21	20	15.4
	Standard Deviation	8	20	16	18	18	15	
No. of cases		4	10	14	12	2	3	45

There is evidently some tendency in the mean of each class to increase with the class-number. Combining classes 5 and 6, the correlation coefficient between the vertical shear and the class-number is +0.22.

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(xi) Relationship of the cloud at Bircham Newton to the vertical change across the frontal zone of the actual wind-component normal to the surface front.

It has been suggested by various writers (e.g. Parker 1949) that the change with height of wind component perpendicular to the direction of a surface front is a measure of the vertical velocity at the frontal zone.

If u, w respectively denote the horizontal and vertical components of wind at a frontal boundary in the vertical plane containing the line of greatest slope of the boundary, and if θ denotes the angle of slope of the boundary, then provided air does not cross the boundary,

$$u = u_b + w \cot \theta \qquad \text{.....(1)}$$

where u_b is the horizontal speed of the boundary.

If the upper and lower boundaries move at the same speed

$$u_2 - u_1 = w_2 \cot \theta_2 - w_1 \cot \theta_1 \qquad \text{.....(2)}$$

where the suffixes 1, 2 refer to the lower and upper boundaries respectively.

If it is assumed that $w_1 = 0$, i.e. the cold air moves horizontally with the lower frontal boundary,

$$u_2 - u_1 = w_2 \cot \theta_2$$

In so far as the existence of cloud implies an upward vertical velocity at the time, $u_2 - u_1$ might thus be expected to have a relation with the class-number of the frontal cloud. The results of such a comparison are shown in Table 6:

Table 6. Relation of frontal cloud to vertical wind-shear normal to the front.

Class-No.		1	2	3	4	5	6	ALL
Increases in mi/hr of the component of wind normal to the surface front from the lower to the upper boundary.	Mean	0	2	7	10	1	3	5.8
	Standard Deviation	8	8	7	13	1	8	
No. of cases		4	10	14	12	2	3	45

There is evidently some increase in the means of each class from class 1 to class 4, but the relation breaks down in classes 5 and 6; this breakdown may be partly due to taking the 950 mb. level for the lower boundary when the frontal zone extends down to the ground. But it is evident that the scatter is large and the correlation coefficient with class-number (combining classes 5 and 6) is only +0.19.

(xii) Relationship between the cloud at Bircham Newton and the difference between the speed of the surface front and the component of wind normal to the surface front at the upper boundary of the frontal zone.

Returning to equation (1) and assuming that $u_b = u_f$ the speed of the surface front, it follows that

$$u_2 - u_f = w_2 \cot \theta_2 \qquad \text{.....(3)}$$

The speed of the front was taken as the average over about 12 hours measured on the D.W.R. charts. u_2 was derived from the serial section of the wind

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at Downham Market, taking the direction normal to the orientation of the surface front. Table 7 below gives the results of relating $u_2 - u_f$ to the class-number of the frontal cloud.

Table 7. Relation of frontal cloud difference between speeds of warm air and of front itself.

Class-No.		1	2	3	4	5	6	ALL
Difference (mi/hr) between the speed of the surface front and the component of wind normal to the surface front at the upper boundary of the zone	Mean	-9	-7	+1	+5	+2	+10	+1.4
	Standard Deviation	4	14	6	16	15	10	
No. of cases		4	9	15	11	2	3	44

There is evidently a tendency for increase in $u_2 - u_f$ with increasing class-number. Combining classes 5 and 6, which are small classes, the relationship is fairly good. The correlation coefficient between $u_2 - u_f$ and the class-number is +0.42.

Referring to equation (3) and multiplying each side by tan θ_2 ,

$$w_2 = (u_2 - u_f) \tan \theta_2$$

The tangent of the angle of slope of each frontal surface had already been determined as described in (vi), and the product of $u_2 - u_1$ and tan θ_2 was accordingly calculated. The correlation of this product with the cloud class-number (again combining classes 5 and 6) was +0.36.

It was considered that the relations expressed in Table 7, though fairly good, might be improved by selecting all cases for which the vertical shear was appreciable. The mean vertical shear for all cases is 15.4 m.p.h. Restricting the cases to those for which the vertical shear was 8 mi/hr, or more, the comparison between $u_2 - u_f$ and the class-number of the frontal cloud was repeated. The correlation was thereby significantly improved to +0.54.

Repeating also for these cases of vertical shear 8 mi/hr or more, the comparison between $(u_2 - u_f)$ tan θ_2 and the class-number, the correlation coefficient of +0.50 was obtained.

(xiii) Relationship between cloud at Bircham Newton and the difference between the speed of the surface front and the component of wind normal to the surface front at the lower boundary of the frontal zone.

Denoting elements referring to the lower boundary by the suffix 1, as before, and with similar approximation, in analogy with equation (3)

$$u_1 - u_f = w_1 \cot \theta_1 \qquad (4)$$

Adopting a technique similar to that in sub-section (xii), the results of the comparison between $u_1 - u_f$ and the class-number were as follows in Table 8:

Table 8. Relation of cloud at front to difference between speeds of cold air and of front itself.

Class-No.		1	2	3	4	5	6	ALL
Difference (in mi/hr) between the speed of the surface front and the component of wind normal to the surface front at the lower boundary	Mean	-9	-6	-7	-5	+1	+7	-5.1
	Standard Deviation	8	12	10	10	13	4	
No. of cases		4	8	14	11	2	3	42

There is a slightly irregular increase in the mean for each class with increasing class-number, but the correlation coefficient is small, namely +0.25.

It is interesting to compare this result with a comparison made by the present writer and reported elsewhere (Matthewman, 1952) between the component of wind at 900 mb normal to the surface warm front and the speed of the surface warm front, restricting the cases to those in which the lower boundary of the frontal zone lay between 910 and 840 mb and the associated T- Ø gram showed a well-marked frontal zone of reduced lapse-rate, with the wet-bulb potential temperature increasing above the lower boundary. Over 37 cases of this kind, the correlation between u_1 and u_f was very high, namely 0.85, with average value of $u_1 - u_f = -0.4$ mi/hr.

The following interpretation of these results is tentatively suggested. Near the ground and up to 900 mb the vertical speed of the cold air, w_1, is usually small, but above this height downward velocities commonly occur, (predominantly negative mean values in Table 8). There is, however, probably a positive correlation between the vertical velocities in the two air masses; thus when cloud amount is small the warm air is probably descending and the cold air descending more quickly, but on the other hand when there is extensive cloud both air masses are likely to be moving upward, the cold air less quickly than the warm air. Thus the difference between the vertical velocity in the two air masses might have little correlation with the cloud structure in conformity with the low correlation found in Table 6.

(xiv) Relationship between cloud at Bircham Newton and the ageostrophic motion of the surface front.

It has been demonstrated that the speed of a warm front is determined by the speed of the cold air mass ahead of it (Matthewman, 1952) and that this speed is often less than the component of the geostrophic wind speed in the appropriate direction because the cold air is undergoing a pronounced acceleration. The warm air mass is usually less accelerated and its motion is likely to be closer to the geostrophic speed, thus providing the conditions for it to rise over the slower moving wedge of cold air. We may thus expect the vertical motion at a warm front to be related to the excess of the geostrophic wind component, u_J, normal to the front over the speed of the front u_f. The relation between the frontal cloud structure and the value of $u_J - u_f$ has therefore been investigated with the results given in Table 9.

Table 9. Relation between cloud at Bircham Newton and the ageostrophic motion of the surface front.

Class-No.		1	2	3	4	5	6	ALL
Difference (mi/hr) between geostrophic component, u_J, normal to the surface front and the speed of the front, u_f. $(u_J - u_f)$	Mean	6	7	7	8	14	6	7.4
	Standard Deviation		7	10	9	10	12	
No. of cases		1	8	17	9	3	3	41

It is clear from Table 9 that there is no close relation between the ageostrophic motion of the front and the thickness of the cloud layers. It is difficult to form reliable estimates of the speed of the front and of the geostrophic wind so that the absence of any clear cut relation may be in part due to errors in measurement and estimation. However, there is some tendency for the mean ageostrophic component of the fronts to be greater for the larger cloud class-numbers but it is doubtful if this is statistically significant.

Conclusions.

From the practical forecasters point of view, the only promising relations so far investigated between the extensiveness of frontal cloud (in the sense of the classification) and various meteorological elements, concern the associated actual wind-field. The testing of the remaining parameters has perhaps fulfilled a purpose in emphasising their unreliability.

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Even the best correlation with class-number here achieved, namely with the difference between the speed of the surface front and the actual wind component at the upper boundary of the zone, directed normal to the surface front, is only of order 0.5, which is probably below the limit of usefulness in the sense of a definite forecasting rule. Nevertheless relationships with no higher correlation have to be employed in some branches of forecasting, e.g. the correlation between actual and advected temperature changes (Craddock 1951): the present relationships may thus be of some aid to the forecaster when nothing better is available.

Extension of the investigation to more cases is not likely to improve these findings, but refinement of technique and restriction to the better-marked or less complex fronts would probably yield higher correlations, which might prove to be of direct use to the forecaster.

The absence of any regular relation between the cloud layers and the frontal zone emphasises the shortcomings of conventional models of the cloud structure at fronts. The relation between the frontal cloud and wind relative to the surface front indicate that extensive cloud is associated with upward motion in both cold and warm air masses. These facts, together with the poor correlation between cloud and parameters concerned with the frontal contrasts, suggest that the cloud in frontal regions may be associated more closely with the general upward motion over a large volume of air including the front rather than with any specific upgliding at the frontal surface.

/References:

References:

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[illegible] at a warm front to be related to the excess of the geostrophic wind component, u_J, normal to the front over the speed of the front u_f. The relation between the frontal cloud structure and the value of $u_J - u_f$ has therefore been investigated with the results given in Table 9.

Table 9. Relation between cloud at Bircham Newton and the ageostrophic motion of the surface front.

Difference ($u_J - u_f$)	Class-No.	1	2	3	4	5	6	ALL
(mi/hr) between geostrophic component, u_J, normal to the surface front and the speed of the front, u_f.	Mean	6	7	7	8	14	6	7.4
	Standard Deviation		7	10	9	10	12	
No. of cases		1	8	17	9	3	3	41

It is clear from Table 9 that there is no close relation between the ageostrophic motion of the front and the thickness of the cloud layers. It is difficult to form reliable estimates of the speed of the front and of the geostrophic wind so that the absence of any clear cut relation may be in part due to errors in measurement and estimation. However, there is some tendency for the mean ageostrophic component of the fronts to be greater for the larger cloud class-numbers but it is doubtful if this is statistically significant.

Conclusions.

From the practical forecasters point of view, the only promising relations so far investigated between the extensiveness of frontal cloud (in the sense of the classification) and various meteorological elements, concern the associated actual wind-field. The testing of the remaining parameters has perhaps fulfilled a purpose in emphasising their unreliability.

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METEOROLOGICAL RESEARCH COMMITTEE

Rainfall in the British Isles resulting from depressions which pass eastward between Scotland and the Faeroes

AIR WEATHER SERVICE
TECHNICAL REFERENCE

by

J.S. Sawyer (Meteorological Office)

1. Introduction

Methods of weather forecasting cover a wide range from those which depend entirely on empiricism and experience to those which are largely based on physical and dynamical principles. The possible methods of forecasting rainfall are similarly varied, but as the physical and dynamical processes which lead to rain are complicated and not fully understood the practical forecaster has relied mainly on empirical methods.

In America considerable effort has been devoted recently to developing methods of forecasting rain which rely entirely on empirical and statistical methods. This technique has been called the "objective method" of forecasting. Its special feature is that the forecaster is provided with criteria or regression equations for the occurrence and amount of rainfall based on the statistical analysis of past observations. The parameters which he inserts into these equations are all known at the time of preparing the forecast, and the forecaster does not need to consider either the changes in the synoptic situation or the physical processes which will result in the rain.

The methods adopted in the Meteorological Office for the forecasting of rain, although not formulated in a systematic manner, are based on a rather different procedure into which a greater element of physical and dynamical argument enters. Basic to the method is the forecasting of the field of surface pressure and the positions of fronts; the expected rainfall is estimated in relation to the forecast pressure field and fronts, as well as by direct extrapolation of the rain area itself.

The approach adopted in forecasting research is largely determined by the technique of forecasting which the research is designed to aid. M.R.Ps. 650 and 676 have been concerned primarily with introducing more physical and dynamical reasoning into the forecasting procedure, but a useful line for research also exists in the codifying of the experience which is the basis of present methods.

Essentially the requirement is to be able to state the distribution and amount of rainfall on the basis of knowledge of the surface pressure field and fronts together with relevant information on the large scale features of the upper air situation (e.g. by means of the thickness pattern). The ability to forecast the surface and upper level weather map is assumed.

In some preliminary experiments which were made to develop empirical methods of relating rainfall to the pressure field a sequence of frontal rain areas was studied. These differed widely in dynamical structure, and it was very difficult to find parameters relating to the synoptic situation which could be objectively determined and related with the rainfall. It was therefore decided to direct further study to the rainfall of depressions on a particular track in the expectation that these would have a more homogeneous structure.

/Depressions

Depressions passing eastward between Scotland and the Faeroes were selected as the subject for the first study along these lines. It was hoped that it would be possible to establish a rainfall distribution typical of depressions on this track and to relate departures of the rainfall from values appropriate to this distribution to features of the accompanying synoptic situation.

The results of the investigation are summarised in the following paragraphs. Although significant correlations were found between the rainfall and various synoptic parameters, the relations did not prove to be of practical forecasting value and many aspects of the investigation will only be briefly mentioned; more complete details are available in a manuscript available in the Forecasting Research Division (M.O. 21 Technical Note No. 52).

2. The average rainfall distribution

The examples for study were taken from the 10 year period 1941 to 1950, and 61 cases were found. In order to provide a reasonably homogeneous set of examples the following criteria were applied:

(a) a depression (with a closed isobar) must have passed between Scotland and the Faeroes with a component of motion towards the east;

(b) the warm sector must have extended across the British Isles as far northward as the Scottish Border. (This criterion serves to eliminate completely occluded systems and cold depressions).

For each selected depression a rainfall chart was plotted from the rainfall amounts published in the Daily Weather Report. The chart covered the whole period during which frontal rain from the depression was falling in any part of the British Isles. For some occasions the totals included contributions from showers in the cold air masses and from drizzle in the warm air mass. Difficulty arose in some cases in which a wave formed on the cold front. Such waves have been regarded as separate disturbances, and their rainfall has been omitted from the totals whenever possible. The uncertainties involved in the arbitrary choice of the rainfall period are not serious, and there is little doubt that in most cases the rainfall totals adequately represent the rainfall of the depression. A greater uncertainty arises from the sparse network of raingauge observations which have been used, and it must be recognised that the rainfall charts do not give an adequate representation of the orographic features of the rainfall pattern. Nevertheless, it is believed that they indicate the broad features of the rainfall distribution over the lower lying areas.

The average rainfall per depression is shown in Fig. 1 for a network of stations and in Fig. 2 is shown the percentage of the 61 depressions which gave less than 0·1 mm of rain. (Lines have been drawn on the charts to help in interpreting the plotted values but these should not be regarded as isohyets because of the absence of values for the hilly areas.) As would be expected the rainfall decreases from an average of more than 10 mm in the Northwest to less than 2 mm in the Southeast. The effect of the hills in the west of Great Britain is suggested, but there is little doubt that with an adequate network of observations this would be much more prominent. On the west coast of Ireland and Scotland rain always accompanies depressions on this track, but towards the southeast the proportion of dry cases approaches 50%.

In order to obtain some idea of the difference between the rainfall amounts in the hilly areas from those on lower ground the average rainfall from the 61 depressions was calculated for three stations which have high annual rainfall totals. The values for Snowdon and Cray Reservoir were taken from the autographic records and that for Eskdalemuir from the Daily Weather Report. The average values are given below for comparison with Figures 1 and 2.

	Average rainfall per depression	Percentage of depressions giving less than 0·1 mm	Normal annual rainfall
Eskdalemuir	16 mm	0	1430 mm (56·3 in)
Snowdon (Cwm Dyli)	33 mm	3	2995 mm(117·9 in)
Cray Reservoir (Breconshire)	10 mm	7	1778 mm (70 in)

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As much of the rain of the depressions was frontal in origin it is convenient to divide the depressions into classes depending on whether or not one or both of the fronts crossed the whole of the British Isles. Charts of the average rainfall and percentage of cases without measurable rain were constructed on the basis of the same network of stations as used in Figures 1 & 2. These are reproduced in Figures 3 to 6. The general similarity is apparent. As would be expected the rainfall amounts are considerably less when neither front crosses all of the country (Class IV); the difference between Class IV and Class I is greater in the southeast of the British Isles but even west Scotland has somewhat smaller amounts with Class IV. The percentage of cases without measurable rainfall is also much greater among the Class IV cases.

3. Departures from the average rainfall

The individual depressions give rainfalls which depart widely from the average rainfall for a depression on the selected track as given by Fig. 1. This is clear from the standard deviations as given in Table I and the histograms of rainfall amounts for two typical areas Fig. 7. (Area rainfalls are based on four selected stations in the area).

Table I - Average rainfall in various areas from depressions passing eastward between Scotland and the Faeroes

Area	W. Scotland	E. Scotland	W. Ireland	E. Ireland	N.W. England	N.E. England	S.W. England	S.E. England
Average rainfall mm	10·1	3·8	7·8	5·0	6·5	2·1	3·1	1·8
Standard Deviation mm	11·4	4·5	10·1	7·4	8·9	3·7	5·5	2·9
Medium mm	8·6	3·2	5·0	3·5	4·5	1·0	1·2	0·4

The departure of the rainfall from the average of Fig. 1 was compared with a considerable number of parameters selected to represent factors in the synoptic situation which were expected to have an influence on the rainfall either for theoretical reasons or from practical experience. Some significant correlations were obtained, and regression equations for estimating the rainfall from the selected parameters were derived. However tests of these prediction equations on data from an independent series of depressions gave little or no success in estimating the rain over most areas of the British Isles, and in view of the errors likely to occur in forecasting the parameters themselves, it was concluded that none of the relationships were sufficiently close to be of direct value in practical forecasting.

The parameters which were investigated are listed in the following table together with some notes on their relation with the rainfall.

Table II - List of parameters investigated and notes on relation with rainfall of depressions passing eastward between Scotland and the Faeroes

Parameter	Remarks
(1) Fall of pressure from preceding ridge to troughline of depression	Significant positive correlation with rainfall in all areas; greatest if fall of pressure measured in latitude 50 to 55N. Correlations range from +0·41 in the north to +0·67 in the south.

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Parameter	Remarks
(2) Sea level pressure on the trough line in latitude 50 & 55N.	Significant negative correlation with rainfall in all sectors - correlation coefficient varies from -0·31 in north to -0·59 in south. Pressure in latitude 60 gives smaller correlations.
(3) Largest downward 3hr pressure tendency at selected stations.	Positive correlations of up to +0·7 with rainfall for pressure tendencies in south and east England.
(4) Overall change in 1000-500 mb thickness with passage of depression.	Sum of rise and fall of thickness at East Anglian stations has significant positive correlation with rainfall. Coefficient ranges from +0·37 in north to +0·60 in southeast. Thickness change at Aldergrove gives lower correlation.
(5) Moisture content of the warm air mass.	Parameters used included (a) surface dew point (b) difference of surface dew point from seasonal normal (c) dew point between 700 & 800 mb. No significant correlation.
(6) Vertical stability of the warm air.	Difficulty was experienced in finding a parameter which was representative of the stability of the warm air mass. Strongest correlation was between rainfall and difference in wet-bulb potential temperature between 950 and 500 mb but this correlation did not exceed 0·40.
(7) Speed of depression.	Significant negative correlation - range from -0·30 in the north to -0·48 in central districts.
(8) Depth of depression.	No significant correlation.
(9) Orientation of surface isobars in the warm sector.	No significant correlation.
(10) Season of the year.	Indications of two maxima of rainfall - one in January and the other in August.

Conclusions

The present investigation has presented some facts about the rainfall associated with depressions passing eastward between Scotland and the Faeroes. These are in accord with synoptic experience; nevertheless it is probably of value to the forecaster to have the information presented in Figures 1 and 2 in numerical form.

The statistical analysis has demonstrated the difficulty of developing statistical methods which will give significantly better forecasts than those made by synoptic experience.

Further empirical investigations along the lines of the present study could serve a useful purpose in establishing the usual rainfall patterns in various synoptic situations; they would present objectively what is now available to the forecaster only as a subjective synthesis of long experience. However it appears that it will be difficult to find simple parameters which will serve to relate departures of rainfall from the average for the synoptic type with features of the prognostic charts. It will probably not be worth while to repeat the extensive statistical study in future empirical investigations of the present type.

Depressions passing Eastward between Scotland and the Faeroes.

FIG 1. Average Rainfall
No. of cases 61.

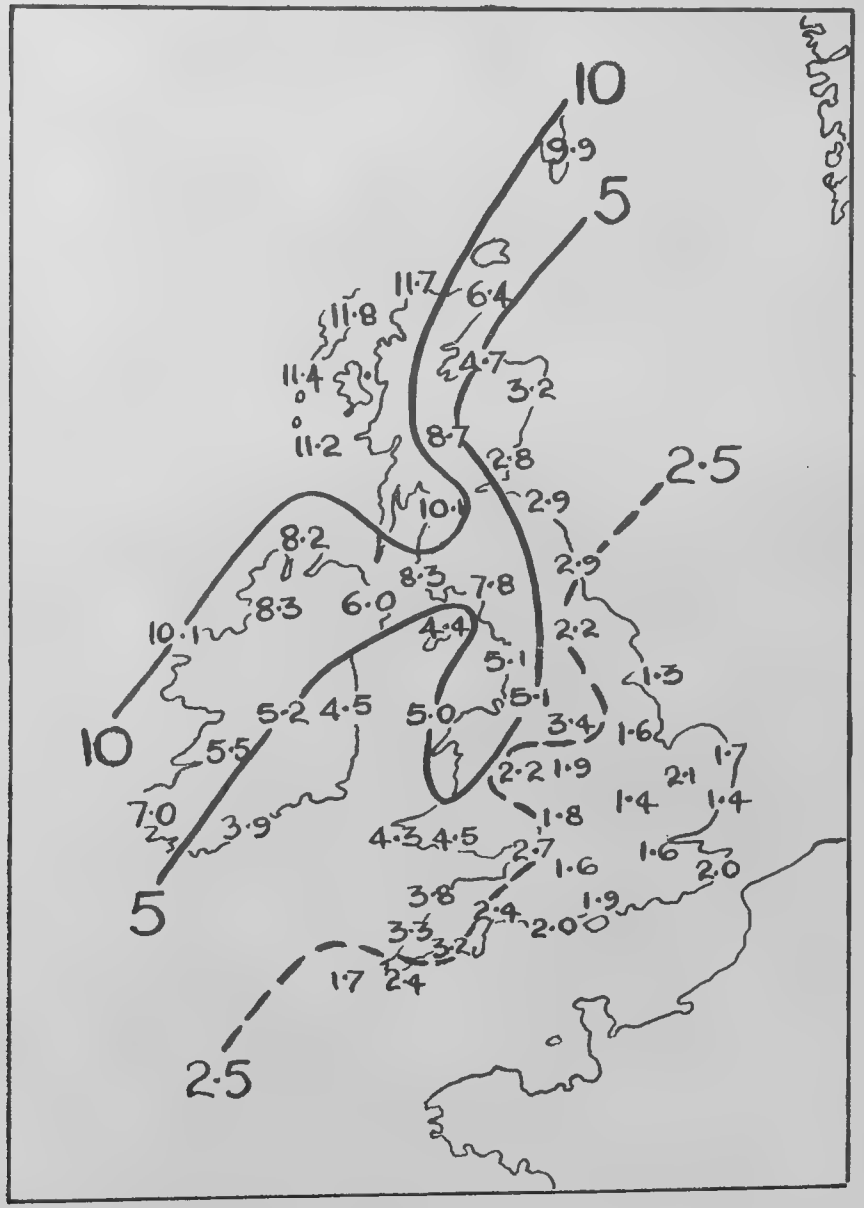
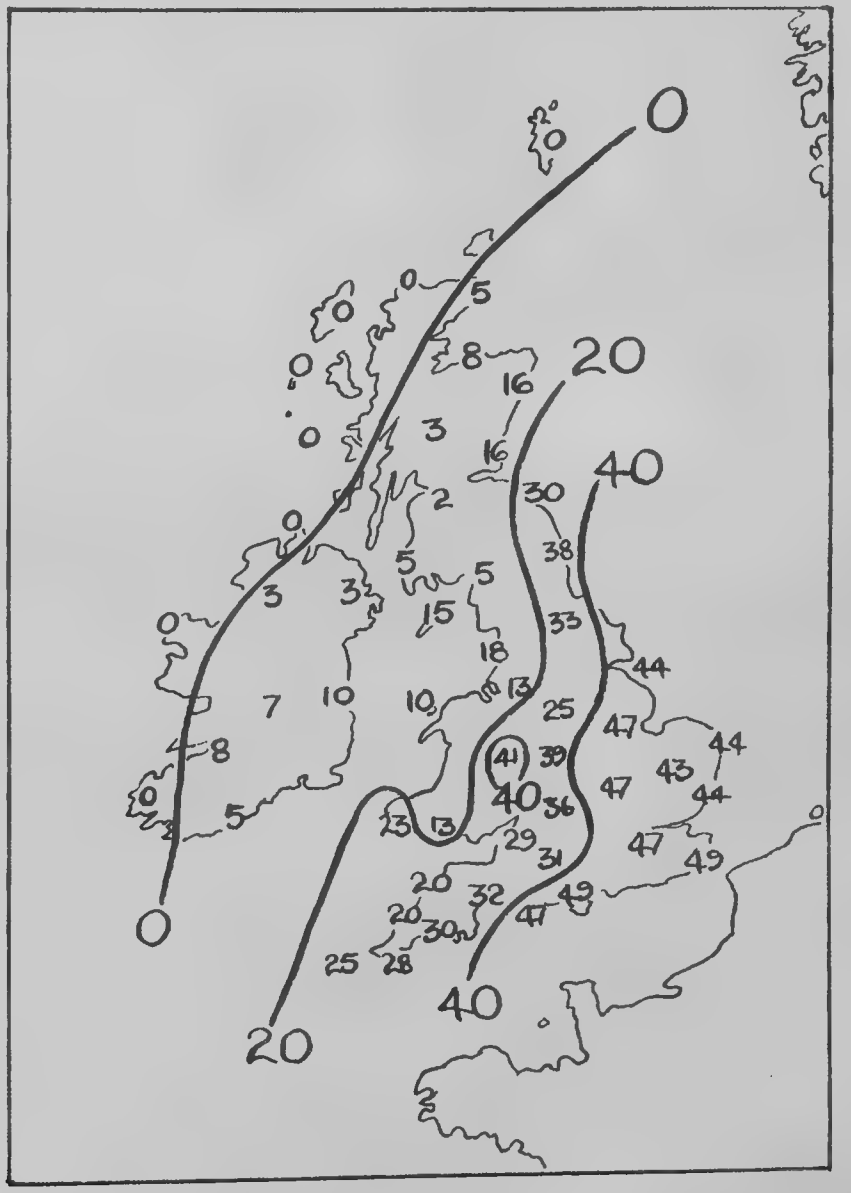
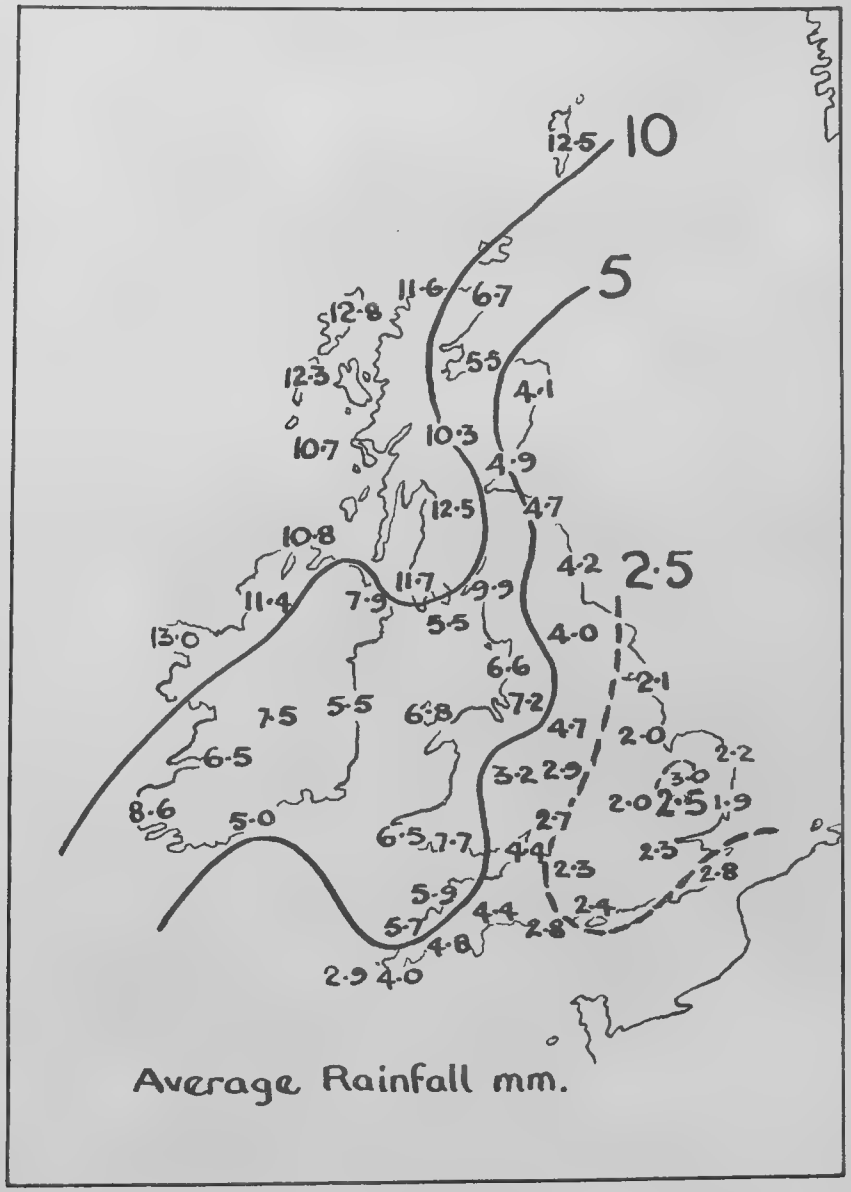


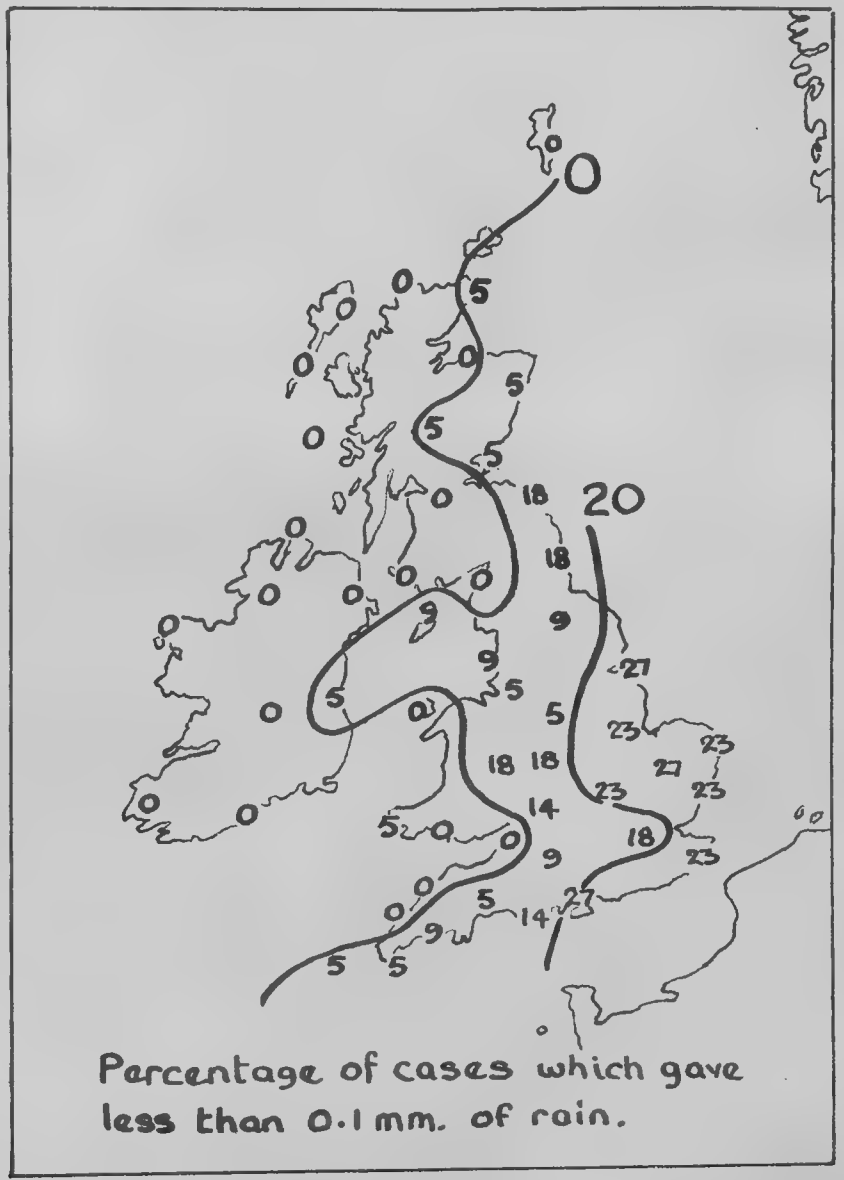
FIG 2. Percentage of cases which gave less than 0·1mm of rain.



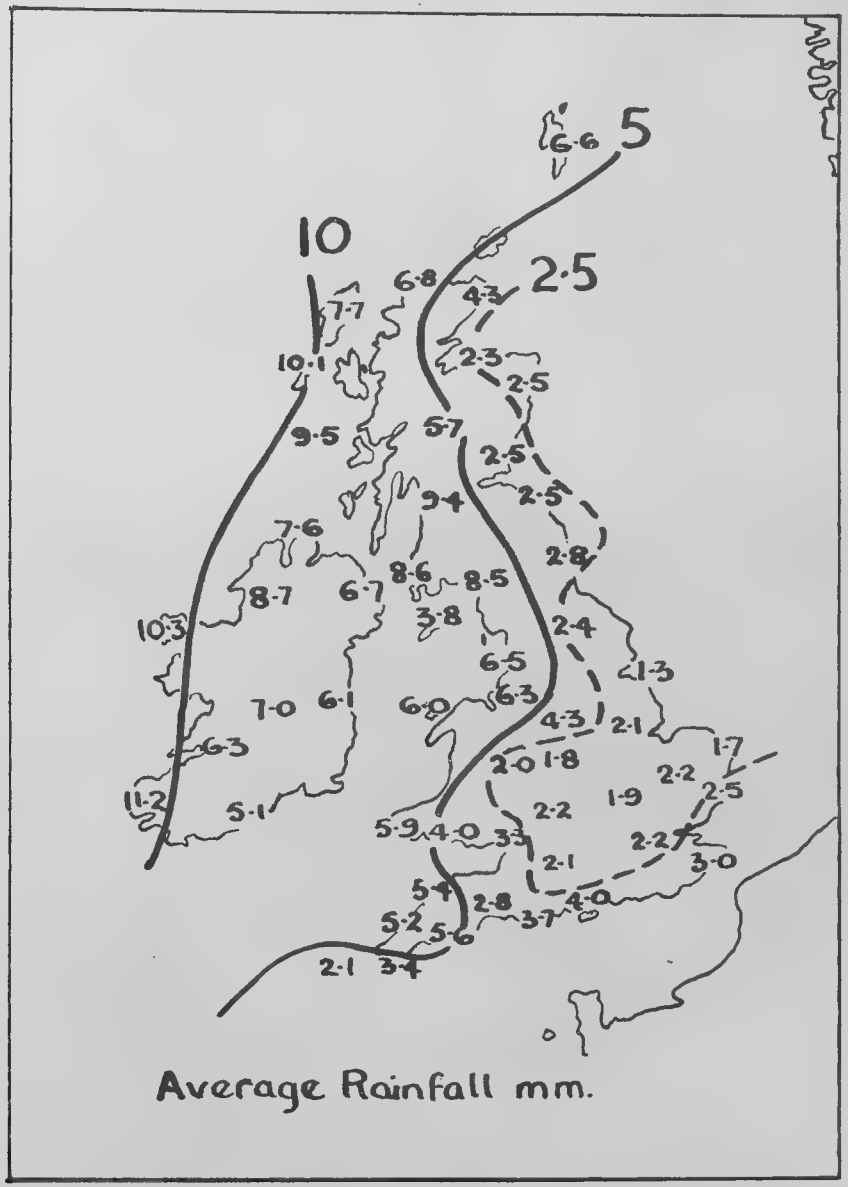
Depressions passing Eastward between Scotland and the Faeroes.

FIG.3. Frontal class I. Both fronts crossed all of British Isles. No. of cases 22.

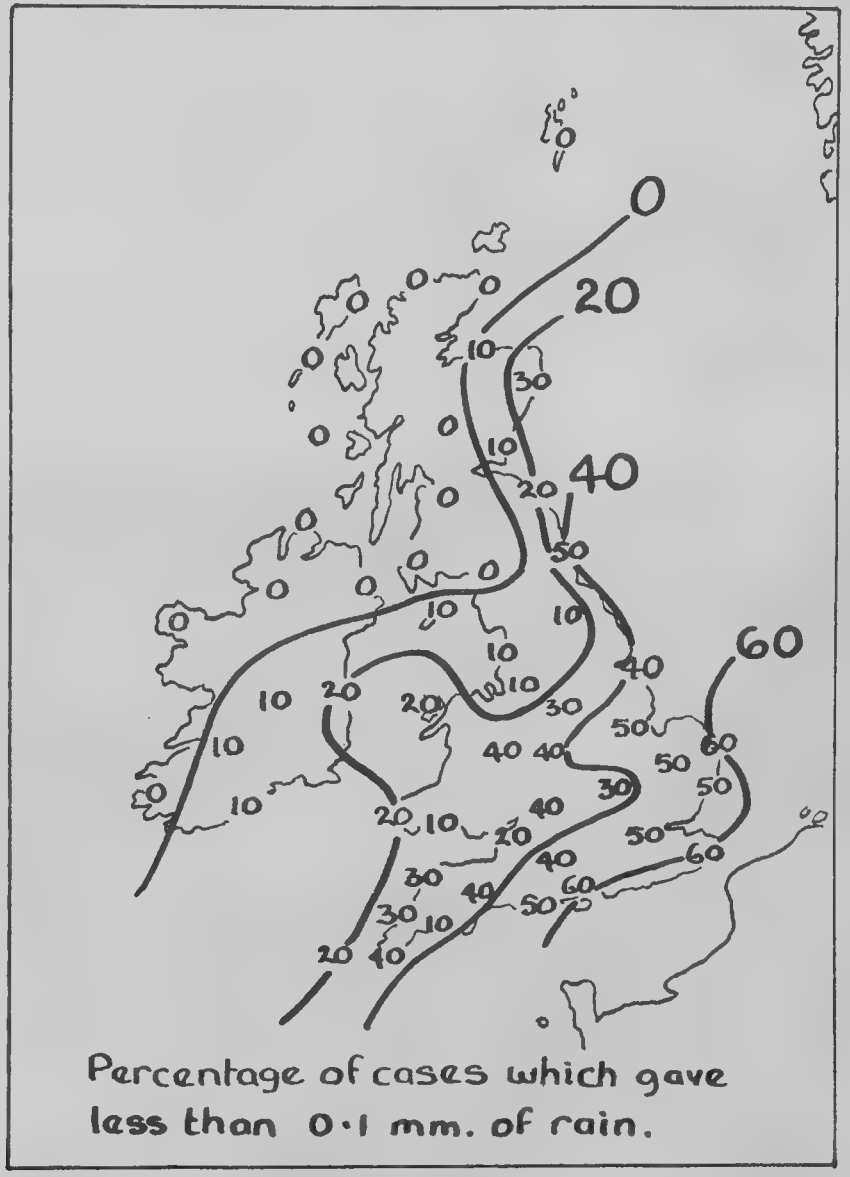




Depressions passing Eastward between Scotland and the Faeroes.
FIG 4. Frontal class II. Warm front only crossed all of British Isles. No. of cases 10



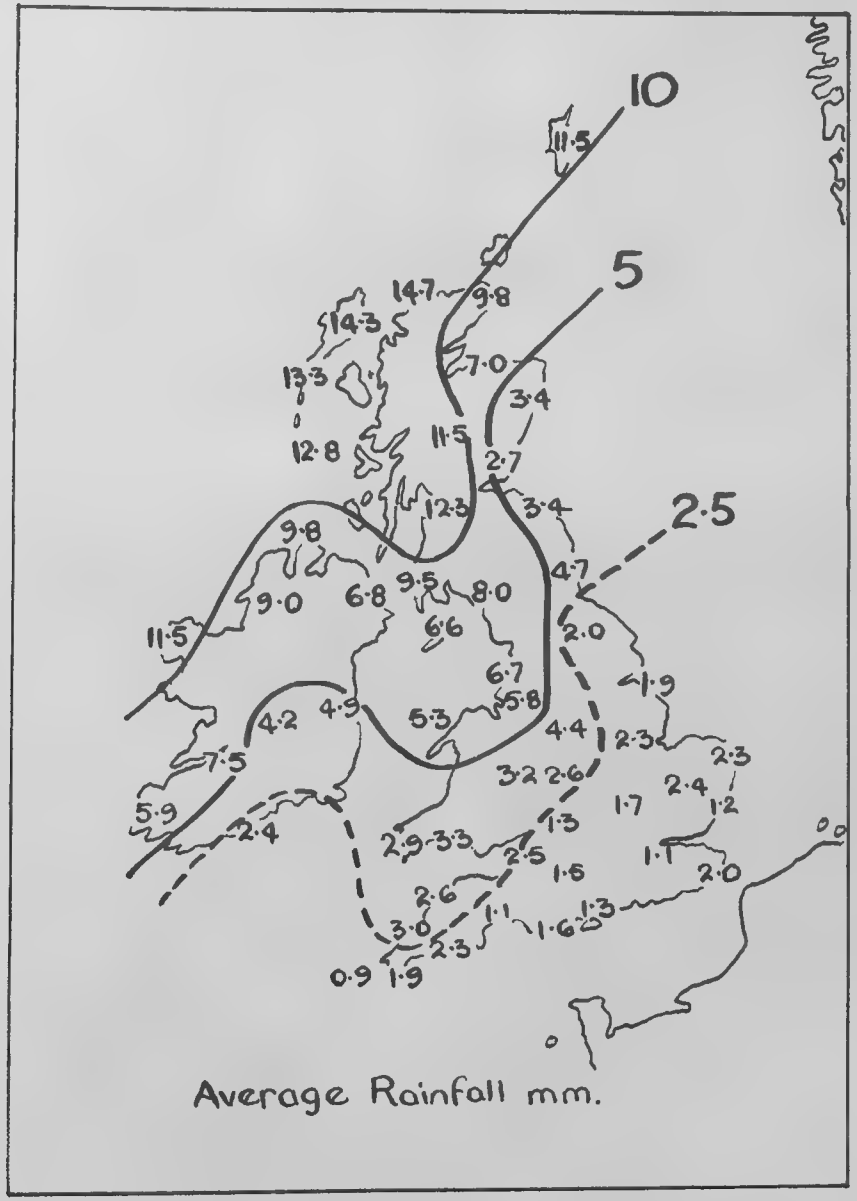
Average Rainfall mm.



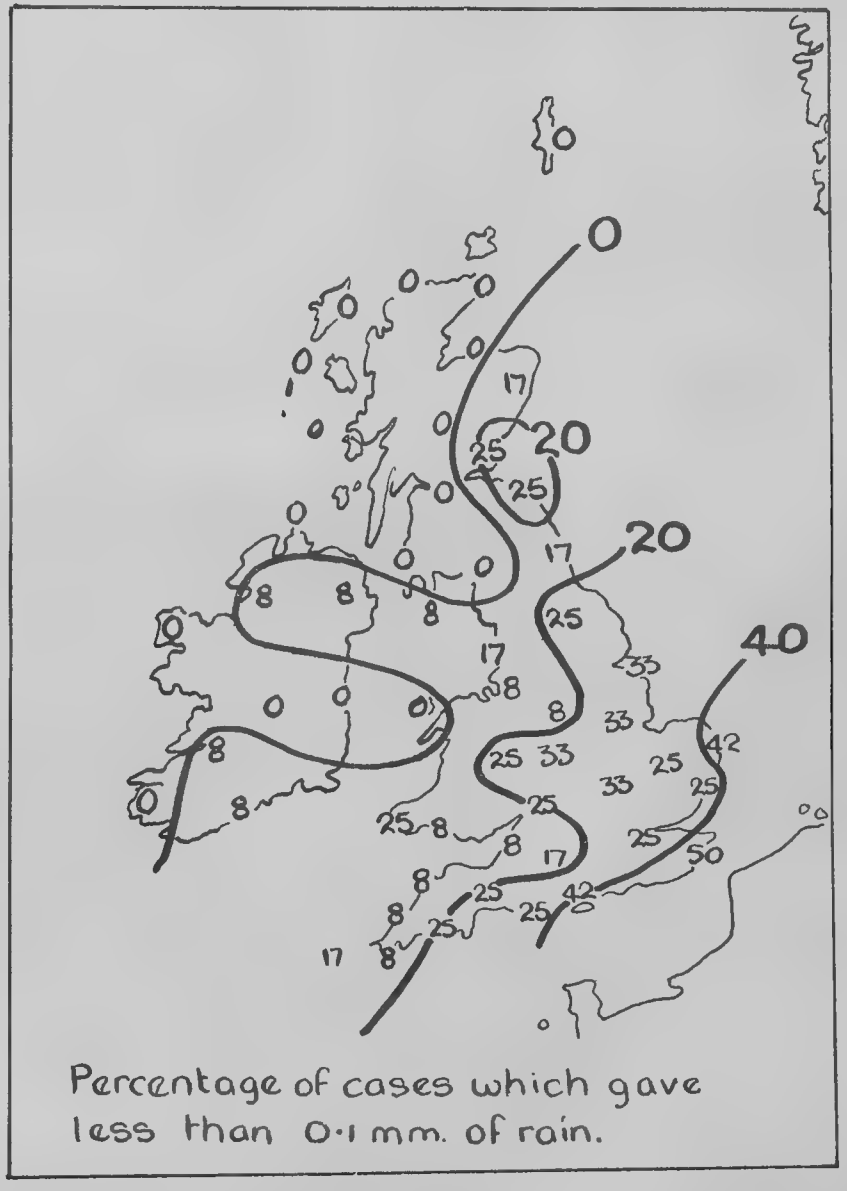
Percentage of cases which gave less than 0·1 mm. of rain.

Depressions passing Eastward between Scotland and the Faeroes.

FIG 5. Frontal class III. Cold front only crossed all of British Isles. No. of cases. 12.



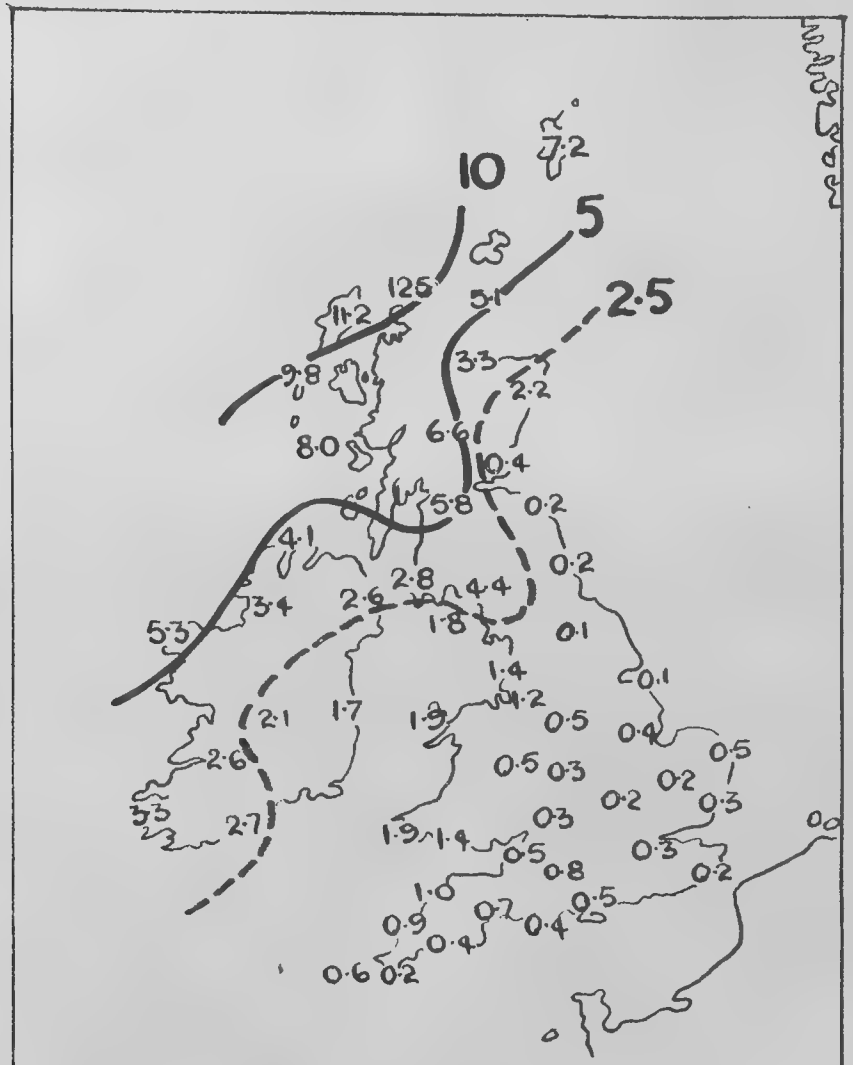
Average Rainfall mm.



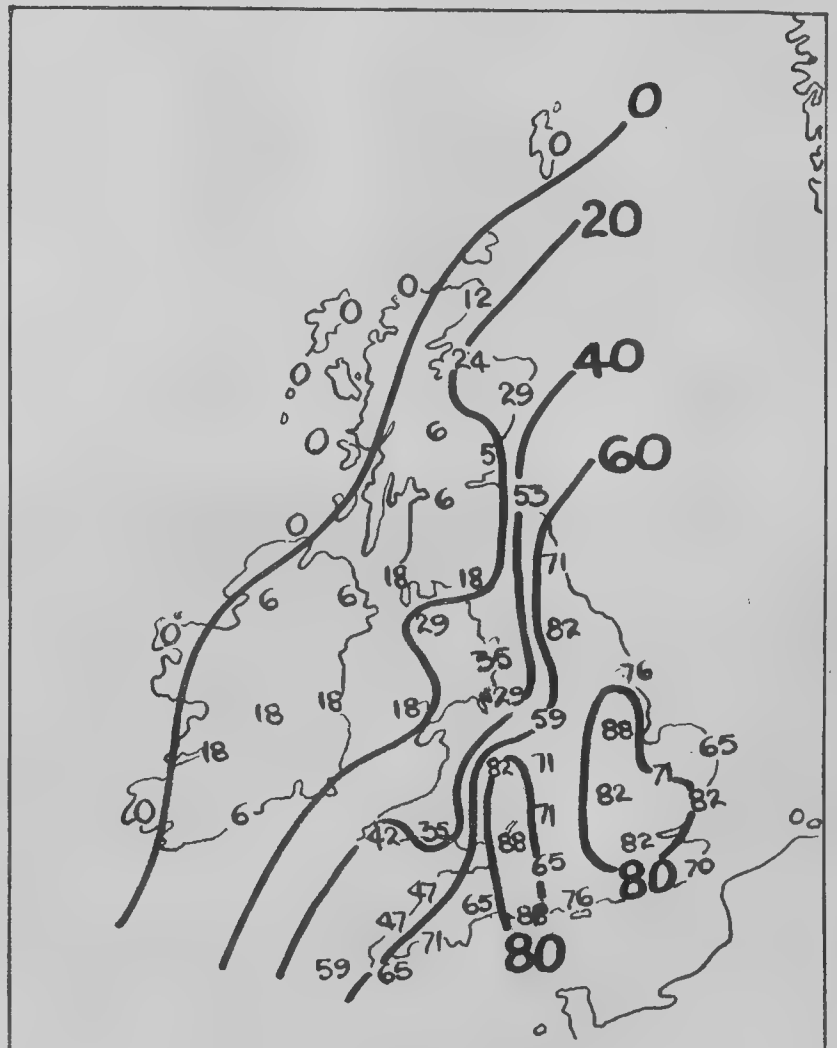
Percentage of cases which gave less than 0·1 mm. of rain.

Depressions passing Eastward between Scotland and the Faeroes.

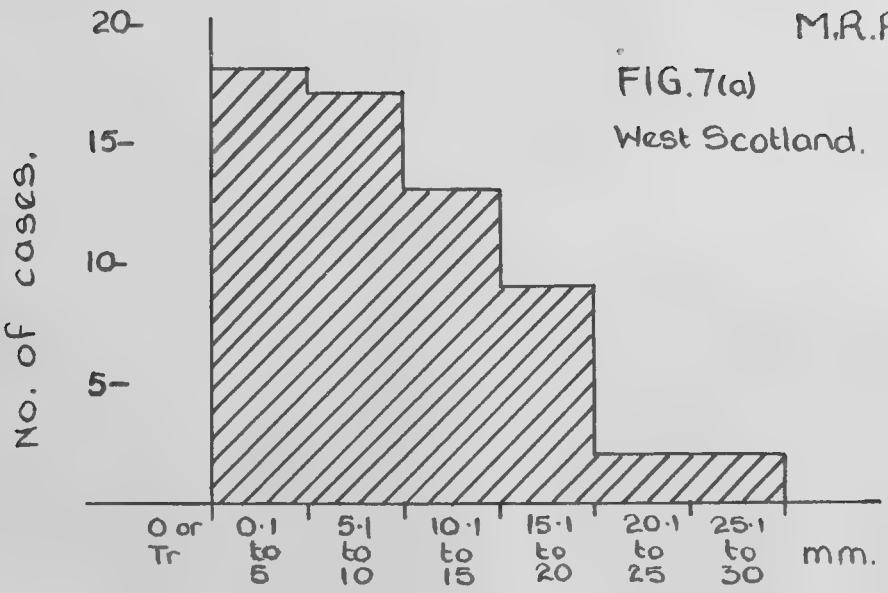
FIG. 6. Frontal class IV. Neither front crossed all of British Isles. No. of cases 17.



Average Rainfall mm.



Percentage of cases which gave less than 0·1 mm. of rain.



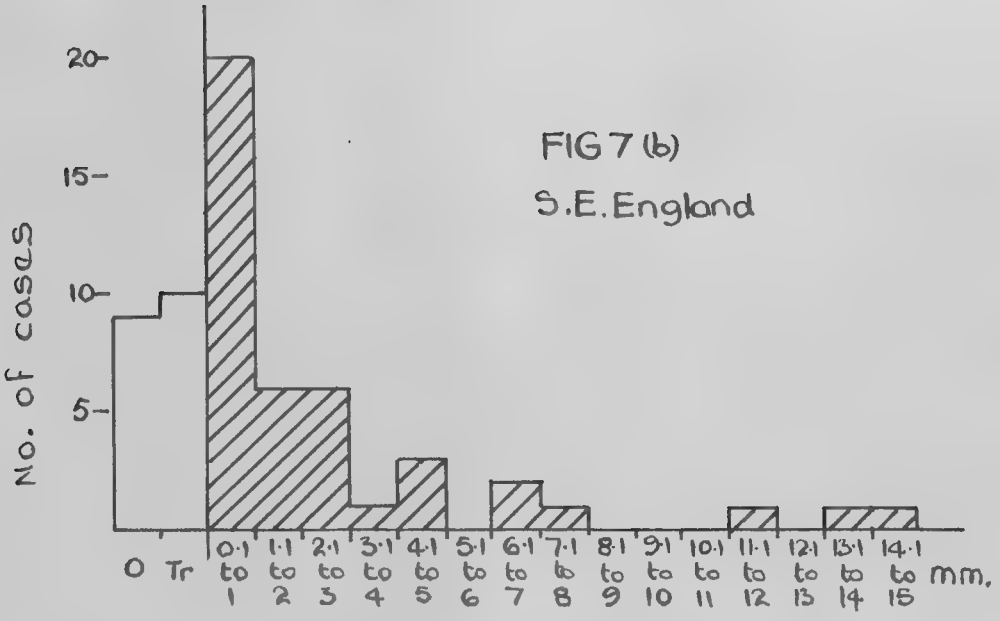


FIG 7. Frequency diagrams of rainfall amounts accompanying depressions moving Eastward between Scotland and the Faeroes. (No of cases 61).

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AIR MINISTRY

METEOROLOGICAL RESEARCH COMMITTEE

ANALYSIS OF EQUIVALENT HEAD-WINDS ON THE GREAT CIRCLE SHANNON-GANDER, AND OF ERRORS IN FORECASTS OF THE SAME, AT LONDON AIRPORT

by

D.G. Harley (Meteorological Office)

Introduction

1. An investigation has been made into the frequency-distribution of errors in equivalent head-winds on the Great Circle air-route Shannon to Gander, as forecast at London Airport. In the course of the analysis the frequency-distribution of 'actual' equivalent head-winds as measured from the synoptic upper-air charts was also examined, and compared with seasonal means already published. A table of monthly extremes is given.

Standard errors of the forecasts and curves of frequency-distribution of errors are reproduced, and the variations discussed.

2. Data

The head-wind components considered here are those measured from contour charts of the 700 mb and 500 mb surfaces prepared at London Airport. The actual charts are drawn twice daily for 0300 and 1500 G.M.T. and forecast charts four times daily, for times which have been altered slightly from time to time, as described below. Additional preliminary charts are also drawn, but these are not considered here. All the forecast charts are composite, i.e. time of validity varies across the chart, in this case from east to west on a basis of 5° longitude per hour, so that the map is valid at Shannon approximately two hours later than at London and at Gander eleven hours later.

The data considered cover the period April, 1949 to May, 1951 inclusive.

3. Measurement of Components

Since the end of March, 1949 the equivalent head-wind has been calculated as routine from these charts for the Great Circle route from Shannon to Gander, this being of particular interest for the most commonly used route across the Atlantic and also a form of numerical forecast check - though it only applies to one part of the map prepared. The measurement is made with a perspex template and specially prepared conversion tables. The template is marked with the Great Circle track, and with pairs of points spaced approximately 100 miles on each side of this track and equidistant along it. Contour height differences for each pair of points are noted, and averaged over the route. Thus an average gradient of the head component of the wind is measured for a belt approximately 200 miles wide centred on the Great Circle. The actual width of the belt is arranged so that a height difference across it of 200 ft. (one contour line) is equivalent to 20 knots. The variation in width of this

/belt

belt between 52° N. and 49° N. is negligible. The effect of the beam component is allowed for by counting the total range of height traversed (up and down) along the route and converting this to average beam component, and then to equivalent head component assuming air speed 200 knots. This is added to the head-component to give equivalent head-wind (hereafter termed E.H.W.). The effect of a beam component is always that of an adverse wind, and because the relation is non-linear the true effect of the beam component is always greater than that calculated as above unless that beam component is constant along the route. But the error involved is not often more than one knot, and is neglected in these measurements. The above measurements are all of geostrophic winds, and therefore depart from reality to some extent, but generally speaking the departure is small over a route of this length, where any one ageostrophic effect concerns only a part of the route.

The E.H.W. figures are plotted on a graph against the relevant time/date; those from actual charts against the time of the chart, and those from forecast charts for the time at mid-way between Shannon and Gander, 32½ deg. W. or 6½ hours after departure from London. Representative sections of summer and winter graphs are reproduced as Figures I and II. The small variations in the summer and the combination of large and small oscillations in winter may be noted. As described in the next section the points on the forecast curves are based on the actual charts of about twenty-four hours before.

4. Basic Data used for Forecasts and Periods of validity of Forecasts

The forecast charts on which the E.H.W. is measured are intended to be used for flights varying in E.T.D. and E.T.A. over periods of six hours; and to be used for flight planning they must be available to operators at least three hours before E.T.D. Thus, since July, 1950 charts have been issued as follows:-

	G.M.T.	G.M.T.	G.M.T.	G.M.T.
Basic Surface Chart used	1800	0001	0600	1200
Basic Upper Chart used	1500	1500	0300	0300
Time of Issue	0100	0700	1300	1900
Valid for E.T.D. London	0400 - 1000	1000 - 1600	1600 - 2200	2200 - 0400
Valid for E.T.A. Gander	1500 - 2100	2100 - 0300	0300 - 0900	0900 - 1500

The forecast charts are in fact drawn for the mid-time of the period of validity, so that the actual periods elapsed between the times of the basic data used and these mid-times of validity at various points on the composite forecast charts are:-

	London	32½°W	Gander	London	32½°W	Gander
Mid-times of periods of validity of forecast charts	0700 and 1900	1330 and 0130	1800 and 0600	1300 and 0100	1930 and 0730	2400 and 1200
Period since basic surface chart	13 hrs.	19½ hrs.	24 hrs.	13 hrs.	19½ hrs.	24 hrs.
Period since basic upper chart	16 hrs.	22½ hrs.	27 hrs.	22 hrs.	28½ hrs.	33 hrs.

The method used in constructing the charts is to build up with thickness patterns from forecast surface charts and is considered the most satisfactory way of using all data and preserving consistency (see also M.R.P.622). The forecast period at 32½° W. may be considered as being about 20 to 24 hours. Though this may appear long, and the basic data unduly old, it is not possible to reduce this as a regular procedure. Twelve and a half hours are accounted for by the flight time, the period of validity, and issue three hours early. The rest of the time is used in handling data and producing the charts.

/About

About four hours are required for the data for the surface actual charts to be received and plotted, and this is dependent on there being no communication troubles. A minimum of one hour is required for analysis of the chart and construction of a forecast surface chart, which is then passed to the upper air forecaster as the basis for construction of forecast upper charts. This with the production of copies for distribution occupies an hour to an hour and a half. The times of issue are therefore the earliest possible regular times in relation to the main surface charts, allowing for reasonable consideration and various other sources of delay.

The time taken for the data for the upper air charts to be received and plotted is more variable, but the major part is usually ready in five and a half to six and a half hours from time of observation.

Scattered data, and, occasionally, a considerable amount of Transatlantic data, may not be plotted until eight hours or more have elapsed. In very bad radio conditions delays may be ten hours or more. Analysis of the charts takes up to two hours, so that the 0300 G.M.T. charts are usually completed by 1100 - 1200 G.M.T. and the 1500 G.M.T. charts by 2300 - 2400 G.M.T. This fits in with the time the surface forecast chart is available. If the 500 mb. chart is analysed first it is ready in time for the surface forecaster to use with Sutcliffe's thickness-pattern methods in making his forecast chart.

For the charts issued at 0700 and 1900 G.M.T. only the European parts of the upper air data are available by 0530 and 1730 G.M.T. when the new forecast charts are begun. These fresh reports not having been analysed thoroughly, can be used only with caution. At the intermediate hours of 0900 and 2100 G.M.T. the majority of stations in the Atlantic area make wind observations only, which are of limited value for contour charts. The same applies to aircraft observations. All these are of course available to the forecaster but are supplementary to the basic chart.

5. Difficulties in using the Graphs as checks on Forecasts

In discussing these figures of equivalent head-wind the difficulties of comparison and various unrealities must be borne in mind.

(1) The E.H.W. measured on the "actual" chart is quite unreal in that the assumption is made either of an infinitely small flight-time, or that the contour systems are not changing with time.

(2) The forecast E.H.W. may correctly differ from the subsequent "actual" for the same nominal time because of the changes in the contour systems taking place towards the end of the route, four or five hours different in time on the composite chart. Such apparent errors are most likely when the E.H.W. is changing rapidly, either through arrival in or departure from the belt considered of a zone of very strong winds, or through the passage across the route of strong circulations.

(3) The "sampling" method of measurement may be expected to lead to errors, but all measurement of gradient involves this kind of sampling, and figures obtained by this method seem to agree well with those made by airline operators using different methods.

(4) The breadth of the belt used must reduce extreme figures of E.H.W. compared with the actual Great Circle, but it is necessary to measure gradients over finite distances and a spacing of about 200 miles usually suffices. For practical use in operations also it gives a figure which is not too susceptible to small changes either in route or in contour patterns. Undoubtedly in the case of a narrow jet-stream aligned along the route, the figure for the broad belt could mislead considerably.

/(5)

(5) The forecast E.H.W. is not that which would be encountered by an aircraft, except when it is substantially constant along the route, as a constant ground speed is assumed.

In fact, an aircraft is relatively delayed by a variable headwind because its ground speed is decreased by the stronger winds, which therefore affect it for longer.

(6) The forecast chart and the "actuals" with which they are compared are drawn by different forecasters, each with his own idiosyncrasies. Over the Atlantic where observations are widely scattered - weather ships are about six hundred miles apart - these general factors can have quite large effects. Faults in the sea-level analysis arising from the same lack of data also have effect, as the upper level charts are constructed using thickness methods. It seems clear that many irregularities in the graphs of "actual" E.H.W. should be smoothed out. In other words, an "error" is not always an error.

(7) Only until 1st June 1949 did the mid-point times of the forecast chart coincide with those of actual charts. As the curves are irregular it is not possible to say from them with certainty what the actual component would have been other than at 0300 and 1500 G.M.T. The solution adopted from 1st July 1949 in measuring errors has been to mean the forecast E.H.W. for before and after the actual chart time to get a figure for that time. The justification for this is that in every case that pair of forecasts is made by the same forecasters on the same shift and from the same basic upper air chart of the day before. On the other hand this procedure reduces the frequency of extreme errors recorded, as the mean is always less than the larger of the two, and increases the number of very small errors when the forecast pair are respectively greater and less than the 'actual'. The magnitude of these effects has varied for different periods as the times of validity of the forecast charts have been altered, so that different periods are not strictly comparable.

Details of the forecast issues made are as follows:-

(a) 1st April to 30th June, 1949

Forecasts were for ETA $31\frac{1}{2}^{\circ}$ W at 2130, 0330, 0930 and 1530 G.M.T. The E.H.W. from the second and fourth forecasts have been compared directly with 'actuals' for 0300 and 1500 G.M.T.

(b) 1st July, 1949 to 30th June, 1950

Forecasts were for ETA $32\frac{1}{2}^{\circ}$ W at 0030, 0630, 1230 and 1830 G.M.T. The E.H.W. from the first pair of forecasts have been counted together, taking a mean weighted in the ratio of 7:5 for first against second and compared with the 'actual' for 0300 G.M.T. The second pair have been similarly treated with regard to 1500 G.M.T.

(c) 1st July, 1950 onwards

Forecasts were for ETA $32\frac{1}{2}^{\circ}$ W at 0130, 0730, 1330 and 1930 G.M.T. The E.H.W. from the first pair have been counted together, taking a mean weighted in the ratio 3:1 for first against second, and compared with the 'actual' for 0300 G.M.T. The second pair have been similarly treated with regard to 1500 G.M.T.

6. Analysis of Graphs

An analysis has been made of frequencies of actual E.H.W. in ranges of 10 knots, together with errors in ranges of 5 knots for each of these, for monthly periods from April, 1949 to May, 1951. The E.H.W. is regarded as

negative when a head-wind is encountered, in conformity with the conventions of airline operators, and errors are taken as positive when less negative than the actuals, and vice versa. The results are presented in the form of tables of mean and standard deviations Tables I and IV, together with tables and graphs of percentage frequencies Tables II, III, V - VII and Figures III - VIII. All have been calculated for three-monthly periods, December to February, etc., for comparison with the figures in M.O 535b - "Equivalent headwinds on some of the principal Air Routes of the World" - which were calculated by an entirely different method.

The frequency tables and graphs are also presented for six-monthly periods May to October, and November to April, these being the summer and winter seasons as considered by airline operators

7. Frequency distributions of 'Actual' E.H.W.

In the calculations of mean and standard deviations the mid-point of each 10-knot range has been used to represent the range. The distribution being similar to normal error distributions with central maxima, this represents a slight flattening and broadening of the curve, thus increasing the standard deviation. It should not however appreciably affect the mean. Table I shows the results and compares them with M.O.535b. That publication gives figures for 10,000 and 20,000 ft. whereas those calculated in this paper are for 700 mb. (9,880 feet) and 500 mb. (18,280 feet).

TABLE I

'Actual' Equivalent Head-Winds Shannon - Gander Great Circle

(D = E.H.W. S.D. = Standard Deviation)
(a) from M.O.535b (b) from London Airport charts

	DEC - FEB		MAR - MAY		JUNE - AUG		SEPT. - NOV.	
	D	S.D.	D	S.D.	D	S.D.	D	S.D.
(a) 10,000 ft.	29	17	15	15	17	11	23	15
(b) 700 mb. 1949					17.5	11.2	22.8	14.0
1950	27.9	13.4	13.3	17.6	19.1	10.2	27.1	15.0
1951	29.3	12.4	11.9	12.5				
	DEC - FEB		MAR - MAY		JUNE - AUG.		SEPT. - NOV.	
	D	S.D.	D	S.D.	D	S.D	D	S.D.
(a) 20,000 ft.	37	23	26	21	23	16	36	19
(b) 500 mb. 1949					26.9	13.9	32.2	20.3
1950	41.6	18.4	19.9	23.1	27.6	12.6	40.6	20.6
1951	42.6	18.2	19.7	16.1				

The actual frequency distributions month by month are given in Table II, and the percentage frequency distribution by seasons in Table III (both Tables are at the end) while results are also plotted as graphs of percentage frequencies of head-wind less than certain figures in Figures III, IV and V. In these figures the plots have been made at the boundary points of the frequency ranges, so that the flattening effect referred to above has only very slight effects indeed and those at the extreme ends of the curves, where the frequencies are in any case very small. In these figures, curves of percentage frequencies derived from M.O.535b as described therein (page 3) are also plotted for comparison.

/8.

8. Deductions

The wide variations both from season to season, and from a month or season one year to the same month or season next year are apparent from these results. Perhaps the most striking are, in 1949 and 1950, the great range of E.H.W. in April followed by a rapid decrease to steadier summer conditions. The autumn months again show wide variation in E.H.W., while the winter has a higher mean, but is rather less variable than autumn. The second year, from June, 1950, was markedly more westerly in character, especially in the autumn quarter.

Considering the variations that do occur from year to year and that only two years are here considered, the agreement with the means from M.O.535b is generally good. From Table I, the following points seem important:-

(1) In December - February the much lower standard deviation found in 1949 - 51 at both levels, together with, at 500 mbs. rather higher means.

(2) In March - May the lower means in both years, especially at 500 mbs., and the large differences in standard deviation between the two years.

(3) At 500 mbs. in June - August the lower standard deviations and higher means.

(4) The constancy of the standard deviations in September - November despite large differences in means between 1949 and 1950.

(5) In some years at least, the maximum standard deviations occur not in the winter but in the spring quarter.

In Figures III and IV there have also been marked the 85% frequency lines, and the points noted at which these cut the curves. This 85% frequency is used in the planning of airline operations in calculating probable loads and regularity. It is interesting that, with regard to the points above, the 85% frequency for December - February for 1949 - 1951 appears significantly lower than the mean (from M.O.535b) at 700 mbs., while there is no real difference at 500 mbs. This follows from (1) above. Also, from Figs. III and IV the assumption of a normal Gaussian distribution of E.H.W. on which the M.O.535b curves are drawn appears to be reasonable.

9. Limits of Variation

Table IV shows the monthly extremes of E.H.W. for the period considered, with the absolute extremes underlined.

TABLE IV

Monthly extremes of equivalent head-wind Shannon - Gander
April, 1949 - May, 1951

	Least favourable		Most favourable	
	700 mb.	500 mb.	700 mb.	500 mb.
APR.	<u>-68</u>	<u>-105</u>	<u>+29</u>	<u>+32</u>
MAY	-40	-50	+20	+24
JUNE	-42	-53	+7	+9
JULY	-35	-45	+11	+5
AUG.	-41	-62	-5	-8
SEPT.	-49	-70	+25	+29
OCT.	-55	-83	+7	+11
NOV.	-59	-81	+16	+22
DEC.	-62	-90	-3	-10
JAN.	-63	-92	0	-4
FEB.	-58	-91	+1	0
MAR.	-54	-67	+23	+28

Both the least favourable and most favourable E.H.W. occurred in April. A sharp decrease in the maximum headwinds occurs from April to May, with a

/minimum

minimum in July. Thereafter there is a fairly rapid increase in late August, continuing gradually to mid-winter. A drop in March precedes the activity of April.

This sequence is clearly related to the cycle of thermal gradients, with its maxima in spring and autumn and minimum in summer.

Easterly E.H.W. may occur in any month but August, December and January, with maxima in spring and autumn. It is interesting that at these seasons the magnitude of the easterly E.H.W. is very much the same at 700 and 500 mbs. being slightly greater at the greater height. As large easterly winds occur with anticyclones to the north of the route, this is presumably related to the low lapse-rate typical of anticyclonic air. The 'quietness' of Atlantic weather in the summer may be seen from the small range of E.H.W. experienced in June, July and August, already indicated by the low standard deviation for the quarter.

10. Frequency distributions of Errors

The means and standard deviation of the errors in the forecasts are shown in Table V.

TABLE V

ERRORS IN FORECAST E.H.W. SHANNON - GANDER GREAT CIRCLE

E = mean error S.D.= Standard deviation

700 mb.	DEC. - FEB. E	S.D.	MAR. - MAY E.	S.D.	JUNE - AUG. E	S.D.	SEPT. - NOV. E	S.D.
1949					+0.6	6.6	+0.1	7.9
1950	-0.9	8.9	-0.1	8.3	-0.1	6.5	-1.5	8.4
1951	-1.4	8.5	-0.8	7.3				
500 mb.	DEC - FEB.		MAR. - MAY		JUNE - AUG.		SEPT. - NOV.	
1949					+0.8	8.5	-1.1	10.1
1950	-0.25	12.3	+0.5	11.4	-0.7	8.5	-1.0	11.0
1951	-0.4	12.7	-1.4	9.7				

The means might be expected to be near zero: they are in fact very small, and vary from season to season. They appear to show a tendency to be negative, though over the two years the mean is less than -0.5 knots. This tendency to overdo the strength of the westerlies could hardly apply to the whole wide area of the North Atlantic which west-bound flight paths may cover, and for which the charts are drawn. It is possible, as the Shannon - Gander Great Circle is the track most used, that more conscious attention is paid by the forecasters to the area adjoining and thereby a slight bias in the drawing of the charts might be introduced. It is, however, difficult to introduce bias consciously and consistently into charts constructed by thickness methods, and the bias may be due as much or more to underestimation of the strength of thermal gradients, or other meteorological factors.

The reason for the maximum of standard deviation of errors in December - February, though the maximum of standard deviation is in September - November, appears from a study of the original graphs to be due to the more frequent rapid changes in the winter, with periods of one or two days and amplitudes of 20 - 40 knots. In the autumn the variations, though larger, are generally of longer period, and easier to follow. On the whole, the standard deviation of error in E.H.W. increases with increase both of mean and standard deviation.

/The

The increase at both levels in standard deviation between the two autumn seasons is associated with increase in the mean. There is a marked minimum in June - August when the E.H.W. are least variable.

Tables VI and VII show the percentage distribution of errors, regardless of magnitudes, for each season; and Figures VI and VII show percentage frequency distribution of errors less than various amounts, regardless of sign. A pronounced bend in the curves is generally to be seen at about 90 - 95% frequency. It is of course the risk of large errors that determines the fuel reserves required for safety in an aircraft. Further investigation into the causes of these large errors is desirable, but it is clear that many of them are matters of timing, in that the positions of a sharp peak or trough in the curves of 'actual' and forecast E.H.W. frequently differ by six to eighteen hours, thereby producing a succession of large errors first of one sign and then of another. The fact of the 'actual' being measured on fixed-time charts also affects the issue, as described above in para.3, as it may be that for Westbound flights the peak of the 'actuals' should be displaced a few hours, altering the apparent errors. In many of these cases, also, an actual flight would not be routed on the Great Circle, as the risk of a large error on that track would be known beforehand, and a track where the probable error would be small and the flight-time not greatly different, could be chosen. (See note, para. 13, on the worst errors).

Normal frequency - distribution curves, calculated from the standard deviations show good fit with the actual frequencies. As errors arise from a variety of causes this is to be expected.

Finally, to show how errors vary with the magnitude of the actual component, Tables VIII and IX show the percentage frequency of each range of error for each range of E.H.W., and the mean error for each range. On the whole, forecasts have positive errors for large negative E.H.W., and negative for positive E.H.W. The cases of large positive error with large positive E.H.W., and vice versa, are generally due to timing error with sharp peaks and troughs, as mentioned in the previous paragraph.

11. Practical Considerations

It was stated in para. 10 that a flight might not be routed on the Great Circle track if the risk of a large error would be known beforehand. Other points that are important to bear in mind in considering the practical application of forecasts are:-

(a) The probable error in measured gradient over a band is proportional to the strength of the gradient. Again the ground speed of an aircraft decreases as the headwind increases so that the importance of an error of given magnitude also increases with increased winds.

(b) Large errors are mainly associated with large values of E.H.W., and on the route concerned a large E.H.W. is generally negative. The present upper limit of E.H.W. on the route, beyond which a direct flight is rarely attempted, is about 60 knots. For that reason many of the larger errors in the lower parts of Tables VIII and IX are of no practical importance.

12. Comparison with other methods

Mr. C.S. Durst has discussed, as follows, the question of the size of forecast errors obtained by the forecast procedure currently used, as against those obtainable by use of either a regression equation or simply by giving the latest actual as a forecast.

He has had statistics calculated, from the graphs, of the temporal correlation of the actual components:-

/TABLE X

TABLE X

	700 mbs. Correlation Co-efficients.		500 mbs. Correlation Co-efficients	
	12 hr.	24 hr.	12 hr.	24 hr.
Spring	+ 0.90	+ 0.79	+0.91	+ 0.78
Summer	+ 0.85	+ 0.70	+ 0.90	+ 0.74
Autumn	+ 0.88	+ 0.76	+ 0.91	+ 0.82
Winter	+ 0.89	+ 0.69	+ 0.79	+ 0.67
Year	+ 0.88	+ 0.75	+ 0.88	+ 0.76

Forecasts may be made using the regression equation:-

$$\overline{V} - V_t = R_t (\overline{V} - V_o) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

where $\overline{V}$ is the normal equivalent head wind, V_o and V_t are those at times 'o' and 't', and R_t is the temporal correlation co-efficient for the interval 't' hours.

The standard error ϵ is then given by:-

$$\epsilon^2 = \sigma^2(1 - R_t^2) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

where σ is the standard deviation.

If, however, the E.H.W. at time 'o' had been used as the forecast for a flight 't' hours later, the standard error would be given by the formula:-

$$\gamma^2 = 2\sigma^2 (1 - R_t) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

The values of R_t given in Table X give the results shown in Table XI.

TABLE XI

Standard 24-hour forecast errors expected to be made by statistical methods (S) and by giving observations only (O) in comparison with standard errors of actual forecasts (A)

	Winter			Spring		
	S.	O.	A.	S.	O.	A.
700 mbs.	9.3	10.2	8.7	9.2	9.7	7.8
500 mbs.	13.5	14.8	12.5	12.6	13.5	10.5

	Summer			Autumn			Annual Mean		
	S.	O.	A.	S.	O.	A.	S.	O.	A.
700 mbs.	7.6	8.3	6.5	9.4	10.0	8.1	8.9	9.5	7.9
500 mbs.	8.9	9.6	8.5	11.8	12.3	10.5	11.7	12.5	10.7

It appears that the standard errors in actual forecasts are about nine-tenths of those made by the use of the regression equations and are about seventeen-twentieths of those that would have been made if no forecasting had been attempted.

/13.

13. NOTE

The two worst errors took place at the time of sharp increases in the E.H.W. That of 49½ knots was for the afternoon of 7th April, 1950. The actual and forecast E.H.W. around that time (forecasts based on actuals of 24-hours before) are:-

Date/Time	6/0300	6/1500	7/0300	7/1500	8/0300	8/1500	9/0300	9/1500
'Actual'	-25	-46	-52	-98	-96	-105	-65	-68
Forecast	-32	-45	-42	-48½	-65½	-105	-73	-67
Error	-7	+1	+10	+49½	+30½	0	-8	+1

Similar figures for the case of 42 knot error on 17th June 1951 are as follows:-

Date/Time	16/0300	16/1500	17/0300	17/1500	18/0300	18/1500	19/0300	19/1500
'Actual'	-28	-41	-73	-83	-68	-54	-32	-28
Forecast	-38½	-37½	-44	-41	-66	-62	-56½	-28
Error	-10½	+3½	+29	+42	+2	-8	-24½	0

Further investigation has yet to be made with these and similar cases but it seems that a powerful development occurred in each case, which was unexpected either in degree or location. Once this had occurred error returned to normal size. Of course, in the case of a jet-stream with rapid decrease in velocity on each side, the exact position of the stream makes all the difference to the E.H.W. on a fixed route, though its development and strength may be sufficiently foreseen.

14. CONCLUSIONS

(a) The seasonal means and standard deviations of equivalent head-winds for the two years June, 1949 - May, 1951 show good agreement with the figures published in M.O. 535b, except that the mean in spring and the standard deviations in summer and winter are lower; and the standard deviation in spring may be higher giving a maximum in that season instead of winter. Extremes, both maxima and minima, occur in April and the lowest monthly maxima and lowest standard deviation occur in the three summer months, after a rapid change from April to May. The assumption, in M.O.535b, of a Gaussian distribution of components about the mean appears to be sound.

(b) The errors in forecasts show a Gaussian distribution about the mean error, which is itself small and variable, but shows a slight tendency to be negative, i.e. too much westerly. About 90% or more of present errors come within a well-defined range but occasional large errors may always occur. The large errors appear to be mainly due to the timing of developments. The standard deviation of errors varies with both mean and standard deviation of head-winds.

(c) The forecasts, which are made using standard thickness-pattern techniques, show an improvement in accuracy over those made by regression equations from the latest 'actuals', and over the use of the latest 'actual' itself as a forecast.

TABLE II

FREQUENCIES OF E.H.W. IN 10 - KNOT RANGES MONTH BY MONTH

700 mb.

	69-60	59-50	49-40	39-30	29-20	19-10	9-0	- 1-10	+ 11-20	+ 21-30
1949 APR.		1	11	13	10	13	6	3	2	1
MAY			1	9	26	20	2	4		
JUNE				4	22	13	15	6		
JULY				5	26	16	11	3	1	
AUG.			2	14	14	27	5			
SEPT.			2	2	19	18	12	3	2	2
OCT.			5	16	16	7	9	7		
NOV.		2	14	21	19	4				
DEC.	1	3	19	21	4	9	5			
1950 JAN.		1	7	16	26	7	5			
FEB.		1	5	14	10	19	6	1		
MARCH		1	5	5	11	12	12	14	1	1
APR.	1	2	6	7	17	12	9	6		
MAY					6	20	14	17	5	
JUNE			3	5	15	18	17	2		
JULY				1	22	29	10			
AUG.			1	20	15	25	1			
SEPT.			7	32	16	5				
OCT.		4	8	17	24	7	2			
NOV.		3	8	14	8	5	9	9	4	
DEC.			2	15	20	22	3			
1951 JAN.	1	6	16	21	13	3	2			
FEB.		2	10	12	20	9	3			
MAR.				9	15	8	18	7	5	
APR.				1	17	23	17	2		
MAY					18	12	18	10	4	

500 mb.

	109-100	99-90	89-80	79-70	69-60	59-50	49-40	39-30	29-20	19-10	9-0	- 1-10	+ 11-20	+ 21-30
APR.					5	10	12	10	12	4	4	0	1	2
MAY						1	16	16	22	5	2			
JUNE							9	17	11	12	5	6		
JULY							6	21	18	7	8	2		
AUG.						4	15	15	18	9	1			
SEPT.						2	4	9	18	15	6	2	0	4
OCT.					1	6	15	14	9	5	3	6	1	
NOV.				4	6	15	19	9	6	1				
DEC.		1	5	5	6	12	13	8	6	6				
JAN.					5	9	18	13	12	3	2			
FEB.				1	3	9	6	12	16	5	4			
MARCH					3	5	6	3	11	11	11	9	2	1
APR.	1	2	0	1	4	5	7	6	15	8	5	5	1	
MAY								2	15	21	10	5	5	4
JUNE						2	7	14	9	19	7	2		
JULY							3	17	29	11	2			
AUG.					1	4	14	18	15	10				
SEPT.				2	6	16	20	13	3					
OCT.			2	3	5	5	19	20	6	2				
NOV.			1	4	3	12	8	6	2	7	7	6	3	1
DEC.					1	8	10	15	19	9				
JAN.		1	3	4	12	17	12	6	7					
FEB.		2	1	2	5	7	10	12	14	3				
MAR.						1	12	13	8	6	12	3	7	
APR.							3	17	19	17	4			
MAY								12	16	10	13	8	3	

TABLE III

PERCENTAGE DISTRIBUTION OF E.H.W. IN 10 KNOT RANGES, BY SEASONS

(A) Quarterly

700 mb. Components in knots

Period	79-70	69-60	59-50	49-40	39-30	29-20	19-10	9-0 (−)	1-10 (+)	11-20	21-30
June - Aug.				1.1	12.5	33.7	30.4	16.8	4.9	0.5	
1949 Sept.- Nov.			1.1	11.6	21.7	29.5	16.7	11.6	5.6	1.1	1.1
1949 Dec.- Feb.		0.6	2.8	17.2	28.3	22.2	19.4	8.9	0.6		
1950 Mar.- May		0.5	1.6	6.0	6.5	18.5	23.9	19.0	20.1	3.3	0.5
1950 June- Aug.				2.2	14.2	28.3	39.1	15.2	1.1		
1950 Sept.- Nov.			3.9	12.6	34.6	26.4	9.3	6.1	4.9	2.2	
1950 Dec.- Feb.		0.6	4.4	15.6	26.6	29.5	18.9	4.4			
1951 Mar.- May					5.4	27.2	23.4	28.4	10.7	4.9	

500 mb. Components in knots

Period	109-100	99-90	89-80	79-70	69-60	59-50	49-40	39-30	29-20	19-10	9-0 (−)	1-10 (+)	11-20	21-30
June- Aug.						2.2	16.3	28.8	25.5	15.2	7.6	4.4		
1949 Sept.- Nov.				2.2	3.9	12.8	21.1	17.8	18.3	11.7	5.0	4.4	0.6	2.2
1949 Dec.- Feb.		0.6	2.8	3.3	7.8	16.7	20.5	18.3	18.9	7.8	3.3			
1950 Mar.- May	0.5	1.1	0	0.5	3.8	5.4	7.1	6.0	22.3	21.7	14.1	10.3	4.4	2.7
1950 June- Aug.					0.5	3.3	13.0	26.0	28.8	21.7	4.9	1.1		
1950 Sept.- Nov.			1.6	5.0	7.7	18.1	25.8	21.5	6.0	5.0	3.9	3.3	1.6	0.5
1950 Dec.- Feb.		1.7	2.2	3.3	10.0	17.8	17.8	18.3	22.2	6.7				
1951 Mar.- May						0.5	8.1	22.8	23.2	17.9	15.7	6.0	5.4	

(B) Six-month periods

700 mb.

May - Oct.				2.7	13.7	33.8	27.7	14.9	6.3	0.8	
1949 Nov.- Apr.			2.8	15.5	23.1	24.0	17.4	10.2	5.8	0.3	0.3
1950 May - Oct.			1.1	5.2	20.4	26.5	28.2	12.0	5.2	1.4	
Nov. - Apr. 1951		0.3	3.0	9.9	19.9	25.7	19.3	14.4	5.0	2.5	

500 mb.

May - Oct.					0.3	3.5	17.7	25.1	26.1	14.5	6.8	4.4	0.3	1.1
1949 Nov.- Apr.	0.3	0.8	1.4	3.0	7.5	15.2	19.0	14.1	18.2	9.4	6.1	3.9	0.8	0.3
1950 May - Oct.			0.5	1.4	3.3	7.3	17.1	22.8	20.9	17.1	5.2	1.9	1.4	1.1
Nov. - Apr. 1951		0.8	1.4	2.8	5.8	12.4	15.2	19.0	19.0	11.6	6.4	2.5	2.8	0.3

TABLE VI

PERCENTAGE FREQUENCY DISTRIBUTION OF ERROR, REGARDLESS OF MAGNITUDE OF E.H.W.

700 mb.

Ranges of error in knots

+ -

	37-33	32-28	27-23	22-18	17-13	12-8	7-3	+2 - -2	3-7	8-12	13-17	18-22	23-27	28-32	33-37
A Quarterly															
June - Aug. 1949				1.1	3.8	6.0	27.7	31.5	18.5	9.8	1.1	0.5			
Sept.- Nov. 1949				0.6	5.0	9.5	22.7	30.0	17.8	7.2	5.6	1.1		0.6	
Dec.- Feb. 1950		0.6		1.1	4.4	11.7	14.4	25.5	21.6	10.6	6.1	3.9			
Mar.- May 1950		0.5		1.1	3.3	12.5	18.0	28.2	20.2	10.3	3.8	0.5	1.6		
June- Aug. 1950					2.2	8.1	21.7	37.0	20.6	6.5	1.6	2.2			
Sept.- Nov. 1950	0.5		0.5	0.5	2.7	6.6	22.5	27.0	20.9	11.0	4.4	3.3			
Dec.- Feb. 1951		0.6		1.7	2.8	7.2	18.9	27.8	17.2	16.6	3.9	2.8	0.6		
Mar.- May					2.7	10.9	18.5	25.6	27.7	7.6	6.5	-	0.5		
B Six-monthly															
May - Oct. 1949				0.8	4.1	6.8	25.6	29.6	19.7	8.7	3.3	0.8	0.3	0.3	
Nov.- Apr. 1950		0.6	0.3	1.4	3.9	12.7	17.1	27.0	19.1	9.1	5.5	2.5	0.8		
May - Oct.			0.3		2.4	7.9	21.7	32.4	23.1	8.2	2.4	1.6			
Nov. - Apr. 1951	0.3	0.3	-	1.1	3.0	7.7	18.8	27.8	20.4	12.8	5.0	2.2	0.6		

TABLE VII

PERCENTAGE FREQUENCY DISTRIBUTION OF ERRORS, REGARDLESS OF MAGNITUDE OF E.H.W.

500 mb.

Ranges of errors in knots

	+ 42-38	+ 37-33	+ 32-28	+ 27-23	+ 22-18	+ 17-13	+ 12-8	+ 7-3	+ 2-2	- 3-7	- 8-12	- 13-17	- 18-22	- 23-27	- 28-32	- 33-37
A Quarterly																
Jan.-May 1949					2.2	6.5	12.5	21.2	22.3	20.6	8.2	4.9	1.1	0.5		
Sept.-Nov.				1.7	1.7	5.0	8.9	19.4	20.0	17.8	12.2	9.4	2.8	-	0.6	0.6
Dec.-Feb.		0.6	1.1	3.3	3.9	5.0	10.6	13.3	21.1	12.8	12.8	8.9	2.2	3.3	1.1	
Mar.-May 1950	(49 kts) 0.5		0.5	0.5	4.4	6.0	12.0	18.0	22.3	15.2	6.5	9.3	1.6	2.7	0.5	
June - Aug.					1.1	4.9	9.8	17.4	27.7	21.7	9.8	3.3	3.3	0.5	0.5	
Sept.-Nov. 1950		0.5	0.5	1.1	1.7	5.0	12.1	14.3	22.0	17.0	13.7	4.4	5.0	1.7	0.5	0.5
Dec.-Mar. 1950	0.5	-	1.1	3.3	3.3	3.9	10.6	13.3	20.0	18.9	0.4	6.7	5.6	1.1	1.6	0.6
Mar.-May 1951				0.5	3.3	4.3	10.9	14.1	18.5	20.6	16.3	7.6	3.3	0.5		
B Six-monthly																
May-Oct.				0.3	2.2	4.6	11.8	20.7	22.4	17.7	8.5	7.9	1.9	0.6	0.6	0.8
Nov.-Apr. 1949	(49 kts) 0.3	0.3	0.8	2.5	3.9	6.6	11.7	16.0	18.7	14.4	10.8	7.7	2.5	3.0	0.8	
May - Oct. 1950			0.3	0.3	1.6	5.2	10.3	15.4	27.1	19.3	11.2	4.9	3.3	0.6	0.3	0.3
Nov.-Apr.	0.3	0.3	0.6	2.2	2.5	3.6	11.0	14.1	20.4	19.6	10.7	7.2	4.7	1.4	1.1	0.3
1951																

DISTRIBUTION OF ERRORS IN EACH RANGE OF E.H.W.

APRIL, 1949 - MARCH, 1951

TABLE IX

700 mb.

Ranges of E.H.W. in Knots	Ranges of Error in Knots																Mean Error	No. of Cases
		37-33	32-28	27-23	22-18	17-13	12-8	+7-3	2-2	-3-7	8-12	13-17	18-22	23-27	28-32	33-37		
+	30/21							25		25		25	25				-8.75	4
+	20/11					5	10	5	15	35	20		10				-3.75	20
+	10/1					2.4	8.5	11.0	29.3	24.4	14.6	6.1	1.2	2.4			-2.7	82
-	0/9			0.6		0.6	6.2	16.9	28.1	19.6	16.9	6.7	3.9	0.6			-1.6	176
-	10/19					1.5	7.3	19.5	29.3	24.4	11.6	4.3	1.8				-1.5	328
-	20/29				1.5	2.8	10.8	25.2	27.8	19.5	7.5	3.6	1.5				+0.5	374
-	30/39	0.3			0.3	4.1	8.2	22.6	29.1	23.3	8.9	2.1	1.0	0.3			+0.1	293
-	40/49		0.8		3.8	18.6	15.2	22.0	31.1	6.8	4.6	0.8	1.5				+4.4	132
-	50/59					15.4	27.0	34.7	15.4			3.9		3.9			+5.2	36
-	60/69		66.6				33.3										+23.3	3

TABLE IX

500 mb.

Ranges of E.H.W. in Knots	Ranges of Errors in Knots																Mean Error	No. of Cases
		42-38	37-33	32-28	27-23	22-18	17-13	12-8	+7-3	2-2	-3-7	8-12	13-17	18-22	23-27	28-32	33-37		
+	30/21						8.3			16.7	8.3	8.3	25.0		8.3	16.7	8.3	-12.9	12
+	20/11							5	15	30	10	25	5	5	5			- 5.0	20
+	10/1						2.1	6.4	19.2	27.6	12.8	10.7	14.9	4.2	2.1			- 3.5	47
-	0/9					4.4		6.6	18.7	23.1	16.5	8.8	9.9	4.4	4.4	2.2	1.1	- 3.7	91
-	10/19					1.1	3.9	11.6	13.8	18.2	22.1	13.3	8.3	5.5	2.2			- 2.7	181
-	20/29				0.3	1.7	4.4	10.8	11.8	27.6	18.6	11.2	8.8	3.0	1.0	0.7	0.3	- 1.8	297
-	30/39		0.3	0.7		2.1	4.8	9.7	22.5	19.4	23.1	9.7	2.8	3.8	0.7	0.7		- 0.1	290
-	40/49				2.3	1.9	7.7	9.3	19.4	22.5	16.2	10.0	8.9	1.2		0.4	0.4	+0.2	259
-	50/59				4.0	5.3	4.7	19.4	18.7	20.0	11.4	6.7	4.7	2.7	2.0	0.7		+2.4	150
-	60/69		1.5		6.1	6.1	12.2	18.2	13.7	18.2	12.2	10.3	1.5					+5.9	66
-	70/79			8.0	8.0	20.0	4.0	20.0	16.0	8.0	12.0		4.0					+10.6	25
-	80/89	8.3 (49 kts)			8.3	8.3	16.6	16.6		16.6	16.6			8.3				+ 8.75	12
-	90/99	16.6		33.3	16.6			16.6			16.6							+20.8	6
-	100/109									100.0								0	1

M.R.P. 749

Figure I

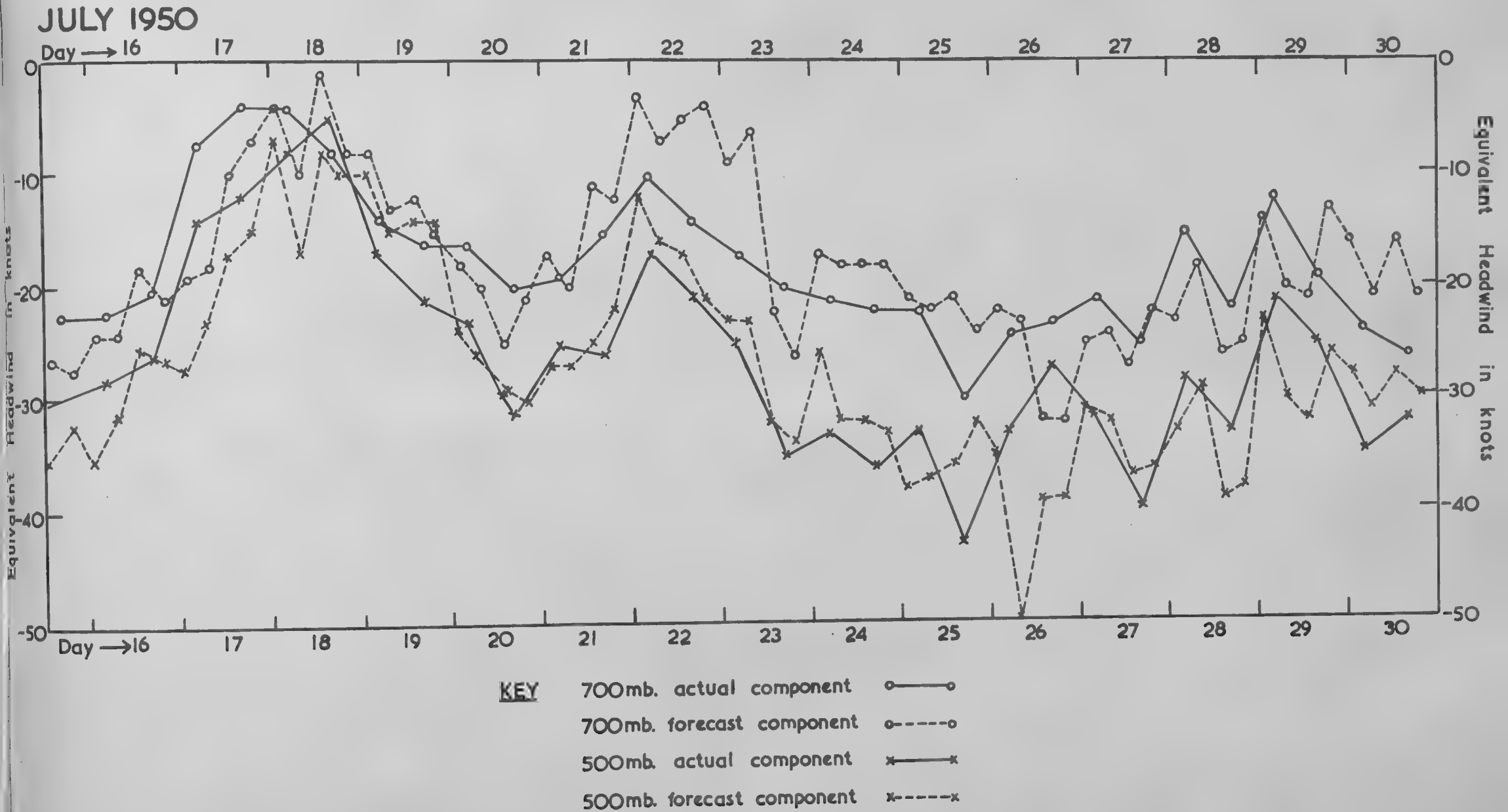


Figure II

M.R.P. 749

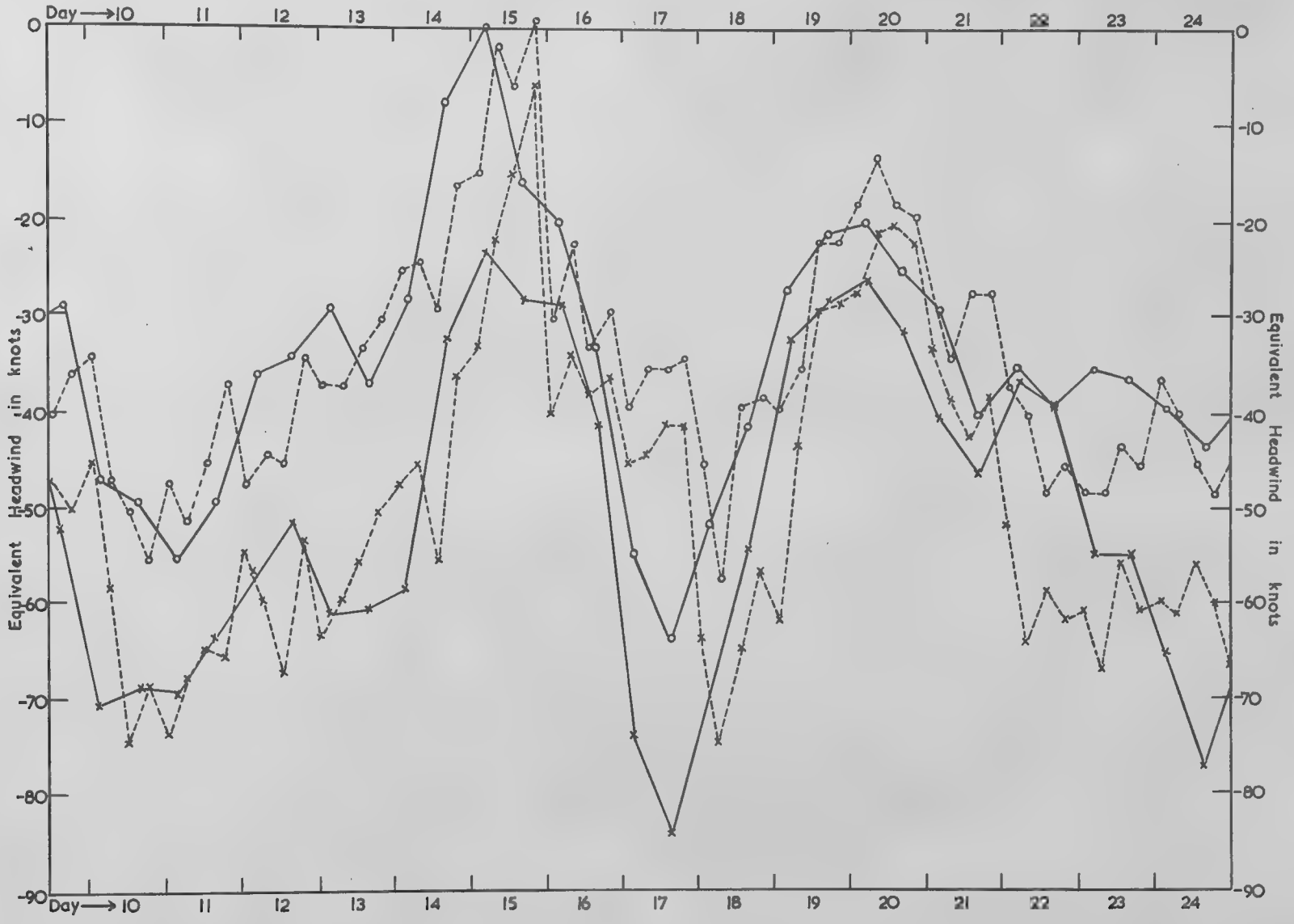
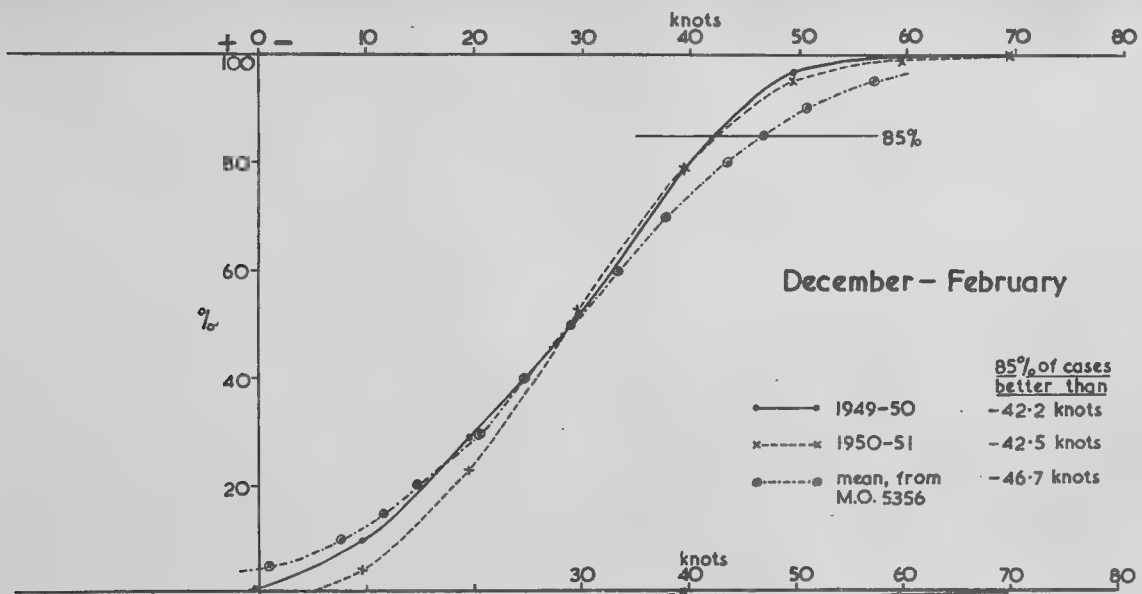
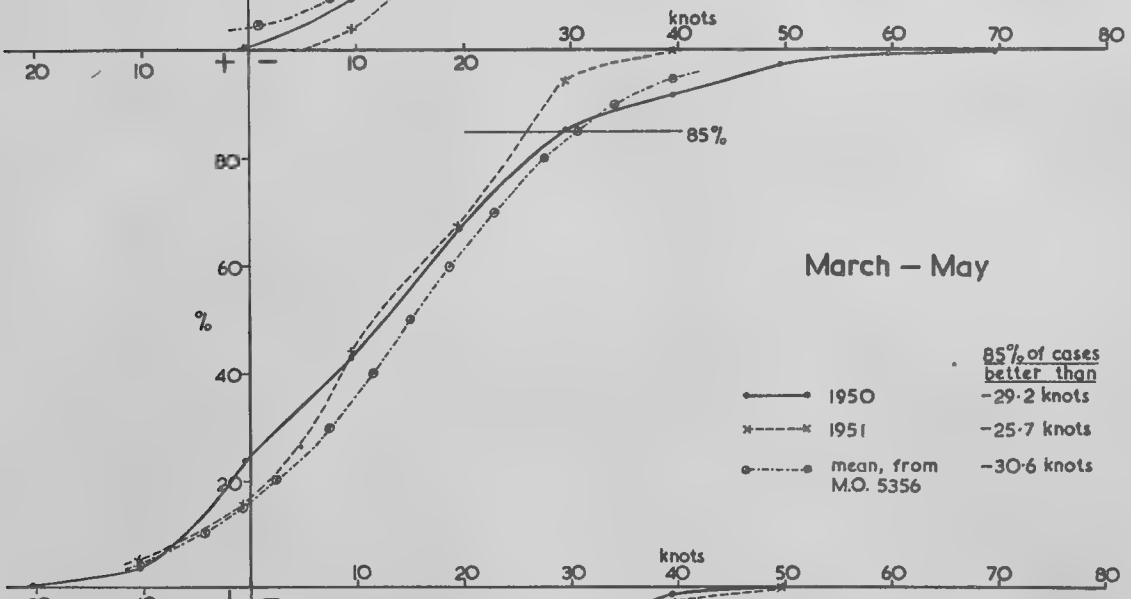
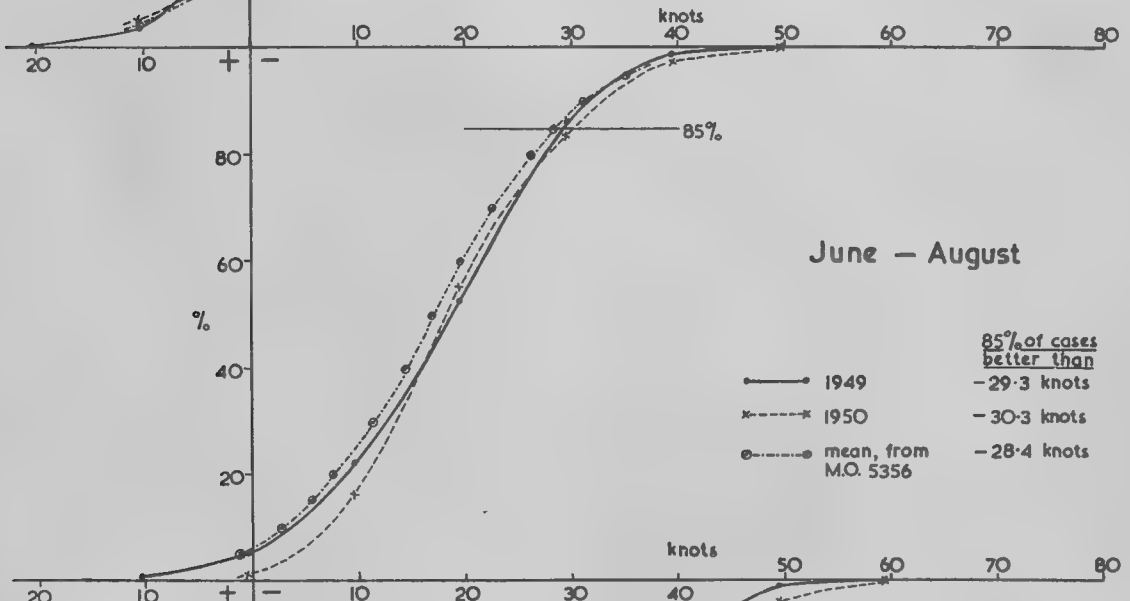


Figure III

Percentage frequencies of E.H.W. better than various figures in each season at 700mbs.







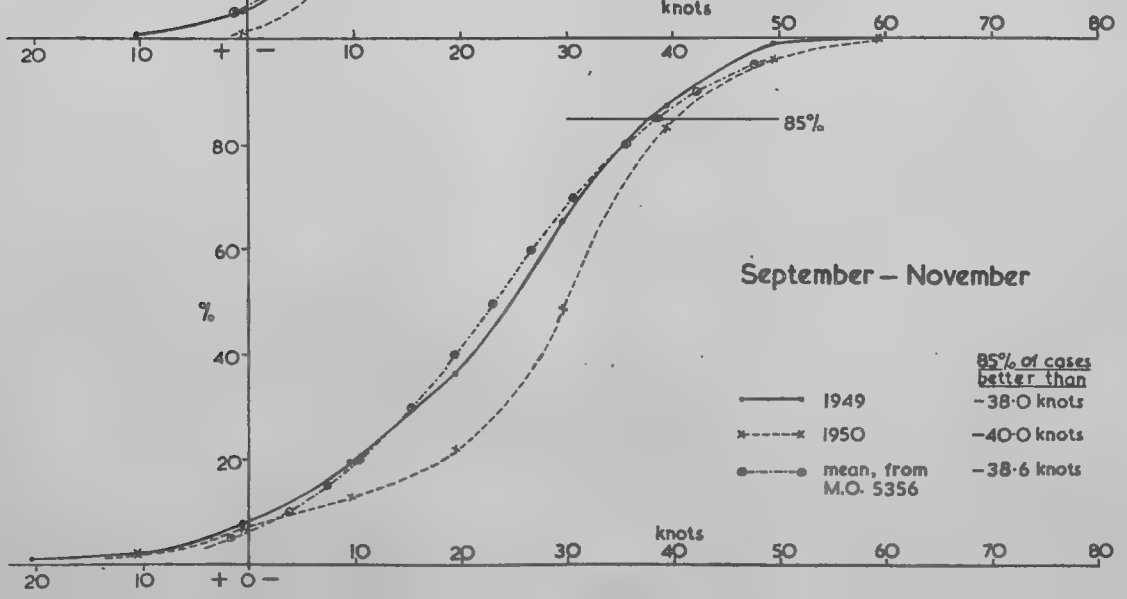


Figure IV

Percentage frequencies of E.H.W. better than various figures in each season at 500mbs.

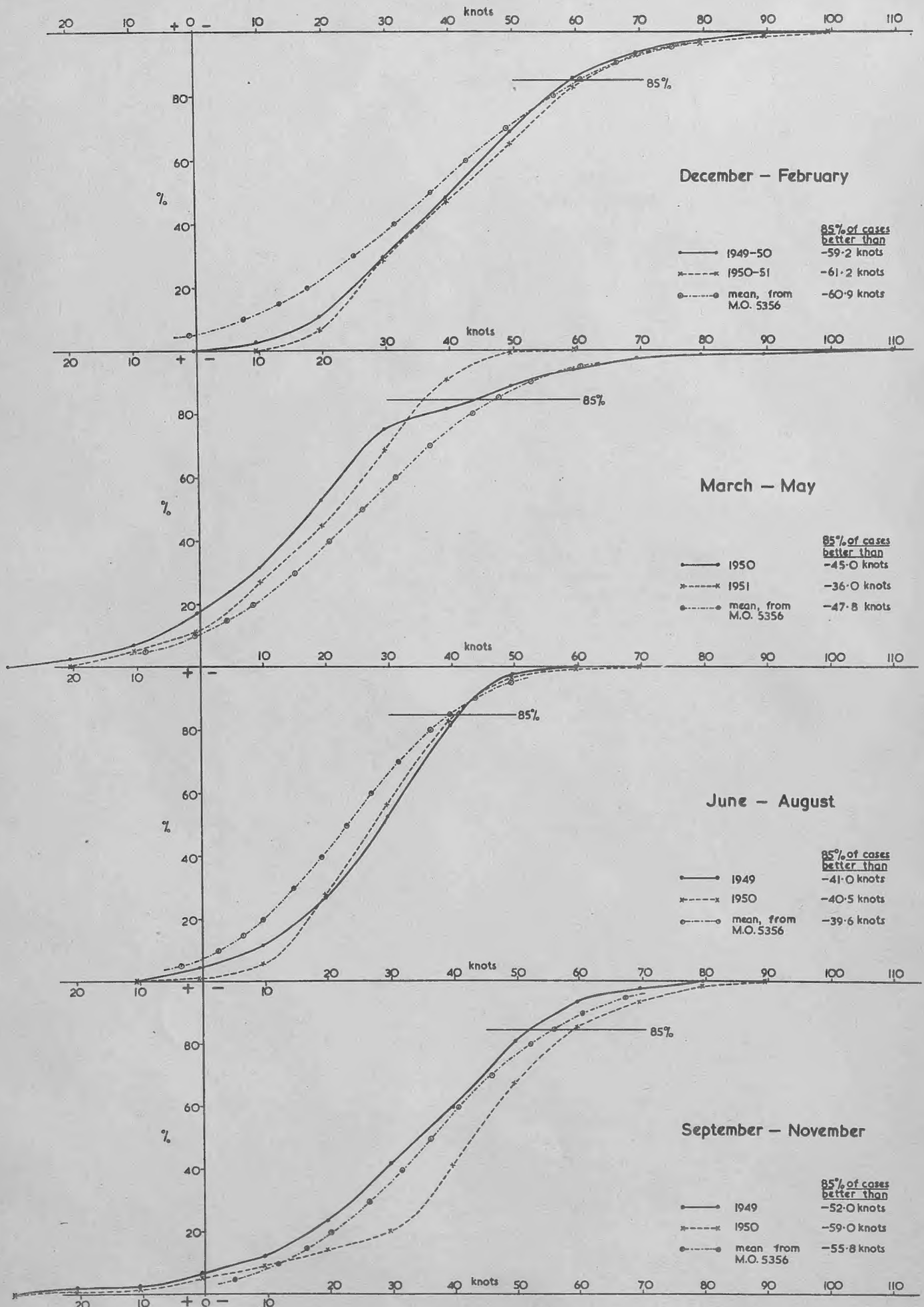


Figure V

Percentage frequencies of E.H.W. better than various figures for six-monthly periods.

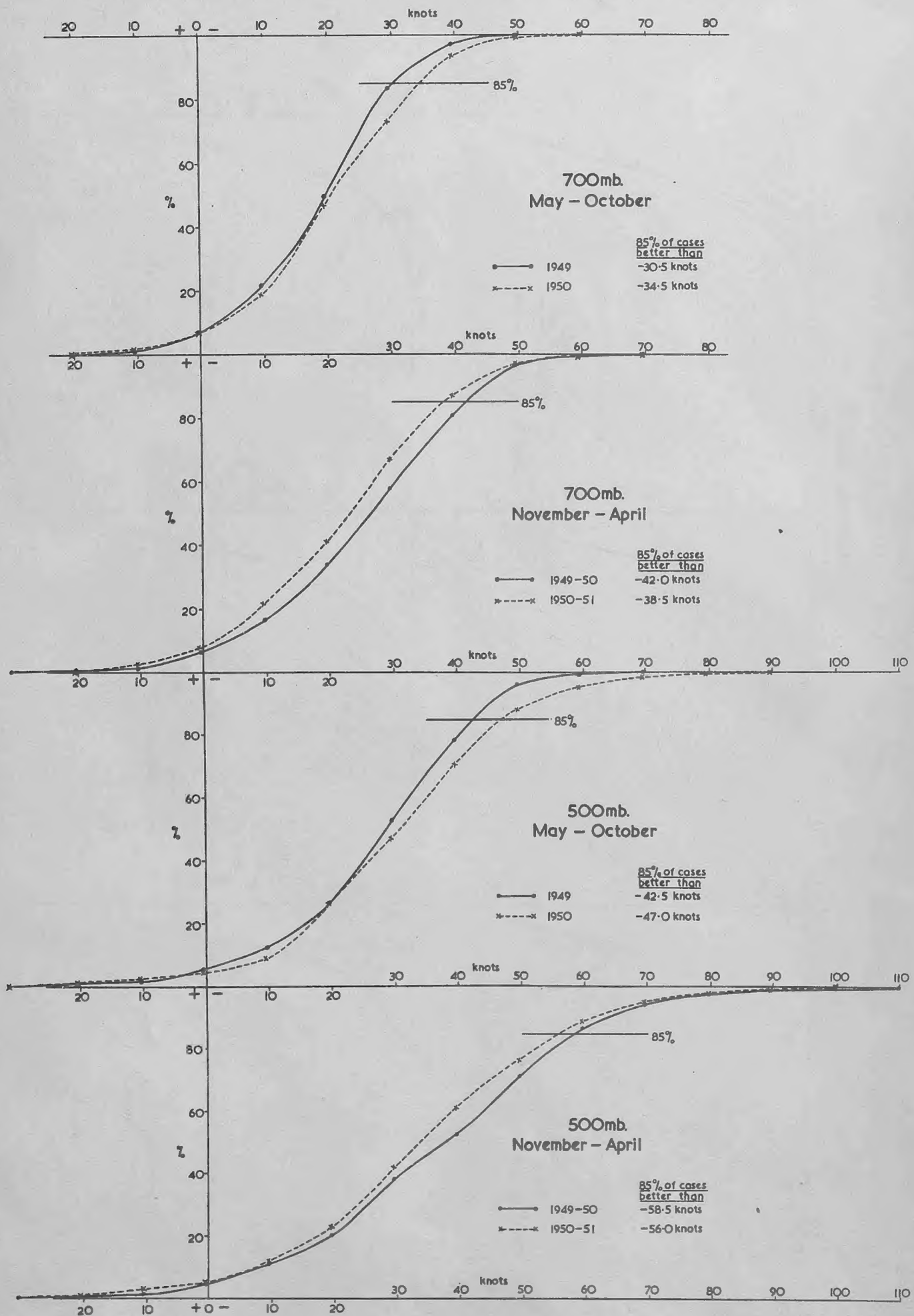


Figure VI

M.R.P. 749

Percentage frequencies of forecast errors less than various amounts at 700 mbs.

in each season

in six-monthly periods

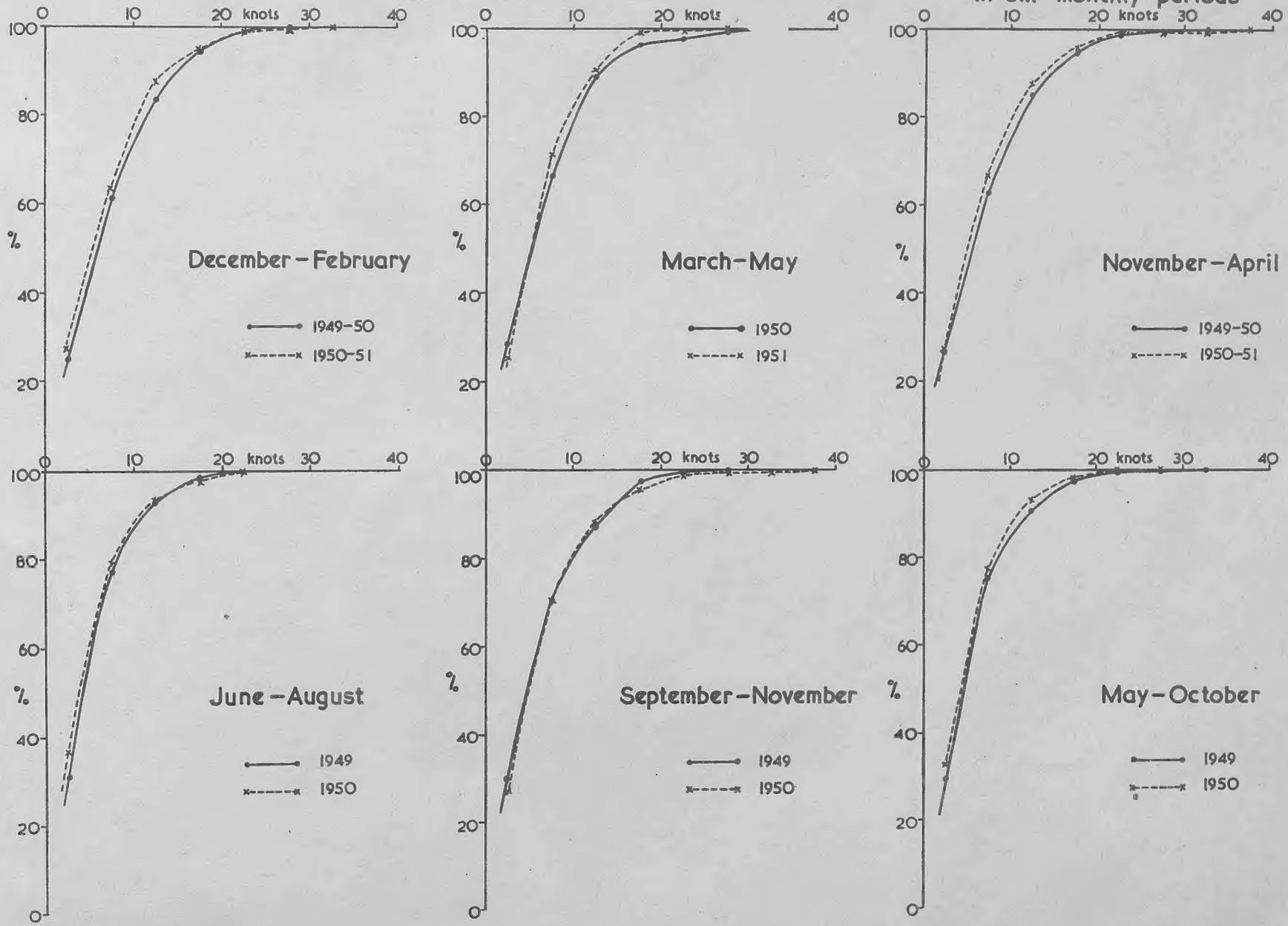
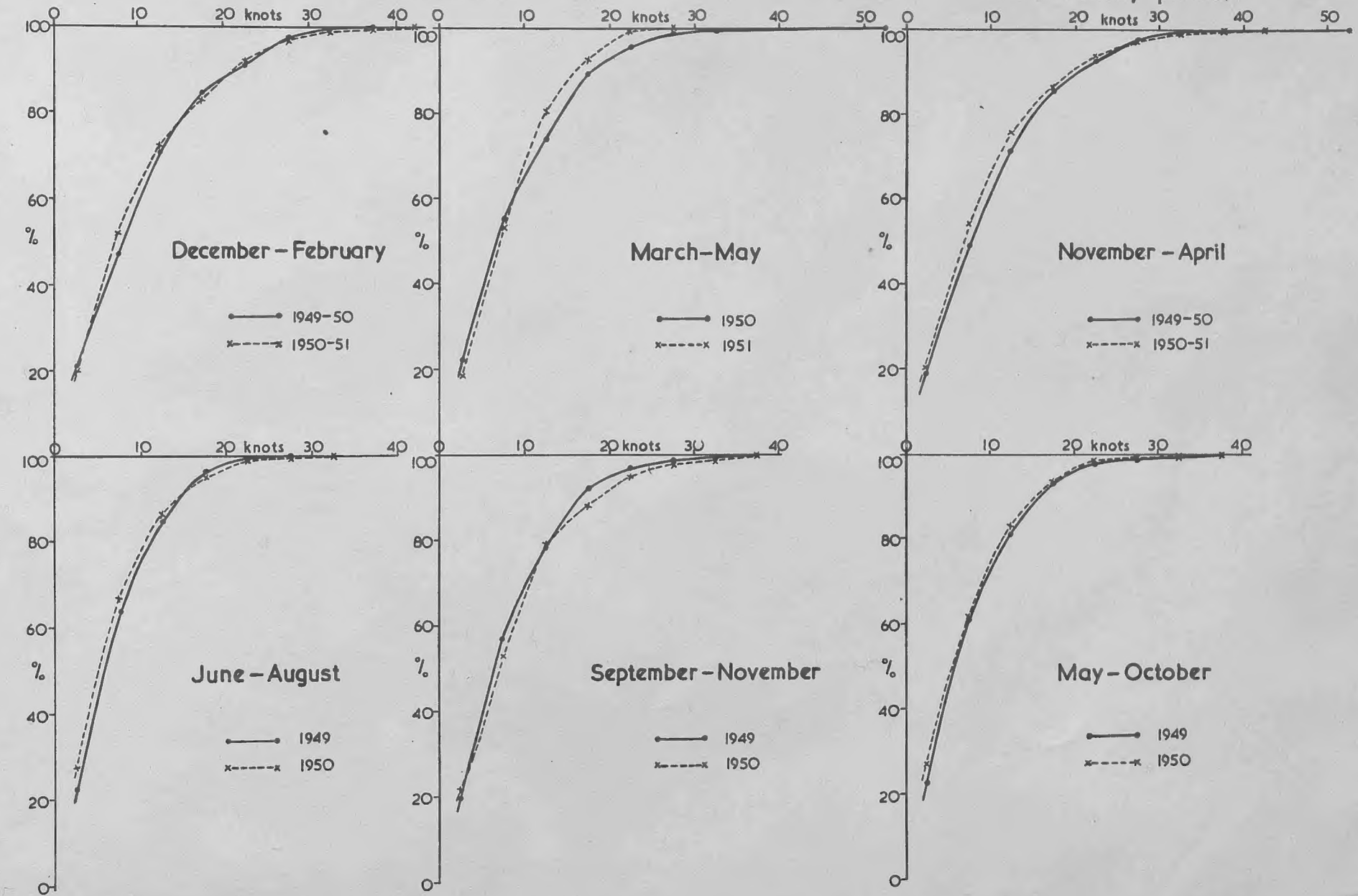


Figure VII

Percentage frequencies of forecast errors less than various amounts at 500 mbs.

in each season

in six-monthly periods